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Model Seismology on Characteristics of Surface Waves Generated from a Sinusoidal Source of a Finite Duration (continued)

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Abstract

Following the previous experiment[1], observations have been made every period at various epicentral distances. The appearance of the wave group becomes more systematic than that of the previous one. A few questions which remained in it have been clarified.

3. Experiments repeated at various epicentral distances.

3.1 The epicentral distance was kept constant 20 cm throughout the previous observation in which following questions remained: i) where was the lower limit of the distance at which usual definition of group velocity $d\omega/d\xi = x/t$, given by (1.11), might be satisfied and what was the phase observed which deviated far from the group velocity theoretical?

In order to obtain the answer for those questions, observations have been repeated at various distances every 1 cm from the epicenter. However, thickness of the surface layer is confined to 10 mm which was used in the previous experiment. The gain of the amplifier was kept constant for the same period, changing the epicentral distance alone. Observations were stopped where amplitudes of waves became too small to be analysed.

The period of the sinusoidal source was varied every 2 μs from 6 μs to 50 μs. Seismic records obtained are exhibited on Fig. 11 where time markes indicate 10 μs. The trace number means the epicentral distance in centi-meter.

3.2 Wave numbers are preserved in a predominant wave group on each trace until the period reaches 20 μs or so which were contained in the wave form generated from the origin. Beyond that period, however, the wave group seems to be divided into two groups which are seen clearly at large distances.

Onset of the former group does not depend on periods but is equal to that of the wave group having a smaller period when no division of the group occured. When the original wave has a period as large as 50 μs, sometimes

Fig. 11(a~d). Seismic records obtained every period.

