[Journal of the Faculty of Science, Hokkaidô University, Japan, Ser. VII, Vol. I, No. 6, 1962]

Relations between the short period changes in geomagnetism and in telluric currents (continued)

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(Received Sept. 30, 1962)

Abstract

Continued from the previous paper[14], relations between the changes in geomagnetism and in telluric currents at Kakioka, Japan are analyzed for comparison with those observed at Dourbes, Belgium. Correlations between geomagnetic and telluric variations observed at the two stations prove to be typical cases where anisotropy of electrical conductivity is an important factor.

Hitherto amplitude ratios and phase differences between geomagnetic and telluric variations have been applied to deduce the electrical state of the earth-interior. However these application of the "magneto-telluric method" hold good for the earth model of isotropic conductivity. Strictly speaking, anisotropy of conductivity should be taken into consideration. The writer avoids the difficulty by analyzing geomagnetic and telluric variations only in the principal directions respectively. As an example, the variation of electrical conductivity with depth at Kakioka is estimated.

6. Observed relations between the changes in the telluric currents and in geomagnetism at Kakioka, Japan

The tellurigrams and magnetograms obtained by the Kakioka Magnetic Observatory, Japan are analyzed in the following. The geographical position of the observatory is 36°14′N latitude and 140°11′E longitude. The span between the E and W electrodes for measuring the telluric potential is 1500 *meters* while the N-S span is 900 *meters*. The present data are those obtained during the year 1958 which were placed at the writer's disposal by the Observatory.

From the tellurigrams and magnetograms, about 30 typical bay-type variations, of which the durations range from 20 *min.* to 60 *min.*, are selected and the vector-diagrams of geoelectric and geomagnetic field in the horizontal plane are obtained together with variation-diagrams of the vertical geomagnetic component. The expression is the same as described in the previous report. Some of the variations in each octant are shown in Fig. 10 by way of examples; their details are listed in Table II where the amplitude ratio is seen to have apparent relationship with the duration. Fig. 10 indicates that geomagnetic variations may occur in all directions while corresponding telluric variations do predominantly in a specific direction. The changes of

