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On the Distribution of Ground Temperature at 1m Depth Influenced by Various Heat Sources (Continued)

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Abstract

In the previous paper,[1] the writer sought the distribution of ground temperature at 1m depth influenced by ascending hot water in a fissure, which intersected at right or any angle with the ground surface. In this paper, discussions were made on the ground temperature in the presence of a hot water flow parallel to the ground surface.

5. Distribution of the ground temperature at 1m depth influenced by a hot water flow parallel to the ground surface when the ground surface temperature is held at zero

In this paper, the writer considered a case in which the heat source was a hot water flow parallel to the ground surface. Previously, as a model of hot water flow, YUHARA[2] selected a cylinder with constant temperature runing parallel to the ground surface. The writer employed an elliptic cylinder instead of a cylinder. The writer is of the opinion that an elliptic cylinder is closer to the natural state than the cylinder. The model adopted in this chapter was an elliptic clyinder with a constant temperature running parallel to the ground surface, while the ground surface temperature was held at zero.

5. 1. *Differential equation and boundary conditions*

When the temperature of the elliptic cylinder is constant, the ground temperature does not change in the direction of the axis of the elliptic cylinder. Accordingly the model adopted in this chapter may be considered as a two dimensional problem as shown in Fig. 5–1. Now, if T is the ground temperature at any point in the ground as in Fig. 5–1, and when T is considered as a steady temperature, the differential equation for T is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \qquad (5\text{–}1)$$

where x and y axes are selected as in Fig. 5–1.

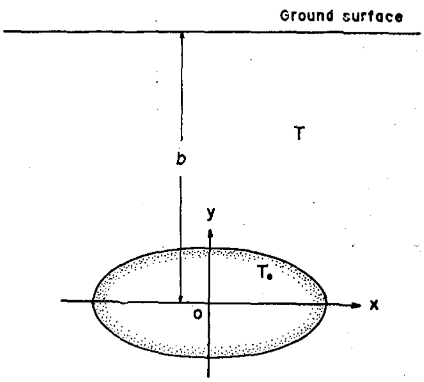


Fig. 5-1.

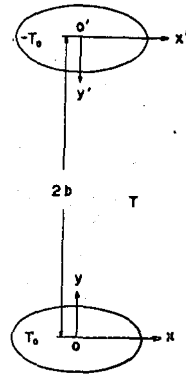


Fig. 5-2.

Fig. 5-1. A schematic map of the model.
Fig. 5-2. A schematic map of model I.

If b is the depth of the center of the ellipse in Fig. 5–1. The boundary conditions are given by

$$T=0 \qquad \text{at} \qquad y=b \tag{5–2}$$

$$T=T_0 \qquad \text{at} \qquad \frac{x^2}{x_0^{\,2}}+\frac{y^2}{y_0^{\,2}}=1 \tag{5–3}$$

where T_0 is the constant temperature of the hot water of the heat source; x_0 and y_0 are respectively major and minor axes of the ellipse.

5. 2. *Approximation of the model*

If the temperatures of two ellipses of model I in Fig. 5–2 are taken respectively as T_0 and $-T_0$,and the distance of the centers of the two ellipses is $2b$, model I satisfies the two boundary conditions (5—2) and (5–3). And if the ground temperature in model I is considered as a steady temperature, model I becomes the same as Fig. 5–1. Here the writer considered model I as the model of Fig. 5–1, and a discussion on model I was made as follows. If one takes the centers O and O' of the two ellipses as the origins of (x, y) and (x', y') coordinate systems as shown in Fig. 5–2, and models II and III may be selected as the following models. Namely, model II has two ellipses with the temperatures T_0 and zero placed at O and O' respectively; model III has two ellipses with the temperatures $-T_0$ and zero placed at O' and O resepctively as shown in Fig. 5–3. The positions of O and O' of the all models denoted in this chapter are

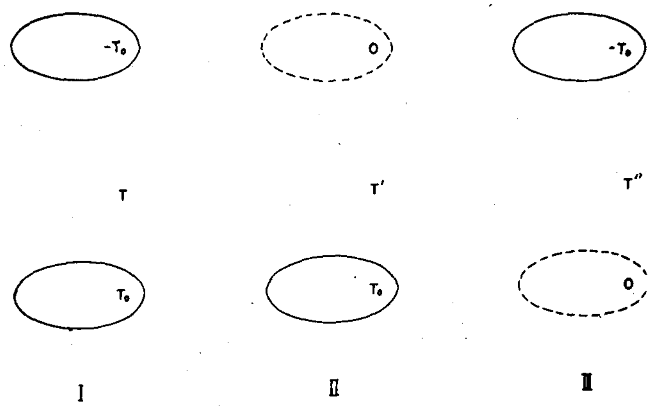


Fig. 5-3. Division of model I.

the same as shown in Fig. 5–2.