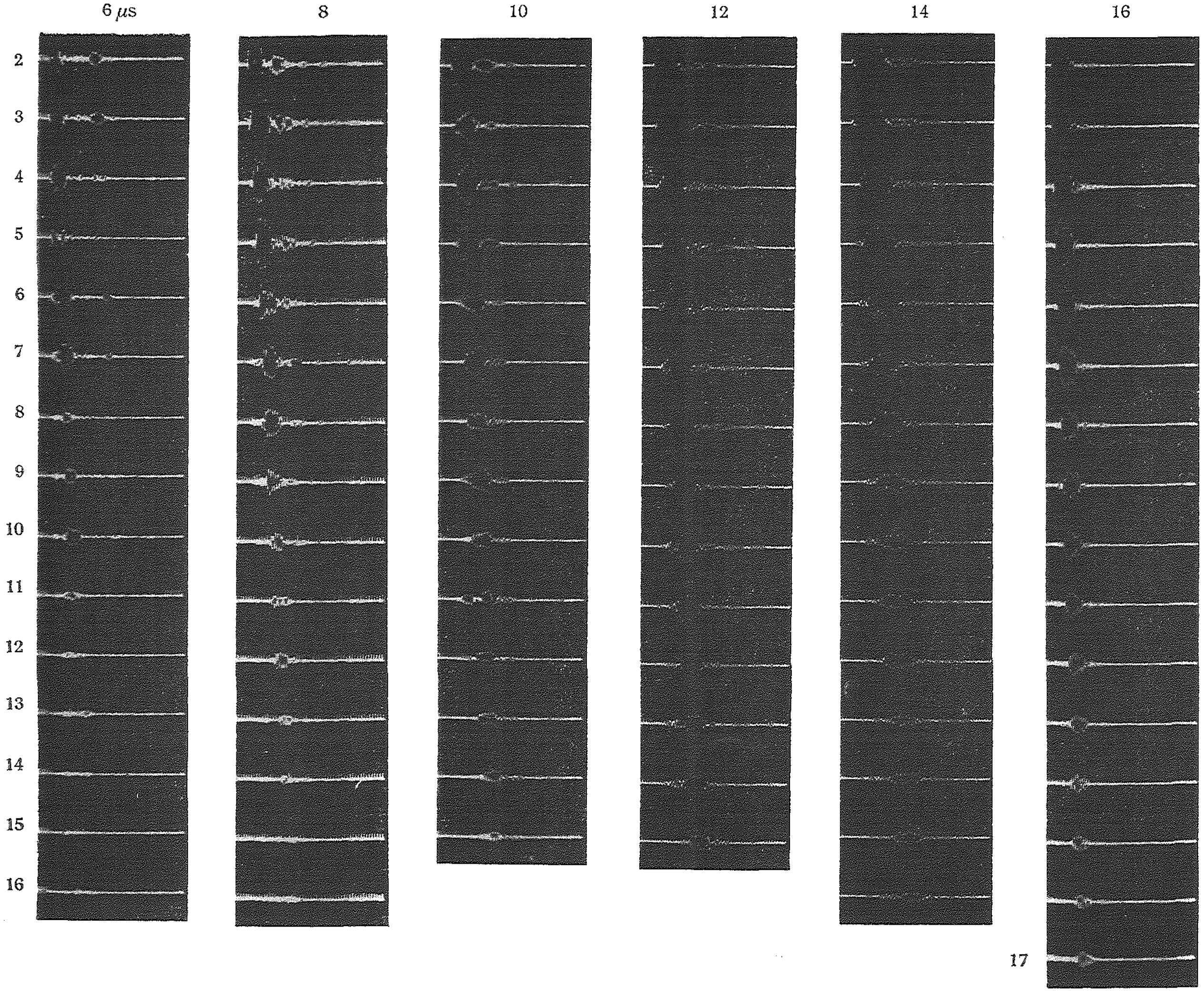

Fig 11(a)

