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LOVE-waves in Stratified Three Layers

(Continued)

Hiroshi OKADA and Kyozi TAZIME

(Received October 22, 1959)

Abstract

At first, some examples are calculated of the dispersion curve, the amplitude function and the amplitude distribution for the first order of LOVE-waves. These calculations remained for cases (A) and (B) in eq. (57) in the previous paper¹⁾.

Next, for cases (C) and (D) in eq. (57), the other examples for the zeroth and the first orders of LOVE-waves are presented.

Lastly, some physical considerations as to these calculations are given.

13. Numerical calculations of the dispersion curve for L_1 in cases (A) and (B)

The dispersion curve for the zeroth order of LOVE-waves, noted by L_0 , were already calculated for cases (A) and (B) of (57) in Sec. 9. The rigidity ratios adopted in the numerical calculation were (a) $\mu_2/\mu_1=4$ and $\mu_3/\mu_2=30$, (b) $\mu_2/\mu_1=30$ and $\mu_3/\mu_2=30$, (c) $\mu_2/\mu_1=1/16$ and $\mu_3/\mu_2=30$ and (d) $\mu_2/\mu_1=1/3$ and $\mu_3/\mu_2=30$.

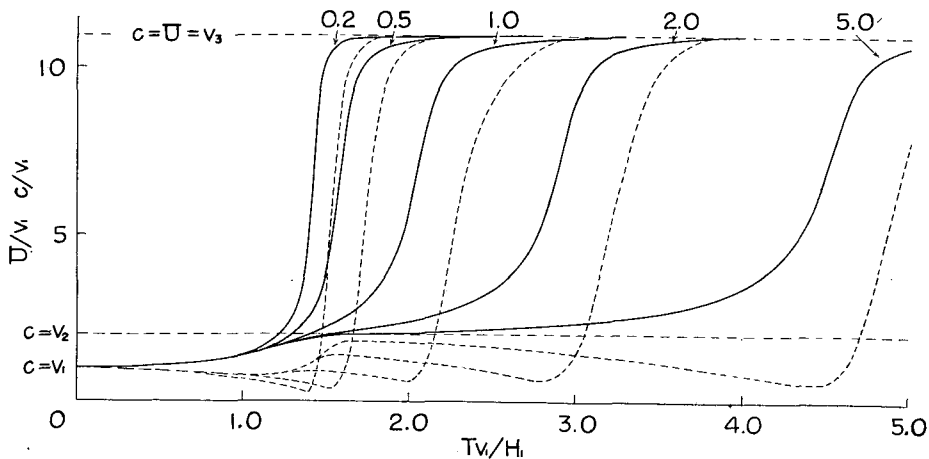
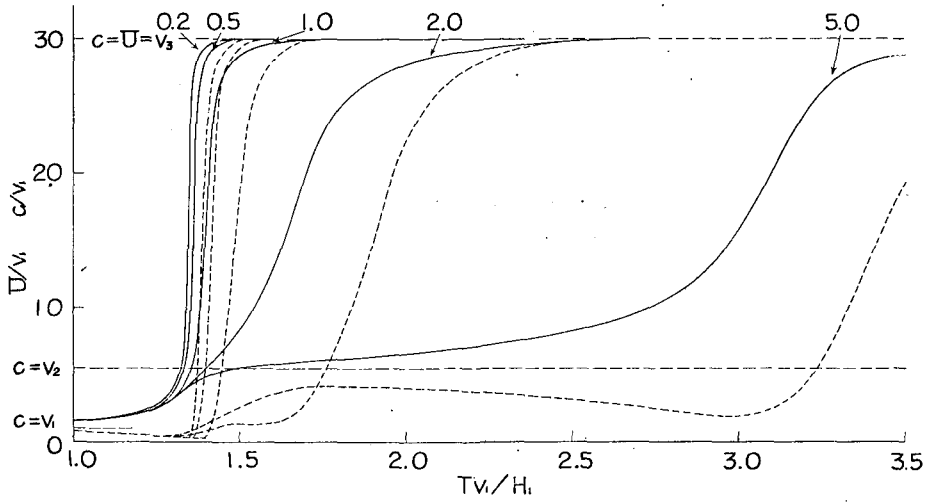
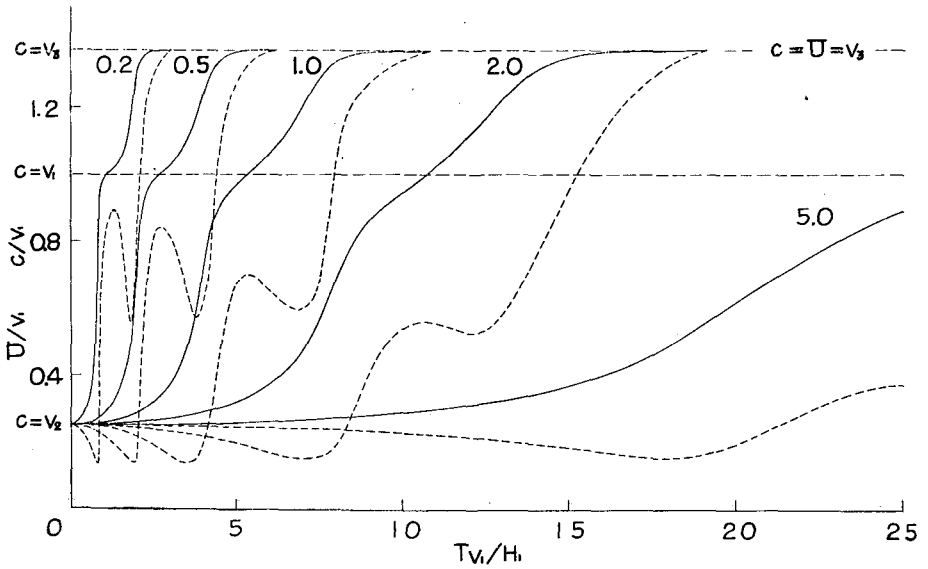


Fig. 7 (a). L_1

Fig. 7 (b). L_1 Fig. 7 (c). L_1

The dispersion curve for the first order of LOVE-waves, noted by L_1 , has been calculated in this section, adopting the same rigidity ratios as before. The results are shown in Figs. 7(a) to 7(d) for various values of the parameter

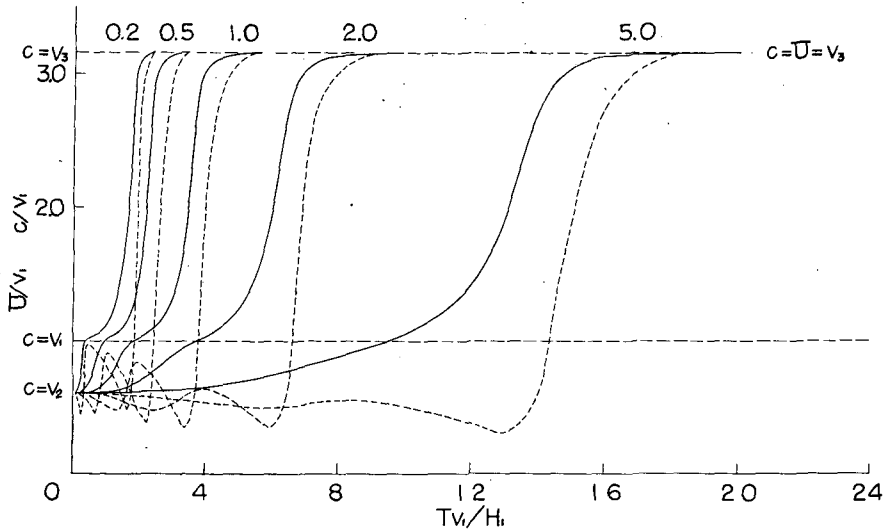


Fig. 7 (d). L_1

Fig. 7. Dispersion curves for L_1 in cases (A) and (B).

- (a) $\mu_2/\mu_1 = 4$ and $\mu_3/\mu_2 = 30$, (b) $\mu_2/\mu_1 = 30$ and $\mu_3/\mu_2 = 30$
- (c) $\mu_2/\mu_1 = 1/16$ and $\mu_3/\mu_2 = 30$, (d) $\mu_2/\mu_1 = 1/3$ and $\mu_3/\mu_2 = 30$

H_2/H_1 .