Now, if the ground temperatures for models II and III are respectively T' and T''. T may be expressed as

$$T = T' + T'' \tag{5-4}$$

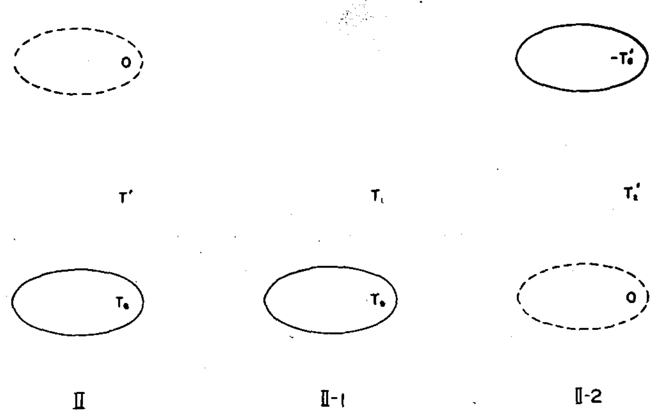
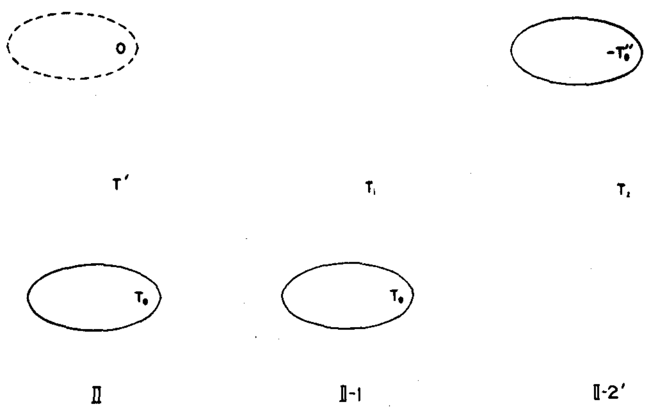


Fig. 5-4. Division of model II.

Furthermore, if models II–1 and II–2 are respectively defined as an ellipse with temperature T_0 placed at O, and two ellipses with temperatures $-T_0'$ and zero exist at O' and O as shown in Fig. 5–4. Let the ground temperatures for models II–1 and II–2 be T_1 and T_2' respectively. T' may be expressed as

$$T' = T_1 + T_2' \qquad (5\text{–}5)$$

The above temperature T_0' is the distribution of the ground temperature on the ellipse with the center O' influenced by the ellipse with the temperature T_0 in model II–1. When the depth of the ellipse is considerably large, the distribution of T_0' approaches a constant temperature. In this case, T_0'' which is the temperature at O' may be employed for T_0'. In model II–2, the distribution of the ground temperature on the ellipse with the center O is defined as zero. But, when b is considerably large, T_0' may be considered small, then the temperature at O influenced by the ellipse with the temperature $-T_0'$ which exists at O' may be considered near to zero. Therefore model II–2



Fig, 5-5. Approximation of model II.

approximats to modle II–2′ in Fig. 5–5; and if T_2 is the ground temperature for model II–2'. (5–5) may be expressed approximately as

$$T' = T_1 + T_2 \qquad (5\text{–}6)$$

In the same way, model III may be approximately divided into models III–1 and III–2 as shown in Fig. 5–6. Models III–1 and III–2 correspond to the cases where the temperatures of the ellipses in models II–2' and II–1 are $-T_0$ and T_0'' respectively. And T'' is given by

$$T'' = T_3 + T_4 \qquad (5\text{–}7)$$

where T_3 and T_4 are respectively the ground temperatures for models III–1 and III–2.

Accordingly, it follows from (5–4), (5–6) and (5–7) that

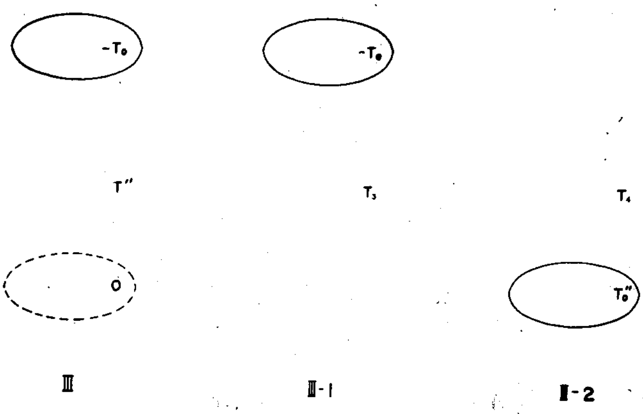


Fig. 5-6. Approximation of model III.

$$T = T_1 + T_2 + T_3 + T_4 \tag{5-8}$$

The region in which (5-8) can be used will be explained below in Section 5. 4.

5. 3. *Solution for the model of Fig. 5-7*

The solution for the model of Fig. 5-1 may be found by solving each of the above four models. Each ellipse of the heat source is placed in the external ground as shown in Figs. 5-5 and 5-6. Let Fig. 5-7 be the model in which ellipse with constant temperature is placed in the uniform ground. The above model are respectively the same as in Fig. 5-7. The solution for the model of Fig. 5-7 will be dealt with first.

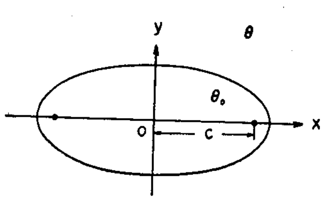


Fig. 5-7. A schematic map of the model.

Now, if x and y axes are selected as in Fig. 5-7, and if the ground temperature θ is considered as a steady temperature, the differential equation and boundary conditions for θ may be obtained by

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \tag{5-9}$$

$$\theta = \theta_0 \qquad \text{at} \qquad \frac{x^2}{x_0^2} + \frac{y^2}{y_0^2} = 1 \tag{5-10}$$

$$\theta = 0 \qquad \text{at} \qquad \frac{x^2}{x_0'^2} + \frac{y^2}{y_0'^2} = 1 \tag{5-11}$$

where θ_0 is the constant temperature of the ellipse, x_0 and y_0 are the same notations as in (5–3), and x_0' and y_0' are very large values compared with x_0 and y_0.

