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On the Change in the Shallow Ground Water Level

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Abstract

It has been considered that a base level governing a ground water depletion curve has remained always constant for a year, but the base level for shallow ground water in Sapporo, Hokkaidô shows a seasonal change. The maximum height of the base level appears in spring and the minimum height in autumn.

By using the values of this base level, the writer corrects an observed rise of the ground water level due to rainfall, and obtains a relation between the corrected rise and the rainfall. That relation is not perfectly linear, since the rise of the water level does not depend only upon the rainfall, but depends also on the intensity of the rainfall, and the time elapsed since the previous rain. But the relation is roughly linear as ,

$$\Delta h = 1.1\,R - 4$$

where Δh expresses the corrected rise and R is the rainfall; their units are respectively in cm and mm.

1. Introduction

Investigations on change in the ground water level have previously been made by some investigators — FUKUTOMI,[1] SENO[2] and KANEKO,[3] for example. The writer also made an observation well of about 4 m depth on the campus of Hokkaidô University, and observed the level of the shallow ground water in the well by a self-recorder during the period from June 3, 1957 to Dec. 26, 1960; he also observed the rainfall by recording rain gage for the periods from May 1, 1958 to Nov. 19, 1958 and from Apr. 27, 1959 to Nov. 23, 1959.

Fig. 1 shows the change in the ground water level expressed as the depth h from the ground surface as represented in Fig. 2. The water levels indicate fluctuations due to rainfall, but in general, during the observation period the water levels in March and April are relatively higher than those of other months; the water levels for the period from July to November show a low height. The maximum water level height is 124 cm on Apr. 7, 1958; this level is higher than the high water level due to the heavy rainfall of 107 mm on Sep. 18, 1957. The minimum is 230 cm, this water level appears several times in the autumns of 1958 and 1960.

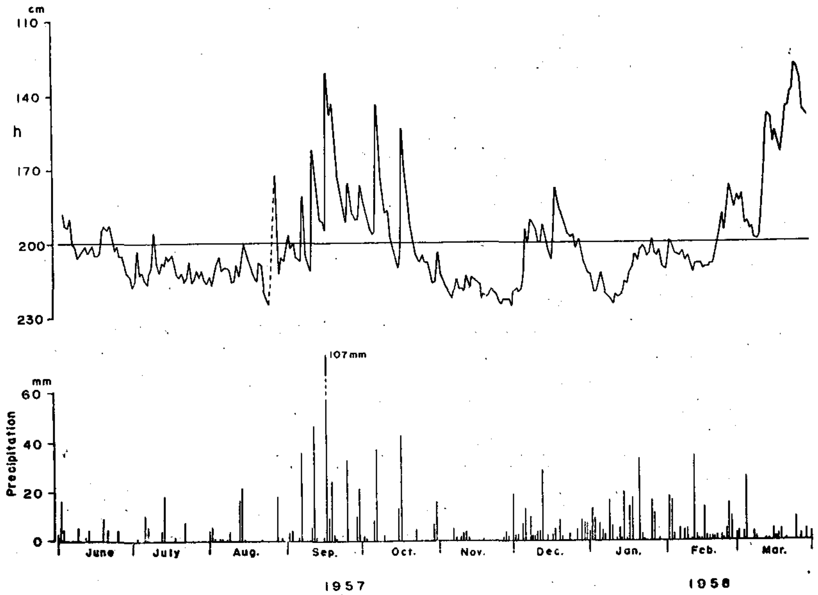
cm
110
140
h
170
200
230
107mm
mm
60
40
20
0
Precipitation
June
July
Aug.
Sep.
Oct.
Nov.
Dec.
Jan.
Feb.
Mar.
1957
1958

Fig. 1-1.

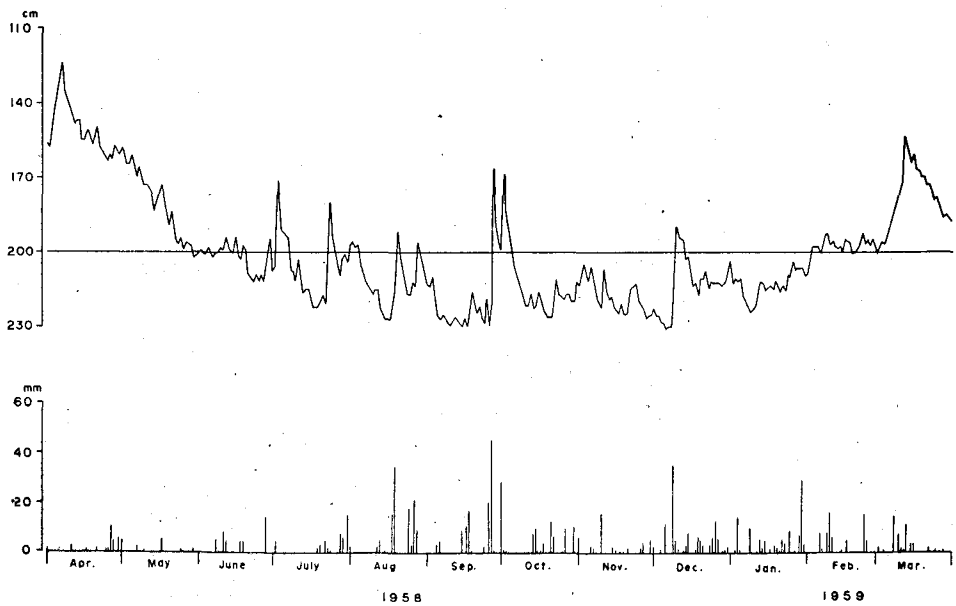
cm
110
140
170
200
230
mm
60
40
20
0
Apr.
May
June
July
Aug
Sep.
Oct.
Nov.
Dec.
Jan.
Feb.
Mar.
1958
1959