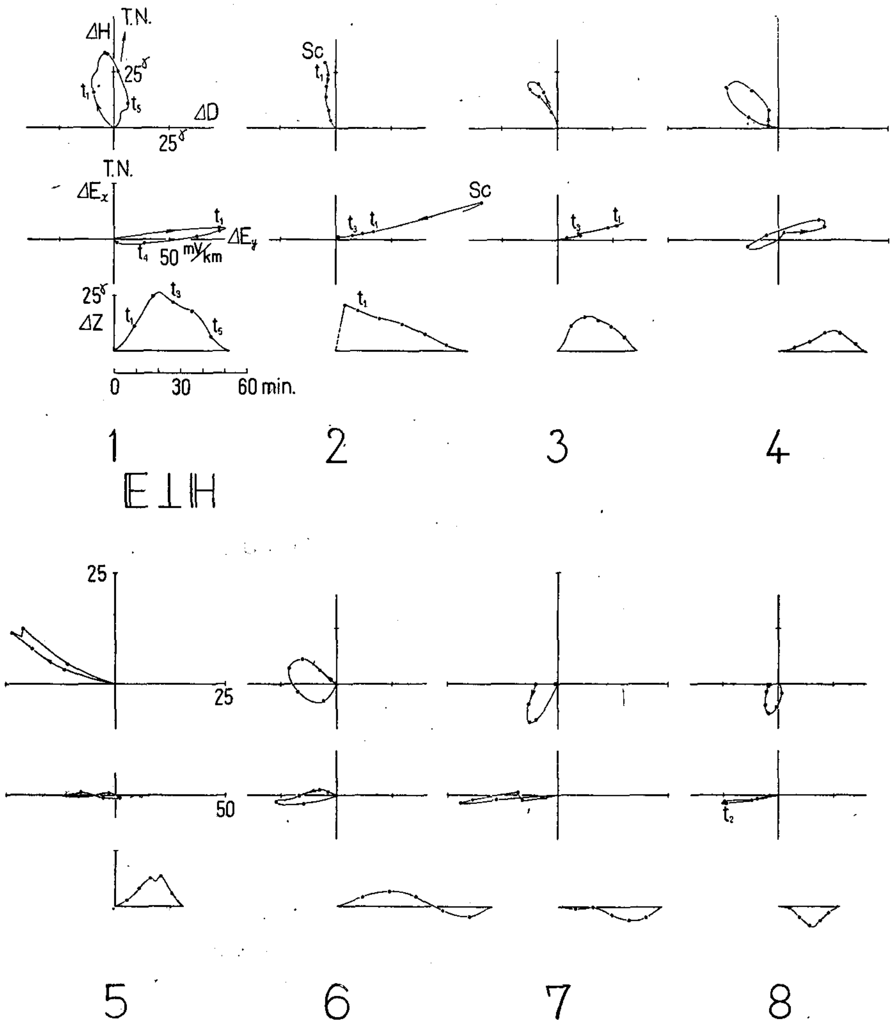
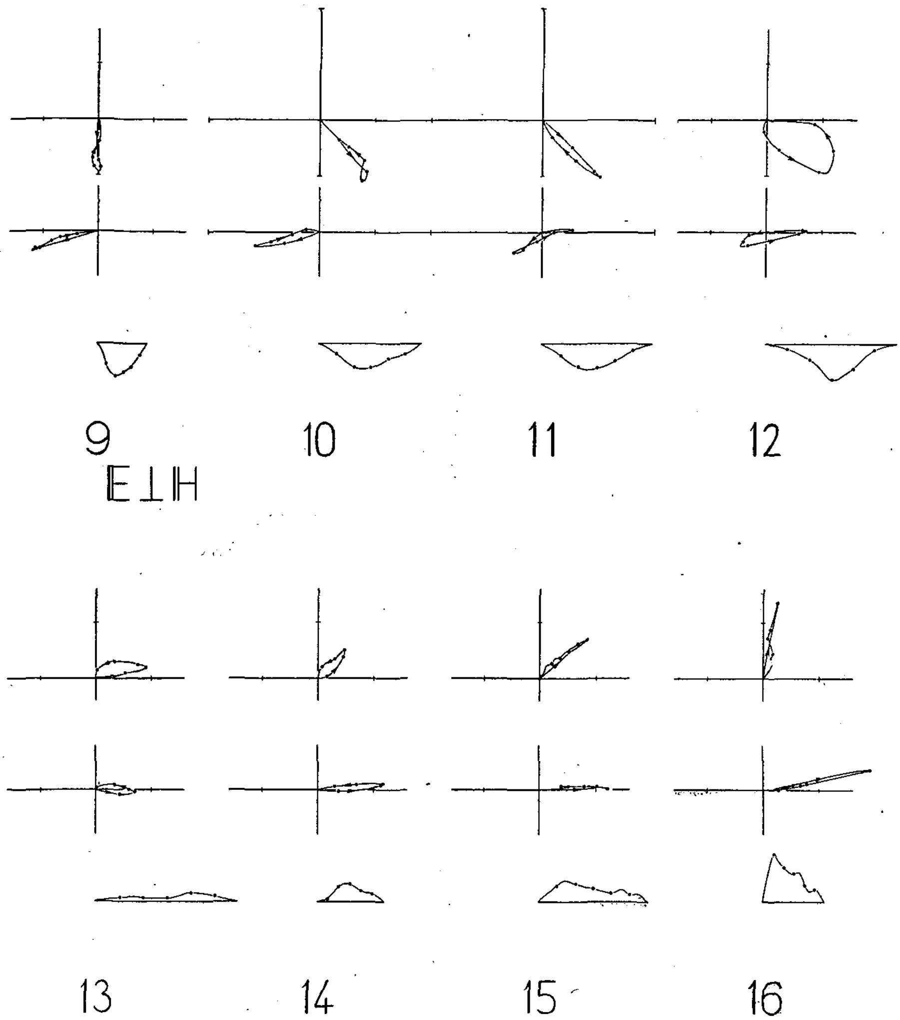


Fig. 10. Examples of the horizontal vector-diagrams simultaneous variation in the vertical geo- sively according to the horizontal octant in

earth-potential shown in Figs. 10–4, 10–11 and 10–12 are transitional. Summarizing the tendency of variations expressed in Fig. 10, one gets the schematic types of variations in series for eacn octant at Kakioka as shown in Fig. 11. Similarly to the results obtained at Dourbes, E-field is not always



of geomagnetic and geoelectric variations and the
magnetic component. The figures are arranged succes-
which the geomagnetic vector changes.

Table II. Variations in earth-potential and in geomagnetism illustrated in Fig. 10.

