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On the Distribution of Ground Temperature at 1 m Depth influenced by Various Heat Sources

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Abstract

When heat source exists underground, the distribution of ground temperature is given by the equation:

$$T = f(x, y, z)$$

where T is ground temperature, while x, y and z represent coordinates of arbitrary positions under the ground. Then, the writer derived theoretically the ground temperature for the function of x, y and z; and calculated the temperature in the case that depth z was equal to 1 m.

1. Introduction

It is a useful method for investigation of hot spring to seek the distribution of ground temperature at 1 m depth.[1)2)] The distribution of the ground temperature for a special heat source under the ground was given by OKAMOTO[3)] and YUHARA.[4)] But the estimation of heat source (hot spring) by the aid of the distribution of the ground temperature has been dealt with almost always qualitatively until the present, and the values themsevles of the ground temperature are not used for the estimation of the size, shape and temperature of the heat source.

Then, the writer wants to make quantitative use of the values themsevles of the ground temperature for the estimation and to derive the distribution of the ground temperature at 1 m depth for special heat sources.

In this paper, the writer considers the following three cases. The 1st case is about the model where the heat source (hot water) extends in parallel with the ground surface at a certain depth and the hot water flows out ground surface through a fissure which connects perpendicularly with the heat source. The 2nd case is when a fissure crosses obliquely to the ground surface. The 3rd case concerns a model where the heat source (hot water) is the same as in the 1st and 2nd cases, and a pipe stands vertically to the heat source. The 3rd case is useful to seek the influence of ascending hot water in the vertical pipe upon the ground temperature near the pipe.

2. Distribution of ground temperature at 1 m depth where hot water flows out the ground surface through a vertical fissure

In hot springs, there are many mechanisms of hot water discharge, but many hot springs flow out through fissures. Then the writer derives the distribution of the ground temperature at 1m depth for certain discharge rates and heat sources of various temperatures and depths.

2.1. *Distribution of ground temperature in a case without any fissure*

When the heat source (hot water) which is parallel to the ground surface spreads infinitely and there is no fissure, distribution of ground temperature is derived as a function of depth only from the ground surface.

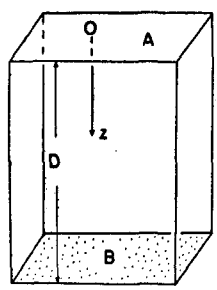

Fig. 2-1. A schematic map of the model.

The above model is illustrated in Fig. 2–1 where z-axis is taken in perpendicularly downward direction from the ground surface, **A** is ground surface, **B** is heat source (hot water), D is depth of the heat source. Now, let T_1 be the ground temperature at depth z from the ground surface and let T_1 be considered a steady temperature. T_1 is a function of z only, the differential equation for T_1 is

$$\frac{d^2T_1}{dz^2} = 0 \tag{2–1}$$

In this paper, zero of temperature is taken as atmospheric temperature. Boundary conditions are

$$\frac{dT_1}{dz} = \lambda\, T_1 \qquad \text{at} \quad z = 0 \tag{2–2}$$

where $\lambda = \dfrac{h}{k}$

$$T = T_0 \qquad \text{at} \quad z = D \tag{2-3}$$

where k is thermal conductivity of the ground, h is the constant of Newton's colling, and T_0 is the temperature of the heat source.

From (2–1), (2–2) and (2–3), the solution is given by

$$T_1 = \frac{T_0}{1+\lambda D}(1+\lambda z) \tag{2-4}$$

Accordingly, the ground temperature is given by the linear equation of z.

2.2. *Differential equation and boundary conditions in the case where there is a fissure*

The model is illustrated in Fig. 2–2 where **A**, **B**, D and z are the same as in Fig. 2–1, **C** is a fissure which is perpendicular to **A** and **B**, y-axis is an intersecting line of the fissure wall and the ground surface, x-axis is perpendicular

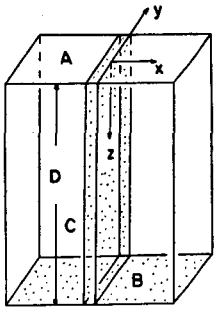

Fig. 2-2. A schematic map of the model.

to the wall. The fissure extends infinitely in positive and negative directions of y-axis. Because the fissure is perpendicular to the ground surface and the heat source, the distribution of the ground temperature is symmetrical to the fissure. Therefore the writer derives the distribution in the positive range of x.

Now, let T be the ground temperature of any point of (x,y,z) and let T be considered a steady temperature. Then, T is indubitably not dependent on y, and the differential equation for T is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{2-5}$$

Boundary conditions are

$$T = T_1 \qquad \text{at} \qquad x = \infty \tag{2-6}$$

$$T = \theta \quad \text{at} \quad x = 0 \tag{2-7}$$

$$T = T_0 \quad \text{at} \quad z = D \tag{2-8}$$

$$\frac{\partial T}{\partial z} = \lambda T \quad \text{at} \quad z = 0 \tag{2-9}$$

where θ is temperature of the hot water in the fissure.

2.3. *Solution*

It follows from (2–5), (2–6) and (2–8) that

$$T = \sum_{s=1}^{\infty} A_s e^{-P_s x} \sin P_s (D-z) + T_1$$

$$= \sum_{s=1}^{\infty} A_s e^{-P_s x} \sin P_s (D-z) + \frac{T_0}{1+\lambda D}(1+\lambda z) \tag{2-10}$$

where A_s and P_s are unknown constants.

From (2–10)

$$T_{z=0} = \sum_{s=1}^{\infty} A_s e^{-P_s x} \sin P_s D + \frac{T_0}{1+\lambda D} \tag{2-11}$$

and

$$\left(\frac{\partial T}{\partial z}\right)_{z=0} = -\sum_{s=1}^{\infty} A_s P_s e^{-P_s x} \cos P_s D + \frac{\lambda T_0}{1+\lambda D} \tag{2-12}$$

By use of (2–11) and (2–12), (2–9) becomes

$$-\sum_{s=1}^{\infty} A_s P_s e^{-P_s x} \cos P_s D + \frac{\lambda T_0}{1+\lambda D} = \lambda\left(\sum_{s=1}^{\infty} A_s e^{-P_s x} \sin P_s D + \frac{T_0}{1+\lambda D}\right)$$

$$\sum_{s=1}^{\infty} A_s e^{-P_s x} (P_s \cos P_s D + \lambda \sin P_s D) = 0$$

When x is finite, because A_s is not equal to 0, the formula in ( ) of the above equation is equal to 0.

$$P_s \cos P_s D + \lambda \sin P_s D = 0$$

$$\tan P_s D = -\frac{P_s}{\lambda}$$

$$= -\frac{P_s D}{\lambda D} \tag{2-13}$$

Putting

$$\mu_s = P_s D \tag{2-14}$$

(2–13) becomes

$$\tan \mu_s = -\frac{\mu_s}{\lambda D} \tag{2-15}$$

And from (2–14) P_s is given by

$$P_s = \frac{\mu_s}{D} \tag{2-16}$$

where μ_s are the roots of (2–15).

By substituting (2–16) into (2–10), T becomes

$$T = \sum_{s=1}^{\infty} A_s e^{-\frac{\mu_s}{D} z} \sin \frac{\mu_s}{D} (D-z) + \frac{T_0}{1+\lambda D} (1 + \lambda z) \tag{2-17}$$

Therefore, it follows from (2–7) and (2–17) that

$$\theta = \sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D} (D-z) + \frac{T_0}{1+\lambda D} (1 + \lambda z) \tag{2-18}$$

Fig. 2–3 indicates a prism which is perpendicular to the heat source. The height, the length (of y-direction) and the width of the prism are respectively D, 1 cm and the same as the fissure. Fig. 2–4 indicates an infinitesimal

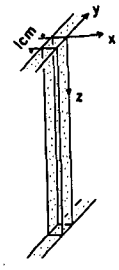

Fig. 2-3 A prism in the fissure.

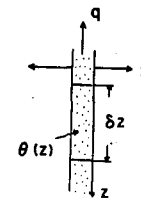

Fig. 2-4. An infinitesimal prism.

prism of which the height is δz, while the width and length are the same as in Fig. 2–3. θ, that is temperature of ascending hot water in the fissure, is considered as function of z only, and is given by (2–18). Fall in the temperature is due to heat conduction to rock around the fissure. Heat conduction of z-direction is negligible, because $k' \, d\theta/dz$ is very small compared with the

heat conduction to the rock, where k' is the thermal conductivity of the hot water.

Then, the following differential equation is derived from income and out go of heat in the infiniteismal prism.

$$-q\rho c\frac{d\theta}{dz}\delta z = 2k\left(\frac{\partial T}{\partial x}\right)_{x=0}\delta z \tag{2-19}$$

where q is the discharge rate of hot water through the prism, ρ and c are respectively the density and the specific heat of the hot water, and k is the thermal conductivity of the rock around the fissure.

From (2-19)

$$\frac{d\theta}{dz} = -K\left(\frac{\partial T}{\partial x}\right)_{x=0} \tag{2-20}$$

where

$$K = \frac{2k}{q\rho c}$$

From (2-17)

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = -\sum_{s=1}^{\infty} A_s\left(\frac{\mu_s}{D}\right)\sin\frac{\mu_s}{D}(D-z) \tag{2-21}$$

Using (2-21), (2-20) becomes

$$\frac{d\theta}{dz} = K\sum_{s=1}^{\infty} A_s\left(\frac{\mu_s}{D}\right)\sin\frac{\mu_s}{D}(D-z) \tag{2-22}$$

Accordingly, it follows from (2-22) that

$$\theta = K\sum_{s=1}^{\infty} A_s\cos\frac{\mu_s}{D}(D-z) + const.$$

and the boundary condition for θ is

$$\theta = T_0 \quad \text{at} \quad z = D$$

Therefore

$$const. = T_0 - K\sum_{s=1}^{\infty} A_s$$

Thus

$$\theta = T_0 - K\sum_{s=1}^{\infty} A_s\left\{1-\cos\frac{\mu_s}{D}(D-z)\right\} \tag{2-23}$$

A_s in equation (2-10) is sought by the following method. Values of θ

given by (2–18) or (2–23) ought to be equal respectively. Then, suppose that θ is nearly equal to T_0 in (2–23), and substituting T_0 into θ of (2–18), A_s in right side of (2–18) is sought. The writer defines this A_s as A_{s1}. Next, substituting A_{s1} into A_s in (2–23), θ is sought. And again, substituting the result into (2–18), One derives A_s of the right side in (2–18). This A_s is defined as A_{s2}. Similarly, A_{sn} is defined as the value of A_s which is obtained by repeating the preceding process n times over. The larger n is taken, the nearer A_{sn} gets to the true value of A_s. But, when accurate temperature is not needed, n may be a few times. The matter will be made clear below.