Fig. 1-2.

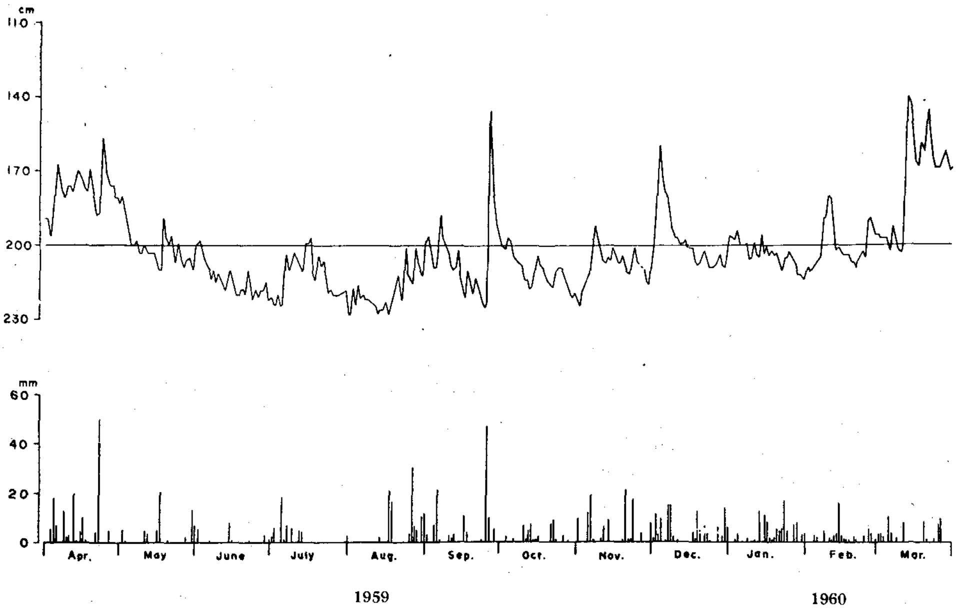

Fig. 1-3.

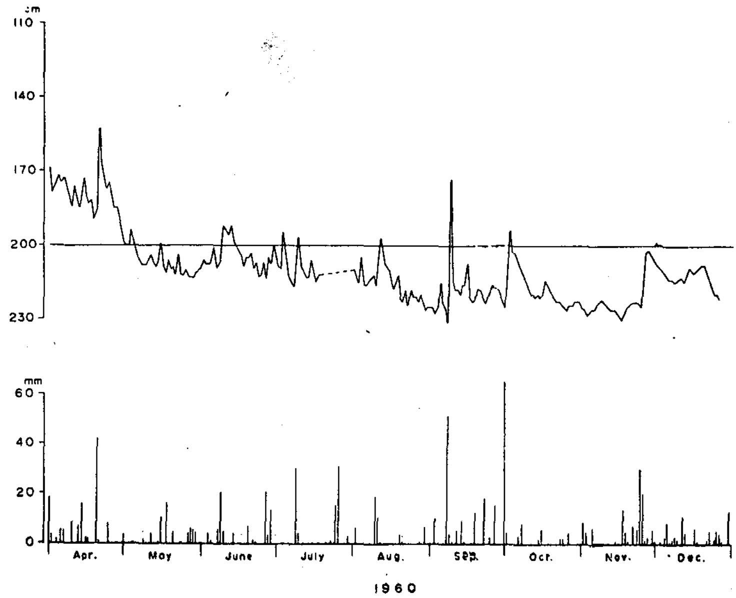

Fig. 1-4.

Fig. 1. Hydrograph of ground water leve in the observation well on the campus of Hokkaidô University in Sapporo, Japan.

From Fig. 1, the writer estimates the change of the base level for the observation period and a corrected rise of the water level due to rain.

In Fig. 1, for values of precipitation except in the periods during which the rainfall was observed by the recording rain gage, use is made of data in "The Weather of Hokkaidô".[4]

2. Change of the base level

2. 1. *Base level*

The ground water table rises due to rainfall or melting snow. When the recharge into the water table is equal to the effect which causes steady lowering of the water table, the ground water level reaches the maximum height and then the water level begins to lower. The manner of depletion of the water level is given by the following formula.[5]

$$\frac{dh}{dt} = \lambda (h_0 - h) \tag{2-1}$$

where t is time, λ is a constant governed by the characteristic of the soil, and h_0 is base level.

