Travel time reliability-based optimization problem for CAVs dedicated lanes

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This paper proposed an optimization problem that determines the deployment pattern of dedicated lanes to connected autonomous vehicles (CAVs) considering the stochastic traffic demand and the stochastic traffic capacity. The difference between CAVs and regular human-piloted vehicles (RHVs) is driving behavior. The driving behavior of CAVs is expected to be more standardized than that of RHVs. Therefore, we assume that when the penetration ratio of CAVs increases in the lane flow, the mean lane capacity will increase, and the lane capacity variance will decrease. The mean and the variance of lane travel time decrease when the penetration ratio increases. Following this assumption, the difference in the stochastic properties between CAVs and RHVs is considered in a traffic assignment model. The traffic assignment model is formulated as a variational inequality problem. The network design problem with equilibrium constraints was solved by a simulated annealing algorithm in a test network.

Keywords: connected autonomous vehicles (CAVs); travel time reliability; stochastic lane capacity; mixed traffic flow

Subject classification codes: include these here if the journal requires them

1. Introduction

It is expected that connected-autonomous vehicles (CAVs) will penetrate the road network in the near future. CAVs can benefit society because they contribute to the improvement of traffic safety and labor productivity. From a policy perspective, it is important to stimulate customers to purchase CAVs and move to the society where CAVs penetrate more. The government can offer policies to give citizens incentives to purchase CAVs. One of the incentives is to deploy dedicated lanes to CAVs. The CAV-dedicate lane is the lane that only CAVs can pass through, while the conventional lane is shared by CAVs and the regular human-piloted vehicles (RHVs). The higher the CAVs penetration rate in society is, the more efficient road network usage is realized thanks to the proficient driving of CAVs (e.g., Levin and Boyles, 2015; Chen et al., 2016; Wang et al., 2019). The CAVs are expected to improve the link or lane capacity and to reduce road congestion. As a network management policy, the deployment of the CAV-dedicated lane can contribute to the faster penetration of CAVs into general consumers.

Some studies address a problem to determine the optimal deployment of CAV-dedicated lanes (Chen et al., 2016, 2017; Zhang and Nie, 2018). The previous studies assume a deterministic traffic demand and travel time, while some other studies focus on the difference in route choice behavior between CAVs and RHVs. The major contribution of CAVs to the whole road network is to improve traffic efficiency owing to their proficient driving behavior. Focusing on the critical headway time, the driving behaviors of CAVs and RHVs are different from each other. It is expected that the critical headway time of CAVs is shorter than that of RHVs (Levin and Boyles, 2016; Wang et al., 2019), and the variation of the headway times of CAVs is smaller than that of RHVs. The CAV technologies such as platooning technology can shorten the headway or the reaction time between the leader and the follower. The driving behaviors of RHVs have more uncertainties because the driving skill of each driver varies. If the number of skillful drivers is large in a link flow, the link capacity becomes large, and link capacity fluctuation becomes small. By contrast, if the number of non-skillful drivers increases in a link flow, the link capacity becomes small, and link capacity fluctuation becomes large. As a result, both the travel time and the fluctuation of travel time increase.

The previous studies address the mixed traffic flow of RHVs and the vehicles with the Advanced Traveler Information System (ATIS). Yang (1998) focuses on the difference in the accessibility to traffic information between ATIS-equipped vehicles and non-equipped vehicles and assumes that one of the two types of vehicles employs a user equilibrium criterion and the other one employs a stochastic user equilibrium criterion for the path choice behavior. Yang et al. (2007) extended the traffic assignment model that is proposed in Yang (1998) to a Stackelberg game and assumed that vehicles equipped with ATIS follow system optimum (SO) and other vehicles follow user equilibrium (UE) or Cournot Nash equilibrium in the path choice behavior.

Some studies recently extend the above studies to the traffic assignment model considering the mixed flows of CAVs and RHVs (Chen et al., 2016; Bagloee et al., 2017; Zhang and Nie, 2018). Some studies also focus on the difference in path choice criteria between CAVs and RHVs. Bagloee et al. (2017) and Zhang and Nie (2018) assume that RHVs and CAVs follow UE and SO in the path choice behavior. Chen et al. (2016) proposed a model that determines the optimal deployment pattern of the CAV-dedicated lanes in which CAVs and RHVs follow UE in the path choice behavior. The model assumed that the incentive for CAVs is the exclusive usage of the CAV-dedicated lane. In the model of Wang et al. (2019), CAVs follow UE, while RHVs follow the cross-nested logit (CNL) model in the path choice behavior. CNL model can relax the assumption of the perfect knowledge of the traffic situation. Seo and Asakura (2017) analyzed the market penetration dynamics of CAVs in terms of the value of time (VoT), travel time, and fare. Table 1 summarizes the path choice criteria and the assumptions for traffic demand and link capacity in the previous studies. Note that in Table 1, the constant capacity means that the link capacity is determined exogenously and independent of the penetration rate of CAVs or vehicles with ATIS. The variable capacity means the link capacity is determined depending on the penetration rate of CAVs.

[Table 1 near here]

The similarity of the above previous studies except for Chen et al. (2016) is that they assume that CAVs and RHVs follow different path choice criteria. It is reasonable to consider the differences in the knowledge level of the real-time traffic situation or VoT because the CAVs can obtain and analyze more real-time traffic information than RHVs. The CAV drivers are expected to enjoy spare time while moving in the vehicle. However, it is not reasonable to impose more additional costs on CAVs than RHVs for realizing the system optimal situation in the whole road network while RHVs behave selfishly in driving. The imposition of the additional costs to CAVs cannot motivate consumers to purchase the CAVs. Incentives should be given to CAV-users to promote CAVs for improving network efficiency.

The difference in driving behaviors can be reflected in stochastic link capacity. Many studies deal with stochastic link capacity in the traffic assignment model. Chen et al. (1999) proposed the concept of capacity reliability, and the concept was extended by Chen et al. (2002). Uchida (2015) proposed a traffic assignment problem that simultaneously considers stochastic traffic demand and stochastic link capacity. However, the stochastic link capacity that considered the mixed traffic flows of CAVs and RHVs is not considered in the previous studies as far as the authors know.

Fewer studies proposed network design problems that consider stochastic traffic demand. Uchida et al. (2011) formulated a network design problem in a multi-modal network considering the stochastic traffic demand and travel time reliability. An and Lo (2016) optimized a transit network based on robust optimization. There are two major trends to consider the stochastic demand in a network design problem. The former assumes the stochastic demand as a random variable, and the latter considers the demand uncertainty by robust optimization.

Each study listed above addresses a static traffic assignment model, while some studies deal with the mixed flow of CAVs and RHVs from the viewpoint of a dynamic approach. van den Berg and Verhoef (2016) investigated the impacts of the penetration of CAVs on the link capacity, the VoT, and changes in the heterogeneity of VoTs. Levin and Boyles (2016) proposed a multi-class cell transmission model for mixed traffic flows. Pan et al. (2019) proposed a multi-class and multilane cell transmission model for the mixed traffic flows. Ye and Yamamoto (2018) analyzed the mixed flow by the fundamental diagram approach and reported that neither the low nor high penetration rate of CAVs contributes to improving network efficiency and that the medium penetration rate may benefit network efficiency.

While the dynamic approach has fruitful advantages to evaluate the impact of the CAV technology on the traffic flow, there are limitations in implementing the network design problem because the solution obtained by the dynamic approach does not rely on the calculation condition. As mentioned in Wang et al. (2019), the dynamic traffic assignment model can describe the effect of the dynamicity of the mixed traffic flows on the path choice behavior (e.g., Levin and Boyles, 2016). However, the model may not be suitable to apply to the network design problem, such as the optimal CAV dedicated lane installation problem, because the solution requires the simulation calculation, and the high computational burden limits the application to the network design problem.

Following the above studies, as far as the authors know, there is no study that deals with the optimal CAVs dedicated lane deployment model considering the stochastic traffic demand and the travel time reliability. In terms of the stochastic aspect of CAVs, some studies address the perception error of drivers by applying the stochastic user equilibrium model or the cross-nested logit model (e.g., Yang. 1998; Wang et al., 2019). However, these studies remain to consider deterministic traffic demand and travel time. The proposed model in this paper considers the stochastic traffic demand and capacity and travel time reliability simultaneously. The path travel time is also formulated as a stochastic variable. Thus, the risk-averse path choice behavior is assumed.

The proposed model in this paper is formulated as a bi-level problem in a road network. The upper-level problem is solved by a heuristic algorithm, a simulated annealing algorithm. The lower-level problem is the traffic assignment problem formulated as a variational inequality (VI) problem like the related studies (Zhang and Nie, 2018; Chen et al., 2016; Wang et al., 2019). To represent the heterogeneous stochastic properties of driving behaviors between CAVs and RHVs, we assumed a stochastic traffic capacity (e.g., Chen et al., 1999; Sumalee and Kurauchi, 2006; Tani and Uchida, 2018) that is calculated by the mixed traffic flow of CAVs and RHVs. The capacity in this study is defined for each lane in a link in the network rather than for each link. The basic model for representing the lane capacity of CAVs and RHVs is based on the model proposed in Wang et al. (2019). We assume that the mean and the variance of the lane capacity increase and decrease, respectively, when the penetration rate of CAVs in the lane increases. The automated driving and platooning technology (Gong and Du, 2018) support this relationship.

The formulation of the stochastic traffic flow and the stochastic travel times was developed from the concept of stochastic traffic flows shown in Tani and Uchida (2018). In the path choice behavior, both CAVs and RHVs are assumed to follow UE, which is the same assumption employed in Chen et al. (2016). The VI problem is solved using successive averages (MSA) by Sheffi (1985).

In general, the network design problem is classified into a continuous network design problem and a discrete network design problem. In this paper, we optimize the location of the CAV-dedicated lane. Thus, our proposed problem is classified into a discrete network design problem. In the literature, many algorithms for a discrete network design problem are proposed, e.g., branch-and-bound technique (LeBlanc, 1975), support-function based algorithm (Gao et al., 2004), active-set algorithm (Zhang et al., 2014), and simulated annealing (Fan and Machemehl, 2006). We adopt a heuristic algorithm, a simulated annealing algorithm for solving the proposed problem to simplify the implementation.

The contributions of this study are threefold. First, this study assumed a traffic assignment model considering the stochastic mixed traffic demand and the stochastic mixed link capacity of CAVs and RHVs. The mean and the variance of lane capacity in this study are dependent on the CAV penetration ratio in a lane flow.

Second, we applied the proposed traffic assignment model to a network design problem that determines the optimal locations of the CAV-dedicated lanes. Third, CAVs and RHVs are assumed to take a risk-averse path choice behavior considering travel time reliability.

The remainder of this paper comprises three sections: Section 2 presents notations, assumptions, and the methodology of the proposed model in this study. Section 3 presents the results of the numerical calculations. Concluding remarks and future research needs are presented in the final section.

2. Methodology

Let $G(N, A)$ denote a general road network. N and A are the sets of nodes and links in the network, respectively.

2.1 Notations and assumptions

The notations used in this paper are summarized below.

N Set of nodes

A Set of links

I Set of OD pairs

$J_{i,h}$ Set of paths between OD pair *i* for type $h \in \{CAV, RHV\}$ vehicles

Q Total traffic demand with the mean of q and the coefficient of variation of cv

Q_i Traffic demand between OD pair *i* corresponding to the portion of p_i

Q_i^h Traffic demand of type h vehicles between OD pair *i* corresponding to the portion of $p_{h,i}$

F_{ij}^h The flow of type h vehicles on path *j* between OD pair *i* corresponding to the portion of $p_{h,i,j}$ with the mean of f_{ij}^h

V_l^h The flow of type h vehicles on lane *l* corresponding to the portion of p_l^h with the mean of v_l^h

C_l The capacity of lane *l* with the mean of c_l and the coefficient of variation of cv_l

C_l^h The capacity of lane *l* with the pure traffic flow of type *h*

p_l The penetration rate of CAVs on lane *l*

L The set of lanes that is defined as $L = L_d \cup L_s$ where L_d is the set of CAV dedicated lanes and L_s is the set of shared lanes

L_c The set of candidates for the dedicated lane or the shared lane

L_n The set of non-candidates for the dedicated lane or the shared lane

T_l Travel time of lane *l*

$T_{h,i,j}$ Travel time of type *h* vehicles on path *j* between OD pair *i*

$\eta_{h,i,j}$ Travel cost of type *h* vehicles on path *j* between OD pair *i*

$\mathbf{\Omega_f}$ The set of mean path flow vector that is represented as $\mathbf{\Omega}_{\mathbf{f}_{CAV}} \times \mathbf{\Omega}_{\mathbf{f}_{RHV}}$ where $\mathbf{\Omega}_{\mathbf{f}_{CAV}}$ and $\mathbf{\Omega}_{\mathbf{f}_{RHV}}$ are the sets of the mean path flow vector of CAVs and RHVs, respectively

$\mathbf{\Omega}_{\mathbf{v}_s}$ The set of mean flows of the shared lanes that is represented as $\mathbf{\Omega}_{\mathbf{v}_{CAV,s}} \times \mathbf{\Omega}_{\mathbf{v}_{RHV}}$ where $\mathbf{\Omega}_{\mathbf{v}_{CAV,s}}$ is the set of mean flows of CAVs on the shared lane and $\mathbf{\Omega}_{\mathbf{v}_{RHV}}$ is the set of mean flows of RHVs

$\mathbf{\Omega}_{\mathbf{v}_d}$ The set of mean flows of the dedicated lanes for CAVs

$\mathbf{x}$ The vector of the policy variables of the upper problem

Without loss of generality, the following assumptions are set in this paper.

(1) The total traffic demand is assumed to follow a lognormal distribution (Tani and Uchida, 2018). The mean and the coefficient of variation of the total traffic demand are given exogenously.

(2) Each OD pair has traffic demands of CAVs and RHVs.

(3) The lane capacity follows a lognormal distribution. The mean and the coefficient of variation of the lane capacity are determined by the penetration rate of CAVs on the lane.

2.2 Network representation

This study presents the special network representation to demonstrate the network design under the mixed traffic flows of CAVs and RHVs. Each link in the road network in this study is composed of two lanes. The possible combination of two lanes in each link is either a CAV dedicated lane and a shared lane or two shared lanes. There is at least one shared lane in each link of the network. Another lane can be converted from the shared lane to the CAV dedicated lane. Note that this assumption is set for simplifying the scheme proposed in this study. If each link has a couple of lanes, the total number of policy variables in the feasible region changes depending on the network setting. For example, when the number of targeted links is *N*, and the number of lanes of each targeted link is m, the total number of policy variables in the feasible region is $(m+1)^N$.

In this study, a path in the network is defined as the sequence of lanes. Therefore, the travel time of a path in the network is calculated as the summation of travel times of lanes that compose the path. All CAVs and RHVs can pass through the shared lanes, while only CAVs can pass through the CAV dedicated lanes. Thus, CAVs have more alternative paths than RHVs, when at least one link has a CAV dedicated lane in the network. The set of lanes in the network is denoted by $L = L_d \cup L_s$ where L_d is the set of CAV dedicated lanes and L_s is the set of shared lanes in the network. Each lane $l \in L$ has its capacity denoted by C_l. The mean and the coefficient of variation of each lane capacity in the network are determined only by the penetration rate of CAVs of the lane. The set of lanes in the network is also denoted by $L = L_c \cup L_n$ where L_c is the set of candidates for the dedicated lane of CAVs and L_n is the set of shared lanes. Each lane in L_c can be the dedicated lane or the shared lane. The vector of policy variables, $\mathbf{x}$ expresses the state of each lane in L_c. The element of $\mathbf{x}$, x_l $(\forall l \in L_c)$ is equal to one if lane *l* is the dedicated lane of CAVs, and to zero if lane *l* is the shared lane.

Figure 1 illustrates an example of the concept of the shared lane and the dedicated lane in a simple network. The network has an OD pair, two paths, and three links. Each link has two lanes. Hence, the road network consists of 6 lanes and three nodes. We assume that one lane on each link is a shared lane, and the other lane is the candidate for the dedicated lane of CAVs. In Figure 1, the vector of policy variables, $\mathbf{x} = (0,0,1)$ shows that only link 3 has a dedicated lane of CAVs. The drivers can change their lanes at each node. The road administrator is assumed to decide where to deploy the CAV dedicated lane(s) in the road network. Thus, the number of alternative policies for the road administrator is $2^{|L_c|}$. As shown above, the link and lane concepts are explicitly distinguished in this paper.

[Figure 1 near here]

2.3 Traffic flow modeling

The methodology applied for modeling the mixed traffic flows with CAVs and RHVs is identical to our previous work (Tani and Uchida, 2018). The appendix shown in Tani and Uchida (2018) mathematically proved that the coefficient of variation of each link flow in a network that is calculated based on independent stochastic OD traffic demands is smaller than that calculated based on correlated stochastic OD traffic demands. This study extends the findings shown in Tani and Uchida (2018) and presents the new formulation of stochastic traffic flows that need no approximate expression in calculating the moment of random network variables.

We assume a stochastic total traffic demand, Q, which follows a lognormal distribution. Its mean and coefficient of variance are represented as q and cv, respectively, and are given exogenously. The mean and variance of total traffic demand are formulated respectively as:

$$E[Q] = q \tag{1}$$

$$\mathrm{var}[Q] = (q \cdot cv)^2 \tag{2}$$

The traffic demand for OD pair i, which is defined by using the ratio of the demand to the total traffic demand, $p_i (\geq 0)$ is shown as:

$$Q_i = p_i \cdot Q \quad \forall i \in I \tag{3}$$

where $\sum_{i \in I} p_i = 1$. The mean and variance-covariance of each OD traffic demand are defined respectively as:

$$E[Q_i] = p_i \cdot q \quad \forall i \in I \tag{4}$$

$$\mathrm{cov}[Q_i, Q_{i'}] = p_i \cdot p_{i'} \cdot (q \cdot cv)^2 \quad \forall i, i' \in I \tag{5}$$

Here, note that (5) expresses also the variance of the traffic demand for OD pair i, when $i = i'$. Next, we set the penetration rate of vehicle type h ($\in \{CAV, RHV\}$) for OD pair i, $p_{h,i}$ across all OD pairs where $\sum_{h \in \{CAV,RHV\}} p_{h,i} = 1$ ($\forall i \in I$) and $p_{h,i} \geq 0$ ($\forall i \in I, \forall h \in \{CAV, RHV\}$). The traffic demand of type h vehicles for OD pair i is defined as:

$$Q_i^h = p_{h,i} \cdot Q_i \tag{6}$$

$$= p_{h,i} \cdot p_i \cdot Q \quad \forall i \in I, \forall h \in \{CAV, RHV\}$$

The penetration rate, $p_{h,i}$ corresponds to the portion of type h vehicles for OD pair i. The mean and the variance-covariance of each traffic demand are defined respectively as:

$$E\left[Q_i^h\right] = p_{h,i} \cdot E[Q_i] \tag{7}$$

$$= p_{h,i} \cdot p_i \cdot q \quad \forall h \in \{CAV, RHV\}\, \forall i \in I$$

$$\operatorname{cov}\left[Q_i^h, Q_{i'}^{h'}\right] = p_{h,i} \cdot p_i \cdot p_{h',i'} \cdot p_i \cdot (q \cdot cv)^2 \quad \forall h, h' \in \{CAV, RHV\}\, \forall i, i' \in I \tag{8}$$

The flow of type h vehicles on path j that serves OD pair i is defined as

$$F_{ij}^h = p_{h,i,j} \cdot Q_i^h \tag{9}$$

$$= p_{h,i,j} \cdot p_{h,i} \cdot p_i \cdot Q \quad \forall i \in I, \forall h \in \{CAV, RHV\}$$

where $p_{h,i,j} (\geq 0)$ is the portion of type h vehicles on path j serving OD pair i and that holds $\sum_j p_{h,i,j} = 1$. The mean and variance-covariance of each path flow are defined respectively as:

$$E\left[F_{ij}^h\right] = E\left[p_{h,i,j} \cdot Q_i^h\right] \tag{10}$$

$$= p_{h,i,j} \cdot p_{h,i} \cdot p_i \cdot q \quad \forall h \in \{CAV, RHV\}\, \forall i \in I, \forall j \in J_{i,h}$$

$$\operatorname{cov}\left[F_{ij}^h, F_{i'j'}^{h'}\right] = p_{h,i,j} \cdot p_{h,i} \cdot p_i \cdot p_{h',i',j'} \cdot p_{h',i'} \cdot p_i \cdot (q \cdot cv)^2 \tag{11}$$

$$\forall h, h' \in \{CAV, RHV\}\, \forall i, i' \in I, \forall j, j' \in J_{i,h}$$

Here, the portions of $p_{h,i}$ and p_i are given exogenously. However, the path choice probabilities of $p_{h,i,j}$ are calculated endogenously by solving the equilibrium problem that is described later in this paper. The path flow and the mean path flow are always non-negative.

$$F_{ij}^{h} \geq 0 \quad \forall h \in \{CAV, RHV\}\, \forall i \in I, \forall j \in J_{i,h} \tag{12}$$

$$E\left[F_{ij}^{h}\right] \geq 0 \quad \forall h \in \{CAV, RHV\}\, \forall i \in I, \forall j \in J_{i,h} \tag{13}$$

For each OD pair, since the sum of path flows must be the corresponding OD demand, the sum of path flows and the mean path flows are respectively preserved as

$$\sum_{j \in J_{i,h}} F_{ij}^{h} = Q_{i}^{h} \quad \forall h \in \{CAV, RHV\}\, \forall i \in I \tag{14}$$

$$\sum_{j \in J_{i,h}} E\left[F_{ij}^{h}\right] = E\left[Q_{i}^{h}\right] \quad \forall h \in \{CAV, RHV\}\, \forall i \in I \tag{15}$$

The flow of type *h* vehicles on lane *l* is represented as:

$$V_{l}^{h} = \sum_{i \in I} \sum_{j \in J_{h,i}} \delta_{i,j,l} \cdot F_{ij}^{h} \quad \forall h \in \{CAV, RHV\}\, \forall l \in L \tag{16}$$

where $\delta_{i,j,l}$ is a variable that equals one if lane l is a part of path $j \in J_{i,h}$ and 0 otherwise. Note that each link in the network comprises two lanes, and thus the two sets of nodes and lanes that compose the network are as described in **2.2**. The mean flow of type *h* vehicles on each lane is represented as:

$$E\left[V_{l}^{h}\right] = \sum_{i \in I} \sum_{j \in J_{h,i}} \delta_{i,j,l} \cdot E\left[F_{ij}^{h}\right] \quad \forall h \in \{CAV, RHV\}\ \forall l \in L \tag{17}$$

In the above equations, if $l \in L_s$ then lane l is a shared lane. If $l \in L_d$ then lane l is a CAV-dedicated lane.