Whereas L_0 has no cutoff period, L_1 has a finite cutoff period at which phase- and group-velocities must coincide with the velocity of S-wave in the lowest layer. Periods of L_1 must be confined between the cutoff and zero.

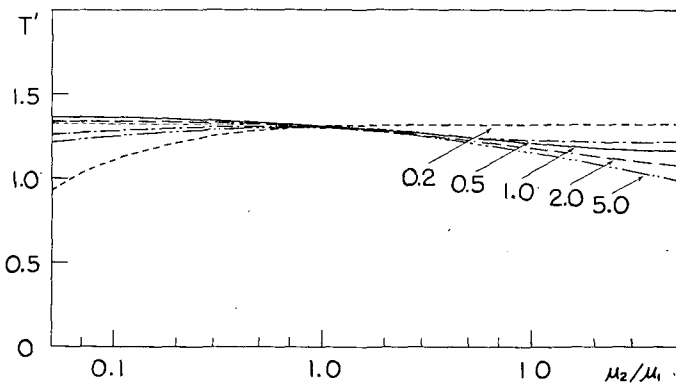


Fig. 8. The period corresponding to the second minimum group velocity for L_1 in cases (A) and (B). $T' = T/(H_1/v_1 + H_2/v_2)$.

Group velocity has two minima as in the case of the zeroth order; one is near $T/(H_1/v_1) = 4/3$ and another is near $T/(H_1/v_1 + H_2/v_2) = 4/3$, for the rigidity ratios (a) and (b). The latter relation will be clear from Fig. 8 where μ_3/μ_2 is kept at 30.

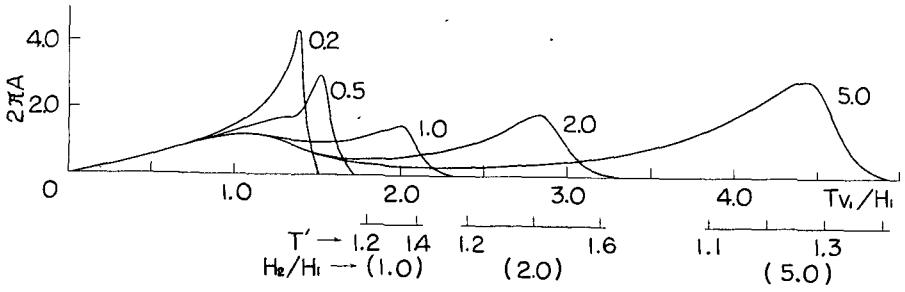


Fig. 9 (a). L_1

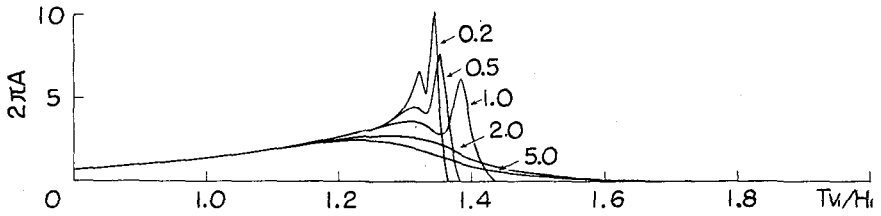


Fig. 9 (b). L_1

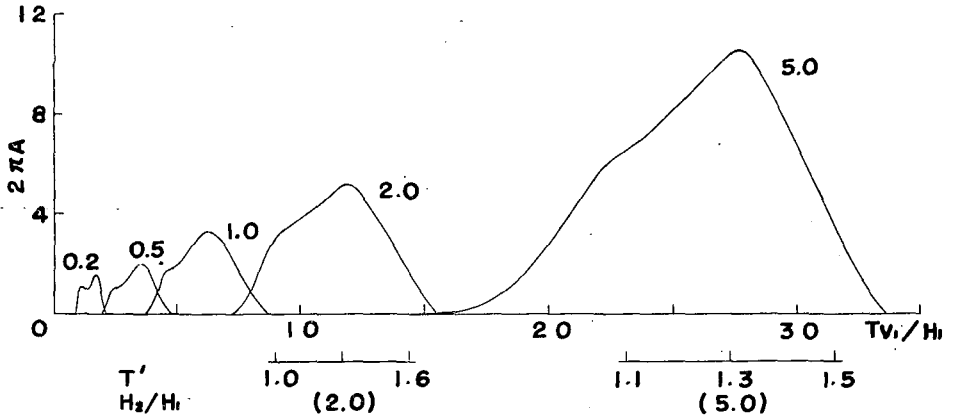


Fig. 9 (c). L_1

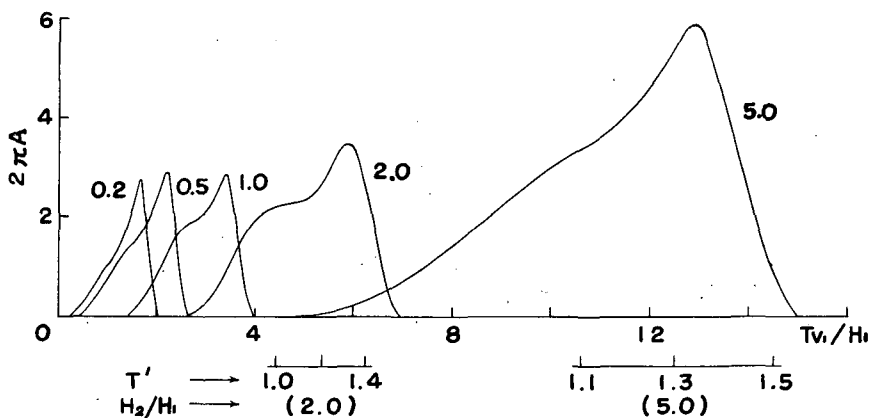


Fig. 9 (d). L_1

Fig. 9. Amplitude functions for L_1 in cases (A) and (B).

$$T' = T / (H_1/v_1 + H_2/v_2).$$

14. Numerical calculations of the amplitude function for L_1 in cases (A) and (B)

The amplitude function $2\pi A(\xi)$ in (45) has been obtained by using the dispersion curve illustrated in Fig. 7. The results are shown in Figs. 9 (a) to 9 (d) as functions of period Tv_1/H_1 .

These amplitude functions may have two maxima in each case. It must be noticed that the first maximum in cases (c) and (d) occurs respectively near the period of the maximum group velocity, whereas the other maxima,

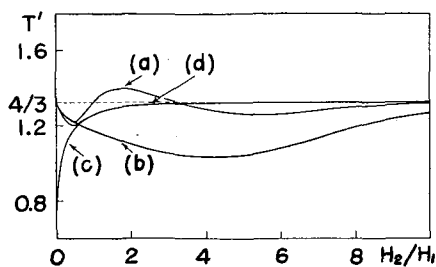


Fig. 10. The period corresponding to the second maximum of the amplitude function for L_1 in cases (A) and (B).

$$T' = T / (H_1/v_1 + H_2/v_2).$$

(a) : (4, 30), (b) : (30, 30), (c) : (1/16, 30), (d) : (1/3, 30)

the first maximum in cases (a) and (b) and the second maximum in all cases, occur respectively near the period of the minimum group velocity.

Fig. 10 shows the relation between the period of the second maximum of the amplitude function and H_2/H_1 , being parameters $(\mu_2/\mu_1, \mu_3/\mu_2)$. It is clear that each curve will approach to the period indicated by $T/(H_1/v_1 + H_2/v_2) = 4/3$ with the increase of H_2/H_1 .

Now it has been found that either L_0 or L_1 must obey the wave-length law in general,

$$T/(H_1/v_1) \text{ or } T/(H_1/v_1 + H_2/v_2) = 4/(2n+1), \quad \begin{cases} n=0 & \text{for } L_0 \\ n=1 & \text{for } L_1 \end{cases} \quad (64)$$

at least qualitatively. The thickness of the layers may have larger influence on L_1 than on L_0 , because the former has shorter wave-length than the latter. Comparing Fig. 10 with Fig. 5, the asymptotic value of (64) seems to be easily attained by L_1 rather than by L_0 . But the ratios of deviation from the asymptotic value are nearly equal to each other for the same value of H_2/H_1 .

This situation may also be seen in the relation between the sharpness of the maximum of the amplitude function and the period. The higher the order of the waves, the narrower the band of the period having considerably large amplitude.

