Analyses of Seismic Waves Generated by Small Explosions

—Experiment at Wakino-machi, Niigata Prefecture—

(Continued)

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Abstract

Seismic waves generated by dynamite explosions were observed in this experiment. In the previous paper, the discussion was confined largely to considerations of body waves alone which were classified into seven wave groups.

In the present paper, surface waves which were classified into wave groups II, III and IV were mainly analyzed.

Dispersive characters of each wave group are shown in Fig. 21 as compared against theoretical dispersion curves.

Wave group IV was identified as $M_{II}$ waves and group III was identified as $M_{III}$ waves on the basis of theoretical dispersion curves. As to wave group II, it was shown that while group II could not be identified with any wave obtained by normal mode solutions, the waves could be identified with PL mode in known earthquakes.

It appears that wave group IV was generated mainly from the direct and/or reflection waves in the superificial layer while the wave group III was generated by the energy supply of the shear waves refracted at the boundary, and it may be said that wave group II was generated by the refraction and/or reflection waves of compressional or composite types which are presented in the previous paper.

"The quarter wave length law" advanced by K. Tazime was also examined for wave group IV.

6. Introduction

Analyses on seismic waves from explosions at Wakino-machi, Niigata Prefecture have been carried out as described in the previous paper, in which the discussion was confined largely to considerations of the propagation of $P$ and $S$ waves. Refraction and/or reflection waves composed $P$ and $S$ waves could clearly be observed especially when the shot-depth was increased.

At the site where the experiments were conducted, the $S$ wave velocity near the surface zone was slightly lower than that near the bottom of the superficial layer, however, the underground structure up to a depth of about 20 m was approximately considered as double layered. Seismic waves recorded in the case of 1 m-depth-shots could be classified into four wave groups
on the basis of periods and phase velocities of respective waves: 1) Waves of group I consisting of direct and refraction $P$ waves, 2) Waves of group II having a phase velocity between the $P$ wave velocity of the superficial layer and that of the substratum, 3) Waves of group III appearing near a point of 50 m distance from the shot hole and developing from that point with the phase velocity similar to the velocity of the $S$ waves in the substratum, and 4) Waves of group IV generated near the shot hole and propagated with the phase velocity of the $S$ waves in the superficial layer. The period of each wave could not be explicitly defined because each wave group except group I had dispersive properties which are peculiar to surface waves in the layered structure. However, roughly estimating the period of each wave group, the results are as seen in Table 1 in the previous paper.\(^1\)

Experiments at the site having such a layered structure have been carried out several times by members of the Seismic Exploration Group of Japan in order to study the surface waves generated by small explosions. The papers\(^4\),\(^5\),\(^6\),\(^7\) on this work have been published as shown in the references.

It has long been considered that such surface waves as that belonging to wave groups II, III and IV are dispersive Rayleigh waves corresponding to the normal mode solutions of the characteristic equation. These surface waves designated as group IV and group III have been identified as $M_{12}$ and $M_{13}$ waves respectively, and the waves of group II, directly following the refracted waves, as have been identified $M_{22}$ waves by K. Tazime\(^4\),\(^5\) and A. Kubotera.\(^6\),\(^7\)

It is noted that such an identification of the seismic waves observed in these experiments into each mode of the dispersive Rayleigh waves has largely been based on “the quarter wave length law” which has been described in detail in papers\(^4\),\(^5\) by K. Tazime. He has pointed out in these papers that each layered medium has its peculiar amplitude characteristics with respect to the periods of waves. As to dispersive Rayleigh waves, however, the amplitude characteristics have not been studied thoroughly as yet, while it has been theoretically presented for Love waves propagated in stratified two or three layers\(^8\),\(^9\),\(^10\) although only a few experimental data for Love wave propagation have been obtained. These amplitude characteristics will also become important in explaining the reason why a single side of the minimum group velocity curve can be found in the observation of earthquakes.

In the recent experiments, on the other hand, many wave groups were observed which could not be favorably explained by “the quarter wave length
law". Surface waves observed in the present experiment will offer an example which cannot be explained by this law.

The object of the present paper is to study these surface waves which remain uncertain in the previous paper and to attempt to explain them.

7. Dispersive character of waves

For each wave group II, III and IV at 65 m distance from the shot hole, the dispersive character was investigated as presented in this section.

Phase and group velocities were measured from records by the method described in SATO’s paper. The results are shown in Fig. 21. In order to

![Diagram](image)

Fig. 21. Observed phase and group velocities for surface waves compared with theoretical dispersion curves and period-amplitude relation for observed surface waves.
examine the identification of waves, the theoretical dispersion curves for the
the first three modes for the two models are superimposed on the figure. The
data used in calculations of theoretical curves are given in Table 5.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
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<tbody>
<tr>
<td>P wave vel.</td>
<td>300 m/s</td>
<td>300 m/s</td>
</tr>
<tr>
<td>S wave vel.</td>
<td>70 m/s</td>
<td>110 m/s</td>
</tr>
<tr>
<td>P wave vel.</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>S wave vel.</td>
<td>140</td>
<td>140</td>
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thickness of superficial layer: 12 m

As seen in Fig. 21, the superficial layer in Model 1 has the velocity of S waves smaller than that in Model 2. The former was more suitable than the latter.