Fig. 2 indicates the relation of h and h_0. The latter is the imaginary level considered under the ground water level, and by the derivation of the base level, the ground water depletion curve can be explained very well. In general, it has been considered that h_0 is the constant level[6] for time. But, in Sapporo, it was found that h_0 changed seasonally as described below in Section 2. 4.

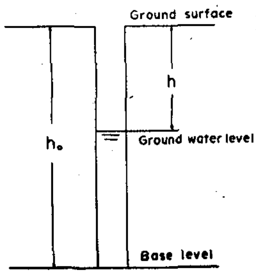

Fig. 2. A schematic map of a base level.

2. 2. *Determination of* λ

For a short period, since h_0 can be considered constant, it follows from equation (2-1) that

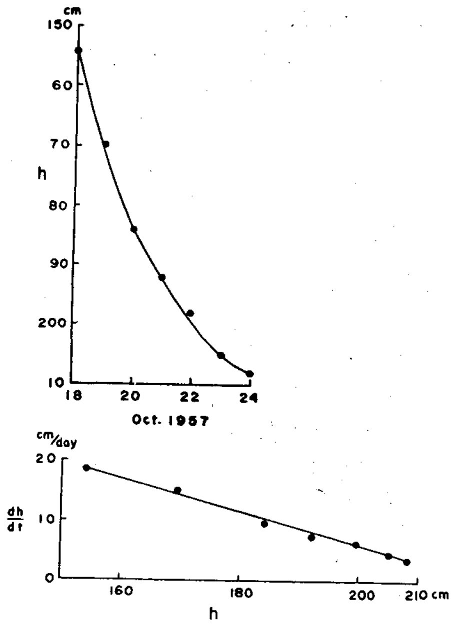

Fig. 3-1.

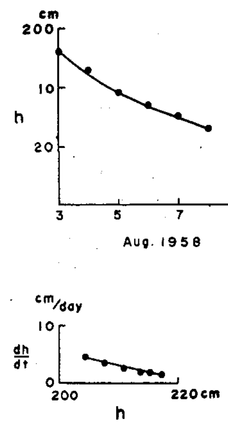

Fig. 3-2.

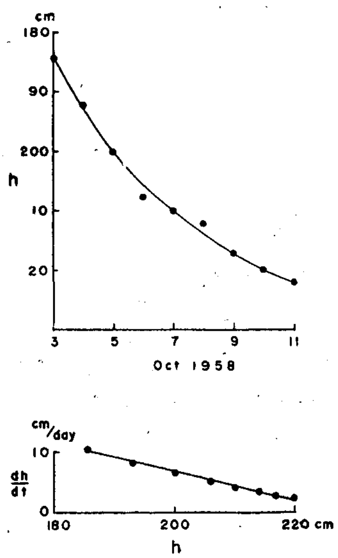

Fig. 3-3.

Fig. 3. Examples of ground water depletion curve and *dh/dt* curve for *h*.

$$\frac{dh}{dt} = -\lambda h + const. \qquad (2\text{–}2)$$

Accordingly, dh/dt is given by a linear equation of h, and the gradient of the straight line expressing equation (2–2) gives the value of $-\lambda$.

During four years observation of the ground water level, it was rare that a period between one rain and a next rain was long; the period was almost always $3\sim5$ days as in Fig. 5. Then the writer selected data from a relatively long period, and drew Fig. 3. These curves show examples of the ground water depletion curve and the relation between h and dh/dt which is derived from the depletion curve. From these curves, it is found that λ is equal to 0.25 1/day.

2. 3. Derivation of h_0 by using $\lambda=0.25$ (1/day)

Comparing with equations (2–1) and (2–2), it is clear that *const.* in equation (2–2) is equal to λh_0, and then the following relation is obtained.

$$\lambda h_0 = \frac{dh}{dt} + \lambda h \qquad (2\text{–}3)$$

From equation (2–3), the value of λh_0 is equal to the value of dh/dt at $h=0$. Then, dividing λh_0 obtained in this way, by λ, h_0 is sought. But, in the case of the short depletion curve, the values of dh/dt needed for drawing dh/dt curve as in Fig. 3 are few, and the dh/dt curve can not be derived from such short depletion curve. Thus, the above method to obtain h_0 needs a considerably long depletion curve, but only a few such curves were obtained during the observation period. Then, the writer used the following method in the effort to find h_0.