From (9), the path flow is represented as a product of a variable, $p_{h,i,j} \cdot p_{h,i} \cdot p_h$, and the total traffic demand, Q. A lane flow summarizes the path flows as shown in (16) and (17). Hence, the lane flow is also represented as a product of a variable, and the total traffic demand shown as:

$$V_l^h = \sum_{i \in I} \sum_{j \in J_{h,i}} \delta_{i,j,l} \cdot F_{ij}^h = p_l^h \cdot Q \quad \forall h \in \{CAV, RHV\} \ \forall l \in L \tag{18}$$

Here, $p_l^h = \sum_{i \in I} \sum_{j \in J_{h,i}} \delta_{i,j,l} \cdot p_{h,i,j} \cdot p_{h,i} \cdot p_h$ is a variable that corresponds to the lane flow V_l^h. This property of the flow of type *h* vehicles on lane $l \in L$ realizes the simple representation of its variance. The variance of the flow of type *h* vehicles on lane $l \in L$ is represented as:

$$\begin{aligned} \mathrm{var}\left[V_l^h\right] &= \mathrm{var}\left[\sum_{i \in I} \sum_{j \in J_{h,i}} \delta_{i,j,l} \cdot F_{ij}^h\right] \\ &= \sum_{i \in I} \sum_{i' \in I} \sum_{j \in J_{h,i}} \sum_{j' \in J_{h,i'}} \delta_{i,j,l} \cdot \delta_{i',j',l} \cdot \mathrm{cov}\left[F_{ij}^h, F_{i'j'}^h\right] \\ &= \sum_{i \in I} \sum_{i' \in I} \sum_{j \in J_{h,i}} \sum_{j' \in J_{h,i'}} \delta_{i,j,l} \cdot \delta_{i',j',} \cdot p_{h,i,j} \cdot p_{h,i',j'} \cdot \mathrm{cov}\left[Q_i^h, Q_{i'}^h\right] \\ &= \left(\sum_{i \in I} \left(\sum_{j \in J_{h,i}} \delta_{i,j,l} \cdot p_{h,i,j}\right) \cdot q_i^h\right)^2 \cdot cv^2 \\ &= \left(v_l^h \cdot cv\right)^2 \quad \forall h \in \{CAV, RHV\}, \forall l \in L \end{aligned} \tag{19}$$

where $v_l^h = E\left[V_l^h\right]$ and $q_i^h = E\left[Q_i^h\right]$. Note that the variance of the total flow on lane *l*, $V_l \left(= \sum_{h \in \{CAV, RHV\}} V_l^h\right)$ is represented as:

$$
\begin{aligned}
\mathrm{var}[V_l] &= \mathrm{var}\left[\sum_{h\in\{CAV,RHV\}} V_l^h\right] \\
&= \mathrm{var}\left[\sum_{h\in\{CAV,RHV\}} \sum_{i\in I} \sum_{j\in J_{h,i}} \delta_{i,j,l} \cdot F_{ij}^h\right] \\
&= \sum_{h\in\{CAV,RHV\}} \sum_{h'\in\{CAV,RHV\}} \sum_{i\in I} \sum_{i'\in I} \sum_{j\in J_{h,i}} \sum_{j'\in J_{h',i'}} \delta_{i,j,l} \cdot \delta_{i',j',l} \cdot \mathrm{cov}\left[F_{ij}^h, F_{i'j'}^{h'}\right] \\
&= \sum_{h\in\{CAV,RHV\}} \sum_{h'\in\{CAV,RHV\}} \sum_{i\in I} \sum_{i'\in I} \sum_{j\in J_{h,i}} \sum_{j'\in J_{h,i'}} \delta_{i,j,l} \cdot \delta_{i',j',l} \cdot p_{h,i,j} \cdot p_{h,i',j'} \\
&\qquad \cdot \mathrm{cov}\left[Q_i^h, Q_{i'}^{h'}\right] \\
&= \left(\sum_{h\in\{CAV,RHV\}} \left(\sum_{i\in I} \left(\sum_{j\in J_{h,i}} \delta_{i,j,l} \cdot p_{h,i,j}\right) \cdot q_i^h\right)\right)^2 \cdot cv^2 \\
&= (v_l \cdot cv)^2 \quad \forall l \in L
\end{aligned}
\tag{20}
$$

where $v_l = E[V_l]$. The variance of lane flow is represented with the mean lane flow and the coefficient of variation of the total traffic demand from the above equation. This formulation is based on Tani and Uchida (2018). The relationship between (19) and (20) is demonstrated with a toy network in the **Appendix**.

The series of the formulation of stochastic traffic flows is advantageous to the numerical computation because it does not require the approximation of the random variables. This property contributes to computing the network design problem constrained to the traffic assignment problem considering the stochastic traffic network and the travel time reliability.

2.4 Stochastic lane capacity in the mixed flow

In previous studies that deal with mixed flows of CAVs and RHVs, a deterministic lane capacity is assumed in the same manner as employed in calculating traffic demands and travel times (e.g., Zhang and Nie, 2018; Wang et al., 2019). The previous studies consider the difference in the headway times between CAVs and RHVs for expressing different driving behaviors of CAVs and RHVs (e.g., Levin and Boyles, 2016; Wang et al., 2019). However, the lane capacity representation in the previous studies misses the difference in the stochastic nature between the driving behaviors. The mean and variance of lane capacity are expected to increase and decrease, respectively, when the penetration rate of CAVs increases in the lane flow. This study considers the relationship between stochastic lane capacity and the penetration rate of CAVs in a lane. In this study, we adopt the lane capacity that is proposed in Wang et al. (2019) shown below:

$$c_l(v_l^{CAV}, v_l^{RHV}) = \frac{1}{p_l(v_l^{CAV}, v_l^{RHV}) \cdot \frac{1}{c_{l,CAV}} + \left(1 - p_l(v_l^{CAV}, v_l^{RHV})\right) \cdot \frac{1}{c_{l,RHV}}} \quad \forall l \in L \tag{21}$$

where

$$p_l(v_l^{CAV}, v_l^{RHV}) = \frac{v_l^{CAV}}{v_l^{CAV} + v_l^{RHV}} \quad \forall l \in L \tag{22}$$

The inverses of lane capacities for pure CAV flows and pure RHV flows approximate headway times for CAVs and RHVs. Here, we assume that the headway times for CAVs and RHVs are equal across the same vehicle type (Wang et al., 2019).

Wang et al. (2019) assumed the deterministic road network and therefore represented the lane capacity as the deterministic variable. The above two equations use the notations defined in this study. $c_{l,CAV}$ and $c_{l,RHV}$ are the deterministic lane capacity when the penetration rates of CAVs are one and zero, respectively. Note that the lane capacity and the penetration rate of CAVs are determined by the deterministic lane flows of CAVs and RHVs, v_l^{CAV} and v_l^{RHV}. Following the above conditions, we extend (21) by substituting the stochastic lane capacity, $C_{l,CAV}$ and $C_{l,RHV}$ following lognormal distributions for $c_{l,CAV}$ and $c_{l,RHV}$ in (21), respectively. The stochastic lane capacity of the mixed traffic flow is then defined as:

$$C_l(V_l^{CAV}, V_l^{RHV}) = \frac{1}{p_l(V_l^{CAV}, V_l^{RHV}) \cdot \frac{1}{C_{l,CAV}} + \left(1 - p_l(V_l^{CAV}, V_l^{RHV})\right) \cdot \frac{1}{C_{l,RHV}}} \quad \forall l \in L \tag{23}$$

where

$$p_l(V_l^{CAV}, V_l^{RHV}) = \frac{V_l^{CAV}}{V_l^{CAV} + V_l^{RHV}} = \frac{p_l^{CAV}}{p_l^{CAV} + p_l^{RHV}} \ \forall l \in L \tag{24}$$

Note that, on the right side of (23), the inverse of lane capacity given exogenously also follows a lognormal distribution, while the summation of the lognormal distributions does not follow a lognormal distribution. Some studies use lognormal distributions (e.g., Sumalee and Xu, 2011; Zhou and Chen, 2008; Zhao and Kockleman, 2002) to express traffic flows. The method of Fenton (1960) is adopted to approximate the summation of independent lognormal distributions as a lognormal distribution. Hence, the lane capacity of (23) is now approximated as a lognormal distribution. In (24), the penetration rate of CAVs is defined by using stochastic lane flows. The lane flow, V_l^h, in this study is defined as the product of a variable, p_l^h, and the total traffic demand Q as shown in (18). Since the total traffic demand appears in both the numerator and denominator of (24), the penetration rate of CAVs is calculated using the corresponding variables of the lane flows. Figure 2 shows the mean lane capacity when the penetration rate of CAVs changes from 0 to 1.

[Figure 2 near here]

2.5 Travel time

We calculate the lane travel time by applying the BPR function (Bureau of public roads, 1964). The BPR function is often used in some previous studies. We substitute the stochastic lane flows and lane capacity into the BPR function and obtain the lane travel time as a random variable (e.g., Lam et al., 2008; Shao et al., 2006; Uchida, 2015).

$$T_l(V_l^{CAV}, V_l^{RHV}) = t_l^0 \cdot \left(1 + \beta_l(V_l^{CAV}, V_l^{RHV}) \cdot \left(\frac{V_l^{CAV} + V_l^{RHV}}{C_l(V_l^{CAV}, V_l^{RHV})}\right)^{n_l}\right) \quad \forall l \in L \tag{25}$$

According to **2.3** and **2.4**, the lane flows and the lane capacity follow lognormal distributions. Here, t_l^0 and n_l are the free-flow travel time and the parameter for the BPR function of lane *l*. The parameter $\beta_l(V_l^{CAV}, V_l^{RHV})$ in (25), which is originally defined in this paper, is a decreasing function with respect to the penetration rate of CAVs. The lane travel time has a deterministic term and a stochastic term. The deterministic term is a constant that represents free-flow travel time, and the stochastic term is a stochastic delay due to congestion. The stochastic term follows a lognormal distribution. Therefore, the lane travel time follows a shifted-lognormal distribution (Srinivasan et al., 2014; Tani and Uchida, 2018). The mean and variance of lane travel time are denoted respectively as:

$$E[T_l] = t_l^0 \cdot \left(1 + \beta_l(V_l^{CAV}, V_l^{RHV}) \cdot E\left[\left(\frac{V_l^{CAV} + V_l^{RHV}}{C_l(V_l^{CAV}, V_l^{RHV})}\right)^{n_l}\right]\right) \quad \forall l \in L \tag{26}$$

$$\mathrm{var}[T_l] = E[T_l^2] - (E[T_l])^2 \quad \forall l \in L \tag{27}$$

where

$$E[T_l^2] = (t_l^0)^2 \cdot \begin{pmatrix} 1 + 2\beta_l(V_l^{CAV}, V_l^{RHV}) \cdot E\left[\left(\frac{V_l^{CAV} + V_l^{RHV}}{C_l(V_l^{CAV}, V_l^{RHV})}\right)^{n_l}\right] \\ +\beta_l^2(V_l^{CAV}, V_l^{RHV}) \cdot E\left[\left(\frac{V_l^{CAV} + V_l^{RHV}}{C_l(V_l^{CAV}, V_l^{RHV})}\right)^{2n_l}\right] \end{pmatrix} \quad \forall l \in L \tag{28}$$

The flow-capacity ratio and its moment are analytically calculated following the property of a lognormal distribution since $V_l^{CAV} + V_l^{RHV} = (p_l^{CAV} + p_l^{RHV}) \cdot Q$ follows a lognormal distribution. Some previous studies that assume other distributions (e.g., normal distribution) as a traffic demand require an approximation method for calculating the moments of random variables such as Isserlis (1918) (e.g., Clark and Watling, 2005; Uchida, 2014). We also note that the adopted method does not require the Taylor series approximation of the lane travel time function, as shown in Uchida (2014). Figure 3 shows the mean lane travel time when the mean lane flows of CAVs and RHVs change from 0 to 1,200 [pcu/hour].

[Figure 3 near here]

The path travel time for traffic demand of type h vehicles is the summation of travel times of the lanes that compose the path, which is shown as:

$$T_{h,i,j} = \sum_{l \in L} \delta_{h,i,j,l} \cdot T_l \quad \forall h \in \{CAV, RHV\} \; \forall i \in I \; \forall j \in J_i, \tag{29}$$

The mean and variance of the path travel time are denoted respectively as:

$$E\left[T_{h,i,j}\right] = \sum_{l \in L} \delta_{i,j,l} \cdot E[T_l] \quad \forall h \in \{CAV, RHV\} \; \forall i \in I \; \forall j \in J_{h,i} \tag{30}$$

$$\mathrm{var}\left[T_{h,i,j}\right] = \sum_{l \in L} \delta_{i,j,l} \cdot \mathrm{var}[T_l] \quad \forall h \in \{CAV, RHV\} \; \forall i \in I \; \forall j \in J_{h,i} \tag{31}$$

For setting the path choice criteria, all drivers in this study are assumed to take risk-averse path choice behavior. The path travel cost in this study is calculated as:

$$\eta_{h,i,j} = E\left[T_{h,i,j}\right] + \gamma \cdot \mathrm{var}\left[T_{h,i,j}\right] \quad \forall h \in \{CAV, RHV\} \; \forall i \in I \; \forall j \in J_{h,i} \tag{32}$$

Note that γ denotes the risk-averse degree and that the drivers in this study are assumed to decide their paths based only on the travel time and the travel time reliability. The covariance of the travel times between two lanes in the network can be defined analytically. However, for simplicity, we calculate the variance of the path travel time without considering the lane travel time covariance.

2.6 Multiclass traffic assignment model

In this section, we show a traffic assignment model for mixed traffic flows. This problem corresponds to the lower problem of the proposed bi-level problem. The traffic assignment problem of drivers who take the risk-averse path choice behavior is formulated as a variational inequality (VI) problem, same as Zhang and Nie (2018) and Wang et al. (2019). The equilibrium conditions of the traffic assignment model are shown as:

$$f_{h,i,j} \cdot \left(\eta_{h,i,j}(\mathbf{f}) - \pi_i\right) = 0 \quad \forall h \in \{CAV, RHV\}\, \forall i \in I\, \forall j \in J_{h,i} \tag{33}$$
$$\eta_{h,i,j}(\mathbf{f}) - \pi_i \geq 0, f_{h,i,j} \geq 0 \;\forall h \in \{CAV, RHV\}\, \forall i \in I\, \forall j \in J_{h,i}$$

where $\mathbf{f}$ represents the vector of the mean path flows. Note that $f_{h,i,j} = E\left[F_{h,i,j}\right]$ and π_i is the minimum path cost between OD pair i. Following the formulations in 2.3, 2.4, and 2.5, note that the mean path flow vector determines the mean and variance of link flow, link travel time, and path travel time respectively and deterministically. Thus, the mean path flow vector determines the path travel cost considering the stochastic path travel time deterministically following the related prior studies (e.g., Shao et al., 2006; Lam et al., 2008; Chen et al., 2010).

We set the convex set of the mean path flow vector, $\boldsymbol{\Omega}_{\mathbf{f}}$ that satisfies both the flow conservation condition (10) and the non-negativity condition (13). If the mean path flow vector, $\mathbf{f}$ is a member of set, $\boldsymbol{\Omega}_{\mathbf{f}}$, the traffic assignment problem is formulated as a nonlinear variational inequality problem shown as:

$$\sum_{h \in \{CAV,RHV\}} \sum_{i \in I} \sum_{j \in J_{h,i}} \eta_{h,i,j}(\mathbf{f}^*) \cdot \left(f_{h,i,j} - f^*_{h,i,j}\right) \geq 0 \quad \forall \mathbf{f} \in \boldsymbol{\Omega}_{\mathbf{f}} \tag{34}$$

The above path-flow-based variational inequality problem follows the related studies that considered the stochastic travel time and the risk-averse path choice behavior (e.g., Shao et al., 2006; Chen et al., 2010). The solution of $\mathbf{f}^*$ and the corresponding minimum path cost $\eta_{h,i,j}(\mathbf{f}^*)$ $\left(\forall h \in \{CAV, RHV\}, \forall i \in I, j \in J_{h,i}\right)$ are obtained by solving (34).