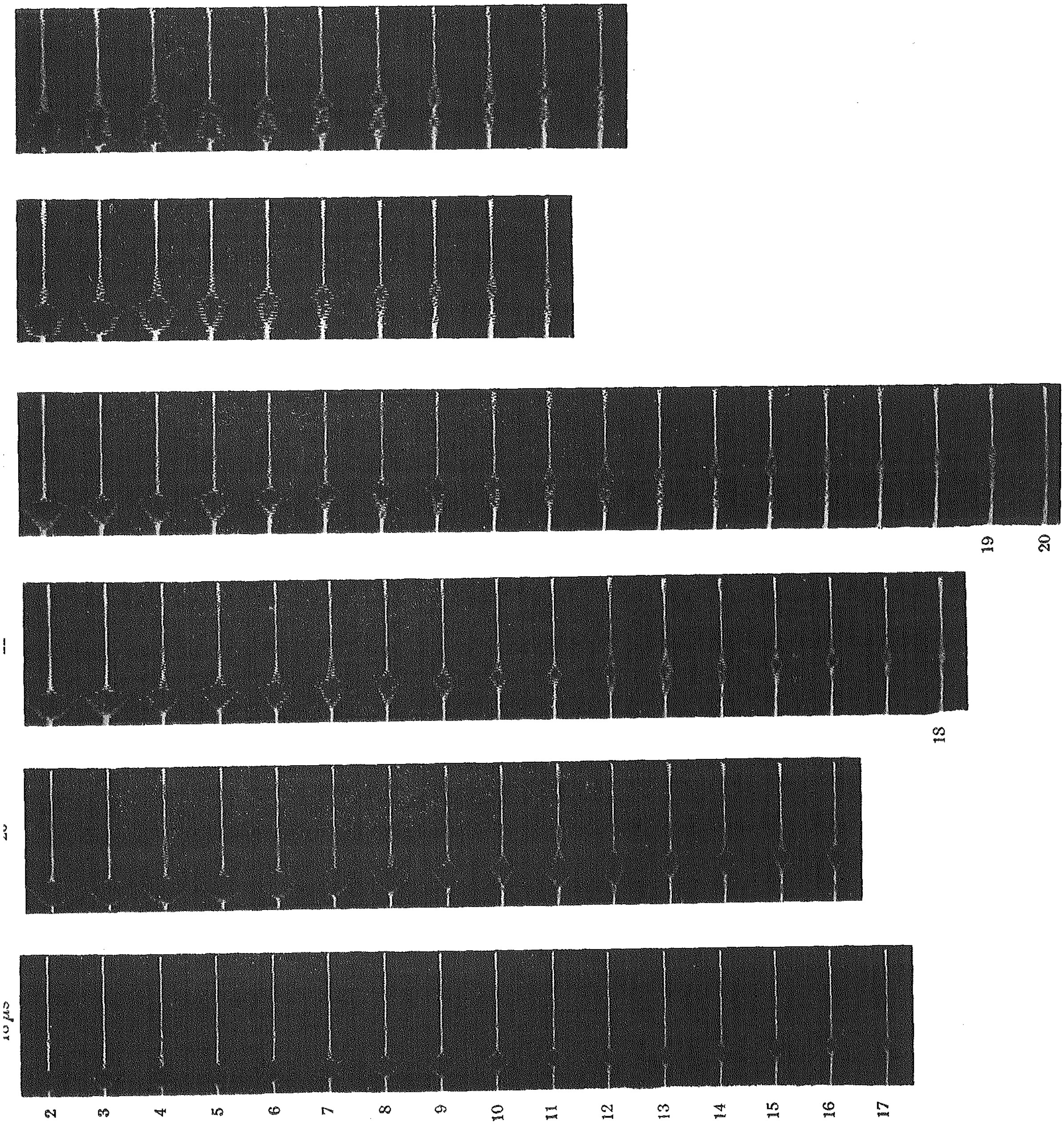

Fig. 11 (b)

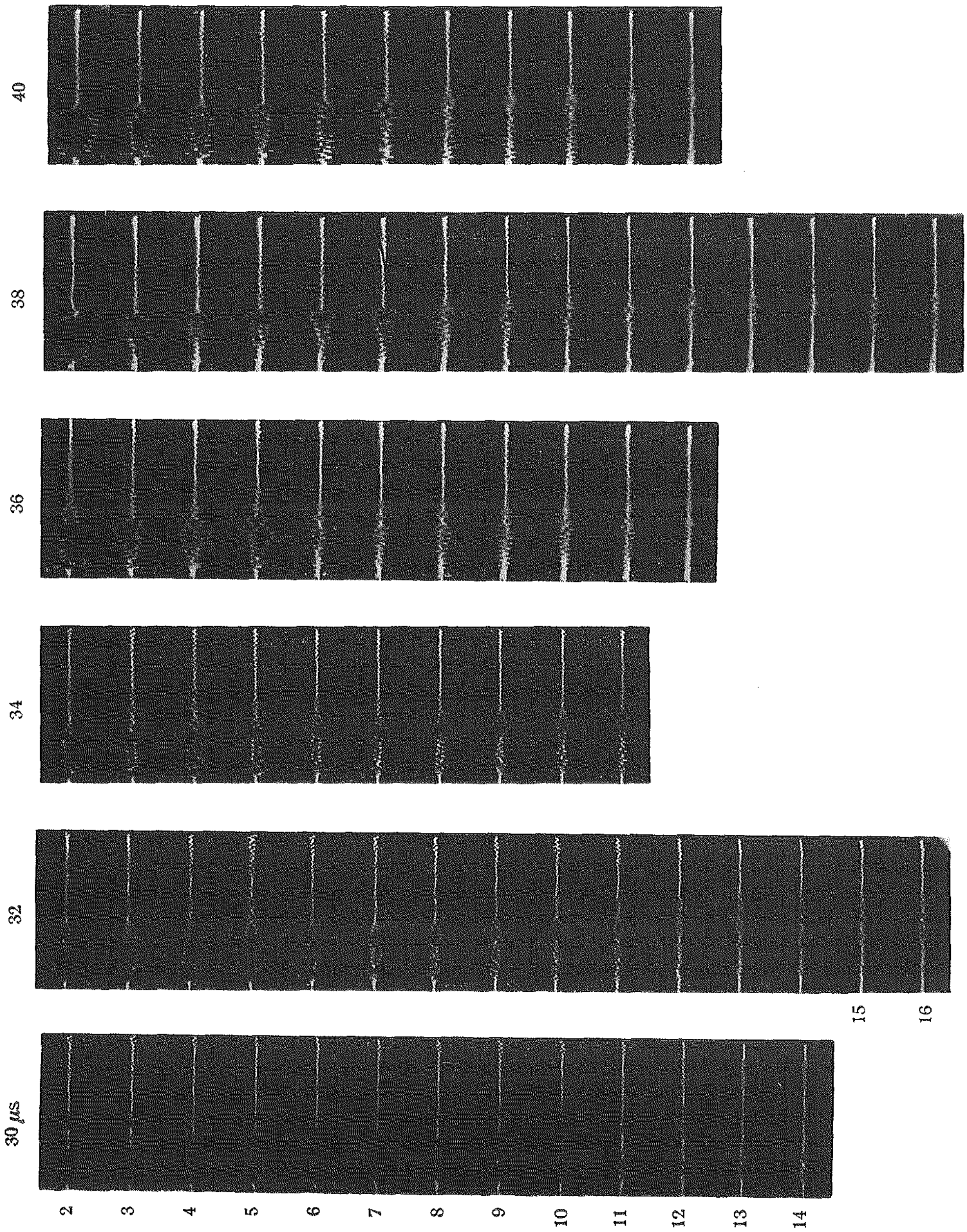

Fig. 11 (c)