From equation (2–1)

$$\log(h_0 - h) = -\lambda t + const. \qquad (2\text{–}4)$$

Equation (2–4) is the relation between h and t in the ground water depletion curve. In Fig. 4, the chain line shows an example of the ground water depletion curve. Now, assuming an arbitrary value of h_0, h_0-h for each value of h can be calculated as indicated by open circles in Fig. 4. The curves of (1), (2) and (3) in this figure are h_0-h curves repersented by logarithmic scale when h_0=230 cm, 223 cm and 217 cm respectively. The curves (1) and (2) are regarded as straight lines, and the gradient of the straight lines of (1) and (2) are respectively 0.18 1/day and 0.25 1/day. The curve of (3) is

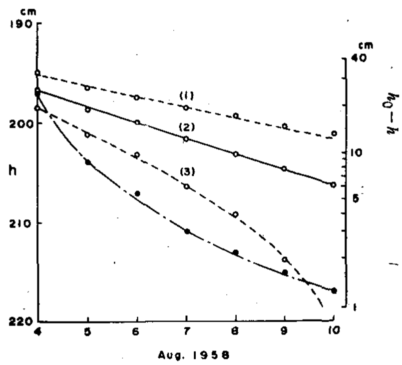

Fig. 4. Decreasing curves of h and $(h_0\text{-}h)$ for time.

not a straight line. Accordingly, for this depletion curve, it is suitable that h_0=223 cm at which the gradient of the h_0-h curve is equal to 0.25 1/day is employed.

By this method, the writer has sought the values of h_0 during the observation period. Fig. 5 shows examples of the employment of this method by which h_0 is obtained. All the gradients of the straight lines in this figure are equal to 0.25 1/day. Numerals on the axis of abscissa are the dates from which the water level begins to lower. For example, 6. 3 indicates that the water level begins to lower from June 3.

2. 4. *Change of the base level during the observation period*

The dashed lines in Fig. 6 show the change of the base level obtained from Fig. 5, while the solid lines show the ground water level.

From Fig. 6, it will be seen that the maximum height of the base level appears in the period from the latter part of March to the first part of April for every year. But, the maximum heights in each year are different; the base level in the first part of April, 1958 is the highest, reaching to about 155 cm. The minimum height of the base level appears in about the period between August and October, the minimum heights in each year being equal; they are about 230 cm, which agrees with the lowest ground water level for the same period.

It is considered that the fluctuation of the ground water level depends mainly on the rainfall, but the base level is mainly governed by the soil bearing the ground water. The ground water level rises temporarily due to a rainfall, after which during the period until the next rainfall the water level

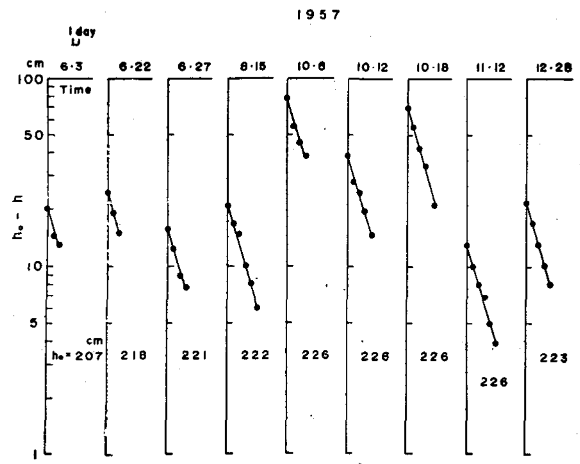
1957
1 day
cm
6·3
6·22
6·27
8·15
10·6
10·12
10·18
11·12
12·28
100
Time
50
$h_o - h$
10
5
1
cm
h_o = 207
218
221
222
226
226
226
226
223

Fig. 5-1.

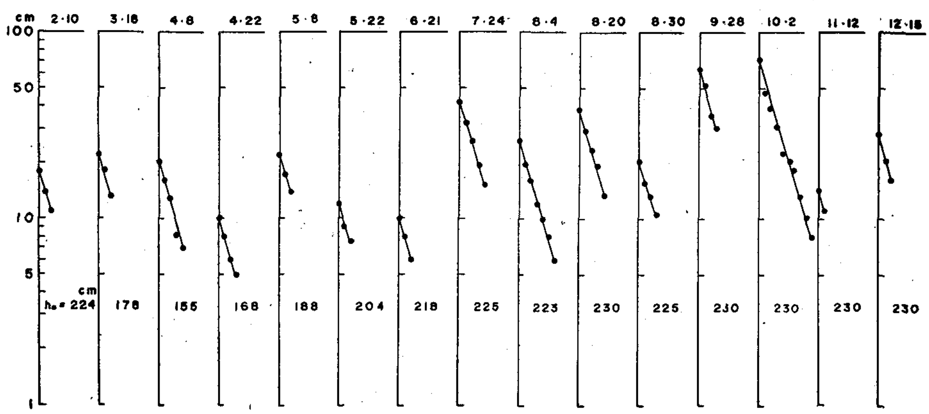
cm
2·10
3·18
4·8
4·22
5·8
5·22
6·21
7·24
8·4
8·20
8·30
9·28
10·2
11·12
12·15
100
50
10
5
1
cm
h_o = 224
178
155
168
188
204
218
225
223
230
225
230
230
230
230

Fig. 5-2.