Next, the variation of wave amplitude with period was obtained for each wave group, as shown in the lower side of Fig. 21 in which the amplitude was adjusted to the character of recording system and the dashed curve represents the envelope of its maximum. As may be seen in this figure the wave group with a longer period has a greater amplitude.

On the other hand, no waves with periods longer than 0.16 s were observed in this experiment. This may depend on the character of the recording system. That is to say, even when a wave with a period longer than 0.16 s was excited, it would not appear on the records. On the other hand, it may be that the waves having such a longer period were essentially not generated under the conditions of this site.

1) Wave group II

The waves designated as group II in Fig. 2 of the previous paper\(^{1}\) are well developed when the shot is fired at the approximate middle of the superficial layer, but they disappear when the shot point is near and/or below the bottom of that layer; the waves lose oscillatory motions but have pulsatory motions with increasing shot-depth. The maximum amplitude appears to shift continuously from earlier to later phases as the distance from the shot hole increases. This is not seen in the case of body waves.

The lines drawn on each equivalent phase of these waves intersect the ordinate in time-distance plots. Therefore, it cannot be considered that wave group II was propagated directly from the origin. The process of the
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construction of wave group II seems to be in such a manner that a wave having a longer period and larger velocity arrives earlier right after the direct or refraction $P$ waves. This group has the maximum amplitude after the first two or three cycles arriving and is followed by several cycles of wave trains with small amplitude. The velocity of each phase ranges from 300 m/s to 800 m/s, that is, it exceeds the velocity of the $S$ waves in the substratum.

In the range of the time when wave group II arrives at an observation point, several kinds of reflection and refraction waves are also expected if the source is near the surface. But both reflection and refraction waves could not be distinctly identified in wave group II; the envelope of wave group II exhibits a fine spindle-like form. If the source is below the boundary, wave group II changes the form of its envelope; it takes on a pulse-like wavelet form as the shot-depth increases.

On the other hand, the wave train of group II starts with a cycle having a velocity larger than that of $P$ waves in the superficial layer and is constructed with successive cycles having a velocity decreasing with time. Therefore, the refraction type as well as the reflection type of waves plays a role on the generation of wave group II.

The dispersive character of wave group II differs from the ordinary character as shown in Fig. 21. The period of each wave hardly varies with times, while the phase velocity shows a marked change. The range of variation of phase velocity is between the velocity of $P$ waves in the superficial layer and that in the substratum. This cannot be expected in normal mode propagation in layered medium, but may be explained by the complex roots in the characteristic equation for the waves. However, the complex roots are not readily found because of difficulties in the analyses of that equation. Wave group II may be taken as a $PL$ mode known from earthquake studies\(^2\),\(^3\) at least within the range of distance of this observation, even though rapid diminishing of the waves was not observed.

For the period-amplitude character, the maximum amplitude is attained when the period is about 0.06 s in which the phase velocity is a little larger than that of $P$ waves in the superficial layer.

2) Wave group III

Wave group III appears at a point of about 50 m distance from the shot hole and were well developed at a considerable distance from that point, while they fall off exponentially as the shot-depth increases. The dispersive character of the waves can be found in the records, although the wave shape is
a little distorted by wave group IV.

The phase and group velocities measured from records are plotted in Fig. 21 as a function of period. The type of dispersion is that seen most commonly in which the group velocity increases with the increasing period. In other words, the observed wave trains begin with the longest period although the step out is not so clear with successive cycles showing shorter periods as the time increases. The phase velocity also increases with the increasing period. The range of phase velocity exists between the velocity of $S$ waves in the superficial layer and that in the substratum. Since this group begins with the phase having the velocity of $S$ waves in the substratum, this group seems to receive the excitation energy mainly from the refraction type of $S$ waves. This will be perhaps recognized in Fig. 22. The origin of the generation of this
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The identification of wave group III with some modes of dispersive Rayleigh waves is also considered.

The periods of the observed waves are within the range which is expected by calculations for $M_{12}$ waves, but phase and group velocities are off the dispersion curves of $M_{12}$ waves. This deviation would be caused by the inhomogeneity of the medium. However, wave group III could not be identified with any other than $M_{12}$ waves, since a satisfactory identification of wave group IV with $M_{21}$ waves was obtained as presented later.

3) Wave group IV

Wave group IV which has greater amplitudes and longer periods appears near the shot hole. The larger the distance from the shot hole, the greater the number of cycles, while the amplitude decreases exponentially with increasing shot-depth. As shown in records, wave trains in this group are composed of two or three cycles with larger amplitudes within a range up to about 40 m from the source. Beyond the distance of about 45 m from the shot hole, wave group III was generated prior to wave group IV. The waves of group IV, however, are transmitted showing a predominance in amplitude and gradually increasing wave cycles. The maximum amplitude appears to shift continuously earlier to later cycles as the distance from the shot hole increases. This will be due to the dispersive character of the waves.

