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<td>OKADA, Hiroshi</td>
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Analyses of Seismic Waves Generated by Small Explosions

Experiment at Ima-machi, Niigata Prefecture —

Hiroshi Okada

(Received Sep. 30, 1964)

Abstract

Seismic waves generated by small explosions were studied at distances from 4 m to about 100 m from the source. The near-surface zone in the present experimental site was characterized by clay and watery sand which were stratified approximately by three layers.

In the present experiment, four types of waves, each of which was characterized by different periods and phase velocities, were observed. They were designated respectively as wave group I, wave group II, wave group III and wave group IV in the order of their arrival.

Wave group I at a distance far from the source was observed to be composed of several kinds of refracted waves.

Wave group II was unfortunately not clarified in the present analysis. However, it appears to have taken the form of reflection waves, in view of the fact that its phase velocity was higher than that of P waves in the substratum.

Wave groups III and IV are considered to be surface waves. In the present paper they do not coincide with the expected type of waves as set forth in the theory of wave propagation in layered media. The results of spectral analyses performed on these wave groups show that wave group III was sensitive to the variation of underground structure, while wave group IV was not.

1. Introduction

Seismic waves generated by explosions are considered as one of the most useful means for conducting experimental studies on the nature of seismic waves generated by earthquakes and for extending the utilization of seismic prospecting. In practice, experiments by explosion are most favorable and essential to the study of physical properties of waves, inasmuch as theories and model experiments mainly deal with idealized conditions.

Numerous experiments\(^1\)-\(^{14}\) on surface waves aside from body waves have been carried out during the past 20 or 30 years. In the majority of these experiments, the main objective has been in the identification of observed surface waves with waves expected in theory.

The body waves from explosions have been studied for the purpose of
elucidation of the mechanism of wave generation\textsuperscript{15,16} and the attenuation of waves\textsuperscript{16–21} by the properties of viscosity and/or absorption in medium.

In the present experiment, an investigation was made on the effect of underground structure and medium on the propagation of seismic waves generally observed in field experiments.

Seismic waves analysed in the present paper were obtained in autumn 1962 in the rice-fields in Ima-machi, Mitsuke, Niigata Prefecture, in a series of experiments provided by the Seismic Exploration Group of Japan. At the site chosen for the experiments, the subsurface structure was mainly composed of clay and watery sand.

2. Experimental procedures

In the present experiment, the vertical component of ground motion was observed by 12 geophones having a natural frequency of 7.5 cps disposed every 2 m along a straight line. The geophones were connected with the refraction seismic amplifiers (Electro-Technical Labs' PRA2–12 type). The recording instrument used was an oscillograph (E.T.L. ER64 type) with galvanometers having a natural frequency of 125 cps.

The frequency-response curve of the recording system, including the geophone, the amplifier and the galvanometer, is shown in Fig. 1.

![Fig. 1. Frequency-response curve for overall system, including geophone, amplifier and galvanometer.](image)

Wave forms obtained by this recording system rather indicate the quantity proportional to the velocity of motion than the displacement.

The three components of ground motion were also observed at the end of every spread which was the same point as that where the 12th geophone (7.5 cps) was placed. The natural frequency of these geophones was 27 cps.
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Fig. 2. Seismic records by 1 m-depth-shots when waves propagate from south to north of the span.

Fig. 3. Seismic records by 1 m-depth-shots when waves propagate from north to south of the span.

Fig. 4. Disposition of geophones and shot holes.

Shot holes were placed at 5 m (A hole), 27 m (B hole), 49 m (C hole) and 71 m (D hole) distance from the nearest geophone. The dynamite explosions at a depth of 1 m in each hole were observed, with the disposition of geophones unchanged. Seismic records thus obtained are shown in Fig. 2. They are reproduced in such a way as to show the ground motion resulting from one
shot.

Similarly, placing four shot holes (A', B', C' and D' hole), in an inverse direction to the above, with an interval of 22 m from the nearest geophone, seismic records which are shown in Fig. 3 were obtained.

In the present experiment, a series of shots were also made at depths of 3 m to 40 m at A, B, C and D holes. The disposition of geophones and shot holes is shown in Fig. 4.

The ends of the span, for the convenience of later description, will be referred to as S and N respectively with S indicating the south and N the north end of the span.

3. Classification of observed waves

Fig. 5(a) and (b) show time-distance plots of individual peaks of remarkable waves of all records exhibited in Figs. 2 and 3.

![Fig. 5. Time-distance plots of every peak.](image)
As may be seen in these figures and records, several wave groups which have different periods and velocities were observed. These records are qualitatively quite similar to those observed in the previous experiment.\(^{22}\)

DOBRIN, \textit{et al}\(^{23}\) observed similar waves as seen in our observations, while the distance range adopted by DOBRIN was about 900 m longer than that in the present experiment.

KELLER\(^{6}\) also studied similar waves, but the entire records were not exhibited in his paper. Time distance curves for all principal seismic waves presented in his paper resemble time-distance plots shown in Fig. 5 in the present paper.

Recently, an investigation of surface waves generated by explosions was made by KUZ'MINA\(^{12}\). In her paper likewise, similar waves were shown.

On the basis of the difference of periods and velocities, the observed waves were classified into four wave groups designated respectively as wave group I, wave group II, wave group III and wave group IV. Notations II, III and IV in the records and time-distance plots indicate the respective wave group.

Wave group I appears to be mainly composed of a body wave such as that refracted from interfaces under the ground. But the composition of each wave cannot be revealed by Figs. 2 and 5 alone. The first arrival of wave group I was hardly observed at points a small distance from the source.

In the present paper, wave groups I, II, III and IV will be mainly analysed.

![Fig. 6. Seismic records at 49 to 71 m distance from shot hole.](image)
The records obtained by the hole shot method are shown in Figs. 6 and 7 where the distance range in Fig. 6 is between 49 m and 71 m and that in Fig. 7 between 71 m and 93 m from the shot hole.

In these figures, the records at shot depths up to 10 m seem to have similar wave patterns to each other in spite of the difference of shot depth. Namely, wave group I was always predominant though it was slightly disturbed, and wave group II, III and IV are also observable though their amplitudes gradually decrease with the increasing of shot depths. The distorted waves between wave group II and wave group III were recorded as in the case of a shot depth of 1 m. The amplitudes showed no influence by shot depths.