The following is a discussion on the determination of A_s. First, let (2–23) be considered approximately as the following equation:

$$\theta \fallingdotseq T_0 \tag{2–24}$$

By use of (2–24), (2–18) becomes

$$T_0 = \sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D}(D-z) + \frac{T_0}{1+\lambda D}(1+\lambda z) \tag{2–25}$$

Therefore

$$\sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D}(D-z) = \frac{\lambda T_0}{1+\lambda D}(D-z) \tag{2–26}$$

When μ_s are the roots of (2–15), the following definite integral is obtained:

$$\int_0^D \sin \frac{\mu_m}{D}(D-z) \sin \frac{\mu_s}{D}(D-z)\, dz = 0 \qquad \text{for } m \neq s \tag{2–27}$$

$$= \frac{D}{2} \cdot C_s \qquad \text{for } m = s \tag{2–28}$$

where

$$C_s = \frac{\mu_s - \sin \mu_s \cos \mu_s}{\mu_s}$$

and

$$\int_0^D \frac{\lambda (D-z)}{1+\lambda D} \sin \frac{\mu_s}{D}(D-z)\, dz = -D \frac{\cos \mu_s}{\mu_s} \tag{2–29}$$

Then, multiplying both sides of (2–26) by $\sin\mu_s(D-z)/D$, and integrating each term from 0 to D, one gets from (2–27), (2–28) and (2–29):

$$A_s \frac{D}{2} C_s = -D T_0 \frac{\cos \mu_s}{\mu_s}$$

Therefore

$$A_s = A_{s1} = -\frac{2T_0}{C_s} \cdot \frac{\cos \mu_s}{\mu_s} \tag{2-30}$$

Substituting (2–30) into (2–23), one gets

$$\theta = T_0 + K \sum_{m=1}^{\infty} \frac{2T_0}{C_m} \cdot \frac{\cos \mu_m}{\mu_m} \left\{1 - \cos \frac{\mu_m}{D} (D-z)\right\} \tag{2-31}$$

accordingly, it follows from (2–18) and (2–31) that

$$T_0 + K \sum_{m=1}^{\infty} \frac{2T_0}{C_m} \cdot \frac{\cos \mu_m}{\mu_m} \left\{1 - \cos \frac{\mu_m}{D} (D-z)\right\}$$

$$= \sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D} (D-z) + \frac{T_0}{1+\lambda D} (1+\lambda z)$$

Therefore

$$\sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D} (D-z) = \frac{\lambda T_0}{1+\lambda D} (D-z) + 2T_0 K \sum_{m=1}^{\infty} \frac{\cos \mu_m}{C_m \mu_m}$$

$$\times \left\{1 - \cos \frac{\mu_m}{D} (D-z)\right\} \tag{2-32}$$

Further the following definite integral is obtained for μ_s which satisfies (2–15).

$$\int_0^D \left\{1 - \cos \frac{\mu_m}{D} (D-z)\right\} \sin \frac{\mu_s}{D} (D-z)\, dz$$

$$= D\left[\frac{1-\cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \left\{1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m\right)\right\}\right]$$

$$\text{for } m \neq s \tag{2-33}$$

$$= D\left[\frac{1-\cos \mu_s}{\mu_s} - \frac{1-\cos 2\mu_s}{4}\right] \qquad \text{for } m = s \tag{2-34}$$

In the same way as in (2–26), multiplying both sides of (2–32) by $\sin \mu_s (D-z)/D$, then integrating each term from 0 to D, one learns from (2–27), (2–28), (2–29), (2–33) and (2–34) that

$$A_s \frac{D}{2} C_s = -DT_0 \frac{\cos \mu_s}{\mu_s} + 2DT_0 K \sum_{m=1}^{\infty} \frac{\cos \mu_m}{C_m \mu_m}$$

$$\times \left[\frac{1-\cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m \right) \right\} \right]$$

Therefore

$$A_s = A_{s2} = -\frac{2T_0}{C_s} \cdot \frac{\cos \mu_s}{\mu_s} + \frac{4T_0 K}{C_s} \sum_{m=1}^{\infty} \frac{\cos \mu_m}{C_m \mu_m}$$

$$\times \left[\frac{1-\cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m \right) \right\} \right] \tag{2–35}$$

Putting

$$B'_s = \sum_{m=1}^{\infty} \frac{\cos \mu_m}{C_m \mu_m}$$

$$\times \left[\frac{1-\cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m \right) \right\} \right]$$

(2–35) becomes

$$A_{s2} = -\frac{2T_0}{C_s} \cdot \frac{\cos \mu_s}{\mu_s} + \frac{4T_0 K}{C_s} B_s' \tag{2–36}$$

Substituting (2–36) into (2–23), (2–23) becomes

$$\theta = T_0 - K \sum_{m=1}^{\infty} \left\{ -\frac{2T_0}{C_m} \cdot \frac{\cos \mu_m}{\mu_m} + \frac{4T_0 K}{C_m} \cdot B_m' \right\} \left\{ 1 - \cos \frac{\mu_m}{D} (D-z) \right\} \tag{2–37}$$

Accordingly, it follows from (2–18) and (2–37) that

$$\sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D} (D-z) = \frac{\lambda T_0}{1+\lambda D} (\lambda D - z) + KT_0$$

$$\times \sum_{m=1}^{\infty} \left\{ \frac{2\cos \mu_m}{C_m \mu_m} - \frac{4K}{C_m} B_m' \right\} \left\{ 1 - \cos \frac{\mu_m}{D} (D-z) \right\} \tag{2–38}$$

In the same way as (2–26) and (2–32), A_s in (2–38) is obtained immediately.

$$A_s = A_{s3} = -\frac{2T_0}{C_s} \cdot \frac{\cos \mu_s}{\mu_s} + \frac{4T_0 K}{C_s} \cdot B_s' - \frac{8T_0 K^2}{C_s} \sum_{m=1}^{\infty} \frac{B_m''}{C_m}$$

$$\times \left[\frac{1-\cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m \right) \right\} \right]$$

$$= A_{s2} - \frac{8T_0K^2}{C_s} \sum_{m=1}^{\infty} \frac{B_m'}{C_m}$$

$$\times \left[\frac{1-\cos\mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2-\mu_m^2} \left\{ 1-\left(\cos\mu_s\cos\mu_m + \frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right] \tag{2–39}$$

From the numerical calculation of the 2nd term of the right side in (2–39), it is known that the 2nd term is very small in comparison with the 1st term. Then, A_{s3} is given approximately by

$$A_{s3} \doteqdot A_{s2} \tag{2–40}$$

Hence, from (2–39) and (2–40), A_{s2} may be used for A_s.

Thus

$$A_s \doteqdot A_{s2} = -\frac{2T_0}{C_s} \cdot \frac{\cos\mu_s}{\mu_s} + \frac{4T_0K}{C_s} B_s' \tag{2–41}$$

$$= -\frac{2T_0\cos\mu_s}{\mu_s - \sin\mu_s\cos\mu_s} + \frac{4T_0K}{\mu_s-\sin\mu_s\cos\mu_s} \sum_{m=1}^{\infty} \frac{\cos\mu_m}{\mu_m - \sin\mu_m\cos\mu_m}$$

$$\times \left[1-\cos\mu_s - \frac{\mu_s^2}{\mu_s^2-\mu_m^2} \left\{ 1-\left(\cos\mu_s\cos\mu_m + \frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right] \tag{2–42}$$

Now, putting

$$B_s = \sum_{m=1}^{\infty} \frac{\cos\mu_m}{\mu_m - \sin\mu_m\cos\mu_m}$$

$$\times \left[1-\cos\mu_s - \frac{\mu_s^2}{\mu_s^2-\mu_m^2} \left\{ 1-\left(\cos\mu_s\cos\mu_m + \frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right] \tag{2–43}$$

(2–42) becomes

$$A_s = 2\left(\frac{-\cos\mu_s + 2KB_s}{\mu_s - \sin\mu_s\cos\mu_s}\right) T_0 \tag{2–44}$$

Substituting (2–44) into (2–10), one gets

$$T = 2T_0 \sum_{s=1}^{\infty} \left(\frac{-\cos\mu_s + 2KB_s}{\mu_s - \sin\mu_s\cos\mu_s}\right) e^{-\frac{\mu_s}{D}x} \sin\frac{\mu_s}{D}(D-z) + \frac{T_0}{1+\lambda D}(1+\lambda z) \tag{2–45}$$

where B_s is expressed by (2–43), and the formula in [ ] of (2–43) for $m=s$ is given by

$$1-\cos\mu_s-\frac{1-\cos 2\mu_s}{4}$$

Let $T_{d=1}$ be the ground temperature at 1 m depth. From (2–45) $T_{d=1}$ is given by

$$T_{d=1}=2T_0\sum_{s=1}^{\infty}\left(\frac{-\cos\mu_s+2KB_s}{\mu_s-\sin\mu_s\cos\mu_s}\right)e^{-\frac{\mu_s}{D}x}\sin\frac{\mu_s}{D}(D-1)+\frac{1+\lambda}{1+\lambda D}T_0 \tag{2–46}$$

The ratio of $T_{d=1}$ to T_0 is obtained immediately from (2–46).

$$\frac{T_{d=1}}{T_0}=2\sum_{s=1}^{\infty}\left(\frac{-\cos\mu_s+2KB_s}{\mu_s-\sin\mu_s\cos\mu_s}\right)e^{-\frac{\mu_s}{D}x}\sin\frac{\mu_s}{D}(D-1)+\frac{1+\lambda}{1+\lambda D} \tag{2–47}$$

$$=\frac{T'_{d=1}}{T_0}+\frac{1+\lambda}{1+\lambda D} \tag{2–48}$$

where

$$\frac{T'_{d=1}}{T_0}=2\sum_{s=1}^{\infty}\left(\frac{-\cos\mu_s+2KB_s}{\mu_s-\sin\mu_s\cos\mu_s}\right)e^{-\frac{\mu_s}{D}x}\sin\frac{\mu_s}{D}(D-1) \tag{2–49}$$

The 1st term of (2–48) is the ground temperature of 1m depth which depends on the ascending hot water in the fissure. However the 2nd term depends on the heat source which spreads infinitely in parallel with the ground surface, and the actual temperature is given by the sum of the above two temperatures.

2.4. *Results of numerical calculation*

The values of $T_{d=1}/T_0$ and $T'_{d=1}/T_0$ are obtained by the employment of (2–47) and (2–49). The constants used are as follows:

$\lambda=\frac{h}{k}=0.15\,\mathrm{m}^{-1}$ [7]

$k=1.7\times10^{-3}$ C.G.S. (thermal conductivity of tuff) [8]

$\rho=1$ g/c.c.

$c=1$ cal/g·°C

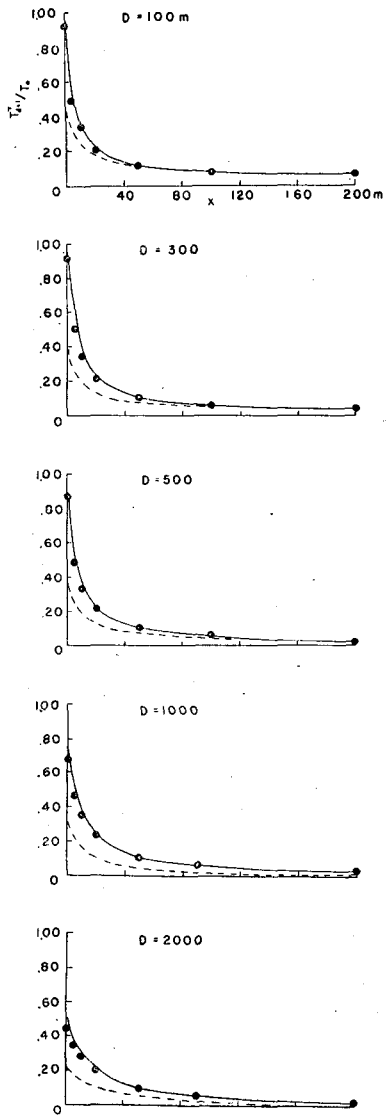

Fig. 2-5. Examples of $T_{d=1}/T_0$ curves for x.

——— ; q=1c.c./sec
● ; =10^{-1}
– – – – ; =10^{-2}

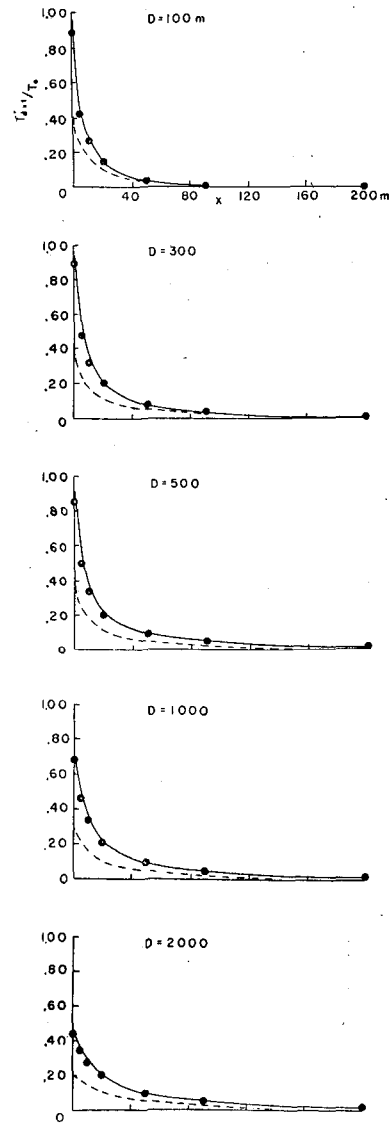

Fig. 2-6. Examples of $T_d'{}_{=1}/T_0$ curves for x.