Fig. Nos.	Time of Occurrence 1958 (G. M. T.)	Duration (*min.*)	Amplitude Ratio ($\frac{mV/km}{\gamma}$) $\frac{\Delta Ex}{\Delta D}$	$\frac{\Delta Ey}{\Delta H}$
10-1	Dec. $04^d 21^h 09^m$	52	−2.3	2.6
2	June 28 07 12	59	−6.8	4.5
3	Apr. 18 11 58	36	−1.4	2.3
4	May 14 14 00	41	−0.15	−0.45
5	May 13 22 19	31	0.00	−1.6
6	Feb. 05 12 15	71	0.33	15
7	May 27 11 55	47	0.52	5.1
8	June 09 11 24	27	1.3	3.6
9	Mar. 30 15 21	23	−29	2.5
10	Feb. 20 21 18	46	−0.59	2.5
11	Mar. 04 04 51	49	−0.71	1.1
12	June 07 09 21	58	−0.49	0.77
13	July 01 11 42	64	−0.091	7.7
14	Feb. 08 09 40	30	0.43	4.4
15	Jan. 17 12 49	49	0.20	3.4
16	Mar. 14 12 12	28	2.6	2.8

perpendicular to H-field in contradiction to Maxwell's law and the variations in earth-potential take a strongly predominant direction, at Kakioka, about 9 *degrees* from east over north including the transitional cases. This predominant direction coincides with that of the maximum resistivity of the earth at the observation point. Concerning the principal direction at Kakioka, T. Yoshimatsu[4),15)] already found it to be 17 and 8 *degrees* from east over north for the diurnal variation and the pulsation respectively, being not very different from the above-obtained value.

At Kakioka, making a difference from Dourbes, the ΔZ variations are not zero for the changes in telluric currents of which vector-diagrams are nearly straight lines, and not loops. This may be interpretable by assuming that electrical conductivity at Kakioka is much more conspicuously anisotropic than at Dourbes: let reference again be made to the equations

$$-\frac{\partial(\Delta Z)}{\partial t} = \Delta E_y \frac{\left|\frac{\partial \rho_{xx}}{\partial x}\right|}{\rho_{xx}} - \Delta E_x \frac{\left|\frac{\partial \rho_{yy}}{\partial y}\right|}{\rho_{yy}}$$