These distorted waves could not be analysed in detail, but they are probably composed of direct waves and/or several types of refracted waves.

When a shot is fired at, or under a depth of 15 m, the appearance of records abruptly changes: i.e. the records become very simple, since wave groups II, III and IV were hardly observed while only wave group I remains unchanged, though its amplitude diminished gradually with the increase of shot depth. The disturbed waves following wave group II almost vanished.

From these findings, it is assumed that the medium changed at a depth of about 15 m.

The phase velocity of each wave showed negligible changes from that seen
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at 1 m shot depth by the increase of shot depths. The period of each wave group showed little or no changes by shot depth.

The relation between the arrival time of each wave group and the shot depth is as follows: Wave group I appeared earlier than that expected from the result of the previous experiment(22), when the shot depth was increased. The same thing can be seen in the distorted waves. On the other hand, the time of appearance of wave groups II, III and IV showed little or no changes by the increase of shot depth.

4. Subsurface structure at experimental site

In order to observe the subsurface structure at the present experimental site, refraction shootings were carried out, resulting in Fig. 8 which shows three stratified layers with a partially sloped interface.

The thickness of the first layer seems to be almost uniform, being some 4 to 5 m, though the velocities of P and S waves gradually varied with increase of depth through the layer. Taking mean values, the velocities of P and S waves were estimated respectively as 400 m/s and 60 m/s.

As to the second layer, the thickness was obtained from refraction shootings as well as from hole shots. The hole shot method is reliable in the determination of the subsurface structure as described in the previous paper(22).

The first arrival time obtained by hole shot at a distance of 39 m, 61 m and 83 m, was plotted in Fig. 9 with respect to the shot depth. The first arrival time employed here was obtained from P waves refracted at the bottom of the second layer. Therefore, the inflection points shown in each curve can only correspond to the interfaces of the media. The curve obtained at a distance of
39 m gives the underground structure near shot hole B, the curve of 61 m gives the underground structure near shot hole C and the curve of 83 m gives the underground structure near shot hole D.

In every curve, the first inflection point was obtained near a depth of 4 m corresponding to the bottom of the first layer. The second inflection point was obtained near a depth of 16 m at shot hole B and C and near a depth of 25 m at shot hole D. Thus, the thickness of the second layer in the south region may be estimated approximately as 12 m.

On the north side of the span, hole shots were not carried out. However, according to experiments of SH wave propagation and refraction shootings, it will be suitable to consider that the thickness of the second layer was about 8 m.

As to the segment of the entire span in which geophones were disposed, the profile of underground structure was determined more minutely. The interface between the second layer and substratum presented in Fig. 8 by a solid line shows a downward grade from north to south. Of course, in the figure, small irregularities were neglected. In order to determine the approximate dip of the interface concerned and the velocity of each layer, the relation between the first arrival time and distance for various shot depths was plotted as shown in Figs. 10(a) and (b).

According to Fig. 10(a), the velocity of 700 m/s was invariably observed at shot depths up to 10 m. When the shot depth was increased further, the
Fig. 10. Time-distance plots of first arrivals for various shot depths. (a): shot hole A. (b): shot hole B.

The time-distance curve gradually took on a hyperbola form with the minimum at a distance of 13 m. The same can be seen in Fig. 10(b). In this figure, the velocity of 1400 m/s was observable when the shot depth was shallower than 15 m.

From these facts and calculations of travel times for various shots, it is considered that the interface between the second layer and substratum is sloping down to the south side at a 10 degree angle. The velocities of the second layer and the substratum were assumed respectively to be about 700 m/s and 1300 m/s.

The velocity and Poisson's ratio of each layer are presented in Table 1, in which the velocity of S wave was obtained at the north segment of the span by the refraction method similar to that described in a previous paper [23]. Accordingly, the velocity of S wave in Table 1 was assumed to be identical to that of the SH wave.
Table 1

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<th>velocity (m/s)</th>
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<td>$P$ wave</td>
<td>$S$ wave</td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>400</td>
<td>60</td>
<td>0.49</td>
</tr>
<tr>
<td>(2)</td>
<td>700</td>
<td>100</td>
<td>0.49</td>
</tr>
<tr>
<td>(3)</td>
<td>1300</td>
<td>220</td>
<td>0.48</td>
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5. Variation of waves with the increase of shot depths

The variation of the maximum amplitude for each wave group with the increase of shot depth was investigated. This variation pattern was valid in the study as to how the explosive energy contributed to the generation of each wave group and as to how the underground structure was formed.

In the previous paper\(^2\), it was shown that the variation pattern of body waves reflected the subsurface structure to a high degree. As to the surface waves, the variation of amplitude indicated that the amplitude of wave group II once became maximal in the superficial layer, and thereafter exponentially decreased with the shot depth, and the amplitude of wave group III and wave group IV diminished exponentially with the shot depth regardless of the depth of interface between the superficial layer and the substratum.

As result, it was clearly demonstrated why the explosive energy had such a minor influence on the generation of these surface wave groups if the shot depth was increased.

In the present experiment, refracted waves were rarely identified on the records which were composed of various $P$ and $S$ waves. In Figs. 6 and 7, the distortion of wave form can be seen in part of wave group I, when the shot was fired at a depth lower than 10 m. This was due to the mixing of several types of refracted waves. Especially in Fig. 6, it is easily seen that the distortion was caused by two different wavelets following the first arrival. Time-distance plots of peaks for these wavelets exhibit a velocity nearly equal to that of the $P$ waves in the third layer. The time interval between the peaks of the first and the second arrival as well as that between peaks of the second and third arrival were invariant even though the shot depths were changed.

From these facts, a wavelet directly following the first arrival is considered to be such a refracted wave as converted from $P$ to $S$ wave in refracting up from the second layer to the superficial layer. If so, the time difference between the first and the following arrival should give the thickness of the superficial layer,
since the velocities of $P$ and $S$ waves in the superficial layer are already known.