It will be physically expected that the amplitude function may arrive at the maximum near the period of the minimum group velocity. Analytically, this phenomenon may be easily recognized by the factor $(v_1/U - v_1/c)$ in (45). On the other hand, it will seem strange that the amplitude function may arrive at the first maximum near the period of the maximum group velocity in cases (c) and (d). Numerically, the present authors have found that the factor $\{1 - (v_1/c)^2\}^{-1}$ in (45) plays the greatest role in these cases.

The latter phenomenon must be accompanied by the "low velocity layer". Considerable energies will escape from the most superficial layer to the low velocity one. Thus the wave of short length cannot keep the law of (64), but ignores that law. Displacement of the wave of short length has the factor $\cosh \hat{\eta}_1 z$ as shown in (iii) of page 126. If c coincides with v_1 , $\cosh \hat{\eta}_1 z$ becomes unity for each depth, resulting in large displacement on the surface of the earth for a certain amplitude on the lower boundary of the most superficial layer.

15. Numerical calculations of the amplitude distribution for L_1 in cases (A) and (B)

Figs. 11(a) to 11(d) illustrate the results calculated for several periods

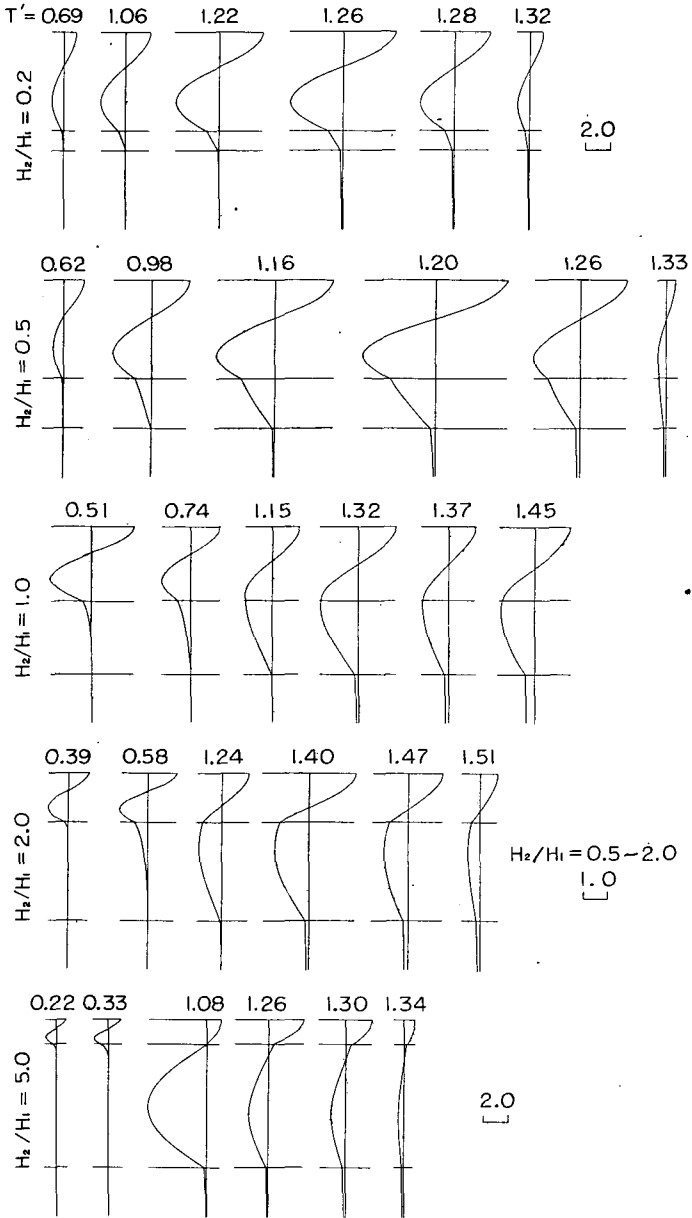