——— ; q=1c.c./sec
● ; =10^{-1}
– – – – ; =10^{-2}

$$q = 1,\ 10^{-1},\ 10^{-2}\ \text{c.c./sec}$$

$$D = 100,\ 300,\ 500,\ 1{,}000,\ 2{,}000\ \text{m}$$

Figs. 2–5 and 2–6 indicate the relation between $T_{d=1}/T_0$, $T'_{d=1}/T_0$ and x. In Figs. 2–5, when discharge rate is more than 10^{-1} c.c./sec, $T_{d=1}/T_0$ decreases suddenly in proporition as x increase, and regardless of the depth of the heat source values of $T_{d=1}/T_0$ are equal to 0.10 at $x=50$ m. In Fig. 2–6, values of $T'_{d=1}/T_0$ for the various depths of the heat source are $0\sim0.03$ at $x=200$m. Then, regardless of the depth of the heat source it becomes clear that the ascending hot water in the fissure almost does not affect the ground temperature of 1 m depth at a distance of 200 m from the fissure.

2.5. *Determination of the temperature and the depth of the heat source according to the distribution of the ground temperature at 1m depth*

When the discharge rate is more than 10^{-1}c.c./sec, from Section 2.3., $T_{d=1}/T_0$ is constant and its value is equal to 0.10 at $x=50$m regardless of the depth of the heat source. Then, let $(T_{d=1})_{x=50}$ be the ground temperature of 1 m depth at $x=50$m. $(T_{d=1})_{x=50}/T_0$ is give by

$$\frac{(T_{d=1})_{x=50}}{T_0} = 0.10 \tag{2–50}$$

Therefore

$$T_0 = 10\,(T_{d=1})_{x=50} \tag{2–51}$$

It follows that the temperature of the heat source is ten times as large as the ground temperature of 1 m depth at $x=50$m.

Next, let θ_0 be the temperature of the hot water at $z=0$. θ_0 is nearly equal to the temperature of the hot water at $z=1$ m.

Therefore, θ_0/T_0 is given by

$$\frac{\theta_0}{T_0} \fallingdotseq \frac{(T_{d=1})_{x=0}}{T_0} \tag{2–52}$$

where $(T_{d=1})_{x=0}$ is the temperature of the hot water at $z=1$ m.

Then, using the relation of (2–52), one may obtain the relation between θ_0/T_0 and D for various discharge rates as shown in Fig. 2–7. If the value of θ_0/T_0 $(=a)$ and discharge rate q is measured, D_a corresponding to a and q is obtained as in Fig. 2–7. But, the above method is applicable in case the discharge rate is more than 10^{-1}c.c./sec.

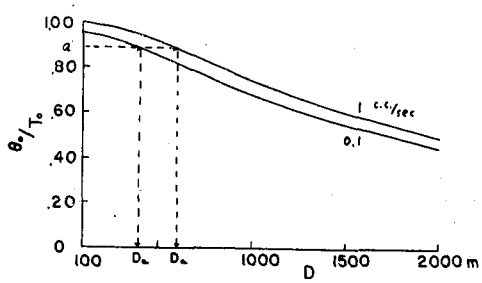

Fig. 2-7. Relation between θ_0/T_0 and D.

3. Distribution of the ground temperature at 1 m depth in case hot water flows out the ground surface through an oblique fissure

The cases where a fissure crosses obliquely with the ground surface are more common than cases in which it is perpendicular to the ground surface. The following is a discussion about the distribution of the ground temperature at 1m depth in case the fissure is oblique.

3.1. *Distribution of the ground temperature in case there is no fissure*

In the same way as in Chapter 2 above, let T_1 be the ground temperature for the model in which there is no fissure and the heat source spreads infinitely in parallel with the ground surface at depth D. **A** and **B** are respectively the ground surface and the heat source (hot water), and x–, x'–, z– and z'-axes are selected as in Fig. 3–1. In Fig. 3–1 z'- axis is perpendicular to the

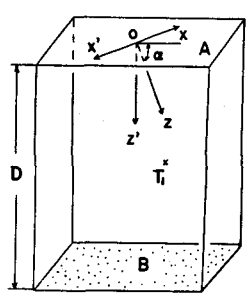

Fig. 3-1. A schematic map of the model.

ground surface, z-axis and the ground surface cross at an angle α, x-and x'-axes are perpendicular to z-axis, and exist on the plane in which z- and z'-axes are included.

Substituting z' for z in (2–4), the equation becomes

$$T_1 = \frac{T_0}{1+\lambda D}(1+\lambda z') \tag{3–1}$$

Now, expressing z' by (x,z) or (x', z)

$$z' = z \sin \alpha - x \cos \alpha \tag{3–2}$$

or

$$z' = z \sin \alpha + x' \cos \alpha \tag{3–3}$$

If (3–2) or (3–3) is substituted into (3–1), T_1 becomes

$$T_1(x, z) = \frac{T_0}{1+\lambda D}\{1+\lambda(z \sin \alpha - x \cos \alpha)\} \tag{3–4}$$

or

$$T_1(x', z) = \frac{T_0}{1+\lambda D}\{1+\lambda(z \sin \alpha + x' \cos \alpha) \tag{3–5}$$

3.2. *Approximation of the model*

The writer proposes next to deal with the model shown as Fig. 3–2 in which the heat source of temperature T_0 spreads infinitely in parallel with the ground surface at depth D and the fissure **C** is not perpendicular to the ground surface. But it is difficult to deal with such a model as the above; then the writer uses the following simple model in place of the above.

Model I in Fig. 3–3 is taken to be the same as Fig. 3–2. Now let the

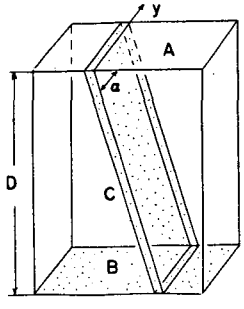

Fig. 3-2. A schematic map of the model.

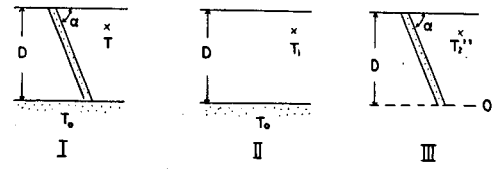

Fig. 3-3. Division of the model.

ground temperature for model I be T, and let the ground temperature for model II or III only be respectively T_1 and T_2''. From (2–48), T is given by

$$T = T_1 + T_2'' \tag{3-6}$$

Next, in Fig. 3–4, the isothermal line of temperature 0 in model *III* is para llel to the ground surface and not perpendicualr to the fissure, while in mode IV it is not parallel to the ground surface and perpendicular to the fissure. But the other boundary conditions of the two models are the same. It is easy to deal with model IV. Therefore the writer substitutes model IV for model III. The propriety of the substitution is explained in below Section 3.6. Now, let the ground temperature for model IV be T_2. In Fig. 3–5, T is considered

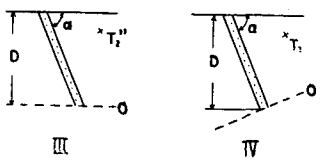

Fig. 3-4. Assumption of the model.

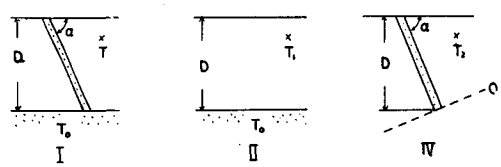

Fig. 3-5. Division of the model.

nearly equal to the sum of T_1 and T_2. Equation (3–6) becomes

$$T \doteqdot T_1 + T_2 \tag{3-7}$$

3.3. *Differential equation and boundary conditions for model IV*

In (3–7), T_1 is obtained by (3–4) or (3–5). Therefore, if T_2 is desired, T is obtained by means of (3–7). Then, in this section the writer gives the differential equation and boundary conditions for T_2. Now, in Fig. 3–6, let the ground temperatures in the right and left sides of the fissure be

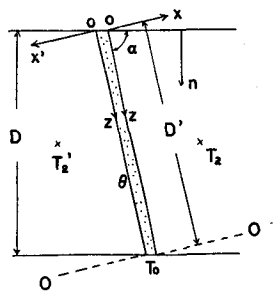

Fig. 3-6. Space coordinates under consideration.

respectively T_2 and T_2'. Similarly, in model I of Fig. 3–5, let the ground temperatures to the right and left sides of the fissure be respectively T and T'. Then T and T' are given by

$$T = T_1 + T_2 \tag{3-8}$$

$$T' = T_1 + T_2' \tag{3-9}$$

Fig. 3-6 shows a cross section perpendicular to the ground surface and the fissure; x-, x'- and z-axes are taken as Fig. 3-6, where z-axis is a line intersecting the wall of the fissure; x- and x'-axes are perpendicular to z-axis. When y-axis is taken as in Fig. 3-2, T_2 and T_2' are independent of y, and the differential equation and boundary conditions for T_2 are

$$\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial z^2} = 0 \tag{3-10}$$

$$T_2 = 0 \qquad \text{at} \quad x = \infty \tag{3-11}$$

$$T_2 = \theta \qquad \text{at} \quad x = 0 \tag{3-12}$$

$$T_2 = 0 \qquad \text{at} \quad z = D' \ (= D/\sin \alpha) \tag{3-13}$$

$$\frac{\partial T_2}{\partial n} = \lambda T_2 \quad \text{at} \quad z = x \cot \alpha \tag{3-14}$$

where n takes the downwards normal direction to the ground surface as in Fig. 3-6, and the relation between n and (x, z) is given by

$$\frac{\partial T_2}{\partial n} = -\frac{\partial T_2}{\partial x} \cos \alpha + \frac{\partial T_2}{\partial z} \sin \alpha \tag{3-15}$$

The equation for T_2' is the same as (3-10); the boundary condition corresponding (3-14) becomes

$$\frac{\partial T_2'}{\partial n} = \lambda T_2' \qquad \text{at} \qquad z = -x' \cot \alpha \tag{3-16}$$

and the other boundary conditions are the same as (3-11), (3-12) and (3-13).

3.4. *Solution*

It follows from (3-10), (3-11) and (3-13) that

$$T_2 = \sum_{s=1}^{\infty} A_s e^{-P_s x} \sin P_s (D' - z) \tag{3-17}$$

where A_s and P_s are unknown constants.

By use of (3-17), (3-15) becomes

$$\frac{\partial T_2}{\partial n} = \sum_{s=1}^{\infty} A_s P_s e^{-P_s x} \sin P_s (D'-z) \cos \alpha$$

$$- \sum_{s=1}^{\infty} A_s P_s e^{-P_s x} \cos P_s (D'-z) \sin \alpha$$

$$= \sum_{s=1}^{\infty} A_s P_s e^{-P_s x} \{\sin P_s (D'-z) \cos \alpha - \cos P_s (D'-z) \sin \alpha\} \tag{3–18}$$

Accordingly, $(\partial T/\partial n)_{z=x\cot\alpha}$ becomes

$$\left(\frac{\partial T_2}{\partial n}\right)_{z=x\cot\alpha} = \sum_{s=1}^{\infty} A_s P_s e^{-P_s x} \left\{\cos \alpha \sin P_s D' \cdot \left(1-\frac{x\cot\alpha}{D'}\right)\right.$$

$$\left. - \sin \alpha \cos P_s D' \cdot \left(1-\frac{x\cot\alpha}{D'}\right)\right\} \tag{3–19}$$

and from (3–17)

$$(T_2)_{z=x\cot\alpha} = \sum_{s=1}^{\infty} A_s e^{-P_s x} \sin P_s D' \cdot \left(1-\frac{x\cot\alpha}{D'}\right) \tag{3–20}$$

Substituting (3–19) and (3–20) into (3–14), ont gets

$$\sum_{s=1}^{\infty} A_s e^{-P_s x} \left\{P_s \cos \alpha \sin P_s D' \cdot \left(1-\frac{x\cot\alpha}{D'}\right) - P_s \sin \alpha \cos P_s D'\right.$$

$$\left. \times \left(1-\frac{x\cot\alpha}{D'}\right) - \lambda \sin P_s D' \cdot \left(1-\frac{x\cot\alpha}{D'}\right)\right\} = 0 \tag{3–21}$$

When x is finite, because A_s is not equal to 0, the formula in { } of (3–21) is equal to 0.