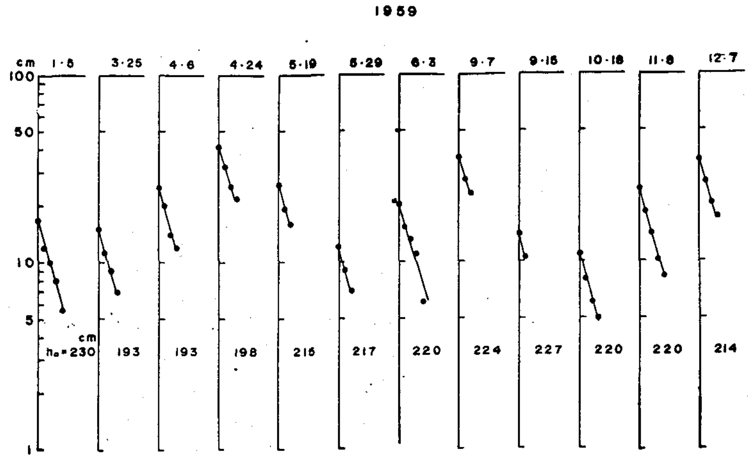

Fig. 5-3.

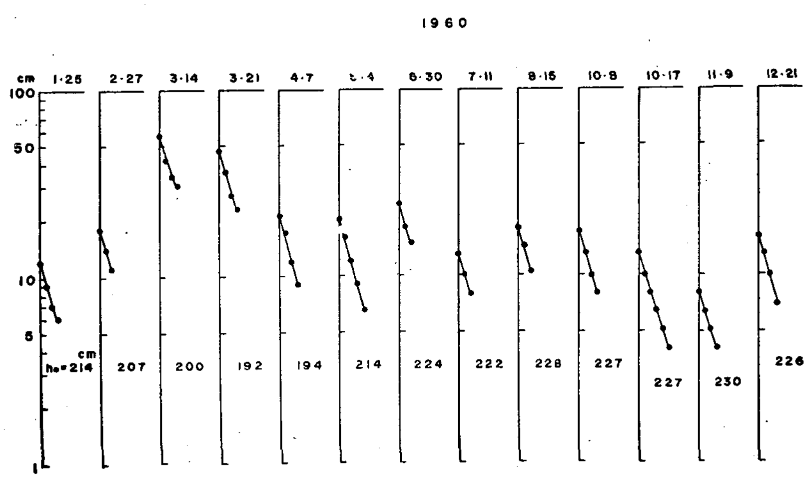

Fig. 5-4.

Fig. 5. $(h_0\text{-}h)$ curves for time.

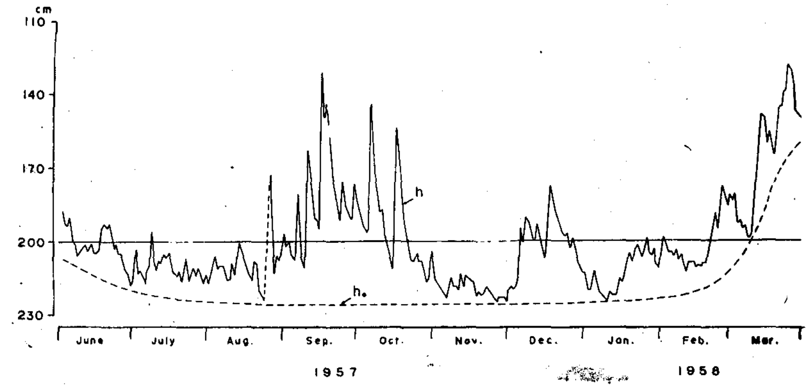

Fig. 6-1.

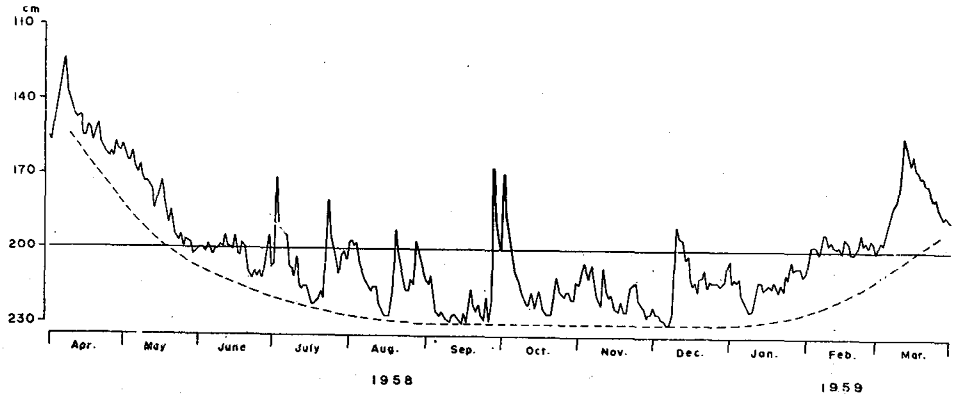

Fig. 6-2.