The line of onset of this wave group passes through the origin of time-distance plots as if the waves are generated directly from the source. The inverse slope of this line gives a value almost equal to the velocity of $S$ waves in the superficial layer.

In order to investigate the dispersive character further, phase and group velocities measured from the records were plotted as a function of period in Fig. 21. The dispersion thus obtained shows as a rare feature: The group velocity decreases with increasing period and the observed wave trains start with the shortest periods, successive phases showing longer periods as time increases on the record, while the phase velocity increases with the increase in period. Superimposed on the observed data are the pertinent portions of the theoretical curves. A good agreement of the plots of wave group IV with the
Theoretical curve of $M_{21}$ waves were found. The period that gives the maximum amplitude of waves is 0.13 s at which the phase and group velocities are about 90 m/s and 50 m/s respectively. As seen in Fig. 21, the amplitude maximum in this group is greater than that of the other wave group.

Waves somewhat similar to those have been reported by M. B. Dobrin et al. However, in this stage, the correspondence of the waves reported in their paper with those observed here could not be discussed.

In most previous observations of surface waves from small explosions, waves apparently similar to those of group IV have been identified as $M_{11}$ waves mainly on the basis of "the quarter wave length law" by K. Tazime et al. Though the dispersive property of the waves has not been thoroughly analysed, he emphasized that this law will be satisfied by the waves corresponding to the Airy phases.

For $M_{21}$ waves, two Airy phases must exist in general. In this experiment, one phase is expected at the period of about 0.14 s and the other is expected at about 0.4 s. Wave group IV seems to correspond to the phase having the shorter period. The maximum amplitude of these waves was obtained near the period which gives the first Airy phase of $M_{21}$ waves as expected from the law.

For the application of "the quarter wave length law" to these waves, since they are identified as waves corresponding to the first minimum of group velocity of $M_{21}$ waves, it would be more reasonable to make use of the following equation:

$$H = \frac{3}{4} V_{p1} T$$

where $H$ is the thickness of the superficial layer, $V_{p1}$ the velocity of $P$ waves in that layer and $T$ the period corresponding to the first minimum of group velocity.

If the value of $V_{p1}$ and $T$ are taken as 300 m/s and 0.14 s respectively, we have $H=32$ m. This exceeds the value ($H=12$ m) obtained already in the previous paper. Such a large difference may be caused by the Poisson's ratio greater than 0.25 in the case of which that law will be most useful.

K. Tazime has pointed out recently that the velocity of $S$ waves must often be used rather than that of $P$ waves in the superficial layer for this law in the case when Poisson's ratio is near 0.5. Based on this fact, wave group
IV may be reexamined by replacing the velocity of $P$ waves with that of $S$ waves. Hence,

$$H = \frac{3}{4} v_{s1} T = 7.4 \text{ (m)}$$

This value is smaller than the expected value. But the difference between the value obtained in such a way against the value obtained by the method of refraction survey will essentially be due to the elastic constants in media. For instance, the smaller the rigidity ratio in media, the harder the law holds. In any event, this law seems to hold better when the velocity of $S$ waves is employed rather than when that of $P$ waves is used, when Poisson's ratio is near 0.5.

"The quarter wave length law" stated in the papers\(^4\),\(^5\) has been considered on each mode of the dispersive Rayleigh waves existing both theoretically and experimentally in the medium, therefore, the coefficient \(= (2n+1)/4\) in which \(n\) means the order number of modes) used in this law are generally not defined without the identification of waves obtained in experiments with those expected in the theory. In applying this law to the observed waves, the dispersive character must be first analyzed.

On the other hand, the envelope of the maximum amplitude of each wave group seems to show a period property peculiar to the layered medium, in which the property of the source must also be contained. Namely, the waves must necessarily pass the layered medium under the control of the period character of the medium. The maximum of this envelope seems to be expected at periods longer than 0.13 s at which the wave amplitude of group IV shows maximum, but it has not been definitely determined.

It is noted that, in this experiment, the thickness of the superficial layer was well estimated by making use of the following relation:

$$H = \frac{1}{4} v_{s1} T_e$$

where \(T_e\) is the period at which the envelope would become maximum. Namely, if \(T_e\) is taken at about 0.14 s, we have

$$H = \frac{1}{4} \times 300 \times 0.14$$

$$= 10.5 \text{ (m)}$$
8. Conclusion

Seismic waves generated by small explosions have been investigated in the previous and the present papers. Some identifications of the observed waves with the waves expected from calculations have been obtained. The study on wave groups may become important in their application to seismic prospecting and also in the investigation of seismic wave propagation.

The application of "the quarter wave length law" to surface waves will be developed more practically by experimental modification. As an example, the envelope of the maximum amplitudes of wave groups was examined here. Namely, it was considered as a period amplitude character peculiar to the layered medium. In this experiment, the possibility of its application was ascertained although only a few analyses were made.

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References

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