Then

$$(P_s \cot \alpha - \lambda) \sin P_s D' \cdot \left(1-\frac{x\cot\alpha}{D'}\right) - P_s \sin \alpha \cos P_s D' \cdot \left(1-\frac{x\cot\alpha}{D'}\right) = 0 \tag{3–22}$$

When the ground temperature of the distant place from the fissure does not have to be known, ratio of $(x \cot \alpha)$ to D' is very small. Accordingly, $(x \cot \alpha)/D'$ takes a very small value in comparison with 1.
Then, (3–22) transforms into

$$(P_s \cos \alpha - \lambda) \sin P_s D' - P_s \sin \alpha \cos P_s D' = 0 \tag{3–23}$$

Therefore

$$\tan P_s D' = \frac{P_s \sin \alpha}{P_s \cos \alpha - \lambda}$$

$$= \frac{P_s D' \sin \alpha}{P_s D' \cos \alpha - \lambda D'} \tag{3–24}$$

Putting

$$\mu_s = P_s D' \tag{3–25}$$

(3–24) becomes

$$\tan \mu_s = \frac{\mu_s \sin \alpha}{\mu_s \cos \alpha - \lambda D'} \tag{3–26}$$

Then, from (3–25) P_s is expressed by

$$P_s = \frac{\mu_s}{D'} \tag{3–27}$$

where μ_s are the roots of (3–26).

Substituting (3–27) into (3–17), one gets

$$T_2 = \sum_{s=1}^{\infty} A_s e^{-\frac{\mu_s}{D'} x} \sin \frac{\mu_s}{D'} (D'-z) \tag{3–28}$$

Therefore

$$T = T_1 + T_2$$

$$= \frac{T_0}{1+\lambda D} \{1 + \lambda (z \sin \alpha - x \cos \alpha)\} + \sum_{s=1}^{\infty} A_s e^{-\frac{\mu_s}{D'} x} \sin \frac{\mu_s}{D'} (D'-z) \tag{3–29}$$

and from (3–12) and (3–29), θ is given by

$$\theta = \frac{T_0}{1+\lambda D} (1+\lambda z \sin \alpha) + \sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D'} (D'-z) \tag{3–30}$$

T' and T_2' are also obtained by the same method as above, namely (3–10), (3–11), (3–12), (3–13) and (3–15) are also satisfactory for T_2'.

Therefore, one gets

$$T_2' = \sum_{s=1}^{\infty} A_s' e^{-P_s' x'} \sin P_s' (D'-z) \tag{3–31}$$

where A_s' and P_s' are unknown constants.

From (3–31)

$$-\frac{\partial T_2'}{\partial x'} = \sum_{s=1}^{\infty} A_s' P_s' e^{-P_s' x'} \sin P_s' (D'-z) \tag{3–32}$$

$$\frac{\partial T_2'}{\partial z} = -\sum_{s=1}^{\infty} A_s' P_s' e^{-P_s' x'} \cos P_s' (D'-z) \tag{3–33}$$

Accordingly, it follows from (3–16), (3–31), (3–32) and (3–33) that

$$\sum_{s=1}^{\infty} A_s' e^{-P_s' x'} \left\{ P_s' \cos \alpha \sin P_s' D' \cdot \left(1+\frac{x' \cot \alpha}{D'}\right) - P_s' \sin \alpha \cos P_s' D' \right.$$
$$\left. \times \left(1+\frac{x' \cot \alpha}{D'}\right) - \lambda \sin P_s' D' \cdot \left(1+\frac{x' \cot \alpha}{D'}\right) \right\} = 0 \tag{3–34}$$

When x' is finite, in the same way as (3–21), equation (3–34) becomes

$$(P_s' \cos \alpha - \lambda) \sin P_s' D' \cdot \left(1+\frac{x' \cot \alpha}{D'}\right)$$
$$- P_s' \sin \alpha \cos P_s' D' \cdot \left(1+\frac{x' \cot \alpha}{D'}\right) = 0 \tag{3–35}$$

For x' whose $(x' \cot \alpha)/D'$ is very small in comparison with 1, (3–35) becomes

$$(P_s' \cos \alpha - \lambda) \sin P_s' D' - P_s' \sin \alpha \cos P_s' D' = 0 \tag{3–36}$$

Therefore

$$\tan P_s' D' = \frac{P_s' \sin \alpha}{P_s' \cos \alpha - \lambda}$$
$$= \frac{P_s' D' \sin \alpha}{P_s' D' \cos \alpha - \lambda D'} \tag{3–37}$$

Putting

$$\mu_s' = P_s' D' \tag{3–38}$$

(3–37) becomes

$$\tan \mu_s' = \frac{\mu_s' \sin \alpha}{\mu_s' \cos \alpha - \lambda D'} \tag{3–39}$$

Comparing with (3–26) and (3–39), the following relation is obtained.

$$\mu_s' = \mu_s$$

Therefore

$$P_s' D' = P_s D'$$

$$P_s' = P_s$$

$$= \frac{\mu_s}{D'} \tag{3–41}$$

Substituting (3–41) into (3–31), one gets

$$T_2' = \sum_{s=1}^{\infty} A_s' e^{-\frac{\mu_s}{D'} x'} \sin \frac{\mu_s}{D'} (D'-z) \tag{3–42}$$

Therefore

$$T' = T_1 + T_2'$$

$$= \frac{T_0}{1+\lambda D} \{1+\lambda (z \sin \alpha + x' \cos \alpha)\} + \sum_{s=1}^{\infty} A_s' e^{-\frac{\mu_s}{D'} x'} \sin \frac{\mu_s}{D'} (D'-z) \tag{3–43}$$

From (3–12) and (3–43), θ is given by

$$\theta = \frac{T_0}{1+\lambda D} (1 + \lambda z \sin \alpha) + \sum_{s=1}^{\infty} A_s' \sin \frac{\mu_s}{D'} (D'-z) \tag{3–44}$$

(3–30) and (3–44) are the equations which express θ, therefore the right sides of (3–30) and (3–44) are respectively equal, and the 1st terms are also equal. Then, the 2nd terms must be equal respectively.

Hence

$$A_s' = A_s \tag{3–45}$$

By use of (3–45), (3–43) becomes

$$T' = \frac{T_0}{1+\lambda D} \{1 + \lambda (z \sin \alpha + x' \cos \alpha)\} + \sum_{s=1}^{\infty} A_s e^{-\frac{\mu_s}{D'} x'} \sin \frac{\mu_s}{D'} (D'-z) \tag{3–46}$$

Fig. 3–7 represents a prism which takes in a fissure; the length is 1 cm, the width is equal to the width of the fissure, the height is D'. Fig. 3–8 shows an infinitesimal part of the above prism, the height being δz, the width and length the same as in Fig. 3–7.

As the writer states in Chapter 2, θ that is temperature of ascending

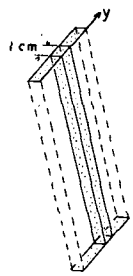

Fig. 3-7 A prism in the fissure.

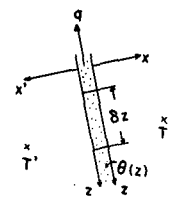

Fig. 3-8. An infinitesimal prism.

hot water in the fissure, is considered as a function of z only, and fall in θ is due to heat conduction to rock around the fissure. Then, the following differential equation is derived from income and outgo of heat in the infinitesimal prism.

$$-q\rho c\frac{d\theta}{dz}\delta z = k\left\{\left(\frac{\partial T}{\partial x}\right)_{x=0} + \left(\frac{\partial T'}{\partial x'}\right)_{x'=0}\right\}\delta z \tag{3-47}$$

where q, ρ, c and k are the same as in (2–19).

From (3–29) and (3–46), one gets

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = -\frac{\lambda T_0}{1+\lambda D}\cos\alpha - \sum_{s=1}^{\infty} A_s \frac{\mu_s}{D'}\sin\frac{\mu_s}{D'}(D'-z) \tag{3-48}$$

$$\left(\frac{\partial T'}{\partial x'}\right)_{x'=0} = \frac{\lambda T_0}{1+\lambda D}\cos\alpha - \sum_{s=1}^{\infty} A_s \frac{\mu_s}{D'}\sin\frac{\mu_s}{D'}(D'-z) \tag{3-49}$$

Substituting (3–48) and (3–49) into (3–47), the latter becomes

$$q\rho c\frac{d\theta}{dz} = 2k\sum_{s=1}^{\infty} A_s\frac{\mu_s}{D'}\sin\frac{\mu_s}{D'}(D'-z) \tag{3-50}$$

Taking

$$K = \frac{2k}{q\rho c} \tag{3-51}$$

(3–50) becomes

$$\frac{d\theta}{dz} = K\sum_{s=1}^{\infty} A_s\frac{\mu_s}{D'}\sin\frac{\mu_s}{D'}(D'-z) \tag{3-52}$$

Accordingly, it follows from (3–52) that

$$\theta = K \sum_{s=1}^{\infty} A_s \cos \frac{\mu_s}{D'} (D'-z) + const\,. \tag{3-53}$$

Boundary condition for θ is

$$\theta = T_0 \qquad \text{at} \qquad z = D'$$

Therefore, *const.* in (3–53) is given by

$$const\,. = T_0 - K \sum_{s=1}^{\infty} A_s$$

Thus

$$\theta = T_0 - K \sum_{s=1}^{\infty} A_s \left\{1 - \cos \frac{\mu_s}{D'} (D'-z)\right\} \tag{3-54}$$

A_s contained in the formulae of T and T' can be found by the method used in Section 2.3. Let A_{sn} be A_s obtained by the process of repeated substitution for n times.