If the paths concerned with these arrivals are assumed to be nearly equal to each other, the thickness of the superficial layer $H_1$ can be roughly calculated by the following equation:

$$H_1 = \Delta t_1 / \left( \frac{\cos \hat{S}_1P_3}{v_{s1}} - \frac{\cos \hat{P}_1P_3}{v_{p1}} \right). \quad (1)$$

In Eq. (1), $\Delta t_1$ is the arrival time difference and $\hat{S}_1P_3$ and $\hat{P}_1P_3$ are given by the equations,

$$\hat{S}_1P_3 = \sin^{-1} (v_{s1}/v_{p3}) \quad \text{and} \quad \hat{P}_1P_3 = \sin^{-1} (v_{p1}/v_{p3}) \quad (2)$$
in which $v_{s1}$ and $v_{p1}$ are respectively the velocities of $S$ and $P$ waves in the superficial layer and $v_{p3}$ is the velocity of the $P$ wave in the third layer.

In the present experiment, though each break of the first and the second arrival could not be identified, the thickness of the superficial layer $H_1$ was obtained from Eqs. (1) and (2), making use of the time difference $\Delta t_1 = 6.8 \times 10^{-2}$s of the wave peaks concerned, which may be expressed as follows:

$$H_1 = 4.0 \text{ m} \quad (3)$$

This coincides with the thickness shown in Fig. 8.

If the third wavelet shown in wave group I is due to such a refracted wave as converted to $S$ wave in the second layer in refracting up from the third layer and also transmitted as an $S$ wave in the superficial layer, the thickness of the second layer $H_2$ may be obtained also by using the following equation:

$$H_2 = \Delta t_2 / \left( \frac{\cos \hat{S}_2P_3}{v_{s2}} - \frac{\cos \hat{P}_2P_3}{v_{p2}} \right) \quad (4)$$
in which the paths concerned with these distortions are assumed to be nearly equal to each other.

In Eq. (4), $\Delta t_2$ is the arrival time difference between the second and the third wavelet following the first arrival, and $\hat{S}_2P_3$ and $\hat{P}_2P_3$ are respectively given by following equations:

$$\hat{S}_2P_3 = \sin^{-1} (v_{s2}/v_{p2}) \quad \text{and} \quad \hat{P}_2P_3 = \sin^{-1} (v_{p2}/v_{p3}) \quad (5)$$
in which $v_{s2}$ and $v_{p2}$ are the velocities of $S$ and $P$ waves in the second layer.

If $\Delta t_2$ is taken as the time difference of peaks of these wavlets, $\Delta t_2$ being $6.5 \times 10^{-2}$s in the records; the thickness of the second layer $H_2$ is obtained from Eqs. (4) and (5) as follows:

$$H_2 = 7.4 \text{ m} \quad (6)$$
This is smaller than the thickness of the second layer obtained in Fig. 8.

On the other hand, the third wavelet is again, another possibility, which may be considered to be due to such a refracted wave as converted to $S$ wave in the second layer in refracting up from the third layer and converted again to $P$ wave in the first layer at the interface. If so, the thickness of the second layer $H_2$ may be obtained approximately by using the following equation:

$$H_2 = \frac{\Delta t_2 - H_1 \left( \frac{(\cos S_1 P_3)/v_{s1} - (\cos S_2 P_3)/v_{s1}}{(\cos S_2 P_3)/v_{s2} - (\cos S_2 P_2)/v_{p2}} \right)}{(\cos S_2 P_3)/v_{s2} - (\cos S_2 P_2)/v_{p2}}$$  \hspace{1cm} (7)

in which $\Delta t_2$ is the arrival time difference between the second and the third wavelet following the first arrival. In Eq. (7), the paths concerned with these wavelets are assumed to be nearly equal to each other.

Thus, the thickness of the second layer is obtained as follows:

$$H_2 = 14.0 \text{ m}.$$  \hspace{1cm} (8)

This coincides with the thickness obtained in Fig. 8.

From these facts, it may be assumed that the distortion by two wavelets in wave group I following the first arrival is due to different types of refracted waves. But, this was not recognized insofar as the records obtained at a shot depth of 1 m were observed.

We propose to refer to these two wavelets in wave group I as the second phase and the third phase in the order of arrival.

In the present experiment, these wavelets were not discernible in the records, when the shot was fired at a shallower depth than 3 m. This was true because each different phase belonging to wave group I had a period nearly equal to the arrival time difference that was taken as $6.5\sim7.0 \times 10^{-2}$ s in the above investigation. As will be seen later, the period of each phase in wave group I was considerably sensitive to shot depth, in other words, the period became shorter with increase of shot depths.

The amplitude variation with the increase of shot depths is shown in Fig. 11 not for each different phase but for wave group I as a whole.

The amplitude became maximal at shot depth of 6 m for wave group I at 43 m and 65 m distance from the shot hole, but the maximum of that wave at 87 m distance may be expected at a shot depth deeper than 6 m. These data are inconsistent with those of the previous experiment$^{22}$ in which the maximum of amplitude was obtained near the interface. Considering the previous results, it was expected that the maximum of amplitude in the present
case would be obtained at a shot depth of about 16 m.

The above discrepancy may be attributed to the fact that the present investigation was not made on each phase. That is, the amplitude variation of wave group I was not that for the first arrival alone. Therefore, it is undesirable to assume the underground structure from this figure.

As for wave group II, the amplitude variation appears to be different from the other wave groups. It became maximal when the shot was fired near a 3 m depth. But the amplitude concerned abruptly decreased with the increase of shot depths as in the case of surface waves. This amplitude variation is quite similar to that of wave group II obtained in the previous experiment.22

The amplitude variations for wave groups III and IV show similar
features resembling each other as shown in Fig. 11. The amplitude was maximal when the shot was fired at 1 m depth. The amplitude concerned abruptly diminished with the increase of shot depths. These amplitude variations seem to indicate that the shallower the shot depth, the more effective to the generation of wave groups III and IV. Accordingly, it may be reasonable to consider wave groups III and IV to be a type of surface wave group respectively.

Investigating the amplitude variation for the disturbed parts of waves directly following wave group II, the results shown by I' in Fig. 11 were obtained. The variation pattern was somewhat similar to that of wave group I, in which the shot depth giving maximal values was a little deeper than that of I.

The three components of ground motion were also observed in the present experiment, in which the natural frequency of geophones was 27 cps, unlike that used in observations of the series of vertical motions.

Fig. 12 shows the records for various shot depths at a distance of 69 m from the shot hole in which the first trace shows the vertical motion, the second trace the transverse and the third trace the radial in each record, respectively.