Next, considering the following conformal transformation

$$z = c \cosh w \tag{5-12}$$

$$\left.\begin{aligned} z &= x + i\,y \\ w &= u + i\,v \end{aligned}\right\} \tag{5-13}$$

where c is the focal distance of the ellipse. From (5–12) and (5–13)

$$x + i\,y = c\,(\cosh u \cos v + i \sinh u \sin v) \tag{5-14}$$

Hence x and y are expressed by

$$\left.\begin{aligned} x &= c \cosh u \cos v \\ y &= c \sinh u \sin v \end{aligned}\right\} \tag{5-15}$$

Therefore, from (5–15) one gets

$$\frac{x^2}{c^2 \cosh^2 u} + \frac{y^2}{c^2 \sinh^2 u} = 1 \tag{5-16}$$

(5–16) is an equation of an ellipse in which major and minor axes are respectively $c \cosh u$ and $c \sinh u$.

Putting

$$\left.\begin{aligned} x_0 &= c \cosh u_0 \\ y_0 &= c \sinh u_0 \end{aligned}\right\} \tag{5-17}$$

and

$$\left.\begin{aligned} x_0' &= c \cosh u_0' \\ y_0' &= c \sinh u_0' \end{aligned}\right\} \tag{5-18}$$

the boundary conditions (5–10) and (5–11) in the x-y plane are transformed to

$$\theta = \theta_0 \qquad \text{at} \qquad u = u_0 \tag{5–19}$$

$$\theta = 0 \qquad \text{at} \qquad u = u_0' \tag{5–20}$$

where u_0' is very large in comparison with u_0. And the differential equation (5–9) is transformed into the u-v plane as in the following equation:

$$\frac{\partial^2 \theta}{\partial u^2} + \frac{\partial^2 \theta}{\partial v^2} = 0 \tag{5–21}$$

where the regions of u and v are $u_0 \leq u \leq u_0'$ and $0 \leq v \leq \pi$ respectively.

In Fig. 5–7, since the ellipse of the heat source is symmetrical for x axis, a tangential line of an isothermal line at $y=0$ must intersect with x axis at right angle, and the positive and negative parts of x axis correspond to $v=0$ and π. Then, from the relation that the w and z planes are conformal to each other, in the u-v plane it is found that isothermal lines at $v=0$ and π are perpendicular to the u axis and the straight line $v=\pi$.

In a two dimensional plane, if an isothermal line is perpendicular to a given line, heat does not flow across the given line. Accordingly the model of Fig. 5–7 may be transformed to the model of the u–v plane as shown in Fig. 5–8. Namely, the model is such that the heat does not flow to the outsides of

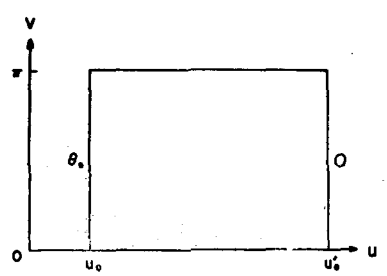


Fig. 5-8. Conformal transformation of the model.

the u axis and the straight line $v=\pi$, and from (5–19) and (5–20) the temperatures at $u=u_0$ and $u=u_0'$ are respectively θ_0 and zero. For the model of Fig. 5–8, it is such that the value of $\partial\theta/\partial v$ in the rectangle $u_0 \leq u \leq u_0'$, $0 < v < \pi$ is equal to zero, and (5–21) becomes

$$\frac{d^2 \theta}{d u^2} = 0 \tag{5–22}$$

From (5–19), (5–20) and (5–22), one gets

$$\theta = \frac{\theta_0}{u_0' - u_0} (u_0' - u) \tag{5-23}$$

where u is expressed from (5-16) by the following function of x and y:

$$u = \sinh^{-1} \left\{ \frac{x^2 + y^2 - c^2 + \sqrt{(x^2 + y^2 - c^2)^2 + 4c^2 y^2}}{2c^2} \right\}^{1/2} \tag{5-24}$$

5. 4. *Solution for the model of Fig. 5-1.*

By use of (5-23) and (5-24), one can immediately get the solutions for models II-1, II-2′, II-3 and II-4. The solutions for T_1 and T_3 are

$$T_1 = \frac{T_0}{u_0' - u_0} (u_0' - u) \tag{5-25}$$

$$T_3 = -\frac{T_0}{u_0' - u_0} (u_0' - u') \tag{5-26}$$

where the origins of u and u' coordinate systems correspond to O and O' in the x-y plane as in Fig. 5-2.

In model II-2′, let u_0'' be defined as the value of u' at which T_2 becomes zero. T_2 and T_4 may also be given by

$$T_2 = -\frac{T_0''}{u_0'' - u_0} (u_0'' - u') \tag{5-27}$$

$$T_4 = \frac{T_0''}{u_0'' - u_0} (u_0'' - u) \tag{5-28}$$

where T_0'' is the ground temperature at O' in model II-1; let u_1 be the value of u at O' in the same model, from (5-25) T_0'' is expressed by

$$T_0'' = \frac{T_0}{u_0' - u_0} (u_0' - u_1) \tag{5-29}$$

Substituting (5-29) into (5-27) and (5-28), T_2 and T_4 become

$$T_2 = -\frac{T_0}{u_0' - u_0} \cdot \frac{u_0' - u_1}{u_0'' - u_0} (u_0'' - u') \tag{5-30}$$

$$T_4 = \frac{T_0}{u_0' - u_0} \cdot \frac{u_0' - u_1}{u_0'' - u_0} (u_0'' - u) \tag{5-31}$$