First, let (3–54) be considered approximately as the following equation.

$$\theta = T_0 \tag{3-55}$$

It follows from (3–30) and (3–55) that

$$T_0 = \frac{T_0}{1+\lambda D} (1+\lambda z \sin \alpha) + \sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D'} (D'-z)$$

Therefore

$$\sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D'} (D'-z) = \frac{\lambda D}{1+\lambda D} T_0 - \frac{\lambda T_0 \sin \alpha}{1+\lambda D} z \tag{3-56}$$

When the depth of the heat source D is very great, because D' is equal to $D/(\sin\ \alpha)$, D' has a still larger. Therefore, for μ_s in which s is not very large, (3–26) becomes

$$\tan \mu_s \doteqdot -\frac{\mu_s \sin \alpha}{\lambda D'} \tag{3-57}$$

When (3–57) is satisfied, the following definite integrals are obtained.

$$\int_0^{D'} \sin \frac{\mu_m}{D'} (D'-z) \sin \frac{\mu_s}{D'} (D'-z)\, dz = \frac{D'}{2} C_s \qquad \text{for} \quad m = s \tag{3-58}$$

$$= 0 \qquad \text{for} \quad m \neq s \tag{3-59}$$

$$\int_0^{D'} \sin \frac{\mu_s}{D'} (D' - z)\, d\, z = \frac{D' (1 - \cos \mu_s)}{\mu_s} \tag{3-60}$$

$$\int_0^{D'} z \sin \frac{\mu_s}{D'} (D' - z)\, d\, z = \frac{D'^2}{\mu_s} \left(1 - \frac{\sin \mu_s}{\mu_s}\right) \tag{3-61}$$

where

$$C_s = \frac{\mu_s - \sin \mu_s \cos \mu_s}{\mu_s}$$

Multiplying both sides of (3-56) by $\sin\mu_s(D'-z)/D'$, then integrating each term from 0 to D', it follows from (3-58), (3-59), (3-60) and (3-61) that

$$\frac{D'}{2} C_s A_s = \frac{\lambda D T_0}{1+\lambda D} \cdot \frac{D' (1-\cos \mu_s)}{\mu_s} - \frac{\lambda T_0 \sin \alpha}{1+\lambda D} \cdot \frac{D'^2}{\mu_s} \left(1 - \frac{\sin \mu_s}{\mu_s}\right)$$

Because D is equal to $(D' \sin \alpha)$, the above equation becomes

$$\frac{D'}{2} C_s A_s = \frac{D'}{\mu_s} \cdot \frac{\lambda D T_0}{1+\lambda D} \left(\frac{\sin \mu_s}{\mu_s} - \cos \mu_s\right) \tag{3-62}$$

Therefore

$$A_s = A_{s1} = \frac{2}{C_s} \cdot \frac{1}{\mu_s} \cdot \frac{\lambda D T_0}{1+\lambda D} \left(\frac{\sin \mu_s}{\mu_s} - \cos \mu_s\right)$$

$$= \frac{2 \lambda D T_0}{(\mu_s - \sin \mu_s \cos \mu_s)(1+\lambda D)} \left(\frac{\sin \mu_s}{\mu_s} - \cos \mu_s\right) \tag{3-63}$$

Substituting (3-63) into A_s of (3-54), θ is obtained. Again, substituting the result into (3-30), one gets

$$T_0 - K \sum_{m=1}^{\infty} A_{m1} \left\{1 - \cos \frac{\mu_m}{D'} (D'-z)\right\}$$

$$= \frac{T_0}{1+\lambda D} (1+\lambda z \sin \alpha) + \sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D'} (D'-z)$$

Therefore

$$\sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D'} (D'-z) = \frac{\lambda D}{1+\lambda D} T_0$$

$$- \frac{\lambda T_0 \sin \alpha}{1+\lambda D} z - K \sum_{m=1}^{\infty} A_{m1} \left\{1 - \cos \frac{\mu_m}{D'} (D' - z)\right\} \tag{3-64}$$

The sum of the 1st and 2nd terms of the right side in (3–64) is equal to the right side of (3–56). Accordingly, from (3–62), the following definite integral is obtained:

$$\int_0^{D'}\left\{\frac{\lambda D}{1+\lambda D}T_0-\frac{\lambda T_0\sin\alpha}{1+\lambda D}z\right\}\sin\frac{\mu_s}{D'}(D'-z)\,dz=\frac{D'}{2}C_sA_{s1}. \tag{3–65}$$

Multiplying both sides of (3–64) by $\sin\mu_s(D'-z)/D'$, then integrating each term from 0 to D', it follows from (2–33), (2–34), (3–58), (3–59) and (3–65) that

$$\frac{D'}{2}C_sA_s=\frac{D'}{2}C_sA_{s1}-K\sum_{m=1}^{\infty}A_{m1}D'$$
$$\times\left[\frac{1-\cos\mu_s}{\mu_s}-\frac{\mu_s^2-\mu_m^2}{\mu_s}\left\{1-\left(\cos\mu_s\cos\mu_m+\frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right] \tag{3–66}$$

Therefore

$$A_s=A_{s2}=A_{s1}-\frac{2K}{C_s}\sum_{m=1}^{\infty}A_{m1}$$
$$\times\left[\frac{1-\cos\mu_s}{\mu_s}-\frac{\mu_s}{\mu_s^2-\mu_m^2}\left\{1-\left(\cos\mu_s\cos\mu_m+\frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right]$$
$$=A_{s1}-\frac{2K}{\mu_s-\sin\mu_s\cos\mu_s}\sum_{m=1}^{\infty}A_{m1}$$
$$\times\left[(1-\cos\mu_s)-\frac{\mu_s^2}{\mu_s^2-\mu_m^2}\left\{1-\left(\cos\mu_s\cos\mu_m+\frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right] \tag{3–67}$$

From (2–34), the value of formula in [ ] of (3–67) for $m=s$ becomes

$$1-\cos\mu_s-\frac{1-\cos 2\mu_s}{4}$$

Let the 2nd term of (3–67) be $-A'_{s1}$. Then, (3–67) becomes

$$A_{s2}=A_{s1}+A'_{s1} \tag{3–68}$$

As a result of substituting (3–68) into (3–54), θ is expressed by

$$\theta=T_0-K\sum_{n=1}^{\infty}(A_{n1}+A'_{n1})\left\{1-\cos\frac{\mu_n}{D'}(D'-z)\right\} \tag{3–69}$$

It follows from (3–30) and (3–69) that

$$T_0 - K \sum_{n=1}^{\infty} (A_{n1} + A'_{n1}) \left\{1 - \cos \frac{\mu_n}{D'} (D' - z)\right\}$$

$$= \frac{T_0}{1+\lambda D} (1 + \lambda z \sin \alpha) + \sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D'} (D' - z)$$

Therefore

$$\sum_{s=1}^{\infty} A_s \sin \frac{\mu_s}{D'} (D' - z) = \frac{\lambda D}{1+\lambda D} T_0 - \frac{\lambda T_0 \sin \alpha}{1+\lambda D} z$$

$$- K \sum_{n=1}^{\infty} A_{n1} \left\{1 - \cos \frac{\mu_n}{D'} (D' - z)\right\} - K \sum_{n=1}^{\infty} A'_{n1} \left\{1 - \cos \frac{\mu_n}{D'} (D' - z)\right\} \tag{3–70}$$

Because the sum of the initial three terms in (3–70) is equal to the right side of (3–64), the following definite integral is obtained from (3–66).

$$\int_0^{D'} \left[T_0 - \frac{T_0}{1+\lambda D} (1+\lambda z \sin \alpha) - K \sum_{n=1}^{\infty} A_{n1} \left\{1 - \cos \frac{\mu_n}{D'} (D' - z)\right\}\right]$$

$$\times \sin \frac{\mu_s}{D'} (D' - z)\, dz = \frac{D'}{2} C_s A_{s2} \tag{3–71}$$

Again, multiplying both sides of (3–70) by $\sin \mu_s (D' - z)/D'$, then integrating each term from 0 to D', it follows from (2–33), (2–34), (3–58), (3–59) and (3–71) that

$$\frac{D'}{2} C_s A_s = \frac{D'}{2} C_s A_{s2} - K \sum_{n=1}^{\infty} A'_{n1} D'$$

$$\times \left[\frac{1 - \cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_n^2} \left\{1 - \left(\cos \mu_s \cos \mu_n + \frac{\mu_n}{\mu_s} \sin \mu_s \sin \mu_n\right)\right\}\right] \tag{3–72}$$

Therefore

$$A_s = A_{s3} = A_{s2} - \frac{2K}{C_s} \sum_{n=1}^{\infty} A'_{n1}$$

$$\times \left[\frac{1 - \cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_n^2} \left\{1 - \left(\cos \mu_s \cos \mu_n + \frac{\mu_n}{\mu_s} \sin \mu_s \sin \mu_n\right)\right\}\right]$$

$$= A_{s2} - \frac{2K}{\mu_s - \sin \mu_s \cos \mu_s}.$$

$$\times \sum_{n=1}^{\infty} \left[1 - \cos \mu_s - \frac{\mu_s^2}{\mu_s^2 - \mu_n^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_n + \frac{\mu_n}{\mu_s} \sin \mu_s \sin \mu_n \right) \right\} \right] \tag{3–73}$$

From the result of numerical calculation, it follows that the 2nd term of the right side in (3–73) is very small in comparison with the 1st term. Then, (3–73) becomes

$$A_{s3} \fallingdotseq A_{s2} \tag{3–74}$$

Hence, A_{s2} may be used for A_s from (3–73) and (3–74) as A_s in Section 2.3. Now, putting

$$B_s = \frac{2\lambda D}{1+\lambda D} \left(\frac{\sin \mu_s}{\mu_s} - \cos \mu_s \right) \tag{3–75}$$

(3–63) becomes

$$A_{s1} = \frac{B_s T_0}{\mu_s - \sin \mu_s \cos \mu_s} \tag{3–76}$$

From (3–67) and (3–76), A_s becomes

$$A_s \fallingdotseq A_{s2} = \frac{B_s T_0}{\mu_s - \sin \mu_s \cos \mu_s} - \frac{2K}{\mu_s - \sin \mu_s \cos \mu_s} \sum_{m=1}^{\infty} \frac{B_m T_0}{\mu_m - \sin \mu_m \cos \mu_m}$$

$$\times \left[1 - \cos \mu_s - \frac{\mu_s^2}{\mu_s^2 - \mu_m^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m \right) \right\} \right] \tag{3–77}$$

Next, putting

$$E_s = 2K \sum_{m=1}^{\infty} \frac{B_m}{\mu_m - \sin \mu_m \cos \mu_m}$$

$$\times \left[1 - \cos \mu_s - \frac{\mu_s^2}{\mu_s^2 - \mu_m^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m \right) \right\} \right] \tag{3–78}$$

(3–77) becomes

$$A_s = \frac{B_s T_0}{\mu_s - \sin \mu_s \cos \mu_s} - \frac{E_s T_0}{\mu_s - \sin \mu_s \cos \mu_s}$$

$$= \left(\frac{B_s - E_s}{\mu_s - \sin \mu_s \cos \mu_s} \right) T_0 \tag{3–79}$$

Thus, using (3–79), equations (3–29) and (3–46) become

$$\frac{T}{T_0} = \frac{1+\lambda(z\sin\alpha - x\cos\alpha)}{1+\lambda D}$$

$$+ \sum_{s=1}^{\infty}\left(\frac{B_s - E_s}{\mu_s - \sin\mu_s\cos\mu_s}\right) e^{-\frac{\mu_s}{D'}x} \sin\frac{\mu_s}{D'}(D'-z) \tag{3–80}$$

$$\frac{T'}{T_0} = \frac{1+\lambda(z\sin\alpha + x'\cos\alpha)}{1+\lambda D}$$

$$+ \sum_{s=1}^{\infty}\left(\frac{B_s - E_s}{\mu_s - \sin\mu_s\cos\mu_s}\right) e^{-\frac{\mu_s}{D'}x'} \sin\frac{\mu_s}{D'}(D'-z) \tag{3–81}$$

Let X- and X'–axes be taken along the ground surface as in Fig. 3–9.

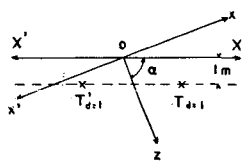

Fig. 3-9. Space coordinates under consideration.