![Fig. 12](image)

*Fig. 12. Records of the three components of ground motion at 69 m from shot hole for various shot depths, in which the first trace is a vertical, the second a transverse component and the third a radial component in each record.*

In these records, no wave was observed to have long period comparable to that of wave groups III and IV. The recorded wave patterns are quite similar to each other for shot depths of 1 m and 3 m, as if the medium were homogeneous up to 3 m depth. However, the wave pattern gradually changed with the increase of shot depth; the waves having shorter periods alone became predominant.
Fig. 13. Particle motion for each wave group at various shot depths. The upper part shows a relation of "up" and "away" and the lower a relation of "up" and transverse motion in the case of rear view of receding motion of waves as it moves away from shot point, in which the scale is arbitrary in each wave group.
Moreover, in these records, it is to be noted that waves observed between wave groups II and III were predominant while the wave groups already classified diminished. Especially, it is interesting that the horizontal components were predominantly observed. From this fact, it seems that the distortional wave energy may be considerably emitted from a dynamite explosion. The travel-time at 69 m distance in Fig. 12 indicates that the waves predominant in horizontal motions correspond to those marked with "I". Hereinafter, these waves will be referred to as wave group I".

The particle motion of each wave group was investigated from records in Fig. 12. Several results for various shot depths are shown in Fig. 13. The upper part of Fig. 13 shows the particle motion related to "up" and "away", and the lower part shows the particle motion related to "up" and transverse motion in the case of rear view of receding motion of waves as it moves away from the shot point. The figures attached to each pattern mean the arrival times in seconds.

As to the particle motion of wave group I, the vertical component was exceedingly predominant, while the radial motion was practically not observed. The entire pattern for wave group I did not change even when the shot depth was increased. From these facts, it was clearly indicated that wave group I was composed of compressional waves.

The particle motion of wave group II was clearly elliptical and progressive in the vertical plane of the profile; the vertical amplitude was more predominant than the radial amplitude.

As for wave group I", the amplitude was predominant in horizontal components as shown in Fig. 12.

The particle motion of wave group III was elliptical and progressive although its pattern presented somewhat irregular form. In these particle motions, the vertical and the transverse motions were greater than the horizontal motions. As the shot depth increased, the pattern changed.

In the case of wave group IV, an elliptical and retrograde pattern was obtained; the vertical amplitude was greater than the radial amplitude. Although the ratio of these amplitudes cannot be exactly obtained, it may be reasonable to identify wave group IV as a dispersive Rayleigh wave. The transverse motion of wave group IV was not observed as might be expected in the theory.
6. Detailed considerations of each wave group

1 Wave group I

Wave group I shows a wave pattern composed of two or three cycles. Near the shot hole, the direct P waves and the waves reflected and/or refracted at shallower interfaces may be contained in wave group I. But, at a distance far from the source, it seemed to be mainly composed of refraction P waves and a limited number of P waves converted to S waves at some interfaces. The amplitude of the first phase abruptly diminished with the increase of distance. Hence, the second and/or third phases became comparatively remarkable in the records, although a decrease in amplitude was seen.

The period of waves directly measured from records for this wave group was $6.0\sim7.0 \times 10^{-2}$ s, but it appears to increase gradually with the distance.

The velocity of the initial phase was about 400 m/s in the range between the shot hole and a 15 m distance. In the intermediate range, it showed about 700 m/s. As the distance was 40 m or more, it became about 1400 m/s which was the greatest in the velocities observed.

This was also seen in the case of inverse shot, but the greatest velocity was observed to be a little smaller than 1400 m/s.

As for the second and the third phase, the velocity was observed in its entirety to be a little less than that of the initial phase.

In the previous paper [22], it was considered that wave group I would be composed of several types of refracted waves. However, in the present experiment, the waves contained in wave group I cannot be definitely identified with each refracted wave in the records obtained at a shot depth of 1 m. That is, it was difficult to investigate separately the property of each considerable wave in wave group I from records at a shot depth of 1 m alone. In order to study these waves more minutely, another procedure of analysis should be designed. If the property of these waves were clarified, it would be useful in considering the generation of surface waves and also in rendering a more precise underground structure, and besides, it would give information on the source and property of the medium in which these waves were propagated.

In order to observe the property of these waves, the application of the spectral analysis to these waves is considered to be one of best methods. However, in practice, it would be difficult to know the spectral structure of each phase, since we cannot observe each phase separately. The most desirable way is analytically to remove unnecessary phases.

The method suitable to our present purpose is presented in Kasahara's
He improved the theory used in the analysis of stationary time series in the information theory to be applied to such a transient wave form as vibrations suddenly begin. Namely, he introduced a weight function $K(t)$ to multiply $f(t)$ which is a wave form concerned and defined it so as to pick up the necessary information, avoiding as much as possible the disturbance caused by multiplication of $K(t)$.

It is considered that a modified function $F(t)$ which is given by

$$
F(t) = f(t) \cdot K(t)
$$

(9)

conserves the spectral structure of the concerned part of $f(t)$, if function $K(t)$ is chosen to satisfy the above conditions.

Function $K(t)$ introduced by him is as follows:

$$
K(t) = \frac{c}{2\sqrt{\pi}} \exp \left\{-c^2(t-t_0)^2/4\right\}
$$

(10)

in which $c$ is an arbitrary constant. According to this function, $f(t)$ in the neighbourhood of $t=t_0$ may be conserved most truly in $F(t)$. Therefore, the spectrum of the wave concerned can be successfully investigated by means of the Fourier analysis, replacing $f(t)$ by $F(t)$.

This procedure can also be applied to the present case, if the constant $c$ and $t_0$ are reasonably given. In the present analysis, $t_0$ is taken as the arrival time of each refracted wave.

Each arrival time concerned cannot be defined exactly in the records obtained at a shot depth of 1 m, but it can be estimated by following the wave at deeper shots up to the records at a shot depth of 1 m. By the above procedure, we obtained Figs. 14 and 15 showing the spectral intensity of each phase for various distances, where the ordinate was given in arbitrary scale. In obtaining these spectra, the decay constant $c$ in function $K(t)$ was taken as 0.05/s. Since the amplitude of $F(t)$ obtained from Eq. (9) for this constant $c$ became very small 0.05 s before and after $t_0$, the influence of the other phases appearing about 0.07 s before and/or after the considerable phase was considered to be negligible on the phase concerned.