The path-flow-based traffic assignment problem of (34) is transformed into a lane-based problem. Set $\mathbf{\Omega}_{\mathbf{f}}$ is represented as $\mathbf{\Omega}_{\mathbf{f}_{CAV}} \times \mathbf{\Omega}_{\mathbf{f}_{RHV}}$ where $\mathbf{\Omega}_{\mathbf{f}_{CAV}}$ and $\mathbf{\Omega}_{\mathbf{f}_{RHV}}$ are the sets of the mean path flow vector of CAVs and RHVs, respectively. With (10), (13) and the lane-path relationship, (17), $\mathbf{\Omega}_{\mathbf{f}_{CAV}}$ and $\mathbf{\Omega}_{\mathbf{f}_{RHV}}$ are projected onto $\mathbf{\Omega}_{\mathbf{v}_{CAV}}$ and $\mathbf{\Omega}_{\mathbf{v}_{RHV}}$ respectively. The vector of the mean lane flows, $\mathbf{v}$ is represented as $[\mathbf{v}_s, \mathbf{v}_d]^T$ where $\mathbf{v}_s$ and $\mathbf{v}_d$ are the vectors of mean lane flows for shared lanes and dedicated lanes, respectively. As for the difference between two-lane types, the sets of the two kinds of mean lane flows are respectively defined as $\mathbf{\Omega}_{\mathbf{v}_s} = \mathbf{\Omega}_{\mathbf{v}_{CAV,s}} \times \mathbf{\Omega}_{\mathbf{v}_{RHV}}$ and $\mathbf{\Omega}_{\mathbf{v}_d} = \mathbf{\Omega}_{\mathbf{v}_{CAV,d}}$ where $\mathbf{\Omega}_{\mathbf{v}_{CAV}} = \mathbf{\Omega}_{\mathbf{v}_{CAV,s}} \times \mathbf{\Omega}_{\mathbf{v}_{CAV,d}}$. The lane-based traffic assignment problem is formulated as a nonlinear variational inequality problem shown as:

$$\text{Find } \mathbf{v}_s^* \in \mathbf{\Omega}_{\mathbf{v}_s}, \mathbf{v}_s^* \in \mathbf{\Omega}_{\mathbf{v}_s} \text{ such that } \hat{\eta}(\mathbf{v}_s) \cdot (\mathbf{v}_s - \mathbf{v}_s{}^*) \geq 0 \quad \forall \mathbf{v}_s \in \mathbf{\Omega}_{\mathbf{v}_s} \tag{35}$$
$$\text{and } \hat{\eta}(\mathbf{v}_d) \cdot (\mathbf{v}_d - \mathbf{v}_d{}^*) \geq 0 \quad \forall \mathbf{v}_d \in \mathbf{\Omega}_{\mathbf{v}_d}$$

where $\mathbf{v}_s = \mathbf{v}_{s,CAV} + \mathbf{v}_{s,RHV}$ and $\mathbf{v}_d = \mathbf{v}_{d,CAV}$. Here, $\hat{\eta}(\cdot)$ in (35) represents the lane cost function defined as the sum of the mean and variance of lane travel time. Following (30)-(32), the mean and variance of path travel time are additively separable in the mean and variance of the travel times of the corresponding lanes that compose the path. The path travel cost is additively separable in the corresponding lane travel costs. Therefore, the total travel cost calculated from the mean path flow and the path travel cost can be calculated from the mean lane flow and the lane travel cost. Thus, a variational inequality problem of (35) is deduced from (34) subject to the two sets of $\mathbf{\Omega}_{\mathbf{v}_s}$ and $\mathbf{\Omega}_{\mathbf{v}_d}$. Needless to say, $\mathbf{\Omega}_{\mathbf{v}_s}$ and $\mathbf{\Omega}_{\mathbf{v}_d}$ are also deduced subject to flow conservation condition, (10) and the non-negativity condition, (13), respectively.

2.7 Network design problem

A network design problem presented in this study is formulated as a bi-level programming problem. The upper-level problem minimizes the total travel cost in the network given by

$$\min Z(\mathbf{x}) = \sum_{l \in L} E[V_l(\mathbf{x}) \cdot T_l(\mathbf{x})] + \gamma \cdot \sqrt{\sum_{l \in L} \mathrm{var}[V_l(\mathbf{x}) \cdot T_l(\mathbf{x})]} \tag{36}$$

$$\text{w.r.t. } \mathbf{x} = (x_1, \cdots, x_l, \cdots x_{L_c}) \tag{37}$$

s.t. (35). The above problem is solved subject to the network equilibrium constraints defined in ***2.6.*** Note that (37) denotes the vector of policy variables for (36), whose component, $x_l\ \forall l \in L_c$ is the binary variable equals 1 when a dedicated lane is provided on the candidate lane *l* and 0 otherwise. The vector of policy variable, **x** changes the states of the set of lanes in (16)-(20) in the lower problem. The above objective function is defined by the sum of the mean total travel time and the weighted standard deviation of total travel time. Note that, in this problem, the penetration rate of CAVs in total traffic demand, p_{CAV} in (4), is always fixed.

2.8 Solution algorithm

We applied the method of successive averages (MSA) introduced by Sheffi (1985) to calculate CAV flows and RHV flows that are the solution to (33). We adopted a heuristic algorithm, a simulated annealing algorithm, to solve the upper-level problem. Some studies adopted a heuristic algorithm such as a simulated-annealing algorithm and a genetic algorithm for solving the discrete network design problem (Fan and Machemehl, 2006) or the continuous network design problem (Friesz et al., 1992). The solution procedure for the upper model in this study is shown in Algorithm 1.

Algorithm 1. Solution algorithm for the upper problem

Step 1: Set an initial feasible solution, $\mathbf{x}^S = \mathbf{x}$ and the initial temperature, t.

Step 2: Generate a tentative solution $\mathbf{x}'$ from the neighbor of the current solution, $N(\mathbf{x})$ randomly. Compute the gap of objective values, $\Delta = f(\mathbf{x}') - f(\mathbf{x})$. If $\Delta > 0$, then set $\mathbf{x} = \mathbf{x}'$. Otherwise, set $\mathbf{x} = \mathbf{x}'$ with the probability of $e^{-\frac{\Delta}{t}}$.

Step 3: If $f(\mathbf{x}) < f(\mathbf{x}^S)$, then set $\mathbf{x}^S = \mathbf{x}$.

Step 4: If the number of replacing the solution vector, $\mathbf{x}$ is larger than ε, then go to Step 2.

Step 5: If the current temperature is larger than the stop temperature $(t > t_s)$, then $\mathbf{x}^S$ is the solution, otherwise go to Step 2 after updating the temperature, t.

3. Numerical experiments

3.1 The network of Nguyen and Dupuis (1984)

3.1.1 Basic settings

We performed numerical calculations in test networks. At first, we used a test network (see Figure 4) shown in Nguyen and Dupuis (1984). The network has 25 nodes, 19 links, 25 paths, and 4 OD pairs. The OD pairs are (1, 3), (1, 4), (2, 3), and (2, 4). Following the assumption of the network described in **2.2**, there are 38 ~~factual~~ lanes in total. In this study, the lane-path sequence is enumerated. The mean and the coefficient of variation of the total traffic demand, Q, are 10,000 [pcu/hour] and 0.3, respectively. The total demand is divided into the total demand for CAVs, $Q^{CAV} = p_h \cdot Q$ and that for RHVs $Q^{RHV} = (1 - p_h) \cdot Q$. The traffic demand for each OD pair and each vehicle type is given as $Q_i^h = p_h \cdot p_{h,i} \cdot Q (i = 1, \ldots, 4)$ where $p_{h,i} = 0.25$. All 19 links are unidirectional, as shown in Figure 4. The free flow travel time of each lane on a link is 0.05 [hour]. Other parameters of the cost function of each lane on a link, α_l, and n_l are 1, and 6, respectively. The parameter of β_l is defined as $\beta_l(V_l^{CAV}, V_l^{RHV})$ =0.1+2 $\cdot \left(1 - p_l(V_l^{CAV}, V_l^{RHV})\right)$ for representing the performance improvement of lane travel time when the CAV penetration rate becomes large.

In this numerical calculation, we prepare two cases in which different parameters are provided to the lane capacity in (23). In case 1, the parameters of the lane capacity are set as $E\left[C_{l,CAV}\right]$=950 [pcu/hour], $E\left[C_{l,RHV}\right]$=950 [pcu/hour], $cv_{l,CAV}$=0.2 and $cv_{l,RHV}$=0.2. In case 2, they are set as 950, 950, 0, and 0.2, respectively. In case 1, only the effects of $\beta_l(V_l^{CAV}, V_l^{RHV})$ on travel times in the network are examined. In case 2, the uncertainty of lane capacity is less than that of case 1. The risk-averse degree in the path cost function that is also the coefficient of the objective function in the upper problem, γ is set as 1. The iteration number of the traffic assignment model is set as 150. In this numerical experiment, the number of feasible solutions for the upper problems is $2^{19} = 524{,}288$. We tried to obtain the minimum solution by a simulated annealing algorithm among the feasible set.

[Figure 4 near here]

3.1.2 Simulation results

We solved eleven upper problems obtained by changing the penetration rate of CAVs of the total traffic demand from 0 to 1 with increments of 0.1. Figures 5, 6, and 7 show the transition of the value of the objective function, the mean total travel time, and the standard deviation of the total travel time. Each figure shows the results of two cases; the left one is of case 1, and the right one is of case 2. In each case, the optimized results and the results without the CAV-dedicated lane are shown. Figure 8 shows the decreasing ratio brought by the optimized CAV-dedicated lane deployment corresponding to the results shown in Figures 5, 6, and 7. The decreasing ratio is defined as the objective value in the case that no dedicated lane is installed in all links divided by that in the optimized case. Thus, if the decreasing value is one, the objective values of the two cases mentioned above are the same.

Figure 5 shows that the values of the objective function in both cases decrease as the penetration rate increases. The same tendencies are observed in the mean and standard deviation values of total travel time (Figures 6 and 7). In all figures, the absolute difference value for each criterion, i.e., the objective function, the mean total travel time, or the standard deviation of total travel time, is very small when the penetration rate of CAVs is between 0 and 0.3. However, the absolute value difference becomes larger when the penetration rate of CAVs is between 0.3 and 0.9. Note that there is no difference in the penetration rate of 1. Especially, the absolute difference value for case 1 is larger than that of case 2. Figure 8 shows the decreasing ratio for each criterion. In terms of the decreasing ratio, the results of case 1 and case 2 are similar.

To summarize the results shown above, the optimized CAV-dedicated lane deployment enhanced the network use efficiency. Especially when the performances of CAVs and RHVs are similar as described in case 1, the network use efficiency is highly enhanced by the optimized CAV-dedicated lane deployment compared with case 2, where there is a large difference in the performances.

[Figure 5 near here]

[Figure 6 near here]

[Figure 7 near here]

[Figure 8 near here]

To verify the results shown above, for each CAV penetration rate, we calculated the values of the objective function for each feasible solution. Figures 9 and 10 respectively show the results for case 1 and case 2, when the penetration rates of CAVs of total traffic demand are 0.1. 0.3, 0.5, 0.7 and 0.9. The horizontal axis and the vertical axis of each histogram represent the value of the objective function and the frequencies in each class interval, respectively. The vertical red line and the vertical black line show the globally optimized value and the value for the case without the CAV-dedicated lane, respectively. The five histograms on the left side show the whole shapes of the distributions depicted by adjusting the range of the horizontal axis to make it easy to see. The horizontal axes of the other five histograms on the right side are the same as that of the histogram at the bottom. In Figure 9, in the two histograms on the left side for $p_{CAV} = 0.7$ and $p_{CAV} = 0.9$, the red line and the black line are located at the left edge and the right edge of the horizontal axis, respectively. Both histograms show unimodal distributions, while the other three histograms on the left side for the other penetration rates show multimodal distributions. The same tendency can be observed in Figure 10.

[Figure 9 near here]

[Figure 10 near here]

Table 2 shows the number of better solutions for case 1. The better solutions defined in this study are the feasible solutions with lower values of the objective function than the solution where no CAV-dedicated lane is deployed in the network. Table 3 shows the same results for case 2. Almost all patterns of CAV dedicated lane deployment increase the value of the objective function when the CAV penetration rate is between 0 and 0.5, while the number of better solutions increases rapidly when the CAV penetration rate is between 0.4 and 0.6. Tables 2 and 3 show a small number of better solutions when the CAV penetration rate is between 0 and 0.3. These results indicate that road network modification can make the network more efficient even in the small CAV penetration rate. The network modification causes the increment of the costs in some lanes, and this cost increment changes the path choice behavior of each driver.

[Table 2 near here]

[Table 3 near here]

Tables 4 and 5 compare the values of the objective function of the global solution and the heuristic solution of case 1 and case 2, respectively. The value shown in each table is the ratio of the value of the objective function from the proposed model to that of the global solution at each penetration rate. It is shown that the proposed model in this study can find a globally optimized solution or an approximation solution to the globally optimized solution at each penetration rate in both cases.

[Table 4 near here]

[Table 5 near here]

In the above conditions, the coefficient of variation of the lane capacity of CAVs, $C_{l,CAV}$ is set as zero. This assumption aims to help readers understand the impact of the penetration of CAVs on the reduction of traffic capacity. However, in the real situation, the coefficient of variation of $C_{l,CAV}$ would be positive but smaller than that of RHVs. Besides, the mean and coefficient of variation of $C_{l,CAV}$ can be changed by adjusting the automated driving performance. Thus, we examined the transition of the value of the objective function of the upper model when the mean capacity and coefficient of variation of $C_{l,CAV}$ varied between 950 and 1,250 and between 0 and 0.2, respectively. The results of the numerical calculation are shown in Figure 11. The other conditions are the same as those described above, but the penetration rate of total traffic demand of CAVs is fixed as 0.5 as an example. Figure 11 shows that the value of the objective function decreases when the mean and coefficient of variation of CAVs increases and decreases, respectively.

[Figure 11 near here]

3.1.3 Comparison of the deterministic network and the stochastic network

Next, we compare the mean total travel times of the deterministic network and the stochastic network. Note that the proposed model can consider the deterministic traffic demands and the lane capacities by setting the corresponding coefficient of variation as zero. The parameters of the lane capacity of the deterministic network are set as $E\left[C_{l,CAV}\right]$=1,250 [pcu/hour], $E\left[C_{l,RHV}\right]$=950 [pcu/hour], $cv_{l,CAV}$=0 and $cv_{l,RHV}$=0. The parameters of the lane capacity of the stochastic network are set as $E\left[C_{l,CAV}\right]$=1,250 [pcu/hour], $E\left[C_{l,RHV}\right]$=950 [pcu/hour], $cv_{l,CAV}$=0 and $cv_{l,RHV}$=0.2. The other settings are the same as the settings in ***3.1.1***. The penetration rates of CAVs of all traffic demands are assumed to be the same, and all of the penetration rates change between 0 and 1 at the same time, like 3.1.1 and 3.1.2.

Figure 12 shows the mean total travel times of the deterministic network and the stochastic network. As the penetration rate of CAVs of total traffic demand increases, two mean total travel times tend to decrease. By considering the stochasticity of traffic demands and lane capacities, there is a difference between the mean total travel times of the two cases. Both curves approach each other as the penetration rates of CAVs increase. This result indicates that the large penetration rate of CAVs reduces the uncertainty of the road network.

[Figure 12 near here]

3.2 The experiment in the Sioux Falls network

In this section, we check the performance of the proposed network design model on a larger test network, the Sioux Falls network shown in Figure 13. We used the network data from the Github dataset, “Transportation Networks for Research,” at https://github.com/bstabler/TransportationNetworks. The network has 24 nodes, 76 links, and 283 OD pairs.

[Figure 13 near here]

Figure 14 shows the histogram of the mean OD traffic demands. The coefficient of variation of total traffic demand is set as 0.2. Figure 15 shows the histogram of the penetration rates of CAVs for all OD traffic demands.

[Figure 14 near here]

[Figure 15 near here]

The parameters of the lane capacity in the case of the stochastic network are set as $E\left[C_{l,CAV}\right]$=3,000 [pcu/hour], $E\left[C_{l,RHV}\right]$=2,500 [pcu/hour], $cv_{l,CAV}$=0 and $cv_{l,RHV}$=0.2. The free flow travel time of each lane on a link is 0.05 [hour]. Other parameters of the cost function of each lane on a link, α_l, β_l and n_l are 1, 2, and 3, respectively. The other conditions are the same as those of **3.1**.

Figures 16 and 17 show the mean flow and mean cost of two lanes on each link in the network. In both figures, the first lane corresponds to a shared lane. On the other hand, the second lane corresponds to either a shared lane or dedicated lane of which state is determined by the policy variable. Figure 18 shows the transition of the value of the objective function for each iteration computed by a simulated annealing algorithm.

[Figure 16 near here]

[Figure 17 near here]

[Figure 18 near here]

4. Concluding remarks

This study proposed a model to find the optimal deployment of the CAV-dedicated lanes in a road network. The model considers the stochastic traffic demand, the travel time reliability, and the stochastic lane capacity of the mixed flows of CAVs and RHVs. This study assumes the lane capacity as a random variable following a lognormal distribution and addresses the stochastic nature of the lane capacity calculated from the penetration rate of CAVs. Because of the automated technology of CAVs, we assume that the mean and the variance of lane capacity increases and decreases, respectively, when the penetration rate of CAVs increases in the lane flow. We assume that the traffic demand follows a lognormal distribution, and thus the corresponding lane travel time follows a shifted lognormal distribution. The driver in this study is assumed to take a risk-averse path choice behavior.

Based on the assumptions, the proposed model is formulated as a bi-level problem. The traffic assignment problem set as the lower-level problem in this study is formulated as a VI problem. The lower model is solved by the MSA. The upper problem that finds optimal deployment of the CAV-dedicated lanes is solved by a heuristic algorithm, a simulated annealing algorithm, subject to the equilibrium constraints from the lower-level problem.

The numerical calculations considering a stochastic lane capacity were performed to demonstrate the proposed model. We used the test network of Nguyen and Dupuis (1984) and the Sioux Falls network. The results show that the model finds an optimized solution. We compare the solutions from the proposed model with the globally optimized solution. All solutions from the proposed model are close to the global solution under the adopted test network. The network efficiency under the optimized deployment of the CAV-dedicated lanes is better than when no dedicated lane is deployed in the network. The results also show that network efficiency is enhanced when the performance of CAVs is improved.

The methodological contributions of the proposed model are twofold. First, by considering the stochastic total traffic demand and the correlated OD traffic demands, the approximate expression for the moment calculation of random network variables is no longer required to formulate the traffic assignment problem considering stochastic network. If the traffic flows follow a lognormal distribution, the previous studies that adopt this assumption applied the approximate expression for the moment calculation of random network variables. However, our method guarantees the reproductive property of random variables, and thus the specific approximation is not required even when lognormal-distributed OD traffic demands are assumed. Besides, any moment of lognormal-distributed lane flow and lane capacity can be calculated analytically. Thus, the mean and variance-covariance of lane travel time or lane delay time also can be calculated analytically.

Second, we developed a scheme that evaluates the stochastic impact of the penetration of CAVs on network efficiency. As we described in the introduction part, we could not find any studies that evaluated the stochastic impact of the penetration of CAVs on network efficiency. These previous studies discussed the difference in the mean lane capacities of the mixed traffic flows of CAVs and RHVs. We defined the stochastic lane capacity considering the mixed traffic flows. The formulation of the stochastic mixed traffic flows is based on our newly developed formulation of the stochastic traffic flow mentioned above.

As future tasks, the difference between the VoT and the value of travel time reliability (VoTR) between CAVs and RHVs should be discussed. The VoT of CAVs is expected to be smaller than that of RHVs because the passengers in CAVs enjoy non-driving activity during their trip. The impact of the assumption of the uncertain VoT on the network flow also can be included in future research. Relating to the difference in

VoT and VoTR, the risk-averse coefficients in the path cost functions also can be different in CAVs and RHVs. The analysis of the different risk-averse behavior is also included in future tasks.

Some studies address the problem of the market dynamics of CAVs (e.g., Chen et al., 2016; Seo and Asakura, 2017). The penetration rate of CAVs on OD traffic demands can change by deploying the dedicated lanes of CAVs. The feedback mechanism from the deployment pattern to the future OD traffic demand must be implemented from a future perspective to analyze the impact of the dedicated lane deployment on the future penetration rate of CAVs.

It is also essential to consider the dynamicity of the mixed traffic flow in the network design problem in future work. The penetration of CAVs will bring about innovative changes in the traffic flow dynamics. This study adopts a static approach to evaluate the macroscopic impact of the mixed traffic flow on network efficiency. The newly proposed BPR function defines the stochastic travel time in the mixed traffic flow. Though the proposed function represents the stochastic property of the mixed traffic flow, it cannot address the dynamicity of the mixed traffic flow. Addressing the dynamicity of the mixed traffic flow is our future work.