Let coordinates of the points of 1 m depth at X and X' be respectively (x, z) and (x', z). Then, (x, z) and (x', z) are given by

$$\left.\begin{aligned} x &= X\sin\alpha - \cos\alpha \\ z &= X\cos\alpha + \sin\alpha \end{aligned}\right\} \tag{3–82}$$

and

$$\left.\begin{aligned} x' &= X'\sin\alpha + \cos\alpha \\ z &= -X'\cos\alpha + \sin\alpha \end{aligned}\right\} \tag{3–83}$$

Now, let the ground temperatures of 1m depth at X and X' be respectively $T_{d=1}$ and $T'_{d=1}$. $T_{d=1}/T_0$ and $T'_{d=1}/T_0$, obtained by substituting (3–82) and (3–83) into (3–80) and (3–81) respectively, are

$$\frac{T_{d=1}}{T_0} = \frac{1+\lambda}{1+\lambda D} + \sum_{s=1}^{\infty}\left(\frac{B_s - E_s}{\mu_s - \sin\mu_s\cos\mu_s}\right) e^{-\frac{\mu_s}{D'}(X\sin\alpha - \cos\alpha)}$$

$$\times \sin\frac{\mu_s}{D'}\{D' - (X\cos\alpha + \sin\alpha)\} \tag{3–84}$$

and

$$\frac{T'_{d=1}}{T_0} = \frac{1+\lambda}{1+\lambda D} + \sum_{s=1}^{\infty} \left(\frac{B_s - E_s}{\mu_s - \sin \mu_s \cos \mu_s} \right) e^{-\frac{\mu_s}{D'}(X' \sin \alpha + \cos \alpha)}$$

$$\times \sin \frac{\mu_s}{D'} \{D' + (X' \cos \alpha - \sin \alpha)\} \tag{3-85}$$

3.5. *Results of numerical calculation*

Fig. 3-10 indicates an example of numerical calculation in the case when the depth of the heat source D is 2,000 m. In the figure, the axis of ordinate takes for convenience $T_{d=1}/T_0$ only and a symbol of $T'_{d=1}/T_0$ is omitted. The constants used for the calculation are as follows:

$k = 1.7 \times 10^{-3}$ C.G.S. (thermal conductivity of tuff)

$\lambda = 0.15\ \mathrm{m}^{-1}$

$\rho = 1$ g/c. c.

$c = 1$ cal/g·°C

$\alpha = 60°,\ 70°,\ 80°,\ 90°$

$q = 0.1,\ 1.0$ c. c. /sec

Let the ground temperatures of 1 m depth at $X=1$ m and $X=X$m be $(T_{d=1})_{X=1}$ and $(T_{d=1})_{X=X}$ respectively. A ratio of the two temperatures M is given by

$$M = \frac{(T_{d=1})_{X=X}}{(T_{d=1})_{X=1}}$$

$$= \frac{(T_{d=1}/T_0)_{X=X}}{(T_{d=1}/T_0)_{X=1}} \tag{3-86}$$

where $(T_{d=1}/T_0)_{X=X}$ is the value of $T_{d=1}/T_0$ at $X=X$m, and is given by (3-84) or (3-85). Fig. 3-11 indicates the relation between M and $(X,\ X')$.

Because T_0 can not be actually measured, $T_{d=1}/T_0$ also can not be measured. On the other hand, $(T_{d=1})_{X=X}$ and $(T_{d=1})_{X=1}$ can be measured, accordingly M can be measured. Then, the relation between M and $(X,\ X')$ may be used more than the relation between $T_{d=1}/T_0$ and $(X,\ X')$.

From Figs. 3-10 and 3-11, it is clear that the larger α becomes, the larger $-d(T_{d=1}/T_0)/dX$ and $-dM/dX$ become at the range of X. Reversely the larger α becomes, the smaller $-d(T_{d=1}/T_0)/dX'$ and $-dM/dX'$ become at the range of X'. Let $M_{X=100}$ be the value of M at $X=100$m. In both cases of $q=1$ c.c./sec and $q=0.1$c.c./sec, $M_{X=100}$ and α are the linear relation as shown in

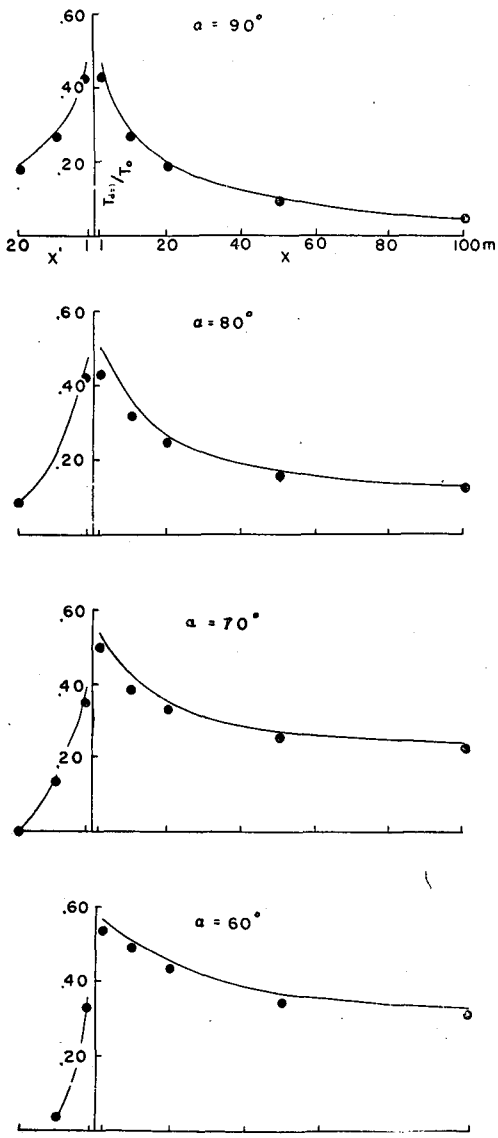

Fig.3-10. Examples of $T_{d=1}/T_0$ curves for X and X'.
——— ; q=1.0c.c/sec
● ; =0.1

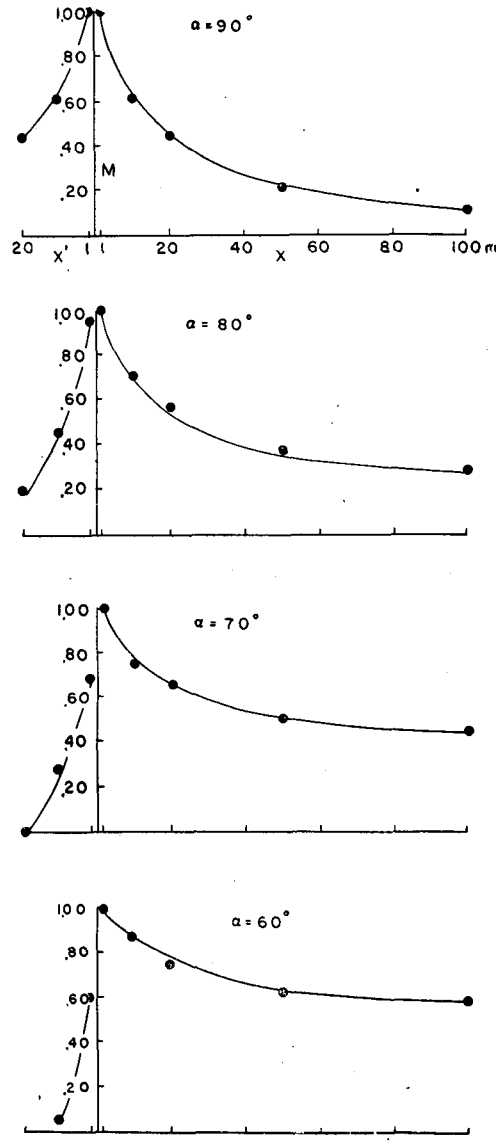

Fig. 3-11. Examples of M curves for X and X'.
——— ; q=1.0c.c./sec
● ; =0.1

Fig. 3–12. The relation is the case at $D=2{,}000$ m, it is not found whether the relation will hold or not at other depths. But it is an interesting relation.

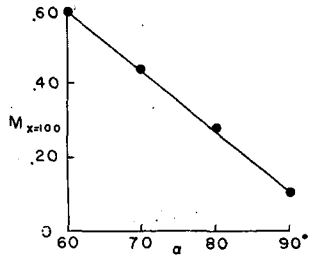

Fig. 3-12. Relation between $M_{x=100}$ and α.

3.6. *Propriety of the above approximate model*

In this section, the writer discusses the propriety of the approximation which in Fig. 3–4 model III is approximated by model IV.

First, let the left side of the fissure be considered. If one employs x'–, z-, L-axes, origin of L-axis and $\overline{AB}$ as in Fig. 3–13, in model III the temperature

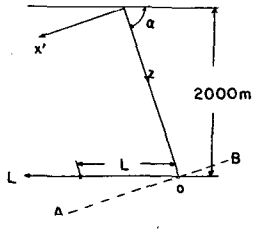

Fig. 3-13. Space coordinates under consideration.

on L-axis is 0 and in model IV the temperature on $\overline{AB}$ is also 0. Accordingly, if the temperature on L-axis for model IV is nearly equal to 0 in the left side of the fissure, it may be proper for model IV to be used in place of model III.

In Fig. 3–13, a coordinate of any point on L-axis (x', z) is expressed by L, D' and α, and the relations are as follows:

$$\left.\begin{aligned} x' &= L \sin \alpha \\ z &= D' - L \cos \alpha \end{aligned}\right\} \qquad (3\text{–}87)$$

Now, let the ground temperature at the point (x', z) expressed in (3–87)

be $(T_2')_{L=L}$. $(T_2')_{L=L}/T_0$ is obtained by substituting (3–87) into the 2nd term in (3–81), and it follows that

$$\frac{(T_2')_{L=L}}{T_0} = \sum_{s=1}^{\infty} \left(\frac{B_s - E_s}{\mu_s - \sin \mu_s \cos \mu_s} \right) e^{-\frac{\mu_s}{D'} L \sin \alpha} \cdot \sin \left(\frac{L \cos \alpha}{D'} \mu_s \right) \tag{3–88}$$

Table 3–1 indicates the values of $(T'_2)_{L=L}/T_0$ for $q=1.0$c.c./sec. In the table, $(T_2')_{L=L}/T_0$ reaches its maximum at $\alpha=60°$, $L=1{,}000$ m, while there is

Table 3-1. Values of $(T_2')_{L=L}/T_0$ for $q=1.0$ c.c./sec.

α \ L	5m	10	20	50	100	500	800	1,000	1,200	2,000
60°	0.00	0.00	0.00	0.01	0.02	0.08	0.09	0.10	0.09	0.06
70	0.00	0.00	0.00	0.01	0.02	0.05	0.06	0.06	0.06	0.03
80	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.03	0.01

very small value for other α and L. Therefore, in the left side, it is ascertained that the above approximation is nearly correct.

About the right side of the fissure, because L-axis continues downside of $\overline{AB}$ line, the temperature on L-axis in the case when the temperature on $\overline{AB}$ is euqal to 0 can not be obtained. In this case, the writer seeks first the value of the ground temperature near $\overline{AB}$ for the model as in Fig. 3–14 (II) in which the fissure is perpendicular to the heat source and D is 2,000 m.

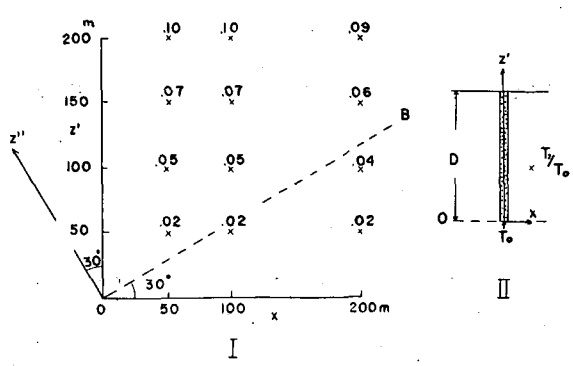

Fig. 3-14. I; Values of T_2/T_0 for the model II.
II; A shematic map of the model.

Now, let x- and z'-axes be taken as in Fig. 3–14 (II). When the temperature

on x-axis is equal to 0, the ground temperature T_2 is expressed by a function of x and z', and is given by the 1st term of (2–45). Fig. 3–14 (I) indicates the values of T_2/T_0. If broken line $\overline{OB}$ and x-axis cross at 30°, $\overline{OB}$ corresponds to $\overline{AB}$ of Fig. 3–13 in the case of $\alpha=60°$. When α is larger than 60°, $\overline{OB}$ gets near to the x-axis, and the ground temperature on $\overline{OB}$ is smaller than that at $\alpha=60°$. Furthermore, in the case of $\alpha=60°$, the position of the fissure is not on the z'-axis but on the z''-axis in Fig. 3–14 (I), and the values of T_2/T_0 are smaller than the case in Fig. 3–14 (I). Accordingly, the aforesaid approximation may be proper on the right side of the fissure, too. But if α is small, the angle between OB and x-axis becomes large, and the ground temperature on $\overline{OB}$ shows a large value. In this case, the approximation is not proper.