However the distance between O and O' is $2b$ in the x-y plane, when x_0' and y_0' are taken much larger than b, it may be considered that the ellipse of the outer boundary condition in model II–1 corresponding to (5–11) agrees with that in model III–1. Accordingly the outer boundaries of model II–1 and III–1, inside which the ground temperatures are given by (5–25) and (5–26) respectively, are expressed by one ellipse, and (5–25) and (5–26) can be summed up in the region $u_0 \leq (u \text{ or } u') \leq u_0'$. Simmilarly the outer boundaries of models II–2' and III–2 are also given by one ellipse, (5–27) and (5–28) can be summed up in the region $u_0 \leq (u \text{ or } u') \leq u_0''$. And, since T_0 is larger than T_0'', u_0' becomes larger than u_0'', (5–30) and (5–31) can not use in the region $u_0'' < (u \text{ or } u') \leq u_0'$. Then (5–25), (5–26), (5–30) and (5–31) can be summed in the region $(u \text{ or } u') \leq u''$, and in thus regions of u and u', it follows from (5–25), (5–26), (5–30) and (5–31) that

$$T = \frac{T_0}{u_0' - u_0}\left(1 + \frac{u_0' - u_1}{u_0'' - u_0}\right)(u' - u) \tag{5–32}$$

where u is expressed by (5–24) and u' is given by

$$u' = \sinh^{-1}\left\{\frac{x'^2 + y'^2 - c^2 + \sqrt{(x'^2 + y'^2 - c^2)^2 + 4\,c^2 y'^2}}{2\,c^2}\right\}^{1/2} \tag{5–33}$$

Now, let the new coordinate axes be selected as Fig. 5–9 in which x-axis is taken along the ground surface and y-axis passes the center of the ellipse of the heat source. By comparing Fig. 5–2 with Fig. 5–9, it is found that the each distance between the origin of the new coordinates system and O or O' is b, all the x-axis of the each coordinates system are in the same direction, and the

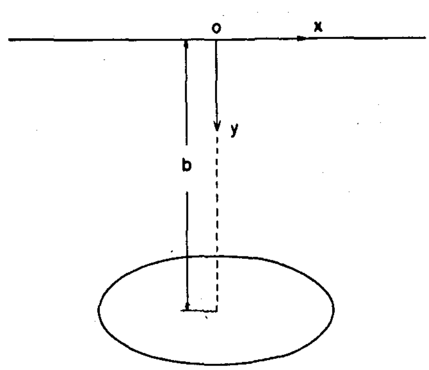


Fig. 5 9. New space coordinates under consideration.

directions of each y-axis are the same or opposite as shown in the figures. Accordingly, from (5–24) and (5–33) u and u' are expressed by the following equation of new coordinates x and y:

$$u = \sinh^{-1}\left\{\frac{x^2 + (b-y)^2 - c^2 + \sqrt{\{x^2+(b-y)^2-c^2\}^2+4\,c^2\,(b-y)^2}}{2\,c^2}\right\}^{1/2} \tag{5–34}$$

$$u' = \sinh^{-1}\left\{\frac{x^2 + (b+y)^2 - c^2 + \sqrt{\{x^2+(b+y)^2-c^2\}^2+4\,c^2(b+y)^2}}{2\,c^2}\right\}^{1/2} \tag{5–35}$$

Let $u_{y=1}$ and $u'_{y=1}$ be respectively the values of u and u' at $y=1\text{m}$. Then, from (5–32) the ground temperature $T_{y=1}$ at 1 m depth becomes

$$T_{y=1} = \frac{T_0}{u_0' - u_0}\left(1 + \frac{u_0' - u_1}{u_0'' - u_0}\right)(u'_{y=1} - u_{y=1}) \tag{5–36}$$

and from (5–34) and (5–35), $u_{y=1}$ and $u'_{y=1}$ are given by

$$u_{y=1} = \sinh^{-1}\left\{\frac{x^2+(b-1)^2-c^2+\sqrt{\{x^2+(b-1)^2-c^2\}^2+4c^2(b-1)^2}}{2\,c^2}\right\}^{1/2} \tag{5–37}$$

$$u'_{y=1} = \sinh^{-1}\left\{\frac{x^2+(b+1)^2-c^2+\sqrt{\{x^2+(b+1)^2-c^2\}^2+4\,c^2(b+1)^2}}{2\,c^2}\right\}^{1/2} \tag{5–38}$$

5. 5. *Results of numerical calculations*

By use of (5–36), the distribution of $T_{y=1}$ can be sought. But, since it is difficult to determine exactly the values of u_0' and u_0'', the writer calculated the value of $(u'_{y=1}-u_{y=1})$ in (5–36) and showed the results of numerical calculations in Fig. 5–10. The focal distance and depth of the ellipse used for the calculations are

$$c = 25,\ 50,\ 100,\ 200\,\text{m}$$

$$b = 50,\ 100,\ 200,\ 300,\ 500,\ 700,\ 1{,}000\,\text{m}$$

In order to clarify the relation between the curves $T_{y=1}$ and $(u'_{y=1}-u_{y=1})$, taking the logarithms of both sides of (5–36), one has