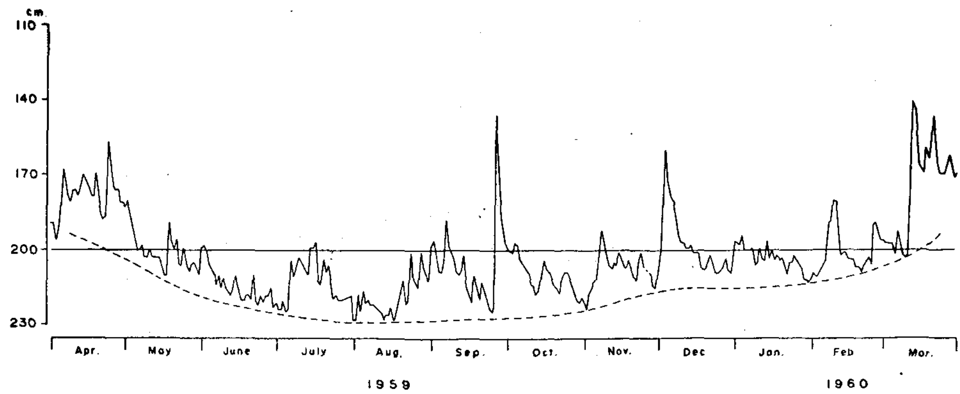

Fig. 6-3.

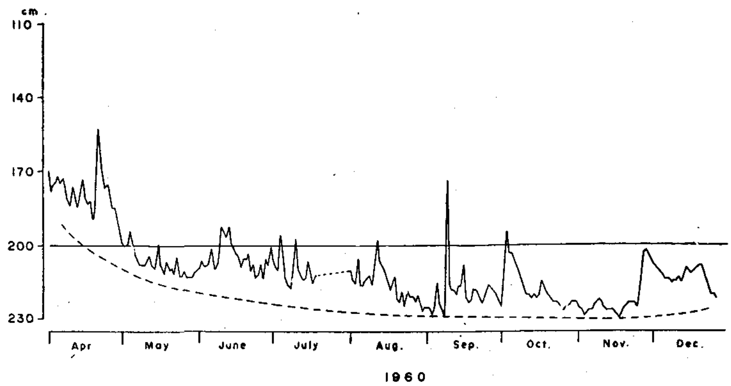

Fig. 6-4.

Fig. 6. Changes of h and h_0 during the period from June, 1957 to Dec., 1960.

continues to lower. From equation (2–1), when h becomes equal to h_0, the value of dh/dt becomes zero, and the depletion ceases. In the case where the solid lines do not agree with the dashed lines as Fig. 6, the water level must lower until it reaches the base leve expressed by the dashed line unless it rains.

In this way, the ground water level has always a tendensy to lower to the base level. Then, the writer considers that one employs the base level to which the water level lowers rather than the observed ground water level as the upper surface of the ground water reservoir. In accord with such consideration, it is found that upper surface of the ground water reservior during the observation period did not go lower than 230 cm.

3. Correction for a rise of the ground water level due to rainfall

The relation between the rise of the ground water level and rainfall is considered an important hydrologic property of the ground water. Then, in Chapters 3 and 4, the writer discusses the relation.

WISLER and BRATER[7] stated that, in the case of a relatively large rainfall or large rain intensity, the air in the soil spaces is entrapped temporarily because of the rapid downward movement of the sheet of water entering into the soil. Therefore, there is an excessive rise of the water level corresponding to the volume of the entrapped air. In such case, the ground water level rises suddenly and then shows a sudden depletion in the first part of the depletion curve. KANEKO[8] elucidated such sudden depletion with the view of that the depletion during the intial period occurs due to the discharge of the above entrapped air in addition to the depletion expressed by equation (2–1).

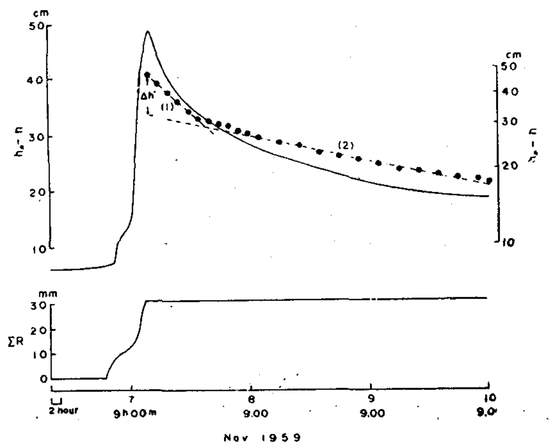

Fig. 7. Change of h_0-h and accumulated rainfall for Nov. 6～10, 1959.