4. Influence of ascending hot water in a pipe upon the ground temperature at 1 m depth near an orifice

The ground temperature near an orifice is affected by ascending hot water in a pipe. To eliminate the influence, it is needed to subtract the ground temperature dependent upon ascending hot water only from the ground temperature measured actually. In this chapter, the writer discusses the influence.

Previously OKAMOTO[10] sought the influence under certain assumptions, But the writer obtains the influence by another method without such assumptions.

4.1. *Differential equation and boundary conditions*

Fig. 4–1 represents a model in which hot water at depth D spreads

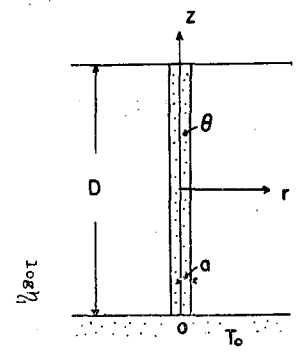

Fig. 4-1. A schematic map of the model.

infinietly in parallel with the ground surface, and a pipe stands perpendicularly on the heat source. Now, let z-, r-axes and origin of z be taken as shown in Fig. 4–1, where z-axis agrees with the center axis of the pipe, and r-axis takes the direction of the radius of the pipe. For this model, the ground temperature may be considered a steady temperature as in Chapters 2 and 3. Then, the differential equation for the ground temperature is given by

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{4–1}$$

where T is the ground temperature at an arbitrary place.

Next, the boundary conditions are as follows:

$$T = T_0 - \frac{\lambda T_0}{1+\lambda D} z \qquad \text{at} \quad r = \infty \tag{4–2}$$

$$T = \theta \qquad \text{at} \quad r = a \tag{4–3}$$

$$T = T_0 \qquad \text{at} \quad z = 0 \tag{4–4}$$

$$\frac{\partial T}{\partial z} = -\lambda T \qquad \text{at} \quad z = D \tag{4–5}$$

where a is radius of the pipe, and θ is the temperature of ascending hot water in the pipe. (4–2) is obtained by substituting D-z into z of (2–4). This is true because directions of z-axis in Chapters 2 and 4 are inverse and the distance between the origins of the two is D.

4.2. *Solution*

The model of Fig. 4–1 is not so complicated as the model in Chapter 3. Accordingly, it is not needful to provide two models as in Chapter 3, and from (4–1), (4–2), (4–4) and (4–5), T becomes

$$T = T_0 - \frac{\lambda T_0}{1+\lambda D} z + \frac{2}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} r\right) \cdot \sin \frac{\mu_s}{D} z \text{ [11]} \tag{4–6}$$

where

$$C_s = \frac{1}{1 + \dfrac{\cos^2 \mu_s}{\lambda D}} \tag{4–7}$$

$$\tan \mu_s = -\frac{\mu_s}{\lambda D} \tag{4–8}$$

and $K_0(\mu_s r/D)$ is the modified Bessel function of the 2nd kind of 0 order, A_s

is an unknown constant.

In the same way as in Chapters 2 and 3, the temperature of ascending hot water in the pipe is considered a function of z only, and the fall in the temperature is considered due to heat conduction into rock around the pipe. Then, the following differential equation is obtained as in Chapters 2 and 3.

$$q \rho c \frac{d\theta}{dz} \delta z = 2 \pi a \delta z k \left(\frac{\partial T}{\partial r} \right)_{r=a} \tag{4-9}$$

where notations of q, ρ, c and k are the same as those in Chapters 2 and 3.

From (4–6) and (4–9), one gets

$$\frac{d\theta}{dz} = - \frac{4 \pi a k}{q \rho c D} \sum_{s=1}^{\infty} C_s A_s \frac{\mu_s}{D} \cdot K_1 \left(\frac{\mu_s}{D} a \right) \cdot \sin \frac{\mu_s}{D} z \tag{4-10}$$

where $K_1(\mu_s a/D)$ is the modified Bessel function of the 2nd kind of the 1st order.

It follows from (4–10) that

$$\theta = \frac{L}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_1 \left(\frac{\mu_s}{D} a \right) \cdot \cos \frac{\mu_s}{D} z + const. \tag{4-11}$$

where

$$L = \frac{4 \pi a k}{q \rho c}$$

Boundary condition for θ is

$$\theta = T_0 \quad \text{at} \quad z = 0$$

Accordingly, *const.* in (4–11) is given by

$$const. = T_0 - \frac{L}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_1 \left(\frac{\mu_s}{D} a \right) \tag{4-12}$$

Substituting (4–12) into (4–11), one gets

$$\theta = T_0 - \frac{L}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_1 \left(\frac{\mu_s}{D} a \right) \cdot \left(1 - \cos \frac{\mu_s}{D} z \right) \tag{4-13}$$

And θ, obtained from (4–3) and (4–6), is

$$\theta = T_0 - \frac{\lambda T_0}{1 + \lambda D} z + \frac{2}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_0 \left(\frac{\mu_s}{D} a \right) \cdot \sin \frac{\mu_s}{D} z \tag{4-14}$$

From the two equations (4–13) and (4–14), A_s can be obtained by the

method used in Section 2.3 or 3.4 As in the previous chapter, let A_{sn} be A_s obtained by repeated substituting process of n times.

First, let (4–13) be considered approximately as the following equation.

$$\theta = T_0 \tag{4–15}$$

Using (4–15), (4–14) becomes

$$T_0 = T_0 - \frac{\lambda T_0}{1+\lambda D} z + \frac{2}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} a\right) \cdot \sin \frac{\mu_s}{D} z \tag{4–16}$$

Therefore

$$\frac{2}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} a\right) \cdot \sin \frac{\mu_s}{D} z = \frac{\lambda T_0}{1+\lambda D} z \tag{4–17}$$

If the relation of (4–8) is satisfied, the following definite integrals are obtained.

$$\int_0^D \sin \frac{\mu_s}{D} z \sin \frac{\mu_m}{D} z \, dz = \frac{D}{2} \cdot \frac{1}{C_s} \qquad \text{for} \quad m = s \tag{4–18}$$

$$= 0 \qquad \text{for} \quad m \neq s \tag{4–19}$$

$$\int_0^D z \sin \frac{\mu_s}{D} z \, dz = \left(\frac{D}{\mu_s}\right)^2 (\sin \mu_s - \mu_s \cos \mu_s) \tag{4–20}$$

where C_s is the constant given by (4–7).

Accordingly, multiplying both sides of (4–17) by $\sin \mu_s z/D$, then integrating each term from 0 to D, it follows from (4–18), (4–19) and (4–20) that

$$\frac{2}{D} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} a\right) \cdot \frac{D}{2} \frac{1}{C_s} = \frac{\lambda T_0}{1+\lambda D} \left(\frac{D}{\mu_s}\right)^2 (\sin \mu_s - \mu_s \cos \mu_s) \tag{4–21}$$

Therefore

$$A_s = A_{s1} = \frac{\lambda T_0}{1+\lambda D} \cdot \frac{1}{K_0\left(\frac{\mu_s}{D} a\right)} \cdot \left(\frac{D}{\mu_s}\right)^2 (\sin \mu_s - \mu_s \cos \mu_s) \tag{4–22}$$

By use of the result which is obtained by substituting (4–22) into (4–13), (4–14) becomes

$$T_0 - \frac{L}{D} \sum_{m=1}^{\infty} C_m A_{m1} \cdot K_1\left(\frac{\mu_m}{D} a\right) \cdot \left(1 - \cos \frac{\mu_m}{D} z\right)$$

$$= T_0 - \frac{\lambda T_0}{1+\lambda D} z + \frac{2}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} a\right) \cdot \sin \frac{\mu_s}{D} z \tag{4–23}$$

Therefore

$$\frac{2}{D} \sum_{s=1}^{\infty} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} a\right) \cdot \sin \frac{\mu_s}{D} z$$

$$= \frac{\lambda T_0}{1+\lambda D} z - \frac{L}{D} \sum_{m=1}^{\infty} C_m A_{m1} \cdot K_1\left(\frac{\mu_m}{D} a\right) \cdot \left(1 - \cos \frac{\mu_m}{D} z\right) \tag{4–24}$$

Because the 1st term of the right side in (4–24) is equal to the right side in (4–17), the following definite integral is obtained from (4–21):

$$\int_0^D \frac{\lambda T_0}{1+\lambda D} z \sin \frac{\mu_s}{D} z \, dz = A_{s1} \cdot K_0\left(\frac{\mu_s}{D} a\right) \tag{4–25}$$

Then, when (4–8) is satisfied, the following definite integral is obtained:

$$\int_0^D \left(1 - \cos \frac{\mu_m}{D} z\right) \sin \frac{\mu_s}{D} z \, dz$$

$$= D\left[\frac{1-\cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \left\{1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m\right)\right\}\right]$$

$$\text{for} \quad m \neq s \tag{4–26}$$

$$= D\left[\frac{1-\cos \mu_s}{\mu_s} - \frac{1 - \cos 2\mu_s}{4\mu_s}\right] \qquad \text{fos} \quad m = s \tag{4–27}$$

In the same way as the case of (4–17), by multiplication of both sides of (4–24) by $\sin \mu_s z/D$, then integrating each term from 0 to D, it follows from (4–18), (4–19), (4–25), (4–26) and (4–27) that

$$\frac{2}{D} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} a\right) \cdot \frac{D}{2} \cdot \frac{1}{C_s}$$

$$= A_{s1} \cdot K_0\left(\frac{\mu_s}{D} a\right) - \frac{L}{D} \sum_{m=1}^{\infty} C_m A_{m1} \cdot K_1\left(\frac{\mu_m}{D} a\right) \cdot D\left[\frac{1-\cos \mu_s}{\mu_s}\right.$$

$$\left. - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \left\{1 - \left(\cos \mu_s \cos \mu_m + \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m\right)\right\}\right] \tag{4–28}$$

Therefore

$$A_s = A_{s2} = A_{s1} - \frac{L}{K_0\left(\frac{\mu_s}{D}a\right)} \sum_{m=1}^{\infty} C_m A_{m1} \cdot K_1\left(\frac{\mu_m}{D}a\right)$$

$$\times\left[\frac{1-\cos\mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2-\mu_m^2}\left\{1-\left(\cos\mu_s\cos\mu_m + \frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right] \tag{4–29}$$

From (4–27), the value of formula in [ ] of (4–29) for $m=s$ becomes

$$\frac{1-\cos\mu_s}{\mu_s} - \frac{1-\cos 2\mu_s}{4\mu_s}$$

Now, let the 2nd term of the right side in (4–29) be $-A'_{s1}$. A_{s2} is expressed by

$$A_{s2} = A_{s1} + A'_{s1} \tag{4–30}$$

By use of (4–30), (4–13) becomes

$$\theta = T_0 - \frac{L}{D}\sum_{n=1}^{\infty} C_n (A_{n1} + A'_{n1}) \cdot K_1\left(\frac{\mu_n}{D}a\right)\cdot\left(1-\cos\frac{\mu_n}{D}z\right) \tag{4–31}$$

Again, substituting (4–31) into (4–14), one gets

$$T_0 - \frac{L}{D}\sum_{n=1}^{\infty} C_n (A_{n1} + A'_{n1}) \cdot K_1\left(\frac{\mu_n}{D}a\right)\cdot\left(1-\cos\frac{\mu_n}{D}z\right)$$

$$= T_0 - \frac{\lambda T_0}{1+\lambda D}z + \frac{2}{D}\sum_{s=1}^{\infty} C_s A_s \cdot K_0\left(\frac{\mu_s}{D}a\right)\cdot\sin\frac{\mu_s}{D}z \tag{4–32}$$