Fig. 11 (a). L_1

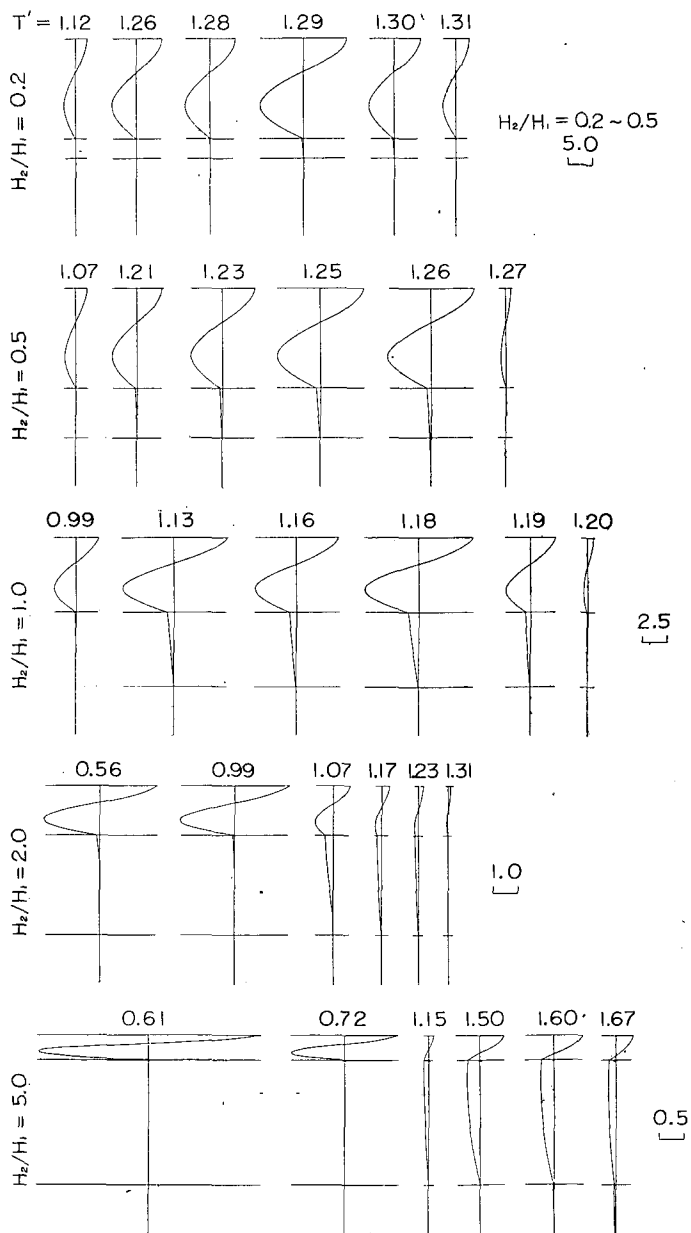


Fig 11 (b). L_1

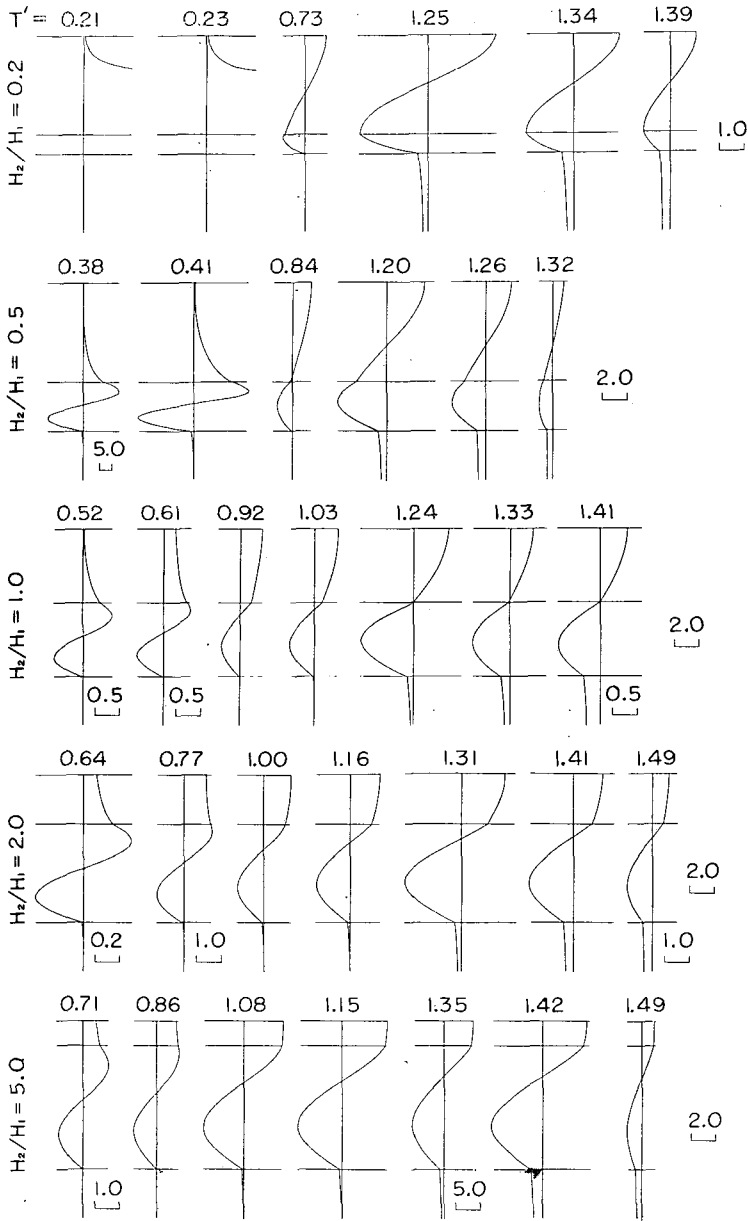


Fig. 11 (c). L_1

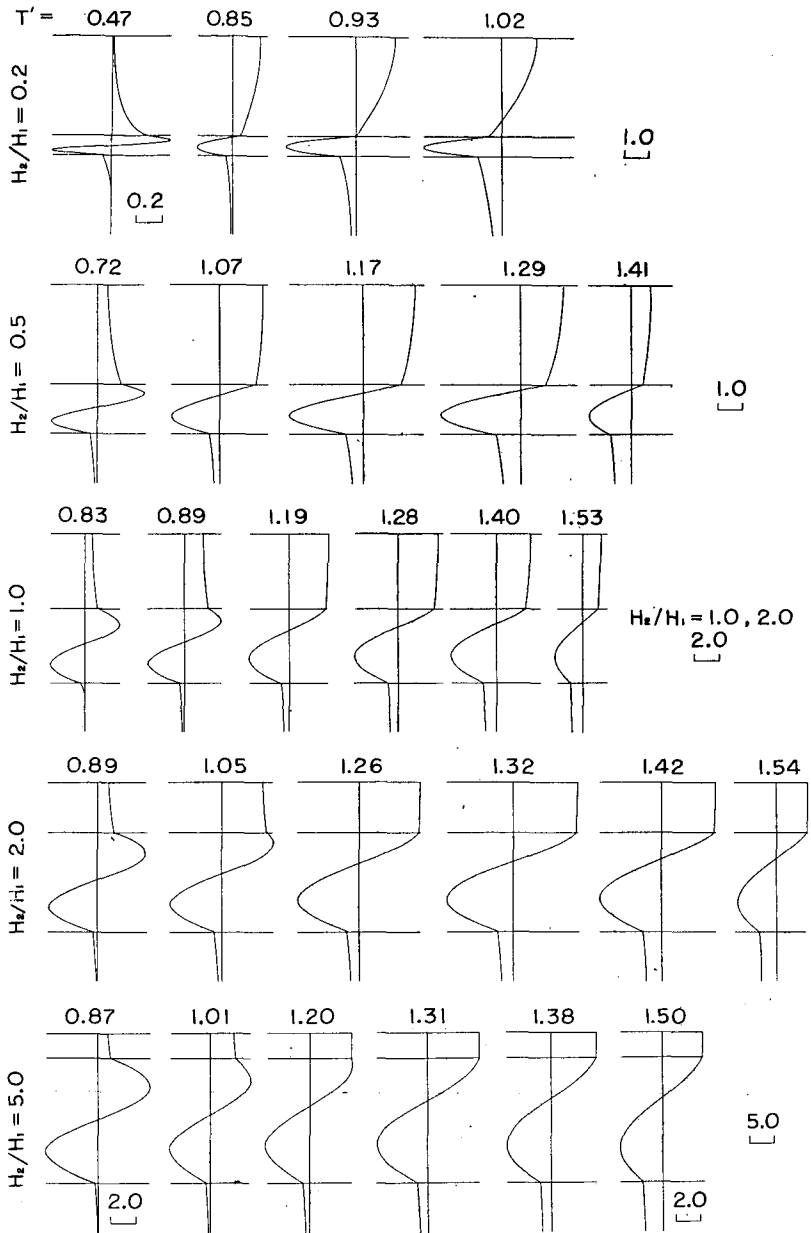


Fig. 11 (d) L_1

Fig. 11. Amplitude distributions for L_1 in cases (A) and (B).
 $T' = T/(H_1/v_1 + H_2/v_2)$.

indicated by $T' = T / (H_1/v_1 + H_2/v_2)$.

Nodes are seen, in Figs. 11(a) and 11(b), in the most superficial layer for all periods. On the other hand in Figs. 11(c) and 11(d), the nodes migrate upwards in the low velocity layer and go into the upper layer with increasing periods.

Actual amplitude distribution does not always show handsome configuration forming 3/4 wave-lengths, whereas (64) is satisfied fairly well at the maximum of the amplitude function.

16. Numerical calculations of the dispersion curve for L_0 and L_1 in cases (C) and (D)

Case (C) from (57) may take the type (iii) in (26). The dispersion curve has been calculated by the characteristic equation (iii) in (27).

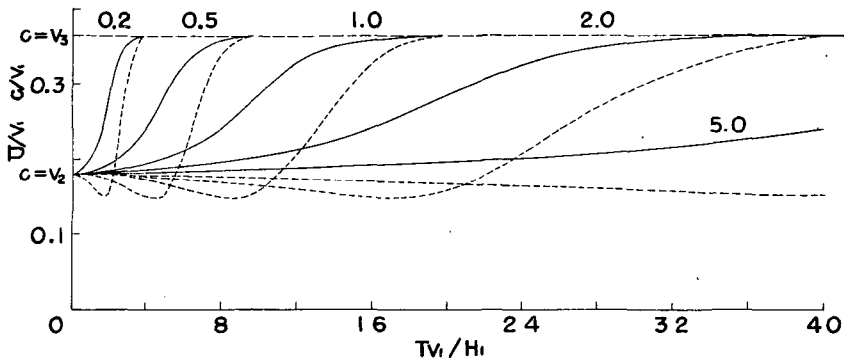


Fig. 12 (a). L_0

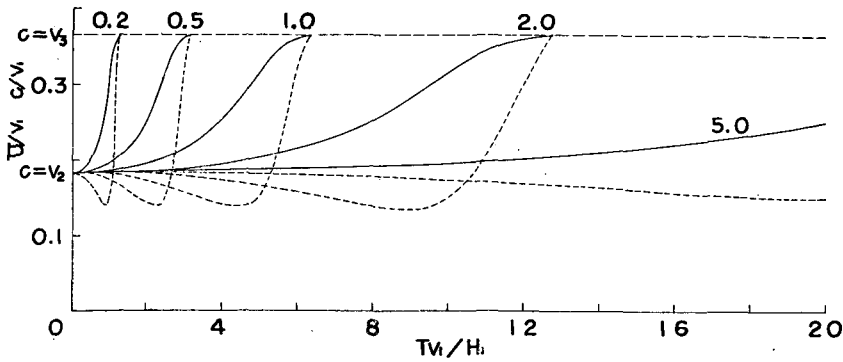


Fig. 12 (a). L_1

Fig. 12 illustrates the first two modes, L_0 and L_1 , in the two cases (a) $\mu_2/\mu_1=1/30$, $\mu_3/\mu_2=4$ and (b) $\mu_2/\mu_1=1/30$, $\mu_3/\mu_2=10$, the parameter being H_2/H_1 .