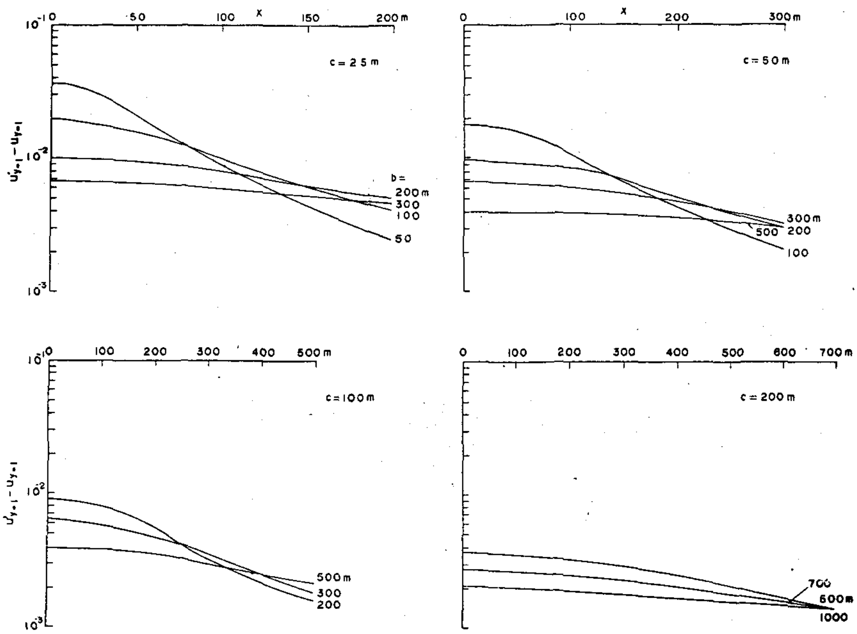


Fig. 5-10. Examples of $u_{y=1}'$-$u_{y=1}$ curves for x.

$$\log_{10}(T_{y=1}) = \log_{10}\left(\frac{T_0}{u_0'-u_0}\right)\left(1+\frac{u_0'-u_1}{u_0''-u_0}\right)+\log_{10}(u'_{y=1}-u_{y=1}) \qquad (5\text{-}39)$$

Hence it is found that the $\log_{10}(u'_{y=1}$ -$u_{y=1})$ and $\log_{10}(T_{y=1})$ curves drawn in the ordinary section paper must be parallel, and the interval of the two curves is $\log_{10}\{T_0/(u_0'\text{-}u_0)\}\{(1+(u_0'\text{-}u_1)/(u_0''\text{-}u_0)\}$. Then, if the curve of observed $\log_{10}(T_{y=1})$ is parallel to the $\log_{10}(u'_{y=1}$ -$u_{y=1})$ curve, b and c for the $\log_{10}(u'_{y=1}$ -$u_{y=1})$ curve are those of the ellipse of the heat source governing the above $\log_{10}(T_{y=1})$ curve. Thus, one can determine the depth and focal distance of the elliptic cylinder of the heat source.

6. Distribution of the ground temperature at 1m depth influenced by the hot water flow parallel to the ground surface when Newton's cooling is held at the ground surface

For the boundary condition at the ground surface, the case holding Newton's cooling is more common than the case described in Chapter 5. The following is a discussion in case Newton's cooling is held at the ground surface and the elliptic cylinder with constant temperature T_0 runs parallel to the ground surface as in Chapter 5.

6. 1. *Differential equation and boundary conditions*

When the elliptic cylinder of the heat source with the constant temperature runs parallel to the ground surface, the model is treated as a two dimensional problem as in Chapter 5. Then, if T is the ground temperature of any point, and if T is considered a steady temperature, the differential equation and boundary conditions for T are

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{6-1}$$

$$T = T_0 \qquad \text{at} \qquad \frac{x^2}{x_0^2} + \frac{(y-b)^2}{y_0^2} = 1 \tag{6-2}$$

$$\frac{\partial T}{\partial y} = hT \qquad \text{at} \qquad y = 0 \tag{6-3}$$

where x and y axes are selected as shown in Fig. 6-1; x_0, y_0 and b are the same notations as used in Chapter 5. And h is

$$h = \frac{h'}{k} \tag{6-4}$$

where h' is the constant of Newton's cooling, and k is the thermal conductivity of the ground.

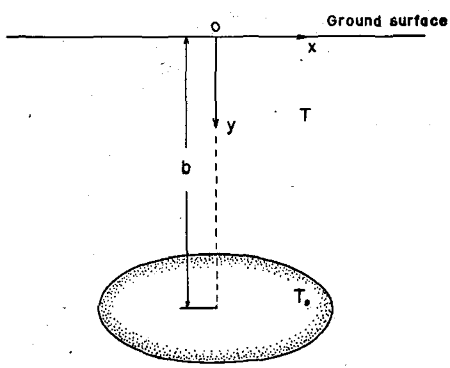


Fig. 6-1. A schematic map of the model.

In the following discussion, temperature zero is taken as the atmospheric temperature.

6. 2. *Approximation of the model*

Now, one calls the model of Fig. 6–1 as model I. And if T_1 and T_2' are respectively the ground temperatures for models II and III in Fig. 6–2, the ground temperature T for model I may be given by

$$T = T_1 + T_2' \tag{6–5}$$

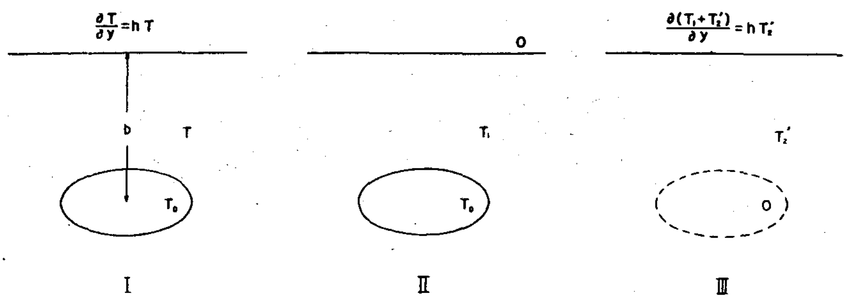


Fig. 6-2. Division of model I.