Acknowledgments

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Appendix

For describing the relationship between (19) and (20), we demonstrate the formulation on the series of the traffic flows by using a test network shown in Figure A1. The test network consists of two OD pairs, four paths, and five lanes. For simplicity, in this example, some of the two nodes are connected by a shared lane. Each OD pair has two different demands of vehicles, i.e., CAVs and RHVs. The flow on lane 2 describes the relationship between (18) and (19) because only lane 2 carries the path flows from both traffic demands. The flows on lane 2 are represented as; $V_2^{CAV} = F_{1,2}^{CAV} + F_{2,3}^{CAV}$ and $V_2^{RHV} = F_{1,2}^{RHV} + F_{2,3}^{RHV}$. Following (18), we rewrite the variance of the flow of type h vehicles on lane 2 shown as:

$$\mathrm{var}[V_2^h] = \mathbf{1_2}^T \mathbf{\Sigma}_2^h \mathbf{1_2}$$

where

$$\mathbf{\Sigma}_2^h = \begin{bmatrix} \mathrm{var}\left[F_{1,2}^h\right] & \mathrm{cov}\left[F_{1,2}^h, F_{2,2}^h\right] \\ \mathrm{cov}\left[F_{2,2}^h, F_{1,2}^h\right] & \mathrm{var}\left[F_{2,2}^h\right] \end{bmatrix}$$

$$\mathbf{1}_2 = [1 \quad 1]^T$$

Hence, the variance of the flow on lane 2 is represented as:

$$\mathrm{var}[V_2] = \mathbf{1}_2{}^T \mathbf{\Sigma}_2 \mathbf{1}_2$$

where

$$\mathbf{\Sigma}_2 = \begin{bmatrix} \mathbf{\Sigma}_2^{CAV} & \mathbf{\Sigma}_2^{CAV,RHV} \\ \mathbf{\Sigma}_2^{RHV,CAV} & \mathbf{\Sigma}_2^{RHV} \end{bmatrix}$$

Note that $\mathbf{\Sigma}_2^{CAV,RHV}$ represents the non-diagonal matrix illustrated as:

$$\mathbf{\Sigma}_2^{CAV,RHV} = \begin{bmatrix} \mathrm{cov}\left[F_{1,2}^{CAV}, F_{1,2}^{RHV}\right] & \mathrm{cov}\left[F_{1,2}^{CAV}, F_{2,2}^{RHV}\right] \\ \mathrm{cov}\left[F_{2,2}^{CAV}, F_{1,2}^{RHV}\right] & \mathrm{cov}\left[F_{2,2}^{CAV}, F_{2,2}^{RHV}\right] \end{bmatrix}$$

$\mathbf{\Sigma}_2^{RHV,CAV}$ is a transpose of $\mathbf{\Sigma}_2^{CAV,RHV}$. If the statistically independent OD demands are assumed, e.g., Lam et al. (2008), the non-diagonal matrix is a zero matrix. Hence, the variance of the flow on lane 2 is represented as:

$$\mathrm{var}[V_2] = \mathbf{1}_4{}^T \mathbf{\Sigma}_2 \mathbf{1}_4$$

$$= \mathbf{1}_2{}^T \mathbf{\Sigma}_2^{CAV} \mathbf{1}_2 + \mathbf{1}_2{}^T \mathbf{\Sigma}_2^{RHV} \mathbf{1}_2$$

where

$$\boldsymbol{\Sigma}_2 = \begin{bmatrix} \boldsymbol{\Sigma}_2^{CAV} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2^{RHV} \end{bmatrix}$$

$$\mathbf{1}_4 = [1 \quad 1 \quad 1 \quad 1]^T$$

Figure A2 shows the series of the traffic flows that are defined in this paper and the relationship between the flow on lane l, V_l and other kinds of traffic flows. As shown in Figure A2, Q^{CAV} and Q^{RHV} are statistically correlated, and its correlation coefficient can be calculated from (5) because the total traffic demand generates both flows. In a similar way, V_l^{CAV} and V_l^{RHV} are statistically correlated with each other. If the total traffic demands of CAVs and RHVs are independent of each other, V_l^{CAV} and V_l^{RHV} are also independent of each other.

[Figure A1 near here]

[Figure A2 near here]

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Table 1. The assumptions on path choice criteria, traffic demand, and link/lane capacity.

			Properties of traffic assignment model		
	Model	Traffic demand	CAVs or vehicles with ATIS	RHVs	Link/lane capacity
Yang (1998)	Traffic assignment model	Deterministic	UE	SUE	Deterministic and constant
Yang et al. (2007)	Traffic assignment model	Deterministic	SO	UE or CN	Deterministic and constant
Chen et al. (2016)	Traffic assignment model	Deterministic	UE	UE	Deterministic and constant
Bagloee et al. (2017)	Traffic assignment model	Deterministic	SO	UE	Deterministic and constant

Zhang and Nie (2018)	Traffic assignment model	Deterministic	SO	UE	Deterministic and variable
Wang et al. (2019)	Traffic assignment model	Deterministic	UE	CNL	Deterministic and variable
van den Berg and Verhoef (2016)	Bottleneck model	Deterministic	-	-	-
Levin and Boyles (2015)	Traffic assignment model	Deterministic	UE	UE	Deterministic and variable
Levin and Boyles (2016)	Cell transmission model	Deterministic	-	-	-
Pan et al. (2019)	Cell transmission model	Deterministic	-	-	-
Ye and Yamamoto (2018)	Traffic flow model	Deterministic	-	-	-
This study	Traffic assignment model	Stochastic	UE	UE	Stochastic and variable

Table 2. The comparison of the solutions (Case 1).

CAV penetration rate	0.0	0.1	0.2	0.3	0.4	0.5
The number of better solutions	2	6	1	7	729	59,512
The ratio of better solutions [%]	0.00	0.00	0.00	0.00	0.14	11.35
CAV penetration rate	0.6	0.7	0.8	0.9	1.0	
The number of better solutions	524,272	524,286	524,279	524,285	524,287	
The ratio of better solutions [%]	100.00	100.00	100.00	100.00	100.00	

Table 3. The comparison of the solutions (Case 2).

CAV penetration rate	0.0	0.1	0.2	0.3	0.4	0.5
The number of better solutions	2	4	1	75	1,017	51,919
The ratio of better solutions [%]	0.00	0.00	0.00	0.01	0.19	9.90
CAV penetration rate	0.6	0.7	0.8	0.9	1.0	
The number of better solutions	476,536	524,279	524,285	524,284	524,287	
The ratio of better solutions [%]	90.89	100.00	100.00	100.00	100.00	

Table 4. The comparison between the global solution and the heuristic solution (Case 1).

CAV penetration rate	0.0	0.1	0.2	0.3	0.4	0.5
The ratio of the value of the objective function	1.00	1.00	1.00	1.00	1.00	1.00
CAV penetration rate	0.6	0.7	0.8	0.9	1.0	
The ratio of the value of the objective function	1.00	1.00	1.00	1.00	1.00	

Table 5. The comparison between the global solution and the heuristic solution (Case 2).

CAV penetration rate	0.0	0.1	0.2	0.3	0.4	0.5
The ratio of the value of the objective function	1.00	1.00	1.00	1.00	1.00	1.00
CAV penetration rate	0.6	0.7	0.8	0.9	1.0	
The ratio of the value of the objective function	1.00	1.00	1.02	1.03	1.00	

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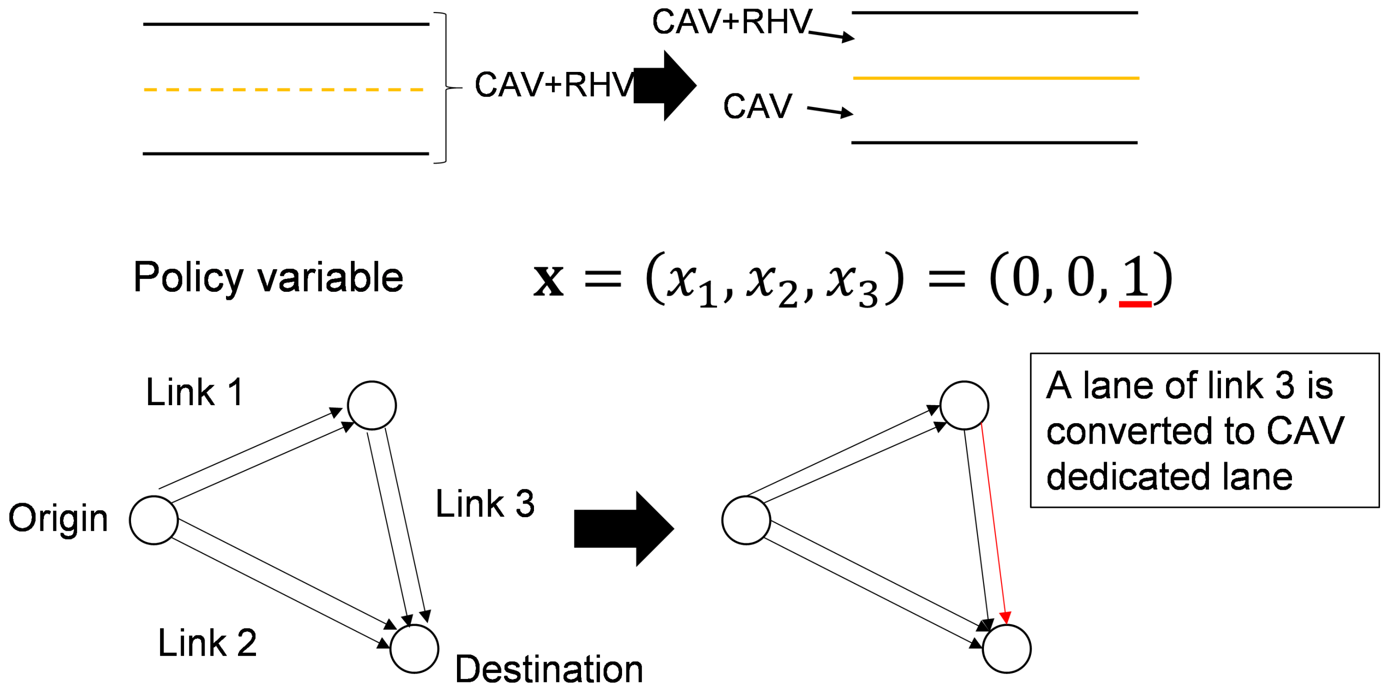
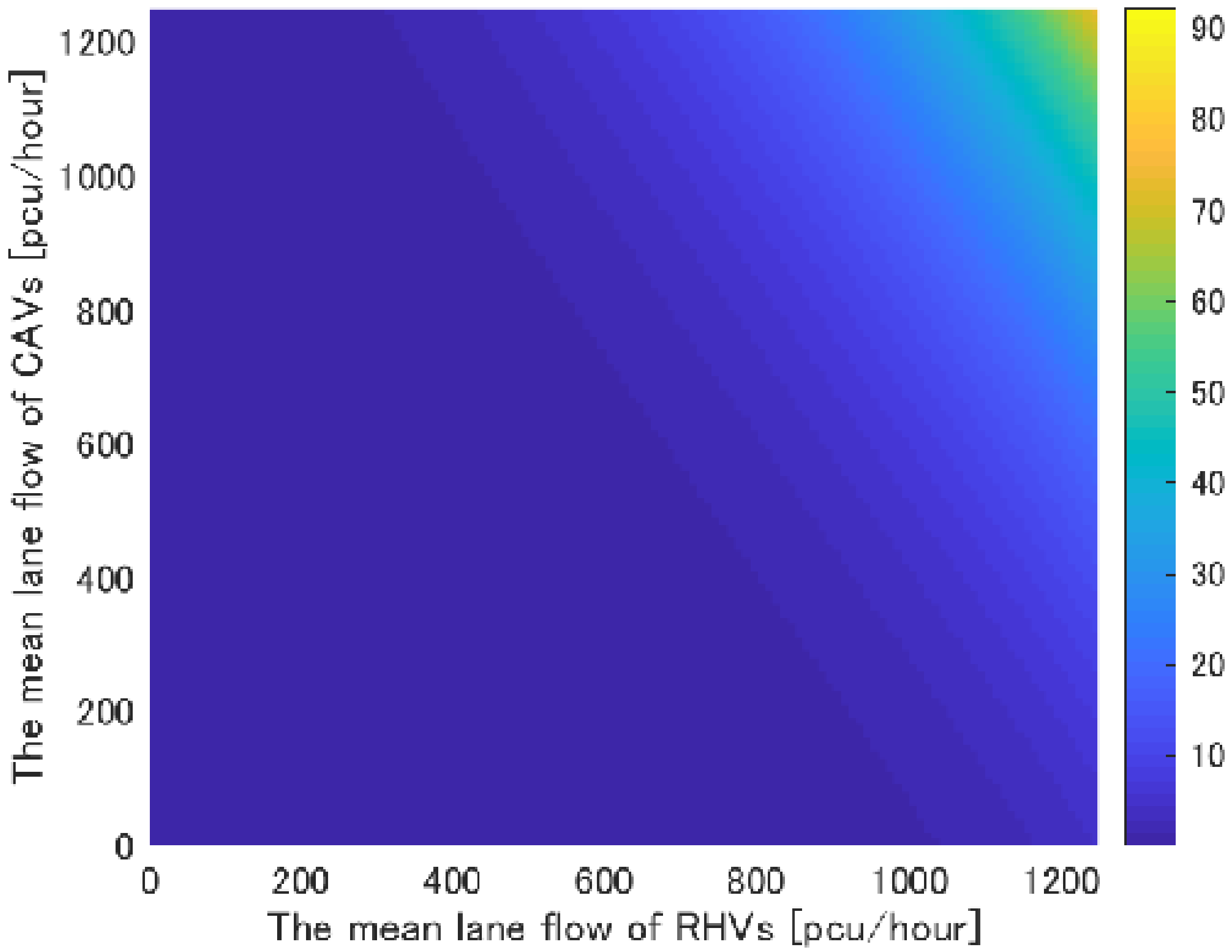
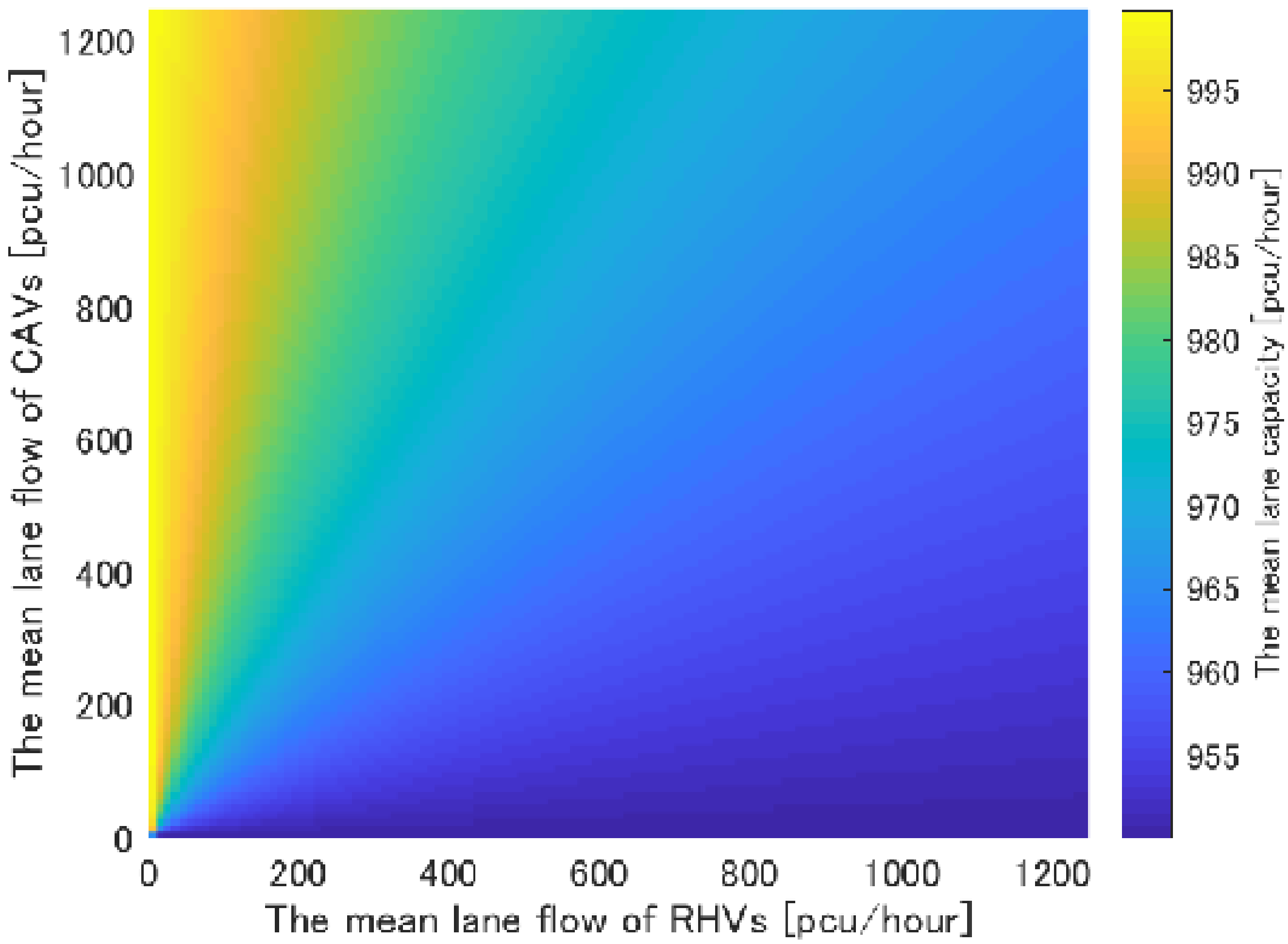
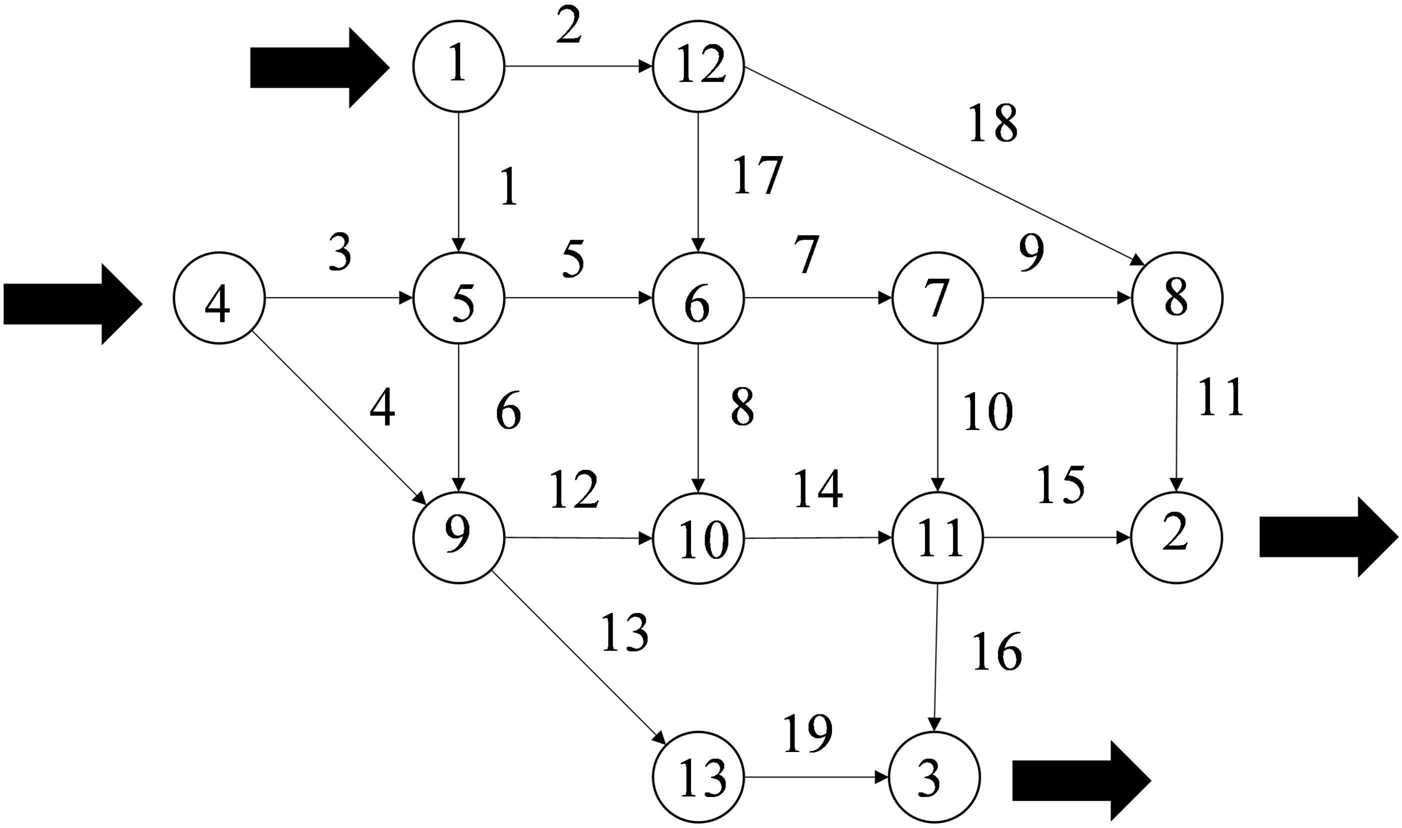