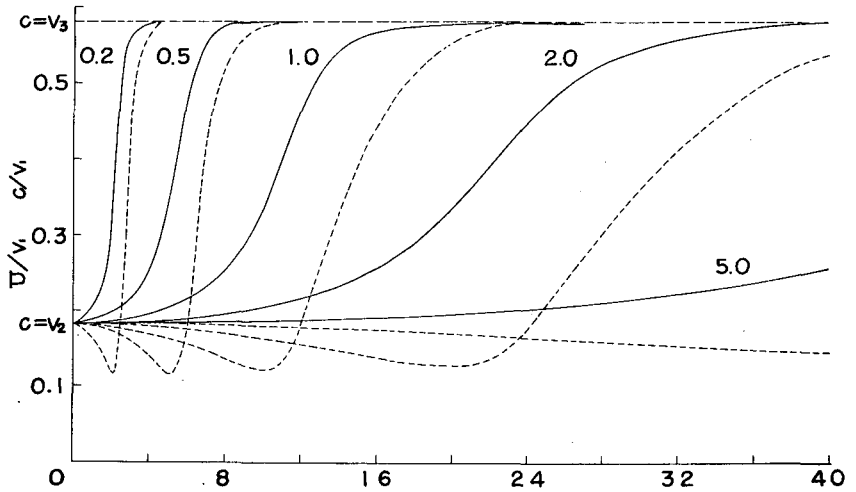


Fig. 12 (b). L_0

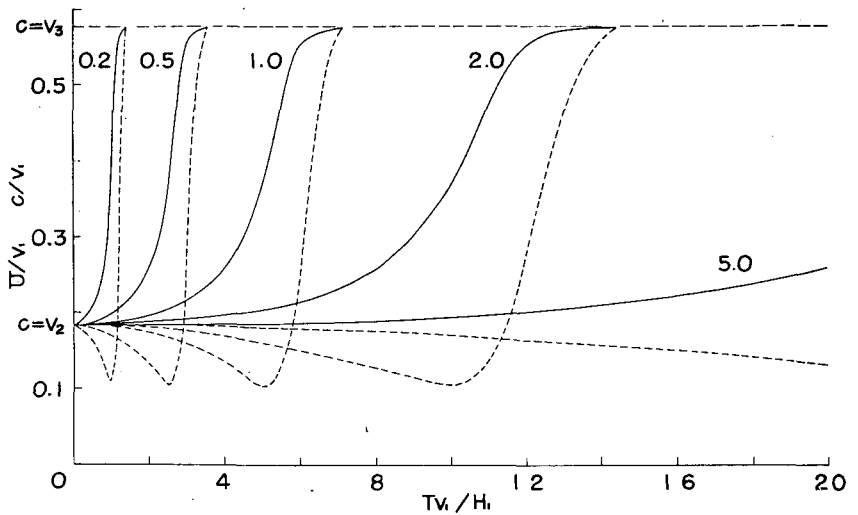


Fig. 12 (b). L_1

Fig. 12. Dispersion curves for L_0 and L_1 in cases (C).

(a) $\mu_2/\mu_1 = 1/30$ and $\mu_3/\mu_2 = 4$, (b) $\mu_2/\mu_1 = 1/30$ and $\mu_3/\mu_2 = 10$.

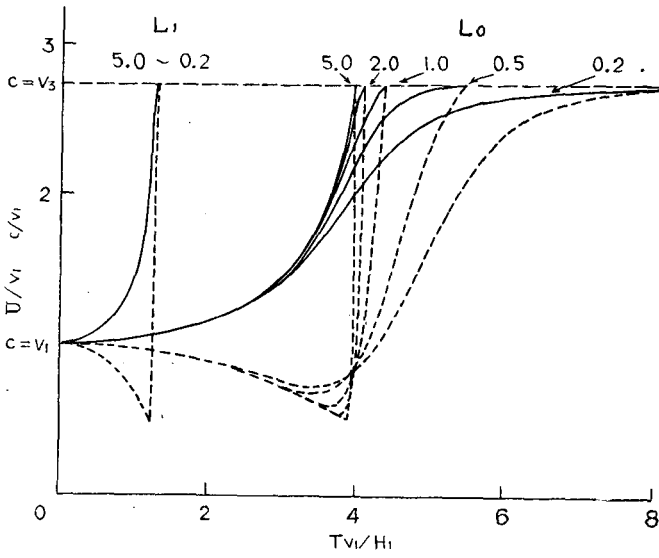


Fig. 13 (a). L_0, L_1

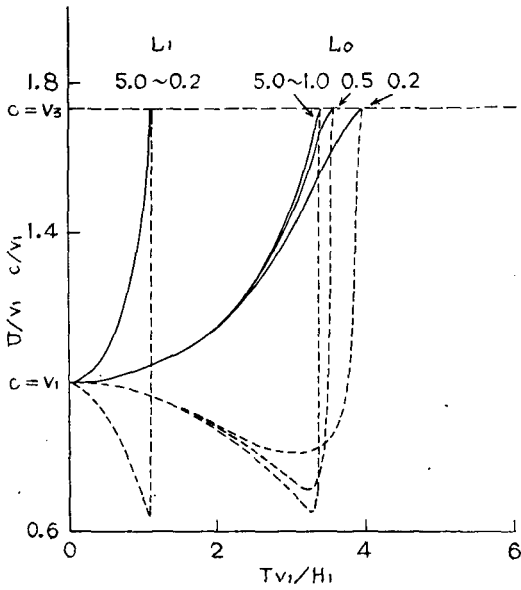


Fig. 13 (b). L_0, L_1

Fig. 13. Dispersion curves for L_0 and L_1 in case (D).

(a) $\mu_2/\mu_1 = 30$ and $\mu_3/\mu_2 = 1/4$, (b) $\mu_2/\mu_1 = 30$ and $\mu_3/\mu_2 = 1/10$.

Case (D) from (57) may take the type (i) in (26). The dispersion curve has been calculated by the characteristic equation (i) in (27). Fig. 13 illustrates the first two modes in the two cases (a) $\mu_2/\mu_1=30, \mu_3/\mu_2=1/4$ and (b) $\mu_2/\mu_1=30, \mu_3/\mu_2=1/10$, the parameter being H_2/H_1 .

In cases (A) and (B), L_0 has no cutoff period. On the other hand in cases (C) and (D), L_0 as well as L_1 has the cutoff period at which phase- and group-velocities coincide respectively with the velocity of S-wave in the lowest layer. The cutoff period must be decreased by increasing the value of H_2/H_1 .

In cases (C) and (D) the dispersion curve has only one group velocity minimum, whereas in cases (A) and (B) the curve has two group velocities minimum.