In the above models, model II is such that an ellipse with the temperature T_0 is placed at depth b and the temperature of the ground surface is held at zero; model III is such that the temperature of the ellipse at depth b is held at zero and the boundary condition at the ground surface is as follows:

$$\frac{\partial(T_1 + T_2')}{\partial y} = hT_2' \qquad \text{at} \qquad y = 0 \tag{6–6}$$

Model II is the same as Fig. 5–1 in Chapter 5, and it is difficult to solve model III exactly. Then the writer considered an approximate model for model III, and employed model IV in Fig. 6–3 as the approximate model. On the boundary condition of model IV, the writer considers only the ground surface, and the condition is

$$\frac{\partial(T_1 + T_2)}{\partial y} = hT_2 \qquad \text{at} \qquad y = 0 \tag{6–7}$$

where T_2 is the ground temperature for model IV.

Hence the difference between models III and IV is the condition at depth b. For modles III and IV; the temperatures of the ellipses at depth b are zero

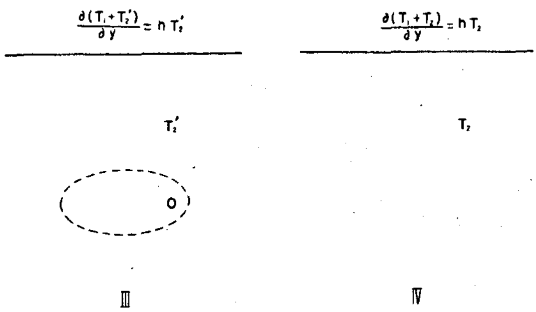


Fig. 6-3. Approximation of model III.

or not zero respectively. But, if the temperature of the ellipse in model IV at depth b is very small in comparison with T_0, model IV may be used for model III. This problem will be made clear below in Section 6. 4.

By the above approximation, T is given by

$$T = T_1 + T_2 \tag{6–8}$$

6. 3. *Solution*

Since the writer considers T and T_1 as steady temperatures, from (6–8) T_2 is also considered as a steady temperature. Then the differential equation for T_2 is

$$\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} = 0 \tag{6–9}$$

From (6–7) the boundary condition is

$$\frac{\partial (T_1 + T_2)}{\partial y} = h T_2 \qquad \text{at} \qquad y = 0 \tag{6–10}$$

In order to be satisfied (6–10), one expresses T_2 in the following definite integral as

$$T_2 = \int_0^\infty A\, e^{-\lambda y} \cos \lambda x \, d\lambda \tag{6–11}$$

where A is an unknown constant. And (6–11) clearly satisfies (6–9).

From (6–11)

$$\left(\frac{\partial T_2}{\partial y}\right)_{y=0} = -\lambda \int_0^\infty A \cos \lambda x \, d\lambda \tag{6–12}$$

$$(T_2)_{y=0} = \int_0^\infty A \cos \lambda x \, d\lambda \tag{6-13}$$

From (5–32)

$$\left(\frac{\partial T_1}{\partial y}\right)_{y=0}$$

$$= 2 s b \left[\frac{x^2+b^2+c^2+\sqrt{(x^2+b^2-c^2)^2+4 b^2 c^2}}{\{(x^2+b^2-c^2)^2+4 b^2 c^2\} \{x^2+b^2-c^2+\sqrt{(x^2+b^2-c^2)^2+4 b^2 c^2}\}} \right]^{1/2} \tag{6-14}$$

where

$$s = \frac{T_0}{u_0' - u_0} \left(1 + \frac{u_0' - u_1}{u_0'' - u_0} \right) \tag{6-15}$$

When c/b is less than $1/2$, $(c/b)^4$ is less than $1/16$. Then in the case $(c/b) \leqq 1/2$, by expanding (6–14) and neglecting terms over the order of $(c/b)^4$, (6–14) becomes approximatly

$$\left(\frac{\partial T_1}{\partial y}\right)_{y=0} \doteqdot \frac{2 s b}{(x^2+b^2-c^2)} \left\{ 1 + \frac{c^2}{2 (x^2+b^2-c^2)} + \frac{2 b^2 c^2}{(x^2+b^2-c^2)^2} \right\} \tag{6-16}$$

Substituting (6–12), (6–13) and (6–16) into (6–10)

$$\frac{2 s b}{(x^2+b^2-c^2)} \left\{ 1 + \frac{c^2}{2 (x^2+b^2-c^2)} + \frac{2 b^2 c^2}{(x^2+b^2-c^2)^2} \right\} - \int_0^\infty \lambda A \cos \lambda x \, d\lambda$$

$$= h \int_0^\infty A \cos \lambda x \, d\lambda \tag{6-17}$$

Therefore

$$\int_0^\infty A (\lambda + h) \cos \lambda x \, d\lambda = \frac{2 s b}{(x^2+b^2-c^2)} \left\{ 1 + \frac{c^2}{2 (x^2+b^2-c^2)} + \frac{2 b^2 c^2}{(x^2+b^2-c^2)^2} \right\} \tag{6-18}$$