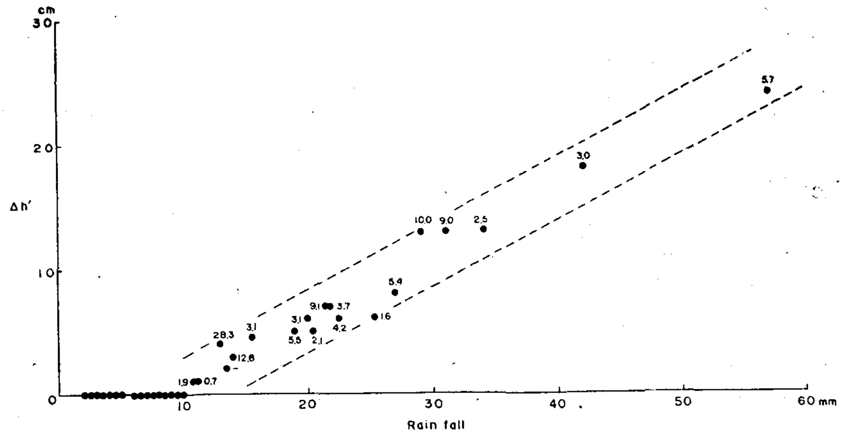

Fig. 8. Relation between $\Delta h'$ and rainfall.

Fig. 7 shows an example of graphs of accumulated rainfall and the ground water depletion curve in which solid line indicates the h_0-h curve expressed on the scale at the left side and solid circles express the values of h_0-h represented by the logarithmic scale of the right side. In this figure, the value of h_0 is 220 cm as in Fig. 5–3. The depletion curve plotted on the logarithmic scale can divides into two straight lines (1) and (2) as this figure. The value of λ for straight line (1) is larger than 0.25 1/day, and the part of (1) is the above sudden depletion. λ for (2) is equal to 0.25 1/day, and (2) is the part of the depletion curve expressed by equation (2–1) only. In the same figure, it is considered that the difference between the height shown by (1) and the height obtained by extending the straight line (2) into the initial time is caused by the entrapped air, and the decrease of the difference is due to the discharge of this air. The maximum difference $\Delta h'$ is considered to represent the rise by the air entrapped at the begining.

Fig. 8 shows the relation between $\Delta h'$ and rainfall, the numerals attached to the solid cricles are the intensity of each rainfall. When the rainfall is less than 10 mm, $\Delta h'$ is equal to 0 cm. And for the same rainfall, the larger the rain intensity is, the greater the value of $\Delta h'$ becomes as shown in this figure. It is seen that $\Delta h'$ is not determined only by the rainfall; it depends also on the rain intensity. But, the relation between $\Delta h'$ and rainfall may be considered roughly linear as shown in this figure.

Now, let h_1 and h_2 denote respectively the hieght of the ground water level from which the rise of the water level begins and that at which the depletion there of begins. h_1-h_2 has been employed as the rise of the water level due to rainfall. But, in general, the observed rise h_1-h_2 is considered the sum of the two rises due to an addition of water to the ground water table and the above described entrapped air. In case of comparing the rises of the water level due to various rainfalls, because the effect of the entrapped air is not constant as shown in Fig. 8, it is suitable to eliminate the rise corresponding to the volume of the entrapped air. In case of discussing the ground water contained in a given soil, the important quality is not the volume of entrapped air, but it is the volume of water. Then, to represent the rise due to rainfall, the writer employs the value Δh given by the following formula which eliminates the rise due to the entrapped air.

$$\Delta h = (h_1 - h_2) - \Delta h' \tag{2–5}$$

By employing Δh, the effect of the entrapped air is eliminated, and the rises for different rainfalls can be compared under the same process as the addition of the water to the water table raises the water level.

4. Relation between the rise of the ground water level and the rainfall

During the periods from May 1, 1958 to Nov. 19, 1958 and from Apr. 27, 1959 to Nov. 23, 1959, rainfall data were obtained by means of the self-recording rain gage. The writer estimated the rise of the ground water level given by

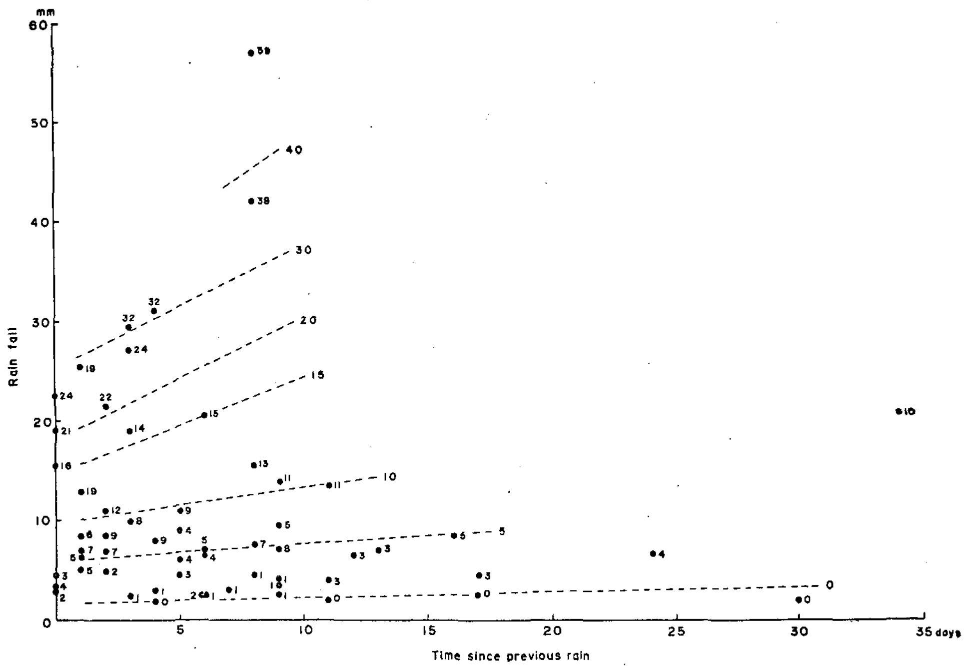

Fig. 9. Relation between Δh, rainfall and time since previous rain.