CAV+RHV
CAV+RHV
CAV
Policy variable
$\mathbf{x} = (x_1, x_2, x_3) = (0, 0, \underline{1})$
Link 1
Link 3
Origin
Link 2
Destination
A lane of link 3 is converted to CAV dedicated lane



The mean lane flow of CAVs [pcu/hour]
1200
1000
800
600
400
200
0
0
200
400
600
800
1000
1200
The mean lane flow of RHVs [pcu/hour]
90
80
70
60
50
40
30
20
10



1200
1000
800
600
400
200
0
The mean lane flow of CAVs [pcu/hour]
0
200
400
600
800
1000
1200
The mean lane flow of RHVs [pcu/hour]
995
990
985
980
975
970
965
960
955
The mean lane capacity [pcu/hour]


1
2
12
18
17
1
4
3
5
5
6
7
7
9
8
4
6
8
10
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12
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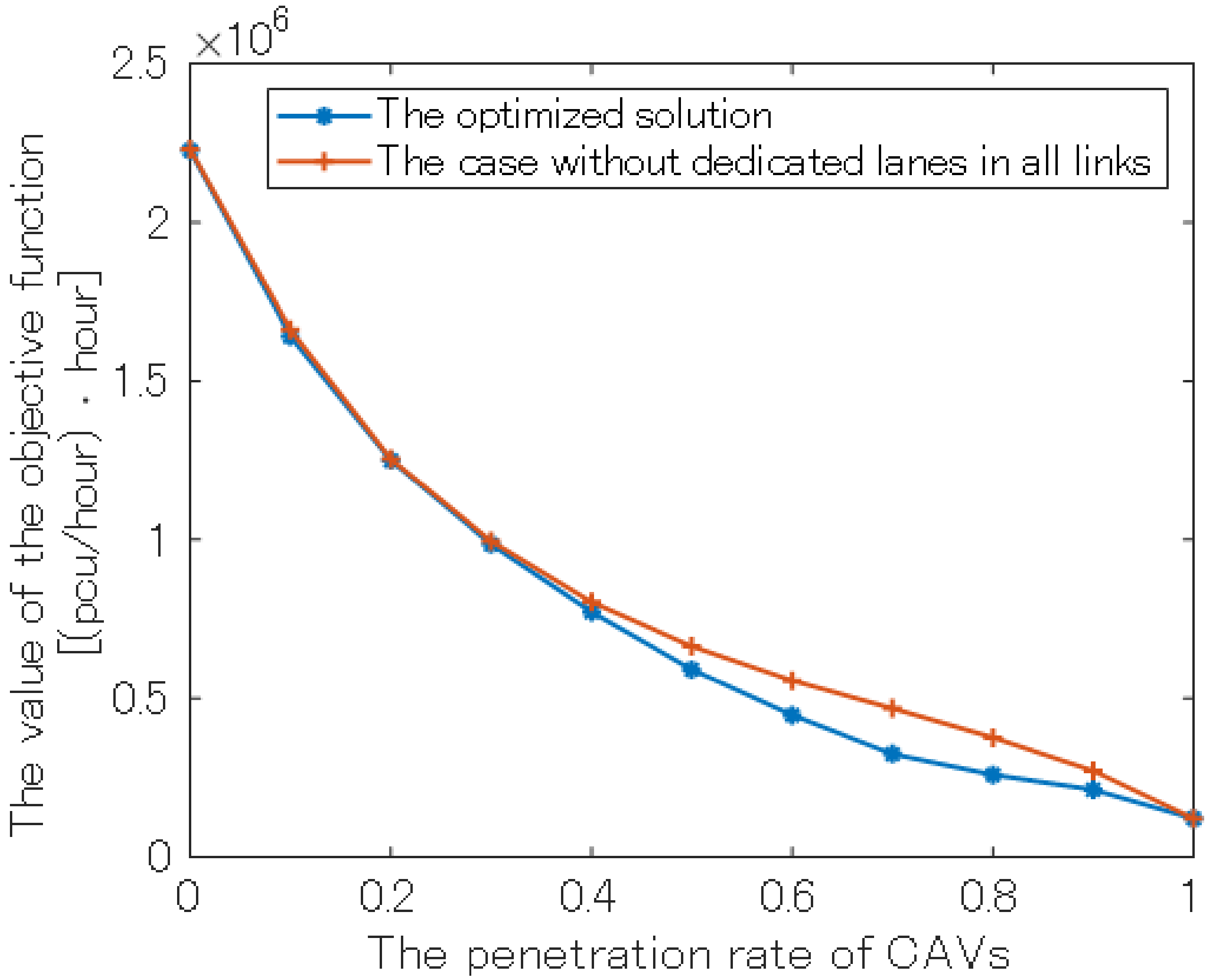
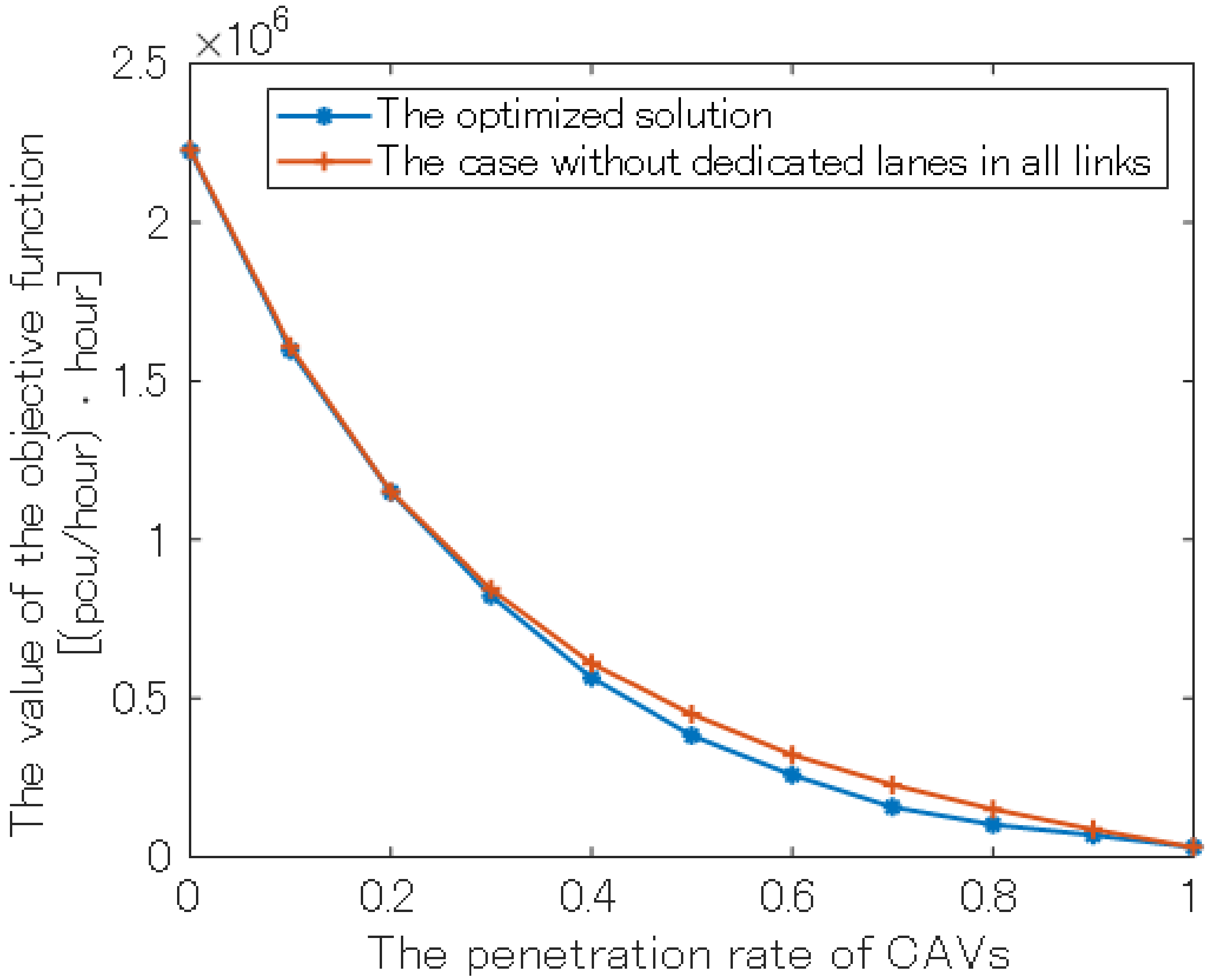
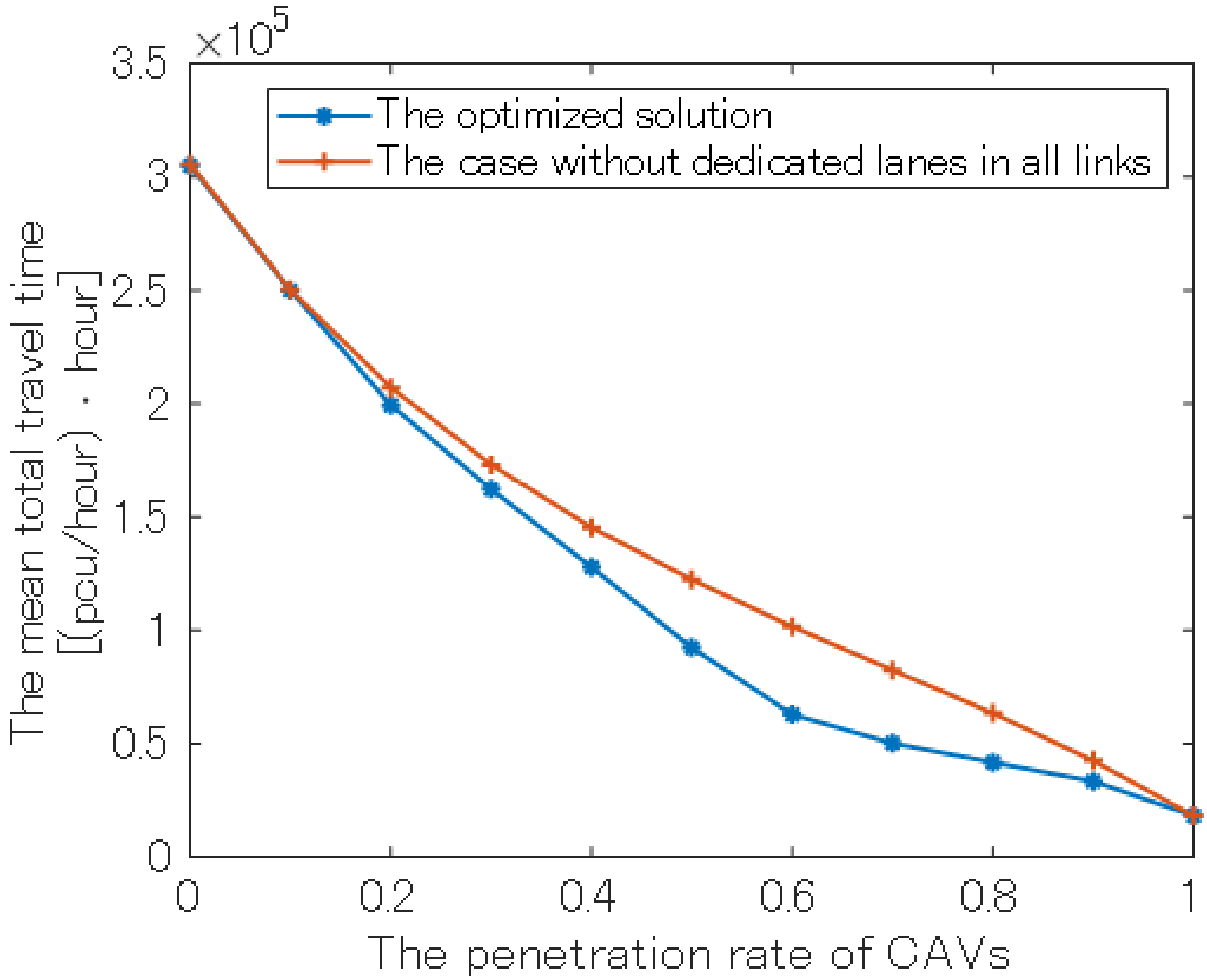
13
16
13
19
3



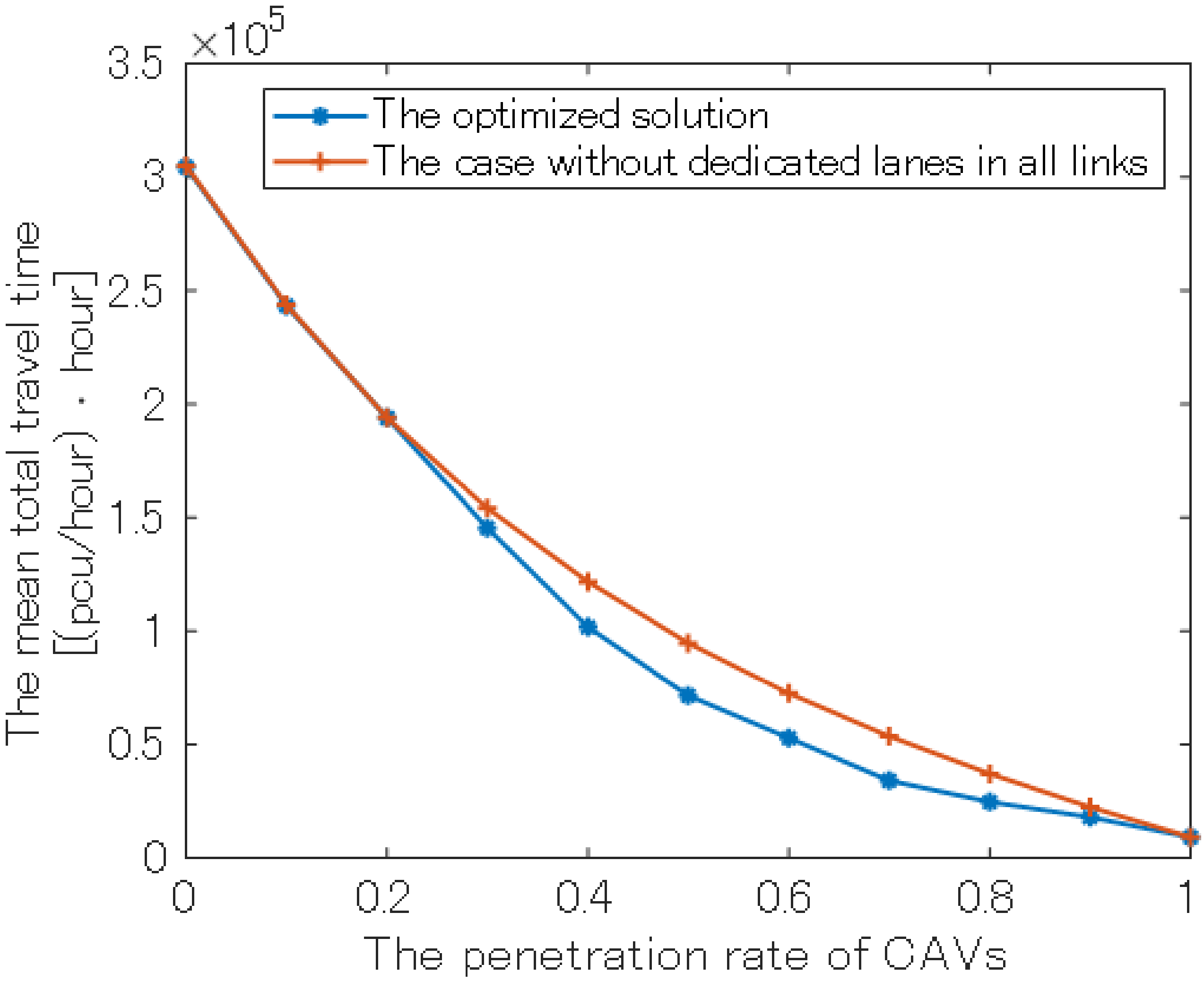
×10^6
2.5
2
1.5
1
0.5
0
The optimized solution
The case without dedicated lanes in all links
The value of the objective function
[(pcu/hour)・hour]
0
0.2
0.4
0.6
0.8
1
The penetration rate of CAVs



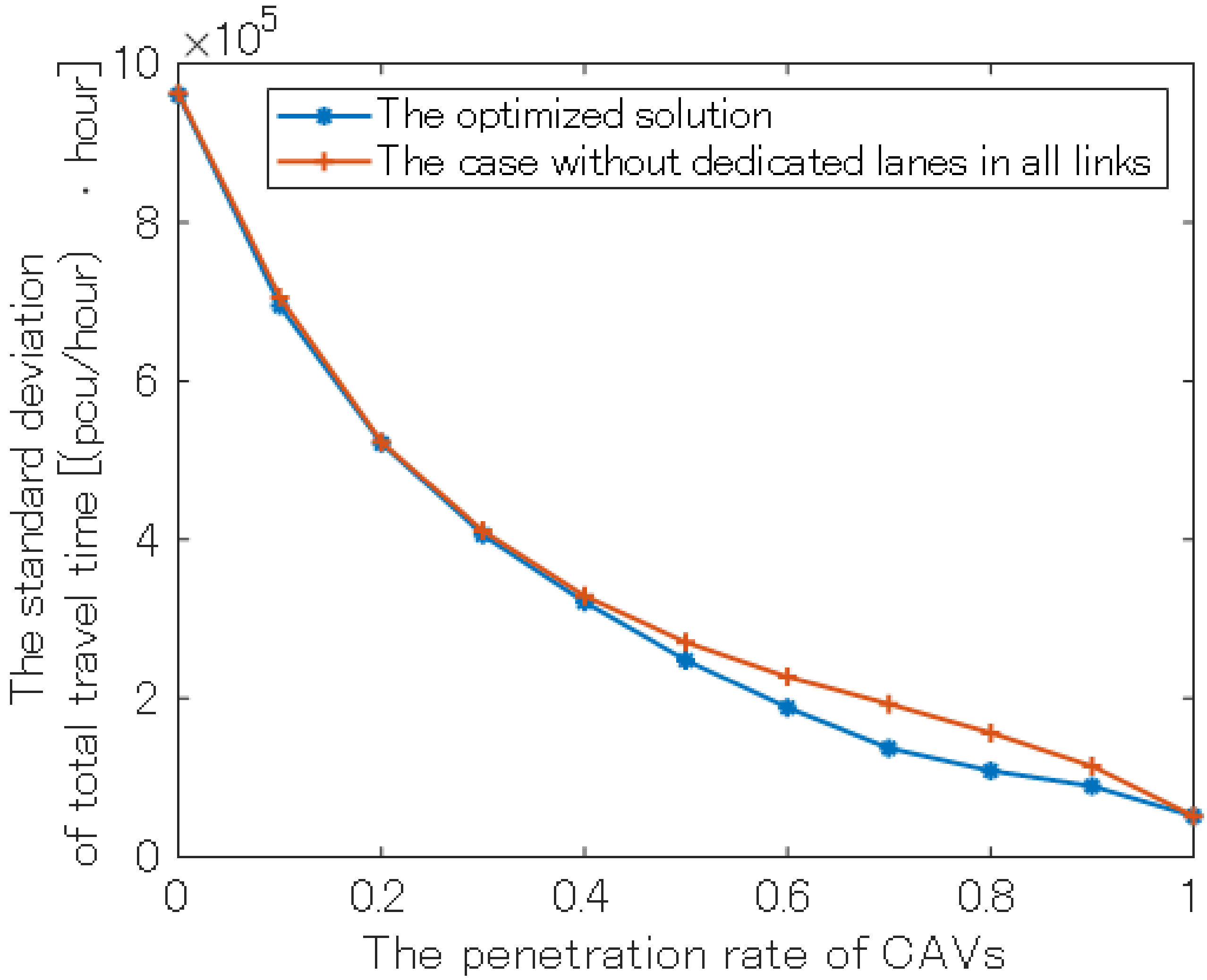
$\times 10^6$
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
The optimized solution
The case without dedicated lanes in all links
The value of the objective function
[(pcu/hour) · hour]
The penetration rate of CAVs