Expressing the right side of (6–18) by Fourier's integral, one gets

$$\frac{2 s b}{(x^2+b^2-c^2)} \left\{ 1 + \frac{c^2}{2 (x^2+b^2-c^2)} + \frac{2 b^2 c^2}{(x^2+b^2-c^2)^2} \right\}$$

$$= \frac{2}{\pi} \int_0^\infty d\lambda \int_0^\infty \frac{2 s b}{(\gamma^2+b^2-c^2)} \left\{ 1 + \frac{c^2}{2 (\gamma^2+b^2-c^2)} \left(1 - \frac{4}{1 + \dfrac{\gamma^2 - c^2}{b^2}} \right) \right\}$$

$$\times \cos \lambda \gamma \cos \lambda x \, d\gamma \tag{6-19}$$

Since the region of γ is from 0 to ∞, and for γ which is much larger than b; the value of ( ) in { } of the right side in (6–19) is considered as 1. Thus, the writer made the following rough approximation:

$$1+\frac{c^2}{2(\gamma^2+b^2-c^2)}\left(1-\frac{4}{1+\dfrac{\gamma^2-c^2}{b^2}}\right)\fallingdotseq 1+\frac{c^2}{2(\gamma^2+b^2-c^2)} \tag{6–20}$$

By using (6–20), the definite integral for γ in (6–19) becomes approximately

$$\int_0^\infty \frac{2\,s\cdot b}{(\gamma^2+b^2-c^2)}\left\{1+\frac{c^2}{2(\gamma^2+b^2-c^2)}\left(1-\frac{4}{1+\dfrac{\gamma^2-c^2}{b^2}}\right)\right\}\cos\lambda\gamma\,d\gamma$$

$$\fallingdotseq \frac{\pi\,s\,b}{4\sqrt{b^2-c^2}}\left\{\frac{4(b^2-c^2)+c^2}{b^2-c^2}+\frac{c^2\lambda}{\sqrt{b^2-c^2}}\right\}e^{-\sqrt{b^2-c^2}\,\lambda} \tag{6–21}$$

Putting

$$\sqrt{b^2-c^2}=B \tag{6–22}$$

it follows from (6–18), (6–19) and (6–21) that

$$\int_0^\infty A\,(h+\lambda)\cos\lambda x\,d\lambda=\frac{s\,b}{2B}\int_0^\infty\left(\frac{4B^2+c^2}{B^2}+\frac{c^2\lambda}{B}\right)e^{-B\lambda}\cos\lambda x\,d\lambda \tag{6–23}$$

Accordingly the unknown constant A is obtained from (6–23):

$$A=\frac{s\,b}{2B}\left(\frac{4B^2+c^2}{B^2}+\frac{c^2\lambda}{B}\right)\frac{e^{-B\lambda}}{h+\lambda} \tag{6–24}$$

Substituting (6–24) into (6–11), T_2 becomes

$$T_2=\int_0^\infty \frac{s\,b}{2B}\left(\frac{4B^2+c^2}{B^2}+\frac{c^2\lambda}{B}\right)\frac{e^{-(B+y)\lambda}}{h+\lambda}\cos\lambda x\,d\lambda \tag{6–25}$$

Expanding $1/(h+\lambda)$, (6–25) becomes

$$T_2=\frac{s\,b}{2B}\int_0^\infty\left\{\left(\frac{4B^2+c^2}{B^2}\right)\frac{1}{h}+\left(\frac{4B^2+c^2}{B^2}-\frac{c^2h}{B}\right)\right.$$

$$\left.\times\left(-\frac{\lambda}{h^2}+\frac{\lambda^2}{h^3}-\cdots\cdots\right)\right\}e^{-(B+y)\lambda}\cos\lambda x\,d\lambda \tag{6–26}$$

Hence, by integrating each term in (6–26) from 0 to ∞, one gets

$$T_2 = \frac{s\,b}{2B}\Bigg[\left(\frac{4\,B^2+c^2}{B^2}\right)\frac{\cos\left(\tan^{-1}\dfrac{x}{B+y}\right)}{h\{(B+y)^2+x^2\}^{1/2}} + \left(\frac{4\,B^2+c^2}{B^2} - \frac{c^2\,h}{B}\right)$$

$$\times\Bigg[-\frac{\cos\left(2\tan^{-1}\dfrac{x}{B+y}\right)}{h^2\{(B+y)^2+x^2\}} + \frac{2!\cos\left(3\tan^{-1}\dfrac{x}{B+y}\right)}{h^3\{(B+y)^2+x^2\}^{3/2}} - \cdots\cdots\Bigg]\Bigg] \tag{6–27}$$

Then it follows from (5–32), (6–8), (6–15) and (6–27) that

$$T = s\,(u'-u) + \frac{s\,b}{2B}\Bigg[\left(\frac{4\,B^2+c^2}{B^2}\right)\frac{\cos\left(\tan^{-1}\dfrac{x}{B+y}\right)}{h\,\{(B+y)^2+x^2\}^{1/2}} + \left(\frac{4\,B^2+c^2}{B^2} - \frac{c^2\,h}{B}\right)$$

$$\times\Bigg[-\frac{\cos\left(2\tan^{-1}\dfrac{x}{B+y}\right)}{h^2\{(B+y)^2+x^2\}} + \frac{2!\cos\left(3\tan^{-1}\dfrac{x}{B+y}\right)}{h^3\{(B+y)^2+x^2\}^{3/2}} - \cdots\cdots\Bigg]\Bigg] \tag{6–28}$$

Putting

$$\left.\begin{aligned} \tan^{-1}\frac{x}{B+y} &= \theta \\ h\left\{(B+y)^2+x^2\right\}^{1/2} &= r \end{aligned}\right\} \tag{6–29}$$

(6–28) becomes

$$T = s\,(u'-u) + \frac{s\,b}{2B}\left\{\left(\frac{4\,B^2+c^2}{B^2}\right)\frac{\cos\theta}{r} + \left(\frac{4\,B^2+c^2}{B^2} - \frac{c^2\,h}{B}\right)\right.$$

$$\left.\times\left(-\frac{\cos 2\,\theta}{r^2} + \frac{2!\cos 3\,\theta}{r^3} - \cdots\cdots\right)\right\} \tag{6–30}$$