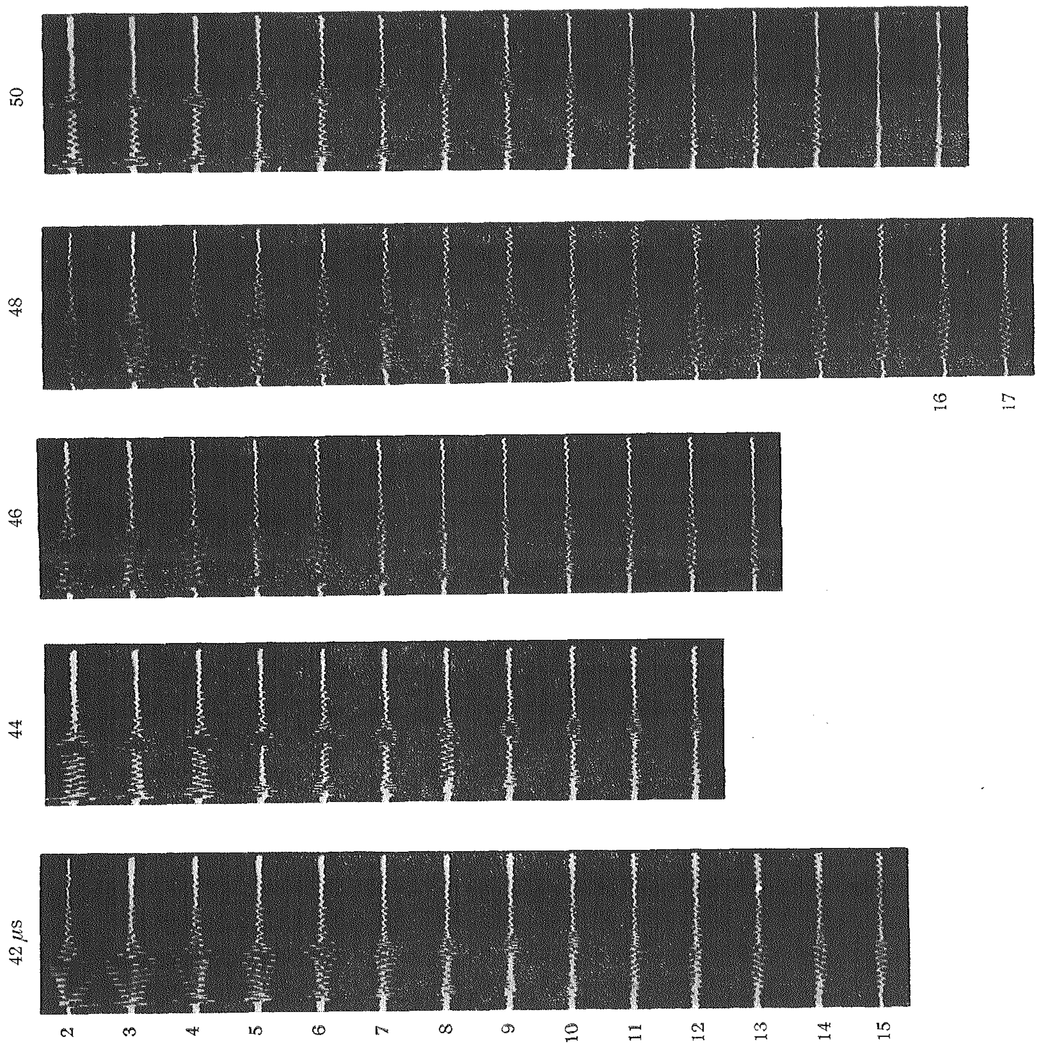

Fig. 11(d)

three wave groups are observed. The first and