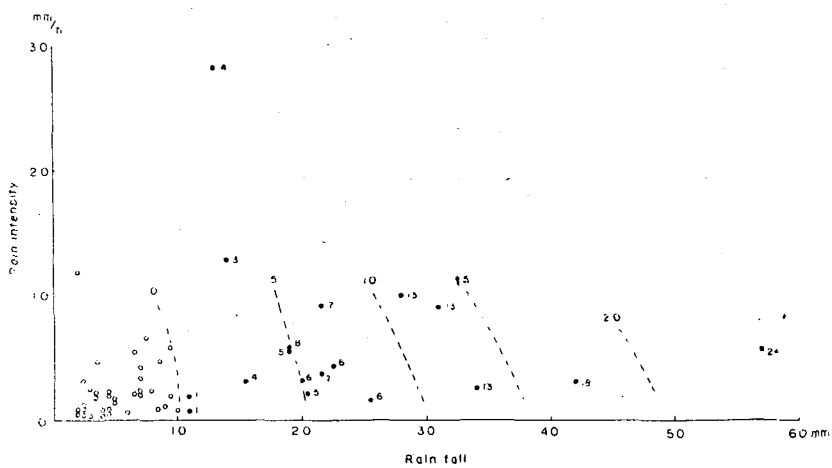

Fig. 10. Relation between Δh, rain intensity and rainfall.

equation (2–5) due to each rainfall for the same periods.

Fig. 9 shows the relation between Δh, rainfall and time since previous rain. Time of the abscissa in this figure may be seen as the degree of dryness in the soil of the upper side of the ground water table. The numerals attached to solid circles indicate the values of Δh. The dashed lines are the lines of equal values of Δh at intervals of 5 cm. The volume of water which is necessary to moisten the soil need to be greater in a case of the long time since previous rain than in a case of a short time. Accordingly, the shorter the time is, the larger the value of Δh becomes for the same rainfall, and the equal line inclines like the gradient of the line is positive as in Fig. 9.

Fig. 10 shows the relation between Δh, rain intensity and rainfall. In this figure, Δh of the open circles is equal to 0 cm, and the numerals attached to solid circles give the value of Δh. In a case of the small rain intensity, recharge to the water table is used rather to hold the water level than to raise the water lavel. Accordingly, for the same rainfall, the larger the rain intensity the greater the value of Δh becomes, and the dashed lines which are the equal lines of Δh at intervals of 5 cm incline like the gradient of their lines negative as in Fig. 10.

As indicated by the above facts, Δh is not a function of rainfall only, it also depends on the time elapsed since previous rain and the rain intensity. But, the relation between Δh and the rainfall is plotted as shown in Fig. 11.

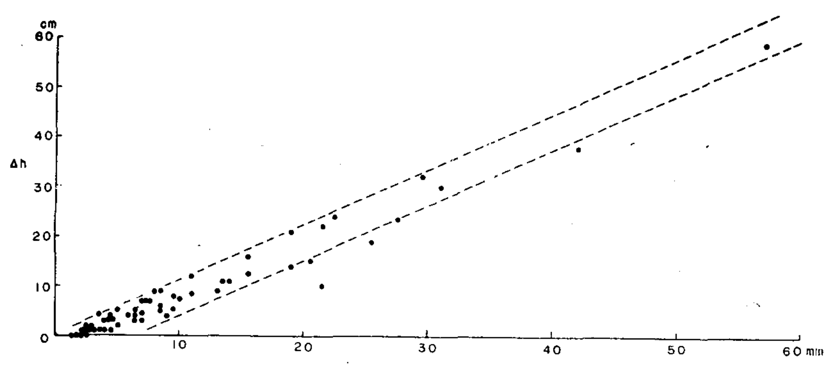

Fig. 11. Relation between Δh and rainfall.

The solid circles expressing the relation are plotted inside the two dashed straight lines in this figure, and is given roughly by the following linear equation for the rainfall.

$$\Delta h = 1.1 R - 4 \qquad (2\text{–}6)$$

where R represents the rainfall, and the units of Δh and R are respectively cm and mm. It may be considered that the solid circles plotted outside the two dashed straight lines fall there because the time since previous rain was very long, or the rain intensity was too small.

Acknowledgements. The writer wishs to express his thanks to Prof. T. FUKUTOMI for his kind advice and discussion concerning this study. Thanks are also due to Mr. K. TAKAHASHI for help in the observation of the ground water level.

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