$$= \Delta E_y \cdot X - \Delta E_x \cdot Y \ . \tag{13}$$

If X comparing with Y can be neglected, one gets

$$\frac{\partial(\Delta Z)}{\partial t} = \Delta E_x \cdot Y \tag{15}$$

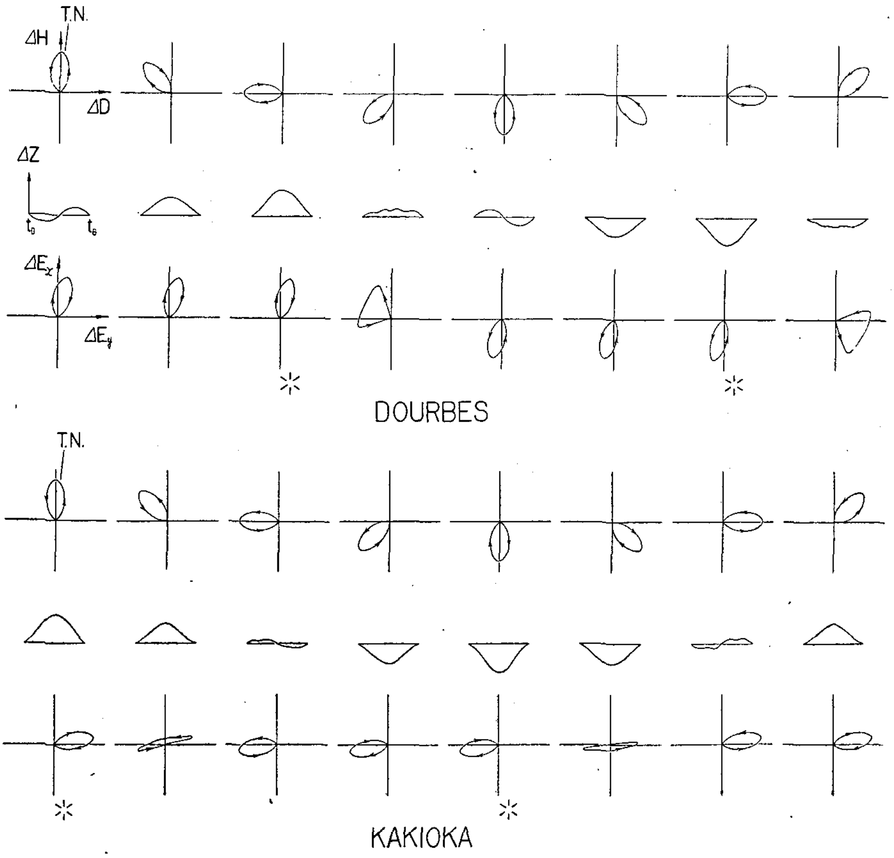


Fig. 11. Schematic types of variations of geomagnetic and geoelectric fields in each octant at Dourbes and Kakioka. Asterisks denote the variations in which both fields are perpendicular each other.

According to this relation, the ΔZ variation depends only on the ΔE_x variation where the x- and y-axes coincide with the principal axes of resistivity. In the previous paper, Y in comparison with X at Dourbes was neglected only for the simplest approximation. By the same method as for Dourbes, one can determine the direction of the principal axis of resistivity as about **9** *degrees* from east over north, and the value of Y, namely, the gradient per unit resistivity as about 0.03/*km*. Here, it should be noted parenthetically that the writer dealt with anisotropy of conductivity as an effective or overall one from the earth-surface to the penetration depth without assuming layer-structures which need ambiguous parameters.

At Kakioka the variation ΔZ shows close parallelism to the variation ΔH on the magnetograms. This is also revealed in Fig. 11; the positive variation ΔZ occcurs in company with the northward geomagnetic variation and its relative amplitude differs according to the azimuth of the magnetic vector in the horizontal plane. To express this anisotropic behaviour of ΔZ at Kakioka, the ratios of ΔZ to the changes of the horizontal geomagnetic resultant vectors ΔR at the time of maximum amplitude in the course of each variation, are shown in Fig. 12. In this directional diagram the radial distance gives the

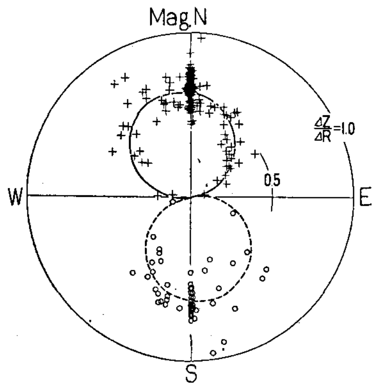


Fig. 12. Anisotropic behaviour of ΔZ at Kakioka.

values of $\Delta Z/\Delta R$ and the azimuth expresses the direction of the geomagnetic variation in the horizontal plane, while the small crosses and circles denote respectively the positive and negative values of ΔZ. Here, one gets also the following empirical formula for dependency of the ratio $\Delta Z/\Delta R$ on the direction of the variation:

$$\frac{\Delta Z}{\Delta R} = 0.63\,\frac{\Delta H}{\Delta R} - 0.10\,\frac{\Delta D}{\Delta R}.$$

This formula is shown in Fig. 12 by means of full and broken lines respectively for positive and negative values of ΔZ.

The anisotropic behaviour of ΔZ at Kakioka is the same in type as that observed at Aburatsubo as already shown in Fig. 7(a), both stations being situated rather near together, about 130 *km* apart; the behaviour makes a complete contrast to that observed at Dourbes shown in Fig. 7(c). At Kakioka and Aburatsubo, the principal direction of telluric currents is nearly in E-W direction, and the ΔZ variations are parallel to the ΔH variations on the magnetograms. However, at Dourbes, the former is nearly in N-S direction and the latter is parallel to the ΔD (declination) variations. As discussed above, the anisotropic behaviours of ΔZ are phenomenally correlated in close connection with those of telluric currents and these are approximately interpreted by anisotropy of conductivity with some plausible assumptions. The writer has analyzed only two, but the most typical cases, one in Belgium and the other in Japan, and believes that there would be intermediate cases between them.

7. The variation of electrical conductivity with depth at Kakioka

In the previous sections, it was proven that anisotropy of electrical conductivity of the earth plays an important role in variations of telluric currents: the usual Maxwell's equations hold good in the limited cases of actual variations, namely, only in the direction of the principal axis of conductivity at that point. Strictly speaking, one must solve Maxwell's equations applied to the medium of anisotropic conductivity. In the following the writer applies the usual Maxwell's equations to the variations which occur in the principal direction instead of considering the effects of anisotropy in general.