17. Numerical calculations of the amplitude function for L_0 and L_1 in cases (C) and (D)

The results are illustrated in Figs. 14 and 15. The maximum value of

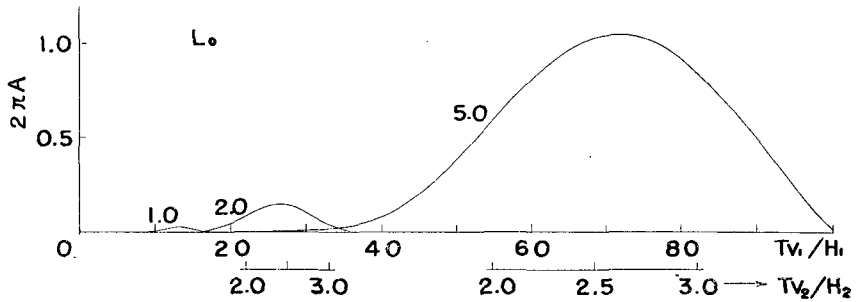


Fig. 14 (a). L_0

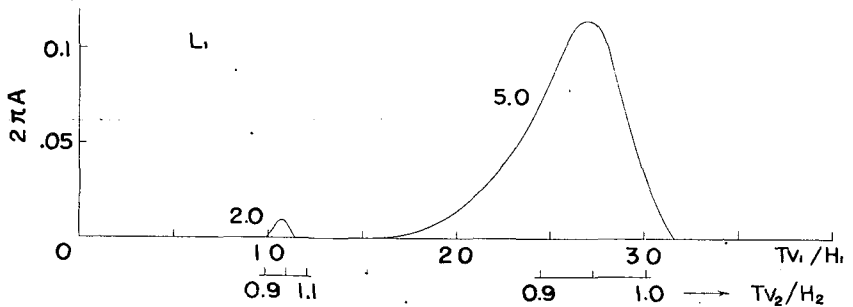


Fig. 14 (a). L_1

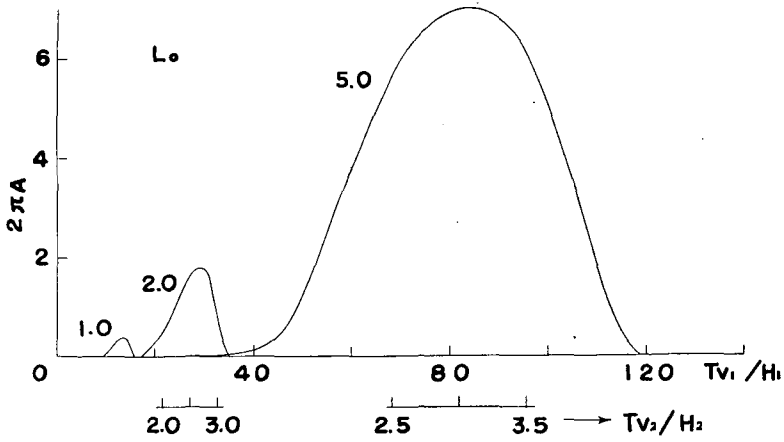


Fig. 14 (b). L_0

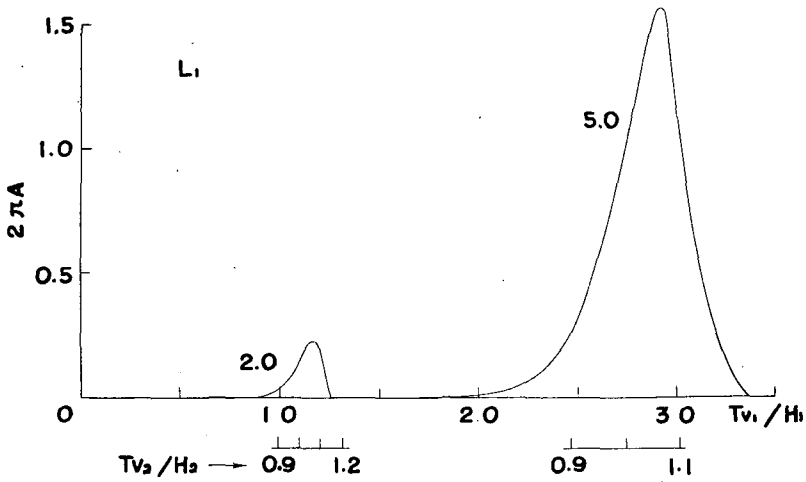


Fig. 14 (b). L_1

Fig. 14. Amplitude functions for L_0 and L_1 in case (C).

the amplitude function is slight when H_2/H_1 is small. However the maximum value increases suddenly with increase of H_2/H_1 .

In case (D) the amplitude function for L_0 , as shown in Fig. 15, has the maximum near $Tv_1/H_1=4$ and that for L_1 near $Tv_1/H_1=4/3$.

It may be curious that the amplitude function for L_1 in case (D) is not influenced by the ratio H_2/H_1 .

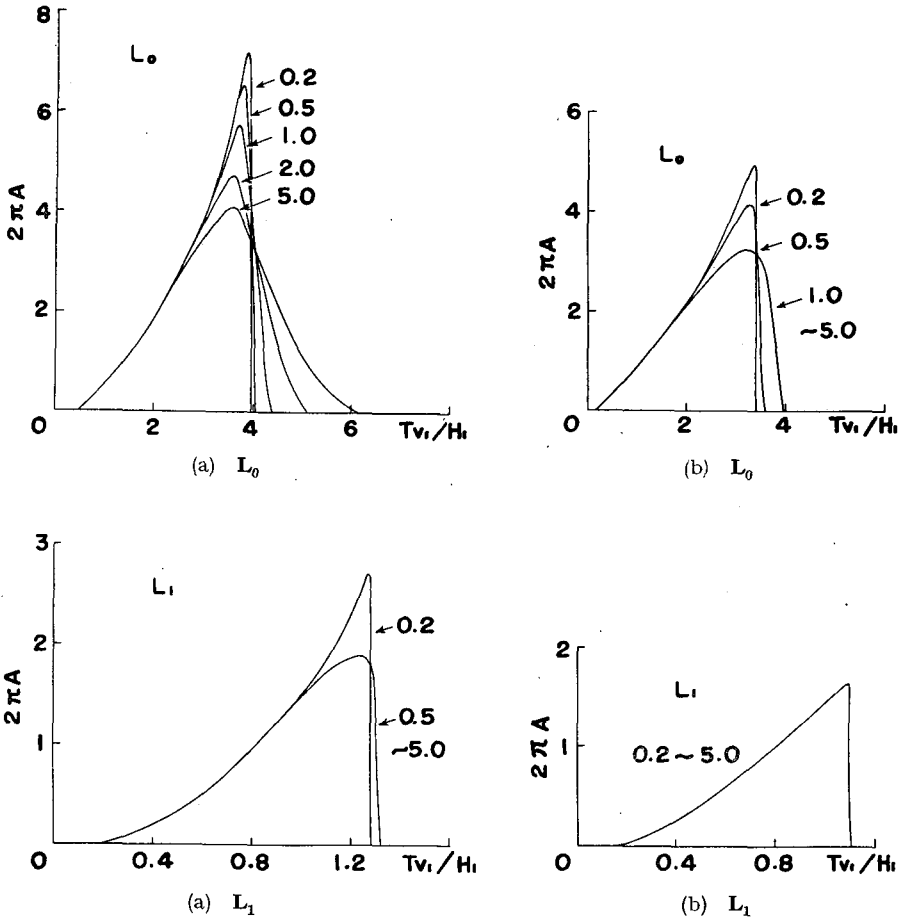


Fig. 15. Amplitude functions for L_0 and L_1 in case (D).

18. Numerical calculations of the amplitude distribution for L_0 and L_1 in cases (C) and (D)

The results are exhibited in Figs. 16 and 17 for several periods indicated by $T/(H_2/v_2)$ or $T/(H_1/v_1)$ respectively.

The amplitude distribution in case (C) may form a type of half wave-length within the second layer, whereas that in case (D) may form a type of quarter wave-length in the first layer.

It must be noted in case (D) that the amplitude in the lowest layer becomes remarkably small, if μ_3/μ_2 decreases or if H_2/H_1 increases.