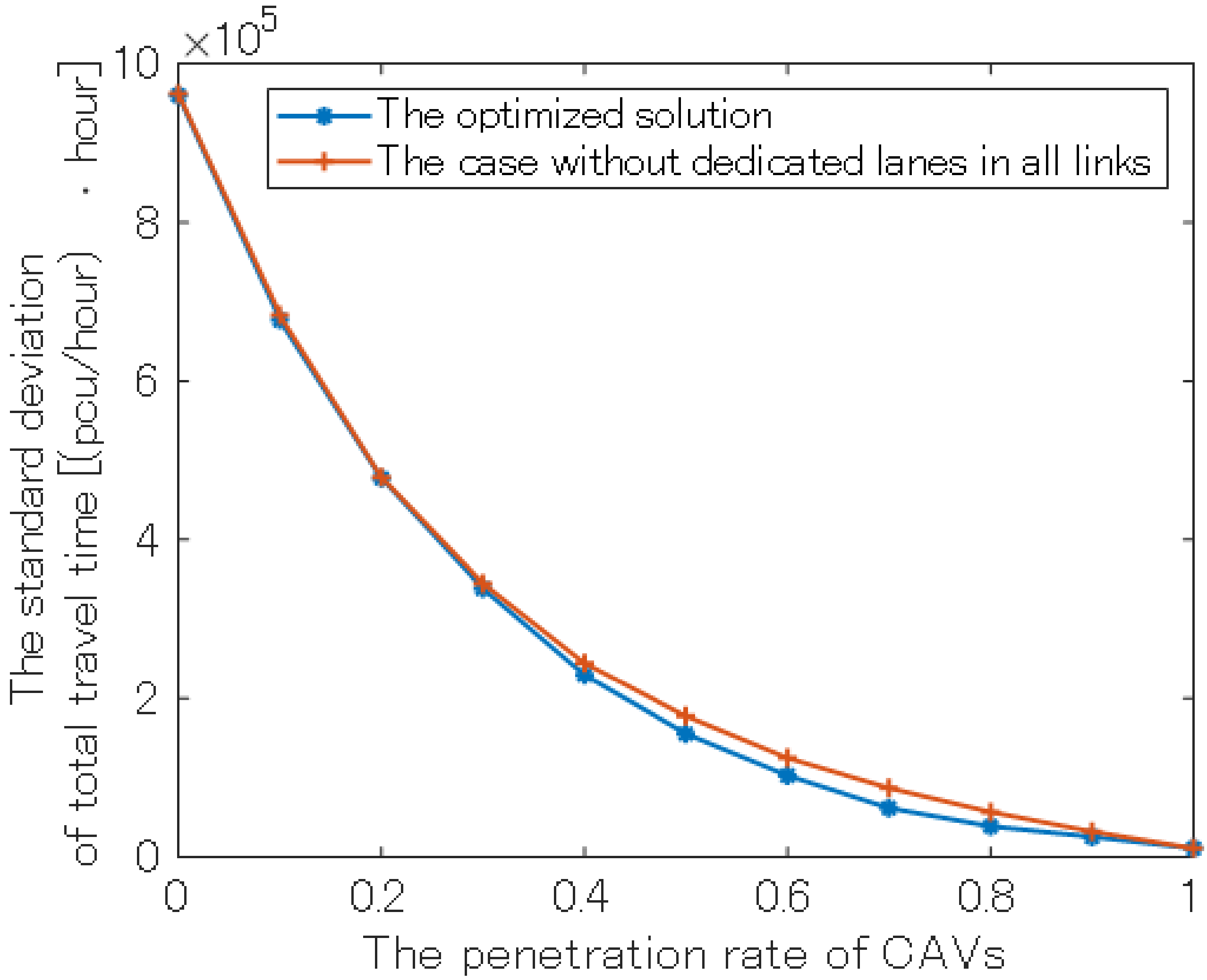
×10^5
The optimized solution
The case without dedicated lanes in all links
The mean total travel time [(pcu/hour) · hour]
The penetration rate of CAVs
3.5
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1



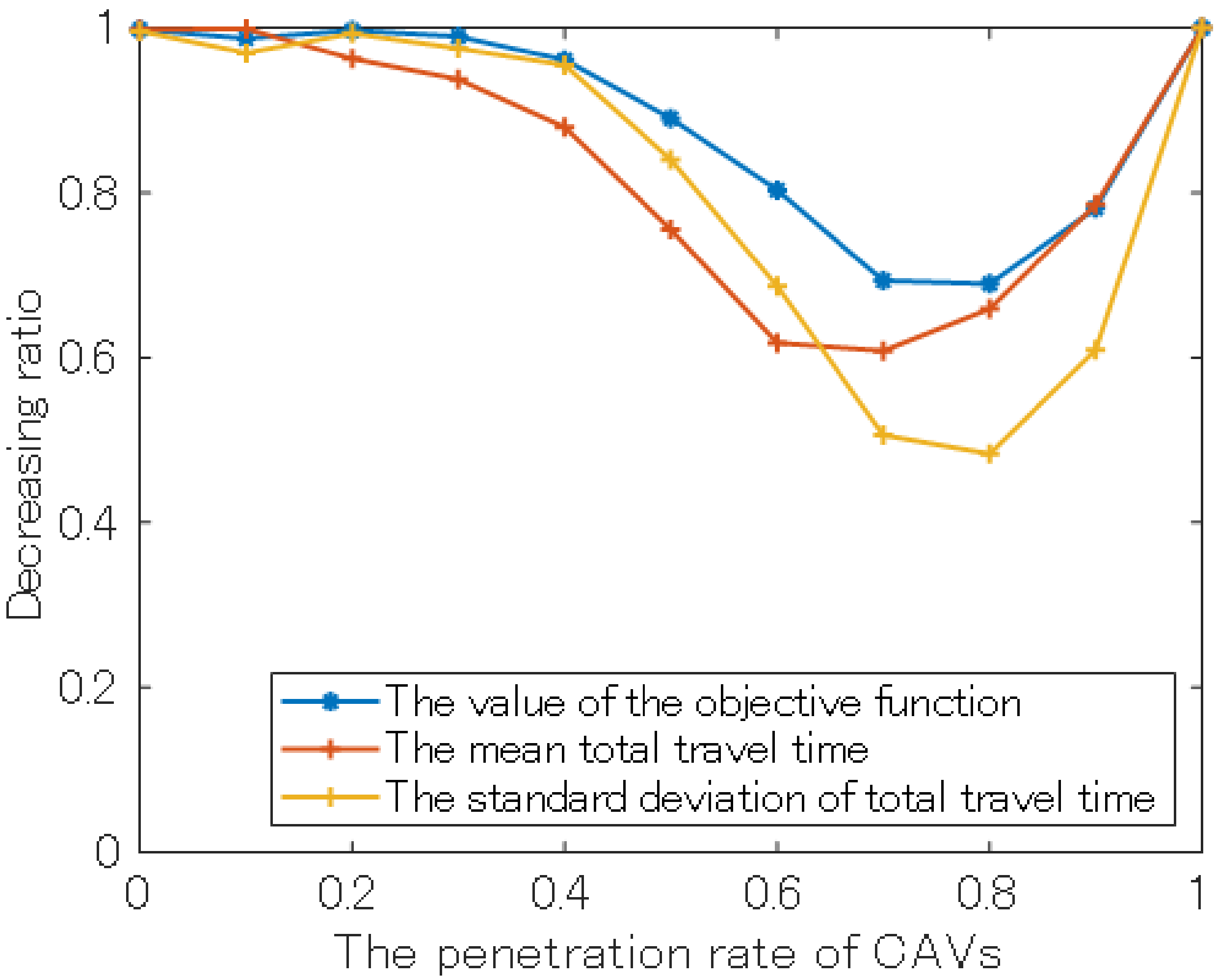
$\times 10^5$
3.5
3
2.5
2
1.5
1
0.5
0
The mean total travel time [(pcu/hour) · hour]
The optimized solution
The case without dedicated lanes in all links
0
0.2
0.4
0.6
0.8
1
The penetration rate of CAVs



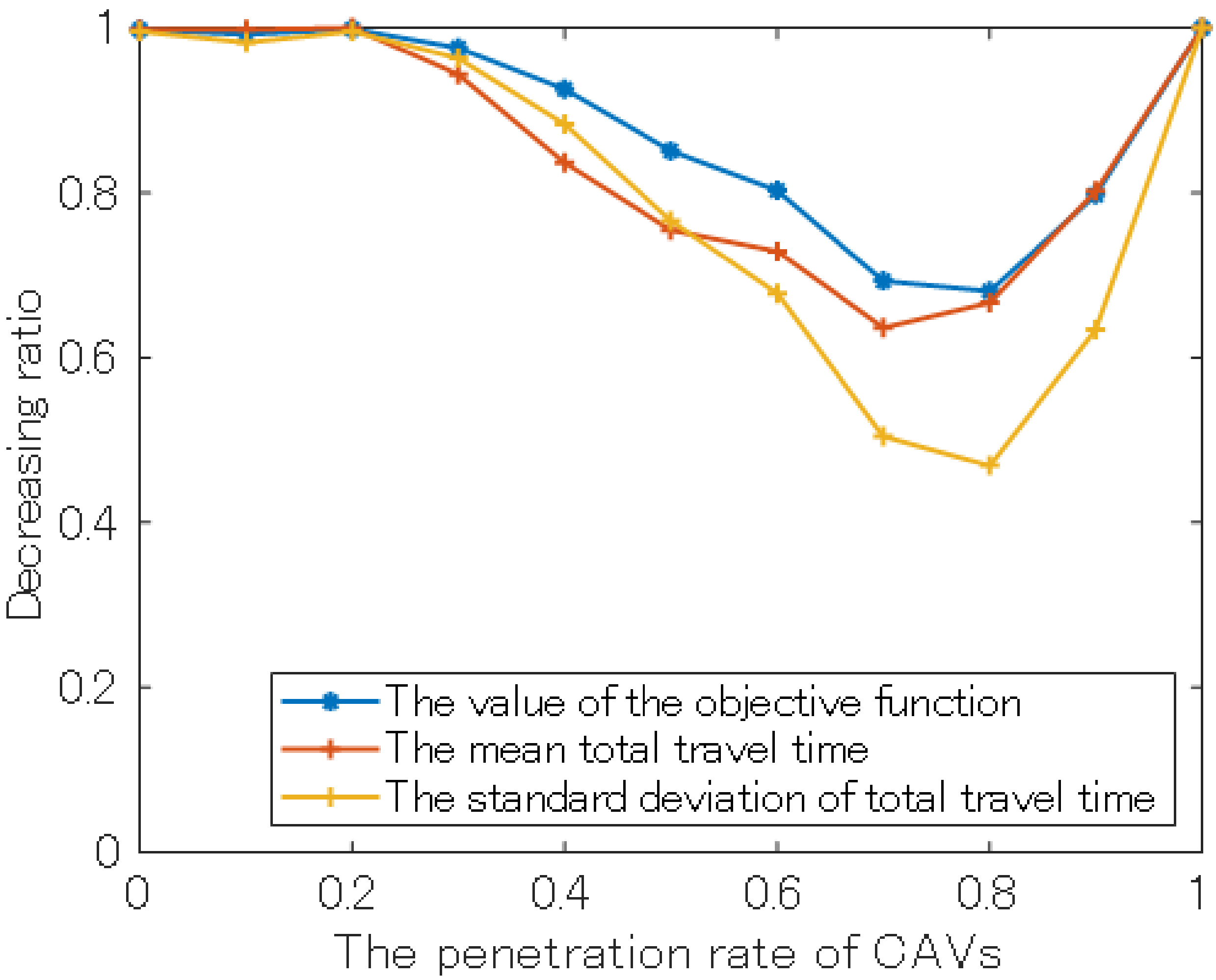
×10^5
The optimized solution
The case without dedicated lanes in all links
The standard deviation
of total travel time [(pcu/hour) · hour]
The penetration rate of CAVs
0
2
4
6
8
10
0
0.2
0.4
0.6
0.8
1



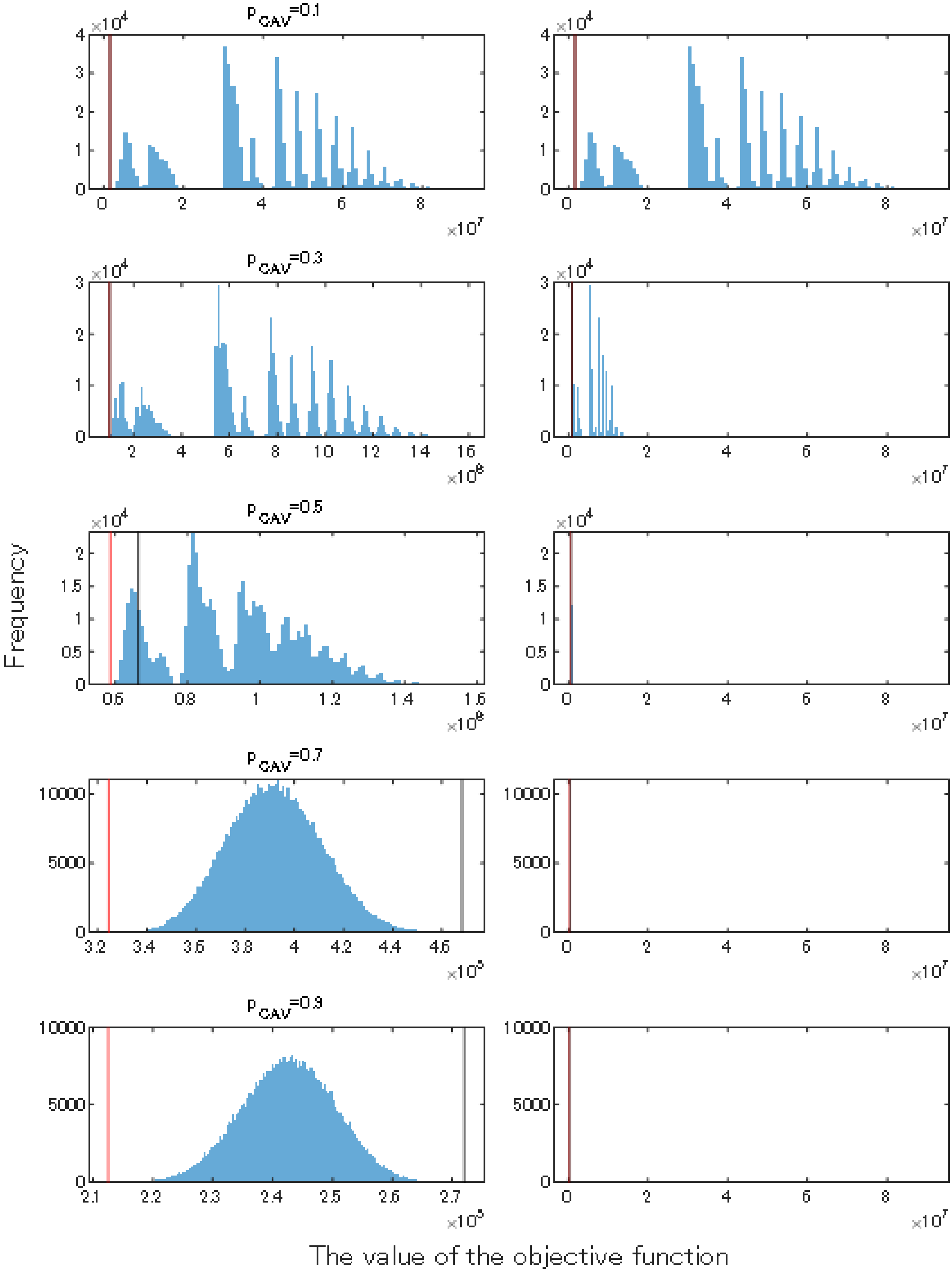
×10^5
The optimized solution
The case without dedicated lanes in all links
The standard deviation of total travel time [(pcu/hour) · hour]
The penetration rate of CAVs
0
2
4
6
8
10
0.2
0.4
0.6
0.8
1



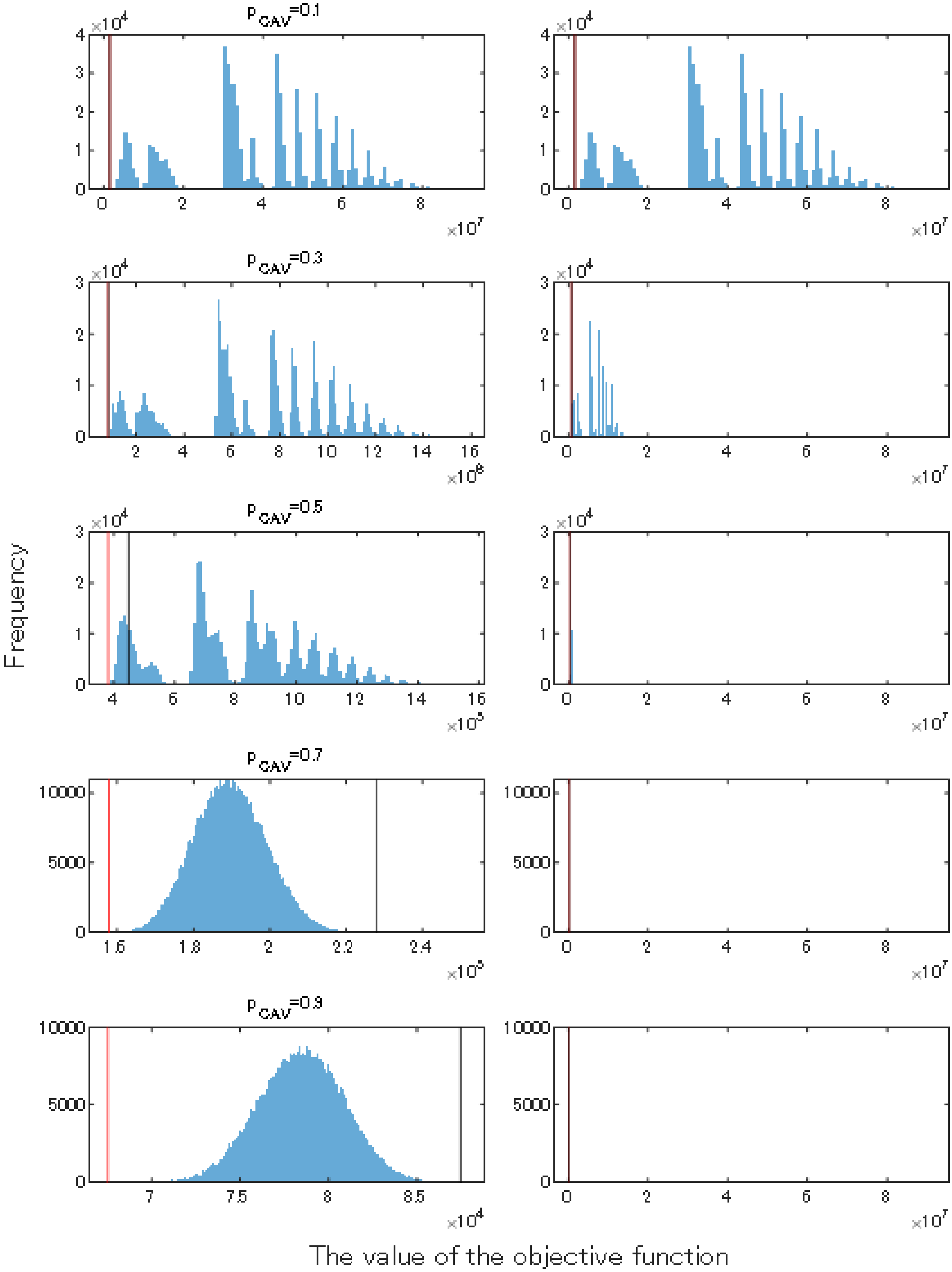
Decreasing ratio
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
The penetration rate of CAVs
The value of the objective function
The mean total travel time
The standard deviation of total travel time