Following the example of E.R. Niblett and C. Sayn-Wittgenstein[16], one can estimate conductivity as a function of depth from observations of the changes in geomagnetism and telluric currents. The equations governing electromagnetic variations expressed in *e.m.u.* are:

$$rot\, \Delta \boldsymbol{H} = 4\pi\, \Delta \boldsymbol{i}\ , \tag{1a}$$

$$rot\, \Delta \boldsymbol{E} = -\frac{\partial \Delta \boldsymbol{B}}{\partial t}\ , \tag{2a}$$

$$\Delta \boldsymbol{i} = (\sigma)\, \Delta \boldsymbol{E}\ , \tag{4a}$$

$$\Delta \boldsymbol{B} = (\mu)\, \Delta \boldsymbol{H}\ . \tag{5a}$$

On the assumption that the horizontal gradients of the field vectors are negligible compared to the vertical gradients, and that time variations are periodic, one may take

$$\partial/\partial x = \partial/\partial y = 0 \ \text{and}\ \partial/\partial t = -2\pi i/T\ . \tag{19}$$

where T is the period. If one takes $\bar{\sigma}$ to denote the effective conductivity to a penetration depth Z, the above equations lead to the following approximations:

$$-\frac{\Delta H_y}{Z_x} = 4\pi \bar{\sigma}_x \Delta E_x , \qquad \frac{\Delta H_x}{Z_y} = 4\pi \bar{\sigma}_y \Delta E_y , \tag{20}$$

and

$$-\frac{\Delta E_y}{Z_y} = \frac{2\pi i \Delta H_x}{T} , \qquad \frac{\Delta E_x}{Z_x} = \frac{2\pi i \Delta H_y}{T} . \tag{21}$$

From (20) and (21), one gets

$$\bar{\sigma}_x = \frac{1}{2\left|\frac{\Delta E_x}{\Delta H_y}\right|^2 T} , \qquad \bar{\sigma}_y = \frac{1}{2\left|\frac{\Delta E_y}{\Delta H_x}\right|^2 T} , \tag{22}$$

and

$$Z_x \approx \frac{1}{2\pi}\left|\frac{\Delta E_x}{\Delta H_y}\right| T , \qquad Z_y \approx \frac{1}{2\pi}\left|\frac{\Delta E_y}{\Delta H_x}\right| T . \tag{23}$$

If electrical conductivity is considered to be a continuous and finite function of depth between the surface and the greatest penetration depth encountered, one can write

$$\sigma = f(z) , \tag{24}$$

and

$$\bar{\sigma} = \frac{1}{Z}\int_0^Z f(z)\, dz = \frac{1}{Z} g(Z) . \tag{25}$$

Then

$$\sigma = \frac{d g(Z)}{d Z} = Z \frac{d \bar{\sigma}}{d Z} + \bar{\sigma} . \tag{26}$$

The value of conductivity at various depths can be estimated from equation (26) provided $\bar{\sigma}$ can be established as a known function of Z.

It must be noted that the penetration depths deduced from the different directions are not in accord with each other if conductivity is anisotropic. In the above equations, strictly speaking, $\bar{\sigma}$ must be treated as a tensor quantity. To avoid complexity of tensor quantities, the writer applies the above equations to the observations in the direction of the minor axis of conductivity. At Kakioka, this direction is nearly east-west as shown in the previous section. Niblett and Sayn-Wittgenstein did not treat σ as tensor nor did they discriminate the principal axis of conductivity at Meanook and thereby they obtained the two different distributions of conductivity with depth.

The writer analyzes the magnetograms and tellurigrams obtained by the Kakioka Magnetic Observatory during 1958 and, in Fig. 13, plots only the amplitude ratio $\Delta E_y/\Delta H_x$ against the period which ranges from 1.2 *min.* to 105 *min.* For the region where $\bar{\sigma}=\sigma=$ constant, a plot of $\Delta E/\Delta H$ vs. $1/\sqrt{T}$ should yield a straight line. So, the $\Delta E_y/\Delta H_x$ data are plotted against $1/\sqrt{T}$ as shown in the Fig. 14 and the equations for two lines are determined by the