Fig. 14 shows spectra of the first three remarkable phases propagated from S to N at various distances. Fig. 15 shows spectra of the second and the third phase in the first three phases propagated from N to S at various distances. Since the initial phase propagated from N to S was not clearly observed, its spectrum was not obtainable.
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Fig. 14. Spectra of the first three remarkable phases propagated from S to N at various distances.

In these figures, the initial phase corresponds to the first arrival of refracted P wave, the second phase to the refracted wave propagated in the last layer as S wave and the third phase to the refracted wave propagated in the second layer as S wave and in the superficial layer as P wave.

The spectra of the initial phase at various distances are reasonably of quite similar features to each other. From this fact, it is clearly shown that
the initial phase has a property which causes the original characteristics to conserve, even though the distance changes. Similar results are also seen in spectra of the second and third phases. Then, it may be considered that the first three phases have the same spectral structure to each other. However, on the other hand, these spectra might be considered to be the results of an analysis of the waves originally being equivalent, since wave group I as a
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whole shows a simple wave pattern. Thereupon, wave group I was investigated with still deeper shots from which each refracted wave was isolated from the other waves as shown in the records of Figs. 6 and 7.

Spectra of the first two phases in question are shown in Fig. 16, in which the parameter indicates shot depth and each spectrum was obtained by the same procedures as described above. Wave trains analysed here are those at a 87 m distance from the shot hole.

As may be seen in Fig. 16, spectra of the initial phase show similar feature to those of the second phase regardless of the shot depth. That is to say, it may be considered that the first phase should generally have a similar spectral structure to that of the second phase.

In Fig. 16, spectra of the third phase could not be obtained since the amplitude was very small. However, spectra of the third phase would take on a feature similar to those of the initial and the second phase.

In the present analysis of wave group I, it must be noted that the period corresponding to the maximum spectral intensity gradually increased from 0.07$s$ to 0.08$s$ within a distance range of 20 m as indicated by arrows in Figs. 14 and 15.

Fig. 17 shows a period of maximal spectral intensity in Fig. 14 plotted against the distance from shot hole. The variation of these period with distance may be caused by an absorption in the medium.
Previously, Ricker\textsuperscript{17}) studied the effect of absorption in a medium on the spectral structure. Taking into account the absorption for a longitudinal plane wave propagated along the positive $x$ axis, he introduced the following expression:

$$x_1 = \exp \left\{ - \left( \frac{\omega}{\omega_0} \right)^q \cdot k \cdot x \right\} \sin \left( 2\pi \omega / v \right) (x - v \cdot t)$$

in which $q$ and $k$ are constants, $\omega$ is a frequency and $\omega_0$ is called the cut-off frequency characterizing the absorption in a medium.

The waves observed in the present experiment were not plane waves but spherical waves, however, his consideration may be applied to the present case so far as the approximate tendency of the absorption is concerned.

These considerations have also been made by Kasahara\textsuperscript{15}) from a standpoint of spectral analysis. He explained the deformation of wave forms as due to the viscosity of the medium.

In order to investigate the increase of period with which the spectrum
became maximal, Ricker's consideration was applied in the present analysis, since this was considered to be due to the absorption in the medium.

If we assume the spectrum of the original wave to be $S(\omega, 0)$, the spectrum $S(\omega, x_1)$ at a point $x=x_1$ can be expressed as,

$$S(\omega, x_1) = \exp \left\{ - \frac{(\omega/\omega_0)^q}{k x_1} \right\} S(\omega, 0)$$

(12)

similarly, the spectrum $S(\omega, x)$ at a point $x(x_1 < x)$ as,

$$S(\omega, x) = \exp \left\{ - \frac{(\omega/\omega_0)^q}{k x} \right\} S(\omega, 0) .$$

(13)

From Eqs. (12) and (13), the relation between the spectra of waves at a point $x_1$ and at a point $x$ is obtained as

$$S(\omega, x) = \exp \left\{ - \frac{(\omega/\omega_0)^q}{k} \cdot \Delta x \right\} S(\omega, x_1)$$

(14)

in which $\Delta x = x - x_1$.

If we assign the spectrum of the initial wave at a distance of 71 m to $S(\omega, x_1)$ in Eq. (14), the spectra at other distances treated in Fig. 14 can be calculated. The spectra thus obtained are shown in Fig. 18 in which $k$ was tentatively assumed as $2.5 \times 10^{-2}/m$ and $q$ as 2. These spectra, as may be seen, represent the tendency of the increase of period corresponding to the maximal spectrum as the distance increases. This fact is similar to that in Fig. 14, that is, Fig. 18 and Fig. 14 coincide with each other, aside from a detail of the spectral form.
According to KUBOTERA\textsuperscript{25}), the values of $k$ in the earth’s crust have been obtained for the propagation of $P$ waves as follows:

- **Granitic layer:** $7.9\times10^{-5}/m$
- **Intermediate layer:** $1.3\times10^{-5}/m$
- **Peridotite layer:** $2.6\times10^{-5}/m$

The value $k=2.5\times10^{-2}/m$ used in the present procedure is extremely larger than those of KUBOTERA’s. However, since the near surface zone in the adjacent areas of the present experimental site is mainly composed of clay and sand (see Fig. 32 in the TAZIME’s paper\textsuperscript{26}), a large value of $k$ might be appropriate to these media. Certainly, in order to interpret the fact that the predominant period is increased by $1.0\times10^{-2}s$ from $6.0\times10^{-2}s$ to $7.0\times10^{-2}s$ in such a short distance range as $20m$, a considerably large value of $k$ must be expected.

KASAHARA\textsuperscript{15}) has presented the spectra of several seismograms obtained by an explosion when the epicentral distance reached $32\text{ km}$ and the charge amounts were $10^3\text{ kg}$.

According to him, the increase of predominant periods was found to be about $3\times10^{-2}s$, i.e. from $11\times10^{-2}s$ to $14\times10^{-2}s$, within the distance range of about $30\text{ km}$, although the type of the medium was not described. He concluded that the predominant period was approximately independent of the propagated distance.

However, it was considered in the present experiment that the predominant period of the spectrum strongly depended upon the propagated distance. That is, the effect of the absorption cannot be neglected in considering wave forms.

This might be supported by the fact that the amplitude of the first arrival abruptly diminished within a short range of several hundred meters in refraction shootings, and also by the fact that it became duller with the increase of distance. These facts can clearly be seen in Figs. 36 and 37 in TAZIME’s paper,\textsuperscript{26}) in which the analyses on the underground structure of the present experimental site were described in detail.