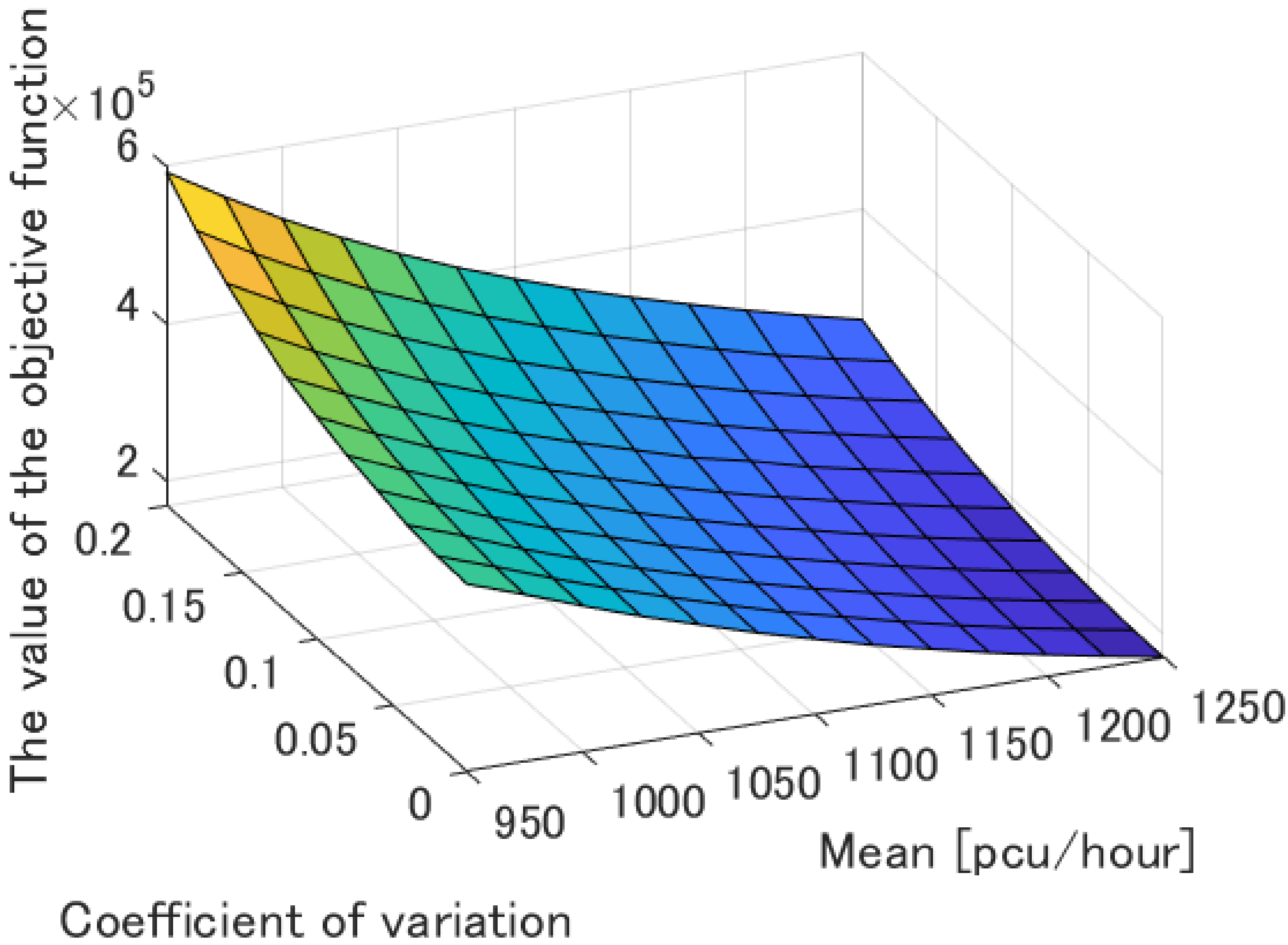
Decreasing ratio
0
0.2
0.4
0.6
0.8
1
0
0.2
0.4
0.6
0.8
1
The penetration rate of CAVs
The value of the objective function
The mean total travel time
The standard deviation of total travel time



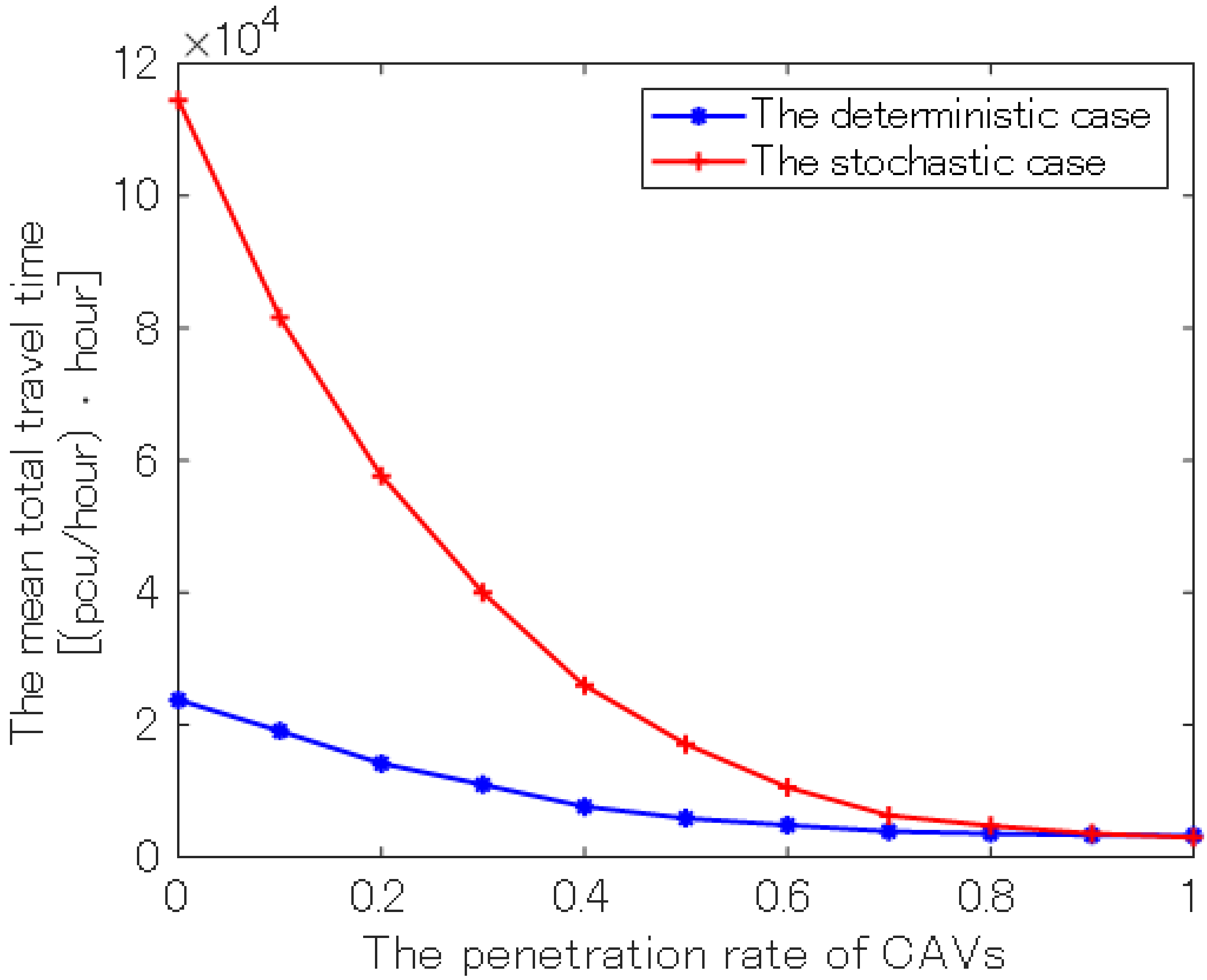
P CAV=0.1
P CAV=0.3
P CAV=0.5
P CAV=0.7
P CAV=0.9
Frequency
The value of the objective function



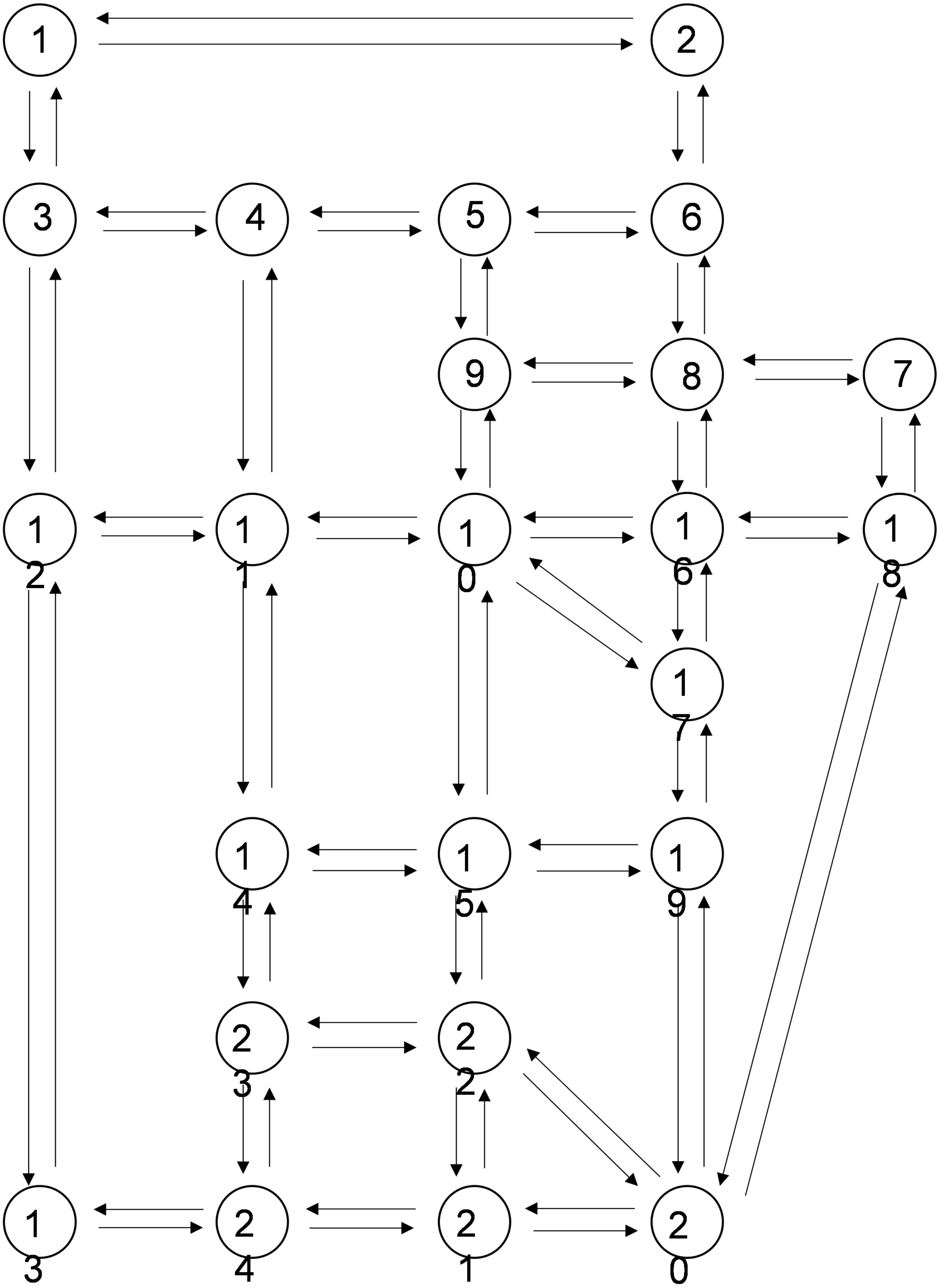
P_CAV=0.1
P_CAV=0.3
P_CAV=0.5
P_CAV=0.7
P_CAV=0.9
Frequency
The value of the objective function

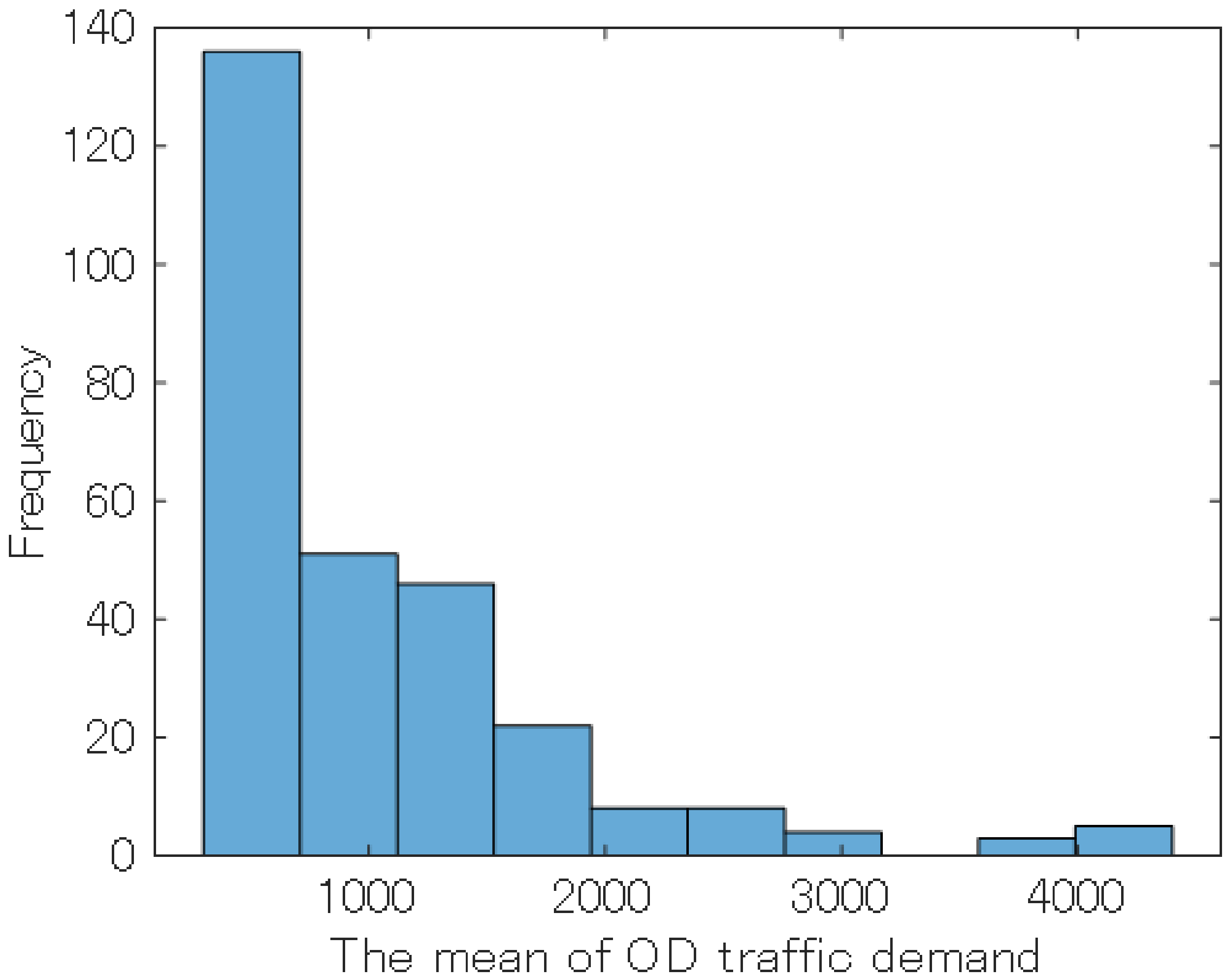


The value of the objective function
$\times 10^5$
6
4
2
0.2
0.15
0.1
0.05
0
950
1000
1050
1100
1150
1200
1250
Mean [pcu/hour]
Coefficient of variation

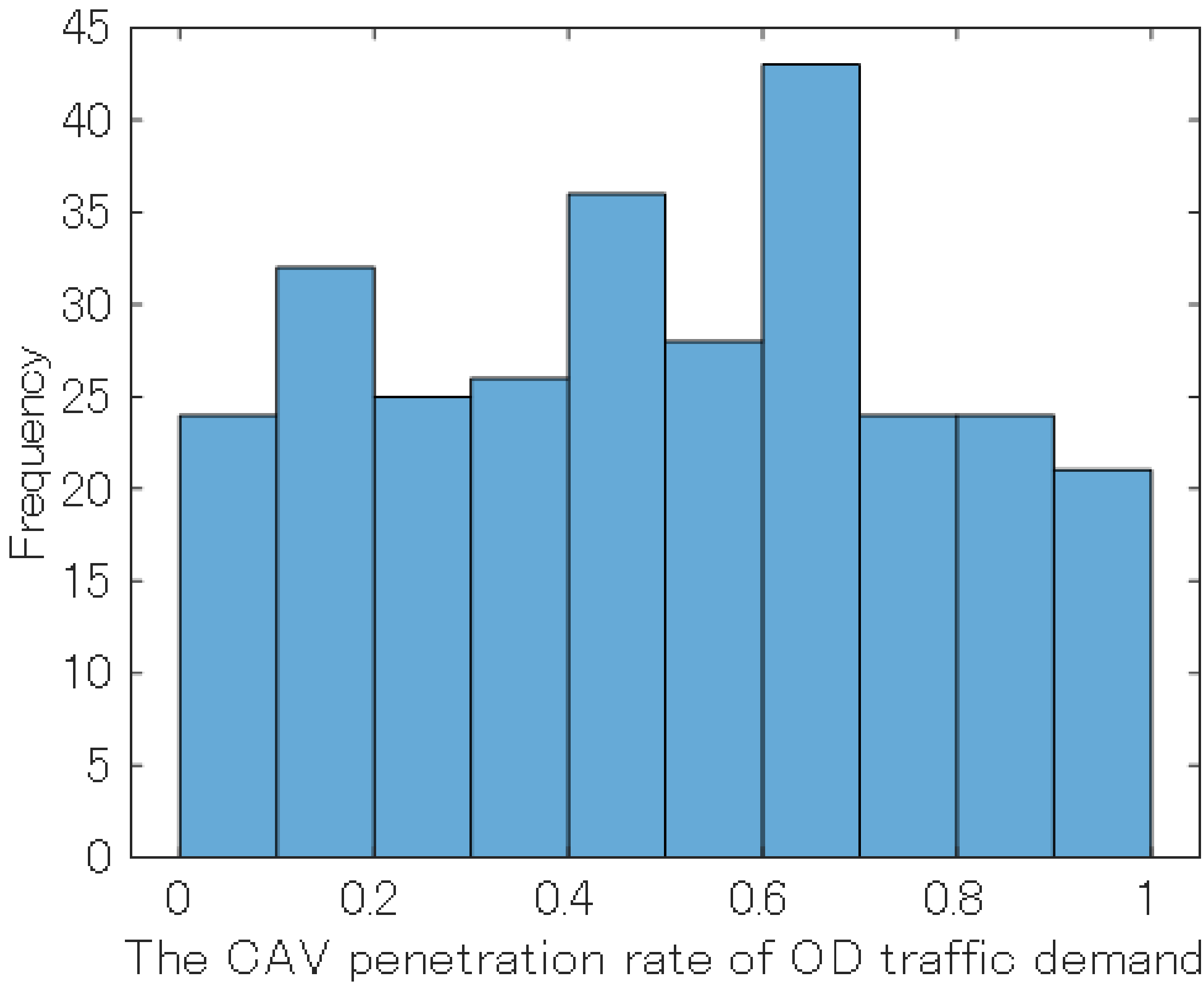


×10^4
12
10
8
6
4
2
0
The mean total travel time
[(pcu/hour) · hour]
0
0.2
0.4
0.6
0.8
1
The penetration rate of CAVs
The deterministic case
The stochastic case