Now, let $T_{y=1}$, $u_{y=1}$, $u'_{y=1}$, $\theta_{y=1}$ and $r_{y=1}$ be respectively the ground temperature at 1 m depth, the values of u, u', θ and r at $y=1$ m. From (6–30) $T_{y=1}$ is given by

$$T_{y=1} = s\,(u'_{y=1} - u_{y=1}) + \frac{s\,b}{2B}\left\{\left(\frac{4\,B^2+c^2}{B^2}\right)\frac{\cos\theta_{y=1}}{r_{y=1}} + \left(\frac{4\,B^2+c^2}{B^2} - \frac{c^2\,h}{B}\right)\right.$$

$$\left.\times\left(-\frac{\cos 2\,\theta_{y=1}}{r^2_{\ y=1}} + \frac{2!\cos 3\,\theta_{y=1}}{r^3_{\ y=1}} - \cdots\cdots\right)\right\} \tag{6–31}$$

where $u_{y=1}$ and $u'_{y=1}$ are expressed by (5–37) and (5–38) respectively, $\theta_{y=1}$ and $r_{y=1}$ are given by

$$
\left.\begin{aligned}
\theta_{y=1} &= \tan^{-1}\frac{x}{B+1} \\
r_{y=1} &= h\left\{(B+1)^2 + x^2\right\}^{1/2}
\end{aligned}\right\} \qquad (6\text{–}32)
$$

6. 4. *Results of numerical calculations*

The value of $T_{y=1}/s$ is calculated from (6–31), and Fig. 6–4 indicates examples of numerical calculations. The constants used for the calculations are as follows:

$h = 0.15\,\mathrm{m}^{-1}$ [3)]

$c = 25,\ 50,\ 100,\ 200\,\mathrm{m}$

$b = 50,\ 100,\ 200,\ 500,\ 700,\ 1{,}000\,\mathrm{m}$

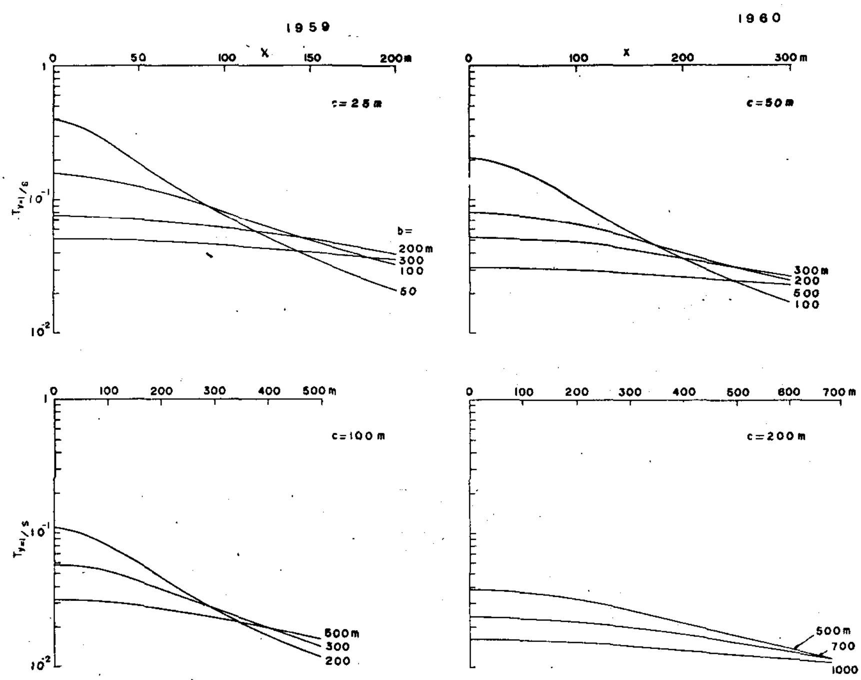


Fig. 6-4. Examples of $T_{y=1}/s$ curves for x.

In the same way as described in Section 5. 5, by comparing the curve of observed $T_{y=1}$ with the $T_{y=1}/s$ curve, the values of b and c of the heat source are sought.

Next, from (6–27) the writer sought a ratio between T_2 and T_0 at the point which corresponds to the center of the ellipse in model III, and ascertained the propriety of the above approximation using model IV for model III. For example, when b and c are 50 m and 25 m respectively, let $(T_2)_{x=0,\ y=50}$ be the value of T_2 obtained by substituting $x=0$ m and $y=50$m into (6–27). $(T_2)_{x=0,\ y=50}$ expresses the ground temperature at the point corresponding to the center of the ellipse in model III of which b and c are 50 m and 25 m respectively, and the value becomes

$$\frac{(T_2)_{x=0,\ y=50}}{T_0} = s \times 0.17$$

$$= \frac{1 + \dfrac{u_0' - u_1}{u''_0 - u_0}}{u_0' - u_0} \times 0.17 \tag{6–33}$$

When u_0' is very larger than u_0, $(T_2)_{x=0,\ y=50}/T_0$ takes a very small value compared with 1. Therefore $(T_2)_{x=0,\ y=50}$ is

$$(T_2)_{x=0,\ y=50} \leqslant T_0 \tag{6–34}$$

Accordingly, when b and c are 50 m and 25 m respectively, it is found that model IV may be used for model III.

In cases of other combinations of b and c in Fig. 6–4, from (6–27) it is also immediately calculated that the value of T_2 at the point corresponding to the center of the ellipse is very small in comparison with T_0.

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