As the shot depth increases, the predominant period of spectrum for each depth becomes shorter as already shown in Fig. 16.

Fig. 19 shows these relations obtained from Fig. 16.

The variation patterns of the initial and the second phase are quite similar to each other, even though the shot depth changes. This variation of period with shot depth seems to reveal the underground conditions near the source. If the formation of the substance near the source becomes compact, the
predominant period will become short in spite of constant charge amounts in a borehole.

In the present experiment, since the charge amount remained constant in the equivalent borehole, it might be considered that the formation up to a depth of 15 m was more loose than that beneath a depth of 15 m. This consideration seems to be reasonable, as may be seen in the datum of a boring near the present experimental site which was shown in Fig. 32 of TAZIME's paper. The predominant period at a shot depth of 1 m was exceptionally short, because the energy loss of explosion necessarily occurred outside the borehole.

2 Wave group II

It is difficult to point out the time and distance of appearance of wave group II. Practically, it was near a distance of 70 m from the shot hole that wave group II was observed separately from the other wave groups (see Fig. 2).

The wave pattern observed beyond a distance of 70 m took on a well-regulated form with three or four cycles. The phases from peak to peak of each wave corresponded with each other even though the distance increased. The velocity of these phases appears to change from 1900 m/s to 900 m/s with the increase in time, being estimated in time-distance plots. The larger the velocity, the greater the amplitude, while the period variation was hardly observed in the records. The wave form, the time of appearance and the phase velocity were almost not varied for wave group II, even though the shot depth changed as shown in Fig. 7 while the amplitude variation against the
shot depth was of such a singular nature that it was not seen in the other wave groups.

In the previous experiment\(^2\), the waves denoted by II were identified as a type of surface wave and interpreted to be the result of a generation of these waves due to several types of single reflection waves. However, the present wave group II could not be identified as surface waves from the standpoint of the phase velocity, because a velocity larger than that of P waves in the substratum cannot be expected in surface waves. Therefore, the most reasonable identification for wave group II would be made with a reflection wave group. However, at the present stage, we do not know to what type of reflection wave it belongs and to which interface it is related.

On the other hand, wave group II was not observed clearly as shown in the records from N to S in Figs. 2 and 4(b). This would be true because the charge amounts were too small to propagate a wave energy such a considerable interface as seen in wave group II in the large charge from S to N experiments.

Spectra for wave group II propagated from S to N were also studied. The analysed time range was between \(4.0 \times 10^{-1}\)s and \(7.2 \times 10^{-1}\)s. The results thus obtained are shown in Fig. 20 for various distances.

![Fig. 20. Spectra of wave group II propagated from S to N at various distances.](image)

These features were simple and similar to each other as might be expected from records. For every spectrum, the period of the maximum was near \(7 \times 10^{-2}\)s regardless of the distance. The variation of the period concerned was hardly seen, differing from that in wave group I. Moreover, even though the shot depth changed, not only the predominant period but also the feature of spectrum were not varied as shown in Fig. 21.

3 Wave group III

The records to a distance of about 20 m from a shot hole were confused, because several types of waves arrive at a point concerned within a short time.
range; consequently, it was impossible to indicate an exact time and a distance of an appearance of wave group III. The records over 20m distance became so simple that the existence of wave group III was discernible. At a point of 70m distance, wave group III was entirely isolated from the other waves, while its wave pattern did not present such a fine sinusoidal wave form as that of the others. Since wave group III was disturbed by the mixing of various waves having different periods and phase velocities, it was a little difficult to follow the phase to phase of each wave when the distance changed.

As to wave group III propagated from S to N, waves having comparatively long periods could not be followed over a distance, by referring records to time-distance plots. A rough estimation of phase velocities of these waves from time-distance plots, shows them scattered between 80 m/s and 100 m/s.

The apparent phase velocities for wave group III in the previous experiment\textsuperscript{22) were nearly equal to the velocity of S wave in the substratum, while the velocities of wave group III, in the present experiment, were approximately
equal to the velocity of S wave not in the substratum but in the second layer.

The wave pattern abruptly changed with a little increase of shot depth, so that waves having relatively short periods became remarkable in the wave group concerned which, however, was still disturbed by the unknown waves. As the shot depth increased beyond 6 m, it became difficult to find wave group III in the records.

The maximal amplitude of wave group III decreased exponentially with the increase of shot depth. This shows that the energy of explosion is not sufficient to generate wave group III when shot depth increases.

The effect of sloping interface between the second layer and the substratum is not well understood on the phase velocity and the period of waves, inasmuch as wave group III is investigated in time-distance plots alone. However, it may be noted in Figs. 2 and 3 that the pattern of waves propagated from N (thin layer) to S (thick layer) was simpler than that from S to N.

When wave group III was propagated from N to S, the amplitude of waves abruptly diminished with the increasing distance from the shot hole, while this could not be seen in waves propagated inversely. This became especially remarkable after wave group III was isolated from wave group IV. This fact might be due to the energy dissipation from thinner to thicker layers.

The phase velocities of waves from N to S were estimated from time-distance plots as 110 m/s to 150 m/s, which were between the velocity of S wave in the second layer and that in the substratum, being a little larger than the phase velocities of waves from S to N. This is inconsistent with the propagation of refracted wave, on which the velocity of the wave propagated in a downslope should be smaller than that in an upslope.

According to the paper of model experiments by Kuo and Thompson\[27\] who investigated the effect of a sloping interface on surface waves, the time-distance plots are no longer linear for waves propagated in media having sloping interface, i.e., the time-distance curves for upslope and downslope become respectively convex upward and convex downward. In the present experiment, however, such phenomena were not observed. It may be true because the sloping interface was relatively short in comparison with the average thickness of the layer concerned. Moreover, it may be difficult in the present analysis to distinguish whether the convex time-distance curve corresponds upward or downward.

The time of appearance of wave group III was earlier in the direction
from N to S than in the direction from S to N. This time difference was approximately 0.2 s near a point of 60 m from the shot hole. This difference may suggest the difference of layered conditions concerning the generation of wave group III. However, such a difference cannot be clarified as long as the mechanism of wave generation is not understood.

If wave group III in the present experiment were identified with wave group III in the previous experiment at Wakino-machi, such a difference as 0.2 s would result in a difference of 20 m layer thickness between the south region and the north region. However, this value was not obtained by refraction method. This discrepancy may not be interpreted without a detailed experiment.