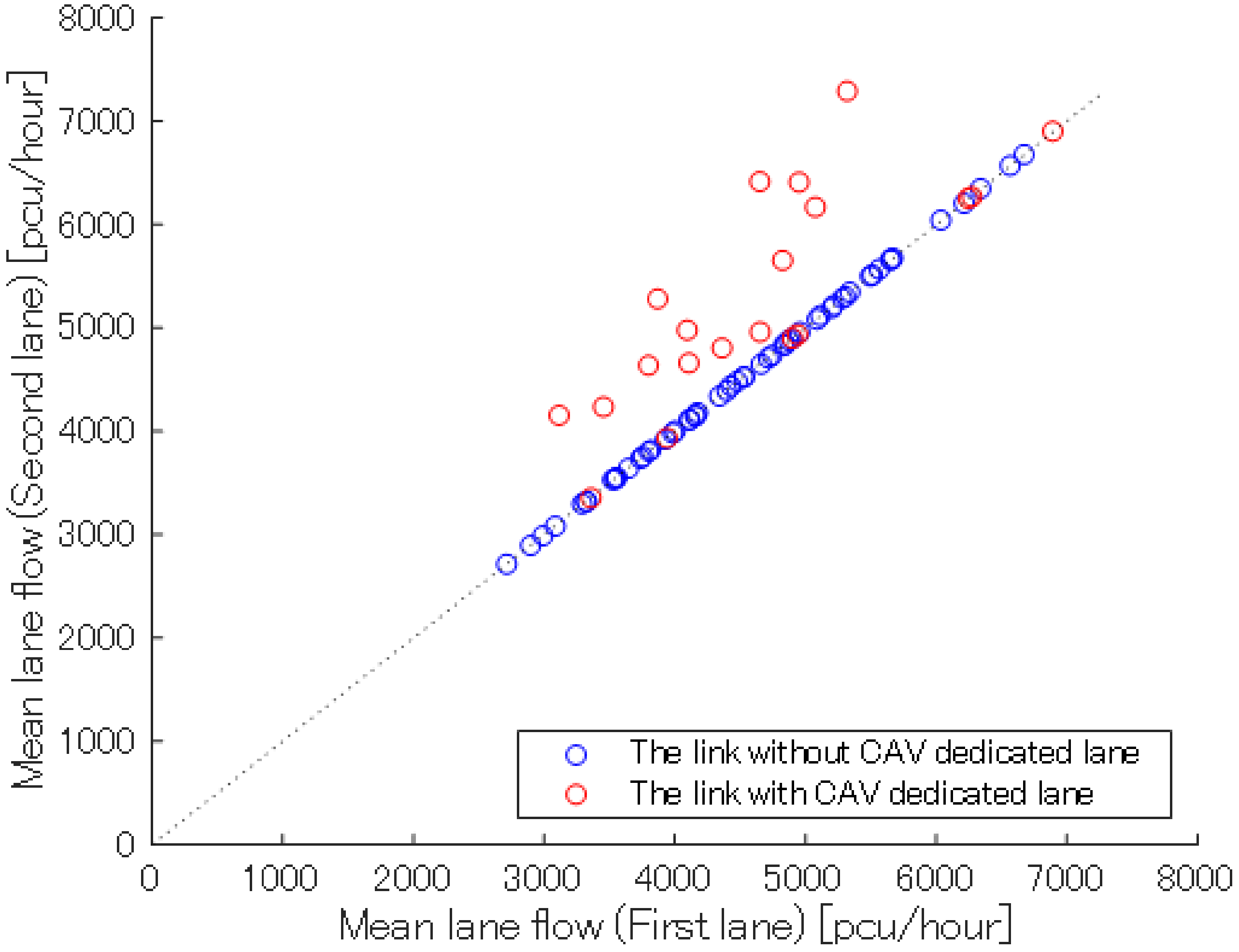

1
2
3
4
5
6
9
8
7
12
11
10
16
18
17
14
15
19
23
22
13
24
21
20



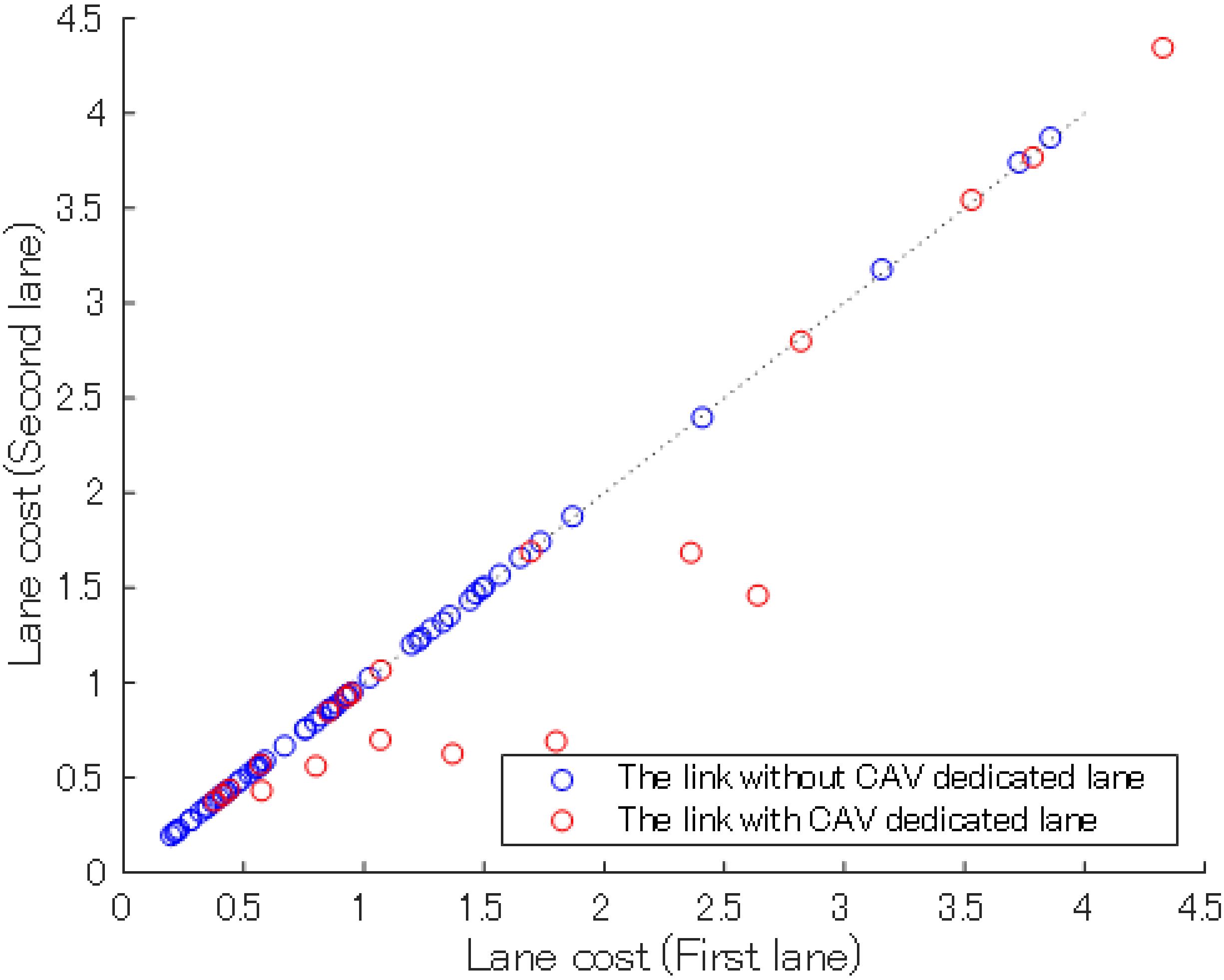
140
120
100
80
60
40
20
0
Frequency
1000
2000
3000
4000
The mean of OD traffic demand



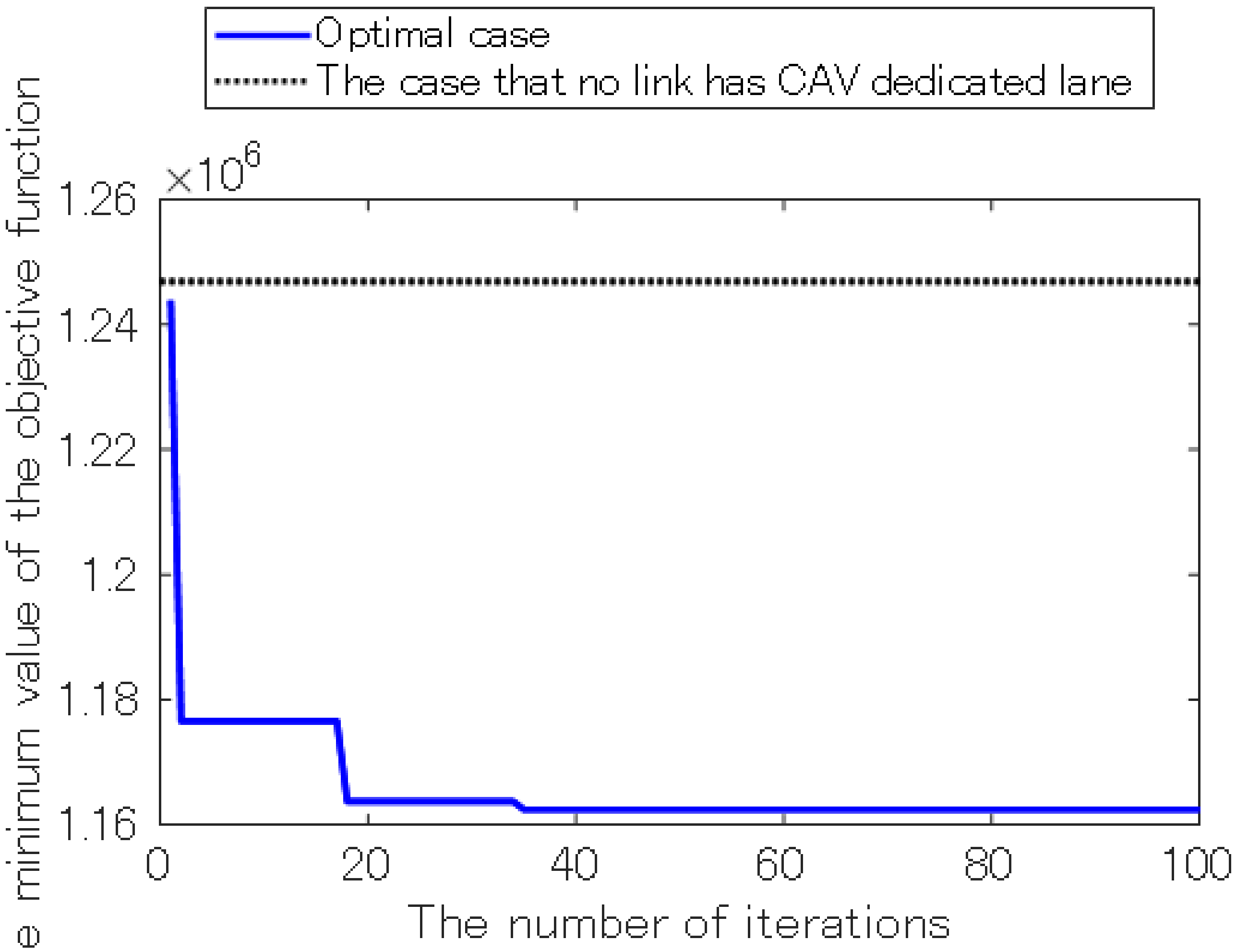
Frequency
45
40
35
30
25
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1
The CAV penetration rate of OD traffic demand



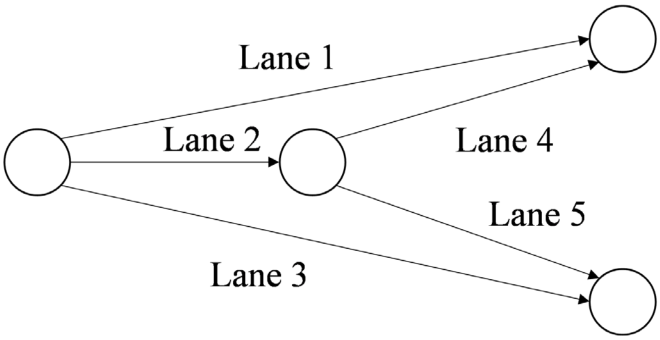
8000
7000
6000
5000
4000
3000
2000
1000
0
Mean lane flow (Second lane) [pcu/hour]
The link without CAV dedicated lane
The link with CAV dedicated lane
0
1000
2000
3000
4000
5000
6000
7000
8000
Mean lane flow (First lane) [pcu/hour]

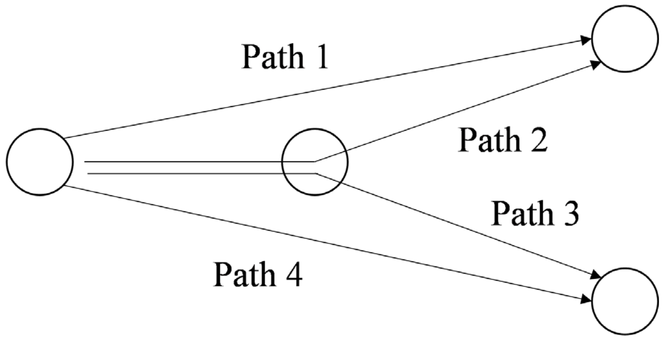


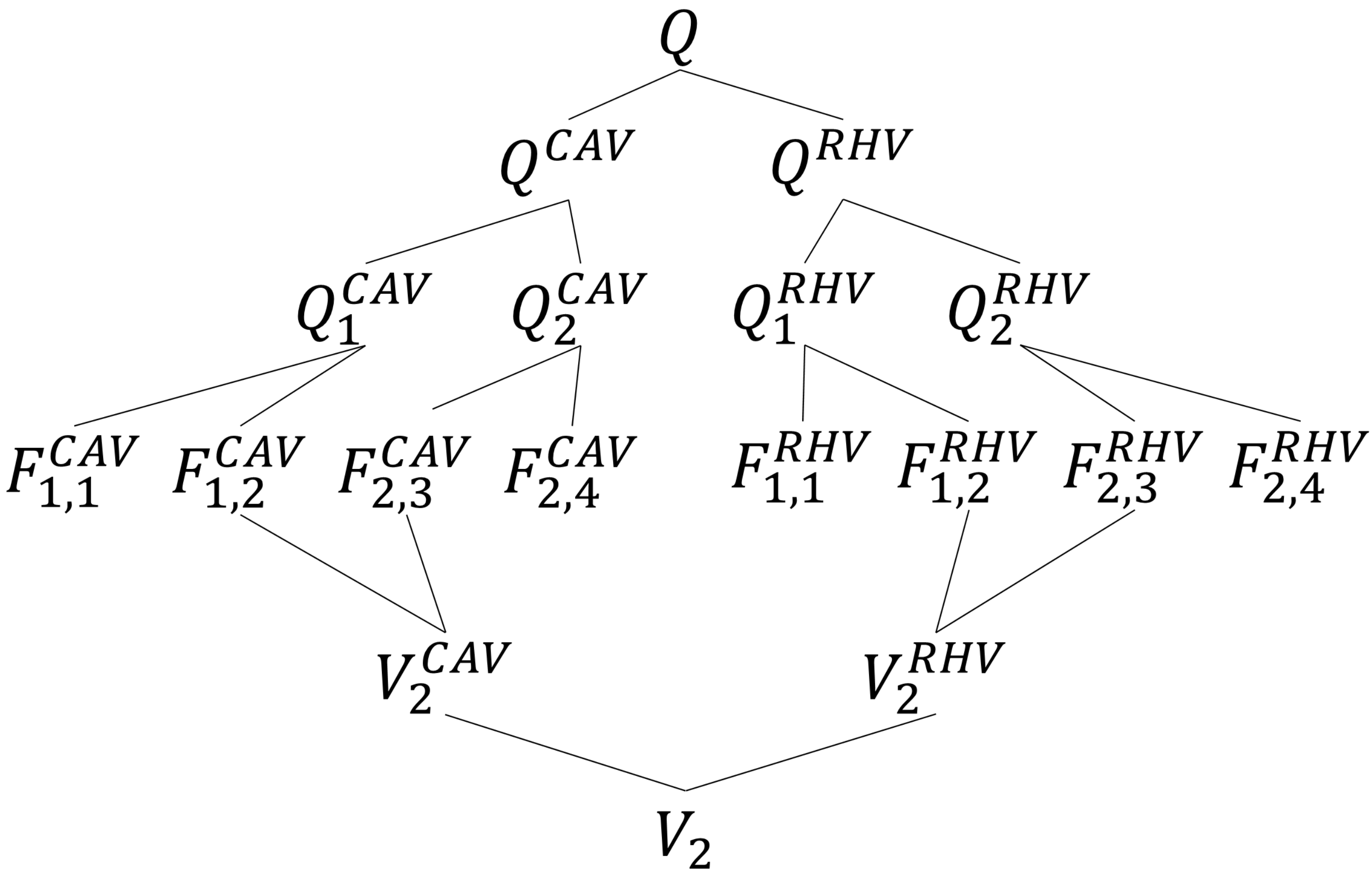
Lane cost (Second lane)
Lane cost (First lane)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
The link without CAV dedicated lane
The link with CAV dedicated lane



Optimal case
The case that no link has CAV dedicated lane
×10^6
1.26
1.24
1.22
1.2
1.18
1.16
0
20
40
60
80
100
The number of iterations
e minimum value of the objective function


Lane 1
Lane 2
Lane 4
Lane 5
Lane 3


Path 1
Path 2
Path 3
Path 4


Q
Q^{CAV}
Q^{RHV}
Q_1^{CAV}
Q_2^{CAV}
Q_1^{RHV}
Q_2^{RHV}
$F_{1,1}^{CAV}$
$F_{1,2}^{CAV}$
$F_{2,3}^{CAV}$
$F_{2,4}^{CAV}$
$F_{1,1}^{RHV}$
$F_{1,2}^{RHV}$
$F_{2,3}^{RHV}$
$F_{2,4}^{RHV}$
V_2^{CAV}
V_2^{RHV}
V_2