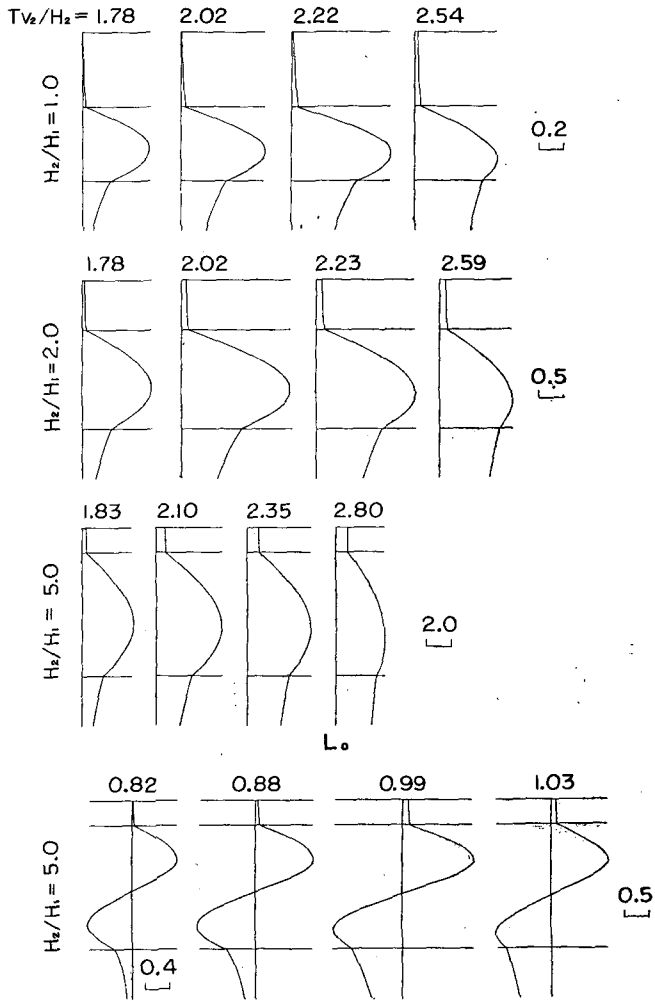


Fig. 16 (a). L_0, L_1

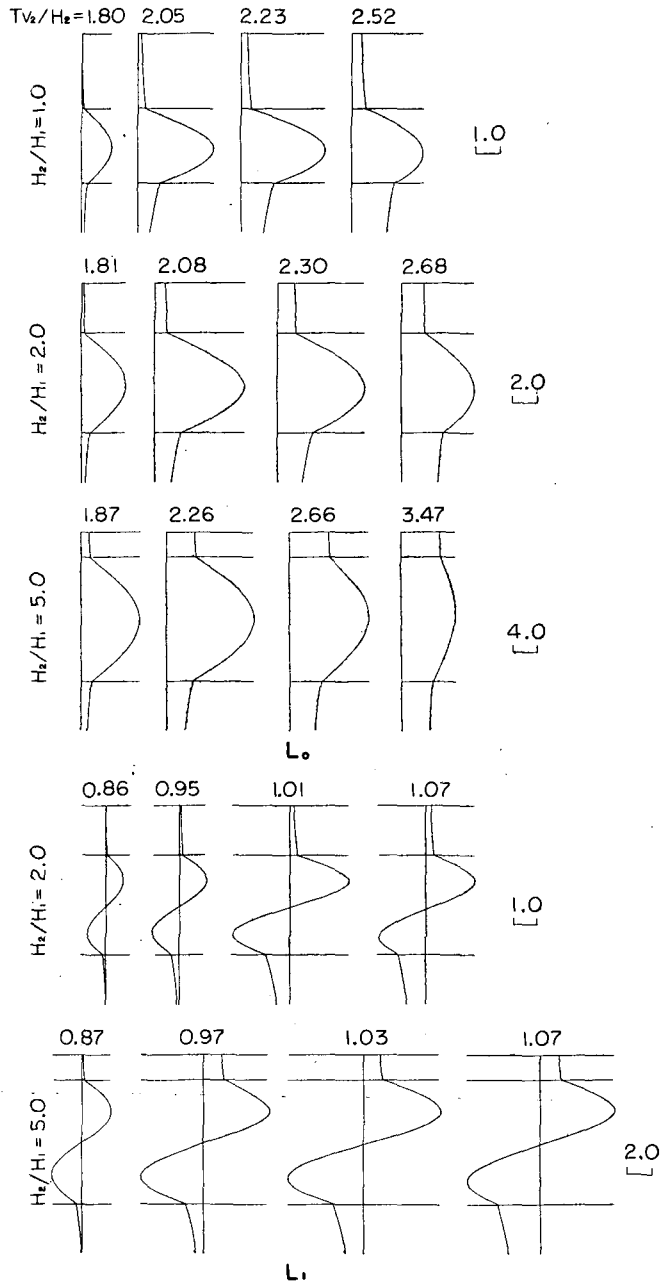


Fig. 16 (b). L_0, L_1
 Fig. 16. Amplitude distributions for L_0 and L_1 in case (C).