Therefore

$$\frac{2}{D}\sum_{s=1}^{\infty} C_s A_s \cdot K_0\left(\frac{\mu_s}{D}a\right)\cdot\sin\frac{\mu_s}{D}z$$

$$= \frac{\lambda T_0}{1+\lambda D}z - \frac{L}{D}\sum_{n=1}^{\infty} C_n A_{n1} \cdot K_1\left(\frac{\mu_n}{D}a\right)\cdot\left(1-\cos\frac{\mu_n}{D}z\right)$$

$$- \frac{L}{D}\sum_{n=1}^{\infty} C_n A'_{n1} \cdot K_1\left(\frac{\mu_n}{D}a\right)\cdot\left(1-\cos\frac{\mu_n}{D}z\right) \tag{4–33}$$

The sum of the 1st and 2nd terms of the right side in (4–33) is equal to the right side of (4–24). Accordingly, the following definite integral is obtained from (4–28).

$$\int_0^D \left\{ \frac{\lambda T_0}{1+\lambda D} z - \frac{L}{D} \sum_{n=1}^{\infty} C_n A_{n1} \cdot K_1\left(\frac{\mu_n}{D} a\right) \cdot \left(1 - \cos \frac{\mu_n}{D} z\right) \right\} \sin \frac{\mu_s}{D} z \, dz$$

$$= A_{s2} \cdot K_0\left(\frac{\mu_s}{D} a\right) \tag{4–34}$$

Again, let both sides of (4–33) be multiplied by $\sin \mu_s z/D$, then let each term from 0 to D be integrated. It follows from (4–18), (4–19), (4–26), (4–27) and (4–34) that

$$\frac{2}{D} C_s A_s \cdot K_0\left(\frac{\mu_s}{D} a\right) \cdot \frac{D}{2} \frac{1}{C_s}$$

$$= A_{s2} \cdot K_0\left(\frac{\mu_s}{D} a\right) - \frac{L}{D} \sum_{n=1}^{\infty} C_n A'_{n1} \cdot K_1\left(\frac{\mu_n}{D} a\right) \cdot D$$

$$\times \left[\frac{1 - \cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_n^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_n + \frac{\mu_n}{\mu_s} \sin \mu_s \sin \mu_n \right) \right\} \right] \tag{4–35}$$

Therefore

$$A_s = A_{s3} = A_{s2} - \frac{L}{K_0\left(\frac{\mu_s}{D} a\right)} \sum_{n=1}^{\infty} C_n A'_{n1} \cdot K_1\left(\frac{\mu_n}{D} a\right)$$

$$\times \left[\frac{1 - \cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_n^2} \left\{ 1 - \left(\cos \mu_s \cos \mu_n + \frac{\mu_n}{\mu_s} \sin \mu_s \sin \mu_n \right) \right\} \right] \tag{4–36}$$

From the result of numerical calculation, the 2nd term of the right side in (4–36) is very small in comparison with the 1st term.

Therefore

$$A_{s3} \doteqdot A_{s2} \tag{4–37}$$

Hence, A_{s2} may be used for A_s from (4–36) and (4–37). Thus, A_s becomes

$$A_s \doteqdot A_{s2} = \frac{T_0 D}{K_0\left(\frac{\mu_s}{D} a\right)} \cdot \frac{\lambda D}{1+\lambda D} \left[\frac{\sin \mu_s - \mu_s \cos \mu_s}{\mu_s^2} - L \sum_{m=1}^{\infty} C_m \frac{K_1\left(\frac{\mu_m}{D} a\right)}{K_0\left(\frac{\mu_m}{D} a\right)} \right.$$

$$\times \frac{\sin \mu_m - \mu_m \cos \mu_m}{\mu_m^2} \left\{ \frac{1 - \cos \mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2 - \mu_m^2} \right.$$

$$\left. \left. \times \left(1 - \cos \mu_s \cos \mu_m - \frac{\mu_m}{\mu_s} \sin \mu_s \sin \mu_m\right) \right\} \right] \tag{4–38}$$

Now, if one puts

$$B_s = \frac{\lambda D}{1+\lambda D}\left[\frac{\sin\mu_s-\mu_s\cos\mu_s}{\mu_s^2} - L\sum_{m=1}^{\infty} C_m \frac{K_1\left(\frac{\mu_m}{D}a\right)}{K_0\left(\frac{\mu_m}{D}a\right)}\frac{\sin\mu_m-\mu_m\cos\mu_m}{\mu_m^2}\right.$$

$$\left.\times\left\{\frac{1-\cos\mu_s}{\mu_s} - \frac{\mu_s}{\mu_s^2-\mu_m^2}\left(1-\cos\mu_s\cos\mu_m - \frac{\mu_m}{\mu_s}\sin\mu_s\sin\mu_m\right)\right\}\right] \tag{4–39}$$

(4–38) becomes

$$A_s = \frac{T_0 D}{K_0\left(\frac{\mu_s}{D}a\right)} B_s \tag{4–40}$$

By use of (4–40), (4–6) becomes

$$\frac{T}{T_0} = 1 - \frac{\lambda z}{1+\lambda D} + 2\sum_{s=1}^{\infty} C_s B_s \frac{K_0\left(\frac{\mu_s}{D}r\right)}{K_0\left(\frac{\mu_s}{D}a\right)}\sin\frac{\mu_s}{D}z \tag{4–41}$$

Let $T_{d=1}$ be the ground temperature at 1 m depth. From (4–41), $T_{d=1}$ is given by

$$\frac{T_{d=1}}{T_0} = \frac{1+\lambda}{1+\lambda D} + 2\sum_{s=1}^{\infty} C_s B_s \frac{K_0\left(\frac{\mu_s}{D}r\right)}{K_0\left(\frac{\mu_s}{D}a\right)}\sin\frac{\mu_s}{D}(D-1) \tag{4–42}$$

As remarked in Section 2.3, the 1st term of (4–42) is the ratio between the ground temperature at 1m depth and T_0 in the case that there is no pipe and the heat source spreads infinitely at depth D. The 2nd term of (4–42) is considered the influence of the ascending hot water upon the ground temperature near the orifice.

Now, let $T'_{d=1}$ be the ground temperature due to the ascending hot water only. Then, $T'_{d=1}/T_0$ is

$$\frac{T'_{d=1}}{T_0} = 2\sum_{s=1}^{\infty} C_s B_s \frac{K_0\left(\frac{\mu_s}{D}r\right)}{K_0\left(\frac{\mu_s}{D}a\right)}\sin\frac{\mu_s}{D}(D-1) \tag{4–43}$$

4.3. *Results of numerical calculation*

Figs. 4–2 and 4–3 indicate the relation between $T'_{d=1}/T_0$ and $r'(=r-a)$.

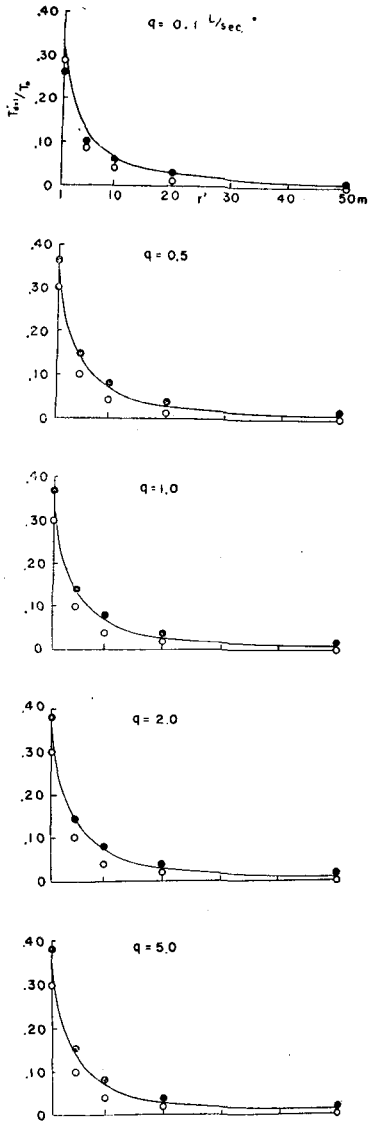

Fig. 4-2. Examples of $T_{d'=1}/T_0$ curves for r'.
○ ; D=50 m
——— ; =200
● ; =500

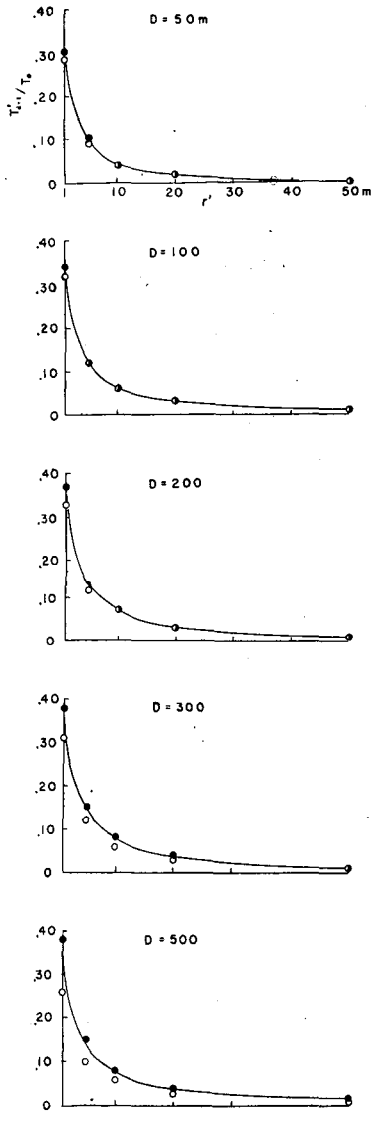

Fig. 4-3. Examples of $T_{d'=1}/T_0$ curves for r'.
○ ; q=0.1 L/sec
——— ; =1.0
● ; =5.0

In this case, Fig. 4–2 holds in the case of constant discharge rate, and Fig. 4–3 in the case of constant depth of the heat source. The constants used for the calculation are as follows:

$k = 1.7 \times 10^{-3}$ C.G.S. (Thermal conductivity of tuff)

$\lambda = 0.15\ \mathrm{m}^{-1}$

$a = 5$ cm

$\rho = 1$ g/c.c.

$c = 1$ cal/g·°C

$D = 50,\ 100,\ 200,\ 300,\ 500$ m

$q = 0.1,\ 0.5,\ 1.0,\ 2.0,\ 5.0$ L/sec

From Figs. 4–2 and 4–3, it becomes clear that the values of $T'_{d=1}/T_0$ at $r'=1$ m are smaller than 0.5 and the values of $T'_{d=1}/T_0$ at $r'=50$ m are included in range of $0\sim0.02$ for any discharge rate and depth of the heat source. In the case of $q\geq0.5$ L/sec, the deeper the heat source is, the larger the value of $T'_{d=1}/T_0$ at $r'\geq1$ m becomes for the same discharge rate and r'. But the values at the distance of more than a certain r' take an equal value regardless of depth of the heat source. When the depth of the heat source is constant, the greater the discharge rate is, the larger the value of $T'_{d=1}/T_0$ becomes for the same r'. But the values of $T'_{d=1}/T_0$ become equal for a discharge rate of more than q_0 which depends on the depth of the heat source as shown in Fig. 4–4.

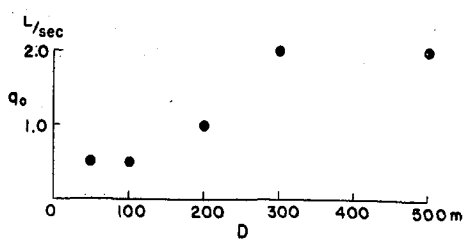

Fig. 4–4. Relation between q_0 and D.

The values of $T'_{d=1}/T_0$ at $r'=20$ m are included in a range of $0.01\sim0.04$ for various discharge rates and depths of the heat source. Hence, it may be considered that the ground temperature of 1 m depth at the distance of

r=20 m is hardly affected by the ascending hot water in the pipe, but the ground temperature nearer than r'=20 m must be affected by the influence of the ascending hot water.

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