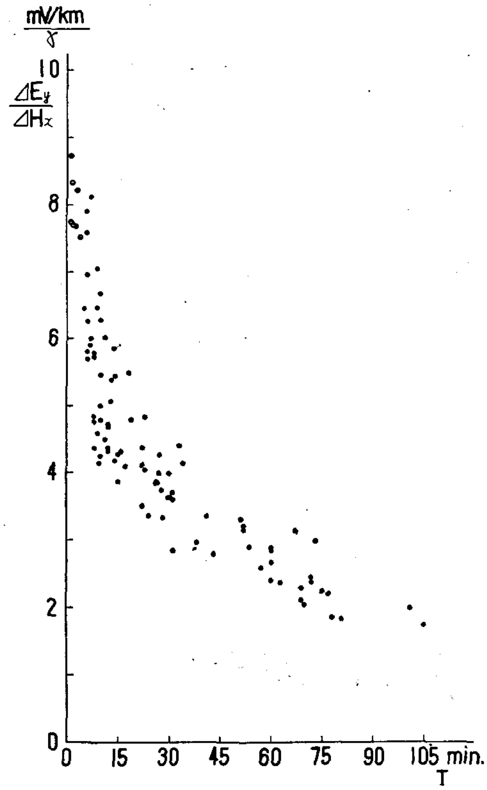


Fig. 13. Amplitude ratio $\Delta E_y/\Delta H_x$ vs. period T.

method of least squares, though the data are not sufficient in number for the second line, assuming that one of them passes through the origin:

$$\frac{\Delta E_y}{\Delta H_x} = 134.50\times 10^5 \frac{1}{\sqrt{T}} \text{ in the range 0.10 to 0.49 of } T^{-1/2},$$

$$\frac{\Delta E_y}{\Delta H_x} = 6.64\times 10^5 + 14.8\times 10^5 \frac{1}{\sqrt{T}} \text{ in the range 0.50 to 0.92 of } T^{-1/2}.$$

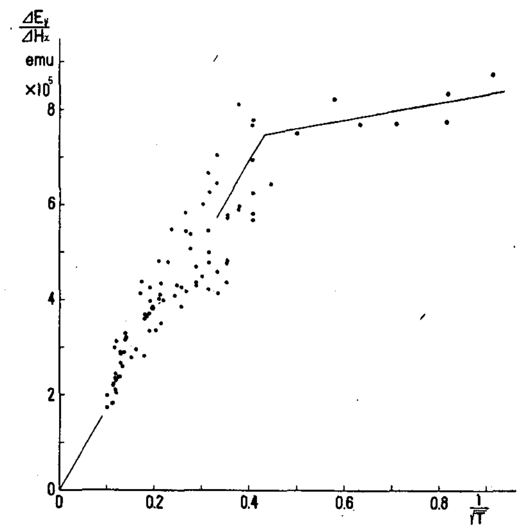


Fig. 14. Amplitude ratio $\Delta E_y/\Delta H_x$ vs. $T^{-1/2}$ (T in *min.*).

On elimination of T from equations (22) and (23) both being replaced by the above values,. these equations give the relation between $\bar{\sigma}$ and Z. Applying the relation to equation (26), one can get the distribution of conductivity as a function of depth as shown in Fig. 15. The discontinuity in the curve at

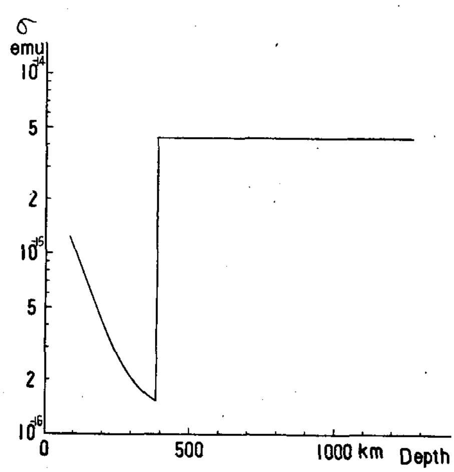


Fig. 15. Variation of electrical conductivity with depth at Kakioka.

$Z=385$ *km* is due to the intersection of the two straight lines in Fig. 14. The discontinuity of electrical conductivity at such a depth has been already pointed out by investigators and may correspond to the boundary between the regions B and C of Bullen's model. For the deep part, which is penetrated by magnetic variations of long period, the results are provisional because such variations are not always sinusoidal. Anyhow, the above is an example of discussion with due consideration for anisotropy of conductivity.

Acknowledgements. The writer is very grateful to the members of the Kakioka Magnetic Observatory who placed their magnetograms and tellurigrams at the writer's disposal. The writer's cordial thanks are also due to Miss M. Chiba for her sincere co-operation in preparing this paper and in drawing many text figures.

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