In order to investigate wave group III further, the FOURIER analysis was carried out. In using the FOURIER analysis, it is desirable that a wave train at the point concerned is isolated from the others. But this was not expected entirely in actual records as shown in Figs. 2 and 3. Near the source, wave group III was completely disturbed by other waves. At a point more than 60 m distant from the source, only the tail of its wave train was disturbed by wave group IV. Here, performing a small artificial distortion at both ends of the wave train and negating a disturbance out of the time interval concerned, we analysed some wave trains. In this analysis, five wave trains of wave group III were picked up every 4 meters. The selected region exists between 75 m and 91 m distance from the shot hole and the time interval is between 1.12 s and 2.12 s for the propagation from S to N. On the other hand, for the propagation from N to S, the selected region exists between 74 m and 90 m and the time interval is between 0.75 s and 1.55 s.

Fig. 22 and Fig. 23 show the amplitude spectra of wave group III obtained at various distances from N to S and those from S to N respectively, in which
the ordinate is given in arbitrary scale.

Spectra shown in Fig. 22 were of similar feature to each other in spite of the difference of their distances from the shot hole; the main feature of the spectrum took on a simple feature and was hardly influenced by the change of the layer thickness concerned. The period corresponding to the maximal spectrum was about 0.14 s independent of the distance.

In general, it is considered on wave propagation in layered media that the predominant period is closely related to the layer thickness under an observation point. However, as to wave group III propagated downslope, the dependency of period on the varying interface could not be seen in the present experiment; the period concerned was obtained as constant regardless of the distance. When wave group III was propagated from the region of thin layer to that of thick layer, the energy of incident waves appears to be almost totally transmitted without any interference, similarly to the waves presented in the paper by Kuo and Thompson. Accordingly, it was found that wave group III was apt to reserve the original characteristics given in generating waves, while the wave amplitude decreased abruptly with distance, since the energy dissipation due to the scattering occurred.

On the other hand, spectra in Fig. 23 show different features from each other for the difference of distance. At a point of 75 m distance, the spectrum was of such a simple feature as to have only one maximum at which the period was about 0.2 s. As the distance increased, the spectrum came to have two maxima as expected from the investigation of the records. The first maximum corresponding to the shorter period was smaller than the second
maximum corresponding to the longer period, in which the shorter period was about 0.15 s and the longer period about 0.23 s for the spectrum near a point of 79 m.

As the distance increased further, the first maximum gradually grew up and finally became larger than the second maximum, while each period corresponding to the first and second maximum did not change.

These spectra which are complex as shown in Fig. 23, are considered to be caused by the following reasons: the wave energy was not totally transmitted to the region of thin layer, but a part of it was converted into reflected waves and/or waves satisfying the condition of a thin layer. Accordingly, the characteristics of the spectrum in the thin layer should partially come into existence in these spectra.

In the present case, it is considered that the development of the first maximum in the spectrum originates from characteristics existing in the thin layer. This interpretation may be supported by the fact that the period corresponding to the first maximum coincides with that of the maximum shown originally in the thin layer (see Fig. 22). Therefore, the spectrum at a point of 75 m is considered to show the characteristics of waves in the thick layer. The second maximum presented at various distances will be the indication of these characteristics. Therefore, the spectra at various distances should be considered as the superposition of two spectra peculiar to the thin and the thick layer.

This may be confirmed by the fact that the ratio of the period corresponding to the first maximum and the period to the second maximum was nearly equal to the ratio of the thickness of the thin layer and that of the thick layer.

Namely,

\[ \frac{\text{period of first max.}}{\text{period of second max.}} = \frac{0.15 \text{ s}}{0.20 \text{ s}} = 0.75 \]

\[ \frac{\text{thickness of thin layer}}{\text{thickness of thick layer}} = \frac{12 \text{ m}}{16 \text{ m}} = 0.75 \]

It may be said that the spectra of wave group III clearly reflect these underground conditions.

The variation of spectrum with shot depth at a point of 87 m is shown in Fig. 24, which is obtained from the waves propagated from S to N. As shot depths are slightly changed near the surface, each spectrum takes on
different feature; each spectrum is no longer similar to that for shot depth of 1 m. Probably, this may be due to distortion by unknown waves that these complex spectra were obtained.

The predominant period became shorter with increasing shot depth. This fact is similar to that for the first arrivals, in which, however, the spectra for various shot depths were of quite similar features to each other.

When any type of waves propagates in media having a sloping interface, the phase velocity may not generally be determined by the usual FOURIER analysis. In this case, the phase velocity $v$ must be determined by

$$v = \omega / \frac{\partial \varphi}{\partial x}$$

(15)

in which $\varphi$ is the phase lag at a point $x$ which is obtained from FOURIER components with respect to a circular frequency $\omega$. Here, $\varphi$ is a function of
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Fig. 25. Phase velocities of wave group III determined formally by the least square method and typical spectra of wave group III, in which black circles and solid line are obtained from waves propagated from S to N, white circles and dotted line from N to S.

If $\partial \varphi / \partial x$ is assumed to be a constant for each period by ignoring that the phase is scattered with respect to the distance, the phase velocities can be obtained by means of the least square method, as shown in the upper part of Fig. 25, in which black circles are obtained from waves propagated from (region of thick layer) to N(region of thin layer) and white circles from N to S. On account of the unreasonable assumption which regards a transition layer to be uniform, the relation of phase velocities and periods presented by black circles was obtained which was highly contrary to common knowledge.

On the other hand, the relation presented by white circles seems to be natural, that is, the phase velocity increases as the period becomes longer; it exists between 140 m/s and 170 m/s.

This fact also reflects that the waves propagated from the thin layer to
the thick layer are apt to reserve the characteristics given in the layer uniformly thin, as was seen in the investigation of amplitude spectrum.

4 Wave group IV

Wave group IV appears lastly among all wave groups observed. It was composed of waves having longer period and larger amplitude which did not easily fall off with the increasing distance. The maximal amplitude seems to shift continuously from earlier to later phases while increasing the wave cycles at the same time, when the distance from the shot hole increases. This may be due to the dispersive property of waves. The wave pattern near the source was not of a fine feature, since wave group IV was disturbed there by other waves. But, after wave group IV was isolated from the others at a greater distance, the wave pattern took on such a fine feature that it appears as sinusoidal waves in which distortion was seldom found. This became remarkable in the wave trains beyond the distance of about 70 m from the shot hole.