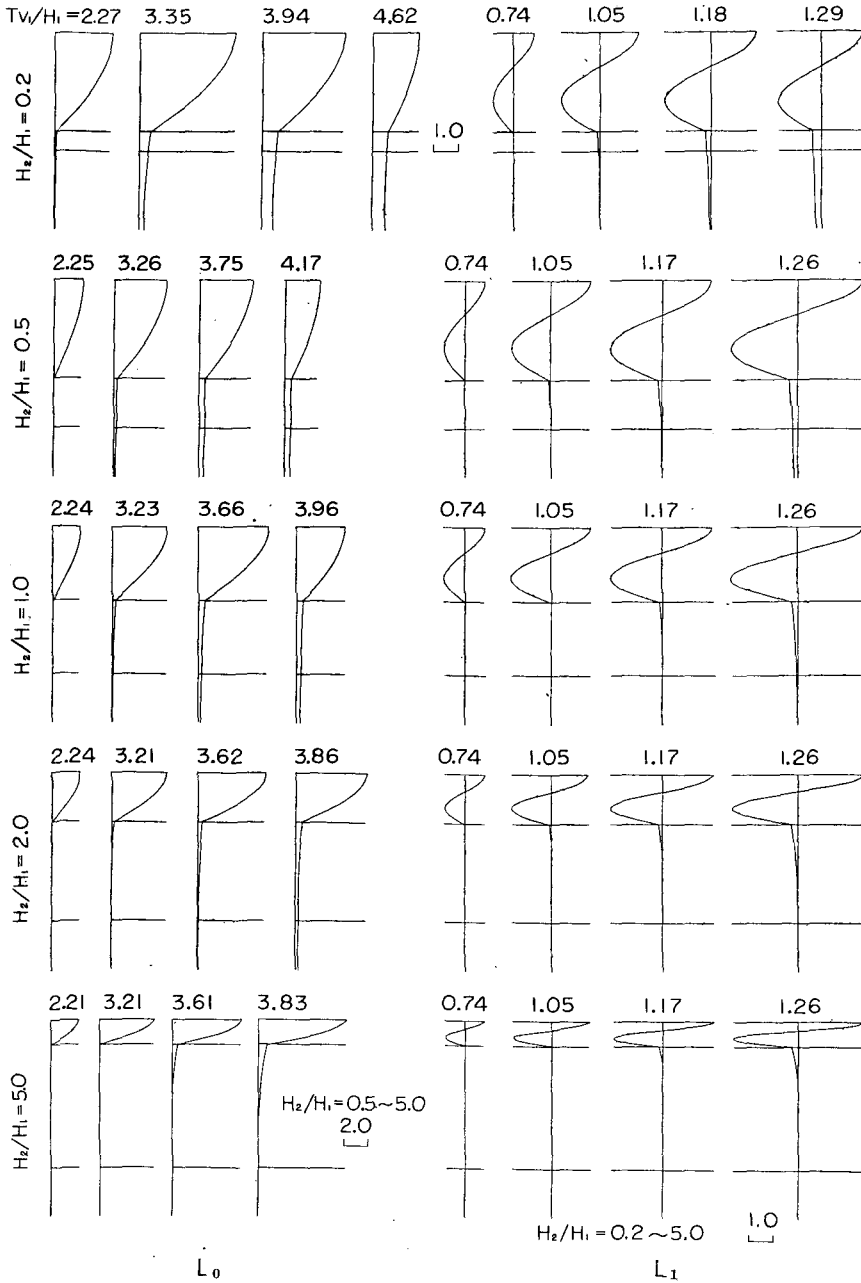


Fig. 17 (a). L_0, L_1

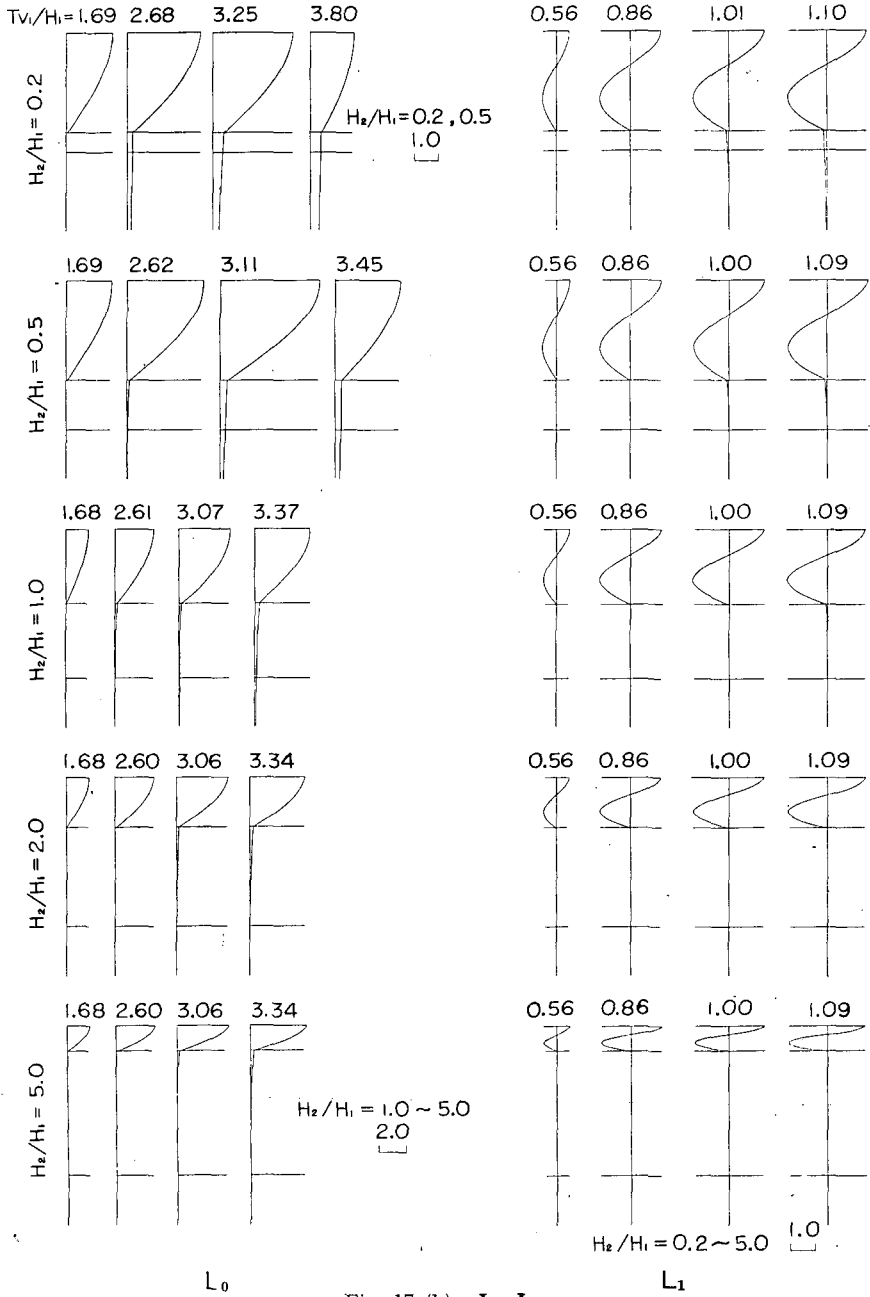


Fig. 17 (b). L_0, L_1
 Fig. 17. Amplitude distributions for L_0 and L_1 in case (D).

19. Conclusions

It is the principal aim of this paper to investigate the basic property of LOVE-waves. In order to make the idea concrete, rather large rigidity ratios are assumed in the numerical calculation, regardless of actual conditions of the earth. These large rigidity ratios will be found only near the surface of the earth in seismic prospecting.

"Quarter wave-length law" has been often experienced in actual observations⁷⁾⁸⁾. It has been expected physically that the amplitude of displacement would be large at the phase corresponding to the minimum group velocity. However this expectation was too abstract to be connected with that law actually experienced.

If a pulse is generated from a source, the observed displacement must have the factor $\{|dU/d\omega|\}^{-1/2}$, as is well known theoretically⁹⁾¹⁰⁾. Indeed the displacement of the stationary phase of group velocity will be large, owing to this factor. But the contribution of this factor is not so effective in making the amplitude of the stationary phase larger than that of the other phase.

The present authors have noticed the amplitude function $2\pi A(\xi)$ itself at periodic motion. They have found that the law of (64) was satisfied by the phase corresponding to the maximum of the amplitude function. Due to the factor $(v_1/U - v_1/c)$ contained in $2\pi A(\xi)$, it will be recognized moreover that the minimum group velocity may give the maximum amplitude.

(64) recalls to our minds the vibration of sound in a tube, one end closed. But it was obscure, with this analogy alone, why the period corresponding to the minimum group velocity would satisfy (64). To tell the truth, a quarter wave-length in z -direction is shown by the amplitude distribution more completely at $Tv_1/H_1 \doteq 0$ than at $Tv_1/H_1 \doteq 4$. It must be remarked here that the true wave-length in z -direction is $2\pi/\bar{\eta}_1$ and Tv_1 means nothing but an apparent wave-length.

In spite of this physical uncertainty, the authors think, the calculated result (64) must be interesting. The observed experience and the abstract consideration, mentioned at the beginning of this section, have been connected well by (64) to each other at least numerically.

In some cases the maximum of the amplitude function is got also near the phase corresponding to the maximum group velocity. This phenomenon is physically less clear than that described already. The low velocity layer must play a grand role in it.

Shielding effect "Quarter wave-length law" is apt to be satisfied by the large

velocity contrast between layers. The nearer the surface of the earth the boundary exists, the larger becomes its effect. If $\mu_2/\mu_1 = \mu_3/\mu_2$, for instance, the third layer cannot play an important role but the second layer plays the greatest part. This phenomenon may be recognized by comparison of Fig. 6 (a) with Fig. 6 (b). In Fig. 6 (b) very small energies penetrate into even the second layer. In this circumstance the boundary between the first and the second layers acts as if it were a shielding screen for the lower layers.

In Fig. 6 (a) the subsurface has two screens; one is the boundary between the first and the second layers and the other is that between the second and the third layers. The latter screen is more effective than the former in Fig. 6 (a). Of course, the thicker the layer acting as a screen, the stronger the shielding effect.

Owing to the shielding effect, it will be easy to deduce, by the observation of surface waves, the geological construction near the surface of the earth. On the other hand, deep constructions cannot be deduced from the observation of surface waves, because deep layers must be masked by shallow layers.

Surface waves generated near the surface must be mainly affected by very small constructions near the surface and they will take nearly whole parts of "noise" on the ground.

If the source of the wave lies deep from the surface, only the waves having large wave-length must be taken into account, because the waves having short wave-length have slight amplitude on the ground in this case. This may be understood by Fig. 6, recalling "reciprocal theorem"¹⁾. LOVE-waves having as large wave-length as the thickness of the earth's crust have been observed in natural earthquakes⁸⁾. Natural earthquakes have so deep sources that the surface waves having short wave-length which might be affected by small construction near the surface cannot be observed on the ground, no matter how large a velocity contrast exists near the surface of the earth.

Now we will conclude that the first boundary which has a large velocity contrast under the source plays the most important role in growth of LOVE-waves. To say it in other words, that first boundary alone will be found by the observation of LOVE-waves.

Acknowledgment The present authors express their thanks to Miss Miyako MUROTA for her help in numerical calculations. The authors wish again, to express their thanks to the Ministry of Education for a grant from the Science Research Fund 1958 by the aid of which the present paper has been published.

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Errata to the previous paper⁶⁾

Page Line or
expression

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|-----|--|---|---|
| 121 | (27) (iii) $\tanh \hat{\eta}_1 H_1 = \tan \bar{\delta}_{12} \dots$ | read | $\tanh \hat{\eta}_1 H_1 = -\tan \bar{\delta}_{12} \dots$ |
| 121 | last $\dots \coth \left\{ \bar{\eta}_1 H_1 \left(\frac{\hat{\eta}_2 H_2}{\bar{\eta}_1 H_1} \right) + \hat{\delta}_{22}' \right\}$ | " | $\dots \coth \left\{ \bar{\eta}_1 H_1 \left(\frac{\hat{\eta}_2 H_2}{\bar{\eta}_1 H_1} \right) + \hat{\delta}_{23}' \right\}$ |
| 122 | (31) $K_{12} K_{23} e^{-2i\eta_2 H_2}$ | " | $K_{12} K_{23} e^{-2i\eta_2 H_2}$ |
| 125 | 16 $\cosh (\hat{\eta}_2 H_2 + \hat{\delta}_{23})$ | " | $\cosh 2 (\hat{\eta}_2 H_2 + \hat{\delta}_{23})$ |
| 126 | (48) $\cot \eta_1 H_1 = \frac{\mu_1 \eta_1}{\mu_3 \eta_3} + \dots$ | " | $\cot \eta_1 H_1 = \frac{\mu_1 \eta_1}{i \mu_3 \eta_3} + \dots$ |
| 127 | 5~8 It is very easy to \dots
\dots of Love-waves. | transfer to below
line 5 of page 128 | |
| 127 | 15 $2\pi A(\xi) = \omega \left(\frac{1}{U} - \frac{1}{c} \right) \cdot \beta'$ | read | $2\pi A(\xi) = 2\pi \omega \left(\frac{1}{U} - \frac{1}{c} \right) \cdot \beta'$ |
| 127 | (52) $2\pi A(\xi) = \left(\frac{1}{U} - \frac{1}{c} \right) \dots$ | " | $2\pi A(\xi) = 2\pi \left(\frac{1}{U} - \frac{1}{c} \right) \dots$ |
| 127 | 23~25 It is very easy to
\dots for $c=v_3$. | transfer to below
line 9 of page 128 | |
| 129 | Fig. 2.(C) Ordinate 2.0 | read | 1.5 |
| 129 | 6 $T(v_1/H_1 + v_2/H_2) = 4$ | " | $T/(H_1/v_1 + H_2/v_2) = 4$ |
| 138 | 5 $R_{1,2}$ | " | $\mathcal{R}_{1,2}^{(1)}$ |