The phase concerned easily corresponds to that at a adjacent point. This can also be seen in time-distance plots shown in Fig. 5. The period directly measured in the records from S to N was about 0.2 s for considerable wave cycles, being independent of the distance. Therefore, it may be considered in contrast to wave group III that wave group IV was not influenced by a sloping interface. The remarkable difference in the wave pattern due to the direction of wave propagation was not seen between wave group IV propagated from N to S and that from S to N.

The period of the considerable phases from N to S was about 0.15 s, which was smaller than that from S to N. In the present case, this period was also invariable even though the distance from the shot hole increased. The time of appearance for the present wave group IV from N to S was earlier than that for wave group IV from S to N. These differences in the period and the time of appearance of wave group IV suggest a difference of the underground structure.

According to time-distance plots obtained from waves propagated from S to N, the apparent phase velocities were estimated between 80 m/s and 95 m/s, which almost coincide with those of wave group III propagated in the same direction. As to the present wave group IV, the later the phases, the larger the phase velocities, while this regularity was not found in wave group III. This is considered to present the difference of waves being free or not free from the sloping interface.

On the other hand, as to wave group IV propagated from N to S, the
apparent velocities were obtained between 70 m/s and 90 m/s from time-distance plots, which are smaller than those of wave group III propagated in the same direction.

Similar to the former case, it was also seen that the phase velocity became larger with the increase of arriving time.

As may be seen in records of Fig. 7, the existence of wave group IV for various shot depths up to 10 m is evidently confirmed. The wave amplitude gradually diminished as the shot depth increased, while the wave pattern for each shot depth as well as for the shot depth of 1 m showed as usual sinusoidal waves without any interference by the other waves. The period for the considerable phases from S to N was about 0.2 s which was the same as that for the shot depth of 1 m. This was one of the most remarkable points different from the other wave groups.

When the shot was fired at a depth deeper than 10 m, wave group IV could not be observed.

The investigation by means of Fourier analysis was also made on wave group IV propagated from S to N for various shot depths and on that from N to S for a shot depth of 1 m. The waves analysed in the present case were those for every 4 meters in the distance range between 71 m and 91 m from S to N and those between 70 m and 90 m from N to S. Each time range analysed

![Fig. 26. Spectra for wave group IV propagated from S to N at various distances.](image-url)
Fig. 27. Spectra for wave group IV propagated from N to S at various distances.

was 2.0 s between 1.8 s and 3.8 s from S to N or between 1.25 s and 3.25 s from N to S.

The amplitude spectra at various distances are shown in Fig. 26 and Fig. 27 for a shot depth of 1 m. As shown in Fig. 26, spectra are of quite similar features to each other, even though the distance from the shot hole is changed; they take on such a simple feature as to become sharply maximal at a period of 0.2 s. No effects of the sloping interface on wave group IV were found in these spectra.

From these facts, it may be understood that wave group IV from S to N has such a well regulated form as to be sinusoidal in spite of the difference of the distance from the shot hole.

On the other hand, as to waves propagated from N to S, the amplitude spectra have slightly complicated forms, which are shown in Fig. 27 for various distances.

The spectrum at a point of 70 m distance was of simple feature similar to the former case, while the period corresponding to the maximum was 0.15 s
less than that for spectrum in Fig. 26. As waves were propagated farther, the
spectrum became gradually complicated; the spectrum corresponding to
longer periods rather than to shorter periods was more disturbed. However,
the period corresponding to the considerable maximum did not change as it
was about 0.15 s. These facts show that wave group IV has a property to
reserve the original characteristics which may be closely related to the point
of wave generation, and is not easily influenced by such a sloping interface as
found in the present experimental site, regardless of the direction of wave
propagation. Namely, it may be considered that wave group IV is such that
its energy is almost totally transmitted without any interference.

The amplitude spectra at a point of 87 m for various shot depths are
shown in Fig. 28. The time range analysed was equal to that for a shot depth
of 1 m. The features of spectra thus obtained are quite similar to each other;
they take on a simple feature which has its maximum without exception at a
period of 0.2 s as well as at that for a shot depth of 1 m.

Accordingly, it is understood for wave group IV that the period at which considerable waves are composed depends not on the source conditions but on the characteristics peculiar to the underground structure in which the waves are developed. The original characteristics related to wave generation is most excellently reserved in wave group IV among all wave groups observed.

In the present analysis for wave group IV, the phase velocity was again taken into account not from time-distance plots but from the phase lag for each period obtained from the Fourier components.

Assuming again \( \partial p/\partial x \) in Eq. (15) to be a constant in wave group IV with respect to the period, the phase velocities of wave group IV were also obtained by means of the least square method. The results are shown in black circles for the waves from S to N and by white circles for the waves from N to S. The phase velocity increases usually with the period as can be seen in the surface wave dispersion within uniformly layered media. As shown in Fig. 29, the phase velocities exist between 70 m/s and 139 m/s regardless of the direction of wave propagation. The remarkable difference due to the direction of wave propagation was not found, although the velocity difference was about 5 m/s at most, which was larger than the error on each phase velocity caused by the calculation.

From these phase velocities, the group velocities were tentatively calculated by using the following relation:

\[
U = c - \lambda (d c/d \lambda)
\]

where \( U \) is the group velocity, \( c \) the phase velocity and \( \lambda \) the wave length. The results are also shown in Fig. 29. For both waves propagated from S to N and those from N to S, the minimum of group velocity was obtained near the period of 0.2 s, at which point the amplitude spectrum for waves propagated from S to N became maximal, while that for those from N to S presented their maximum at a period shorter than 0.2 s. The typical spectra for each direction of wave propagation are again presented in Fig. 29. In this figure, it is seen that wave group IV is predominant near the period corresponding to the minimum group velocity.

It can be said from the above investigations that the waves free from underground structure complications have simpler spectrum, resulting in an usual dispersion.

In the analysis of these waves, it is also important to compare the
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Fig. 29. Phase and group velocities and typical spectra of wave group IV, in which black circles and solid line are obtained from waves propagated from S to N; white circles and dotted line from N to S.

dispersion obtained from observations with that expected theoretically in the same underground structure as a matter of routine, though it was not carried out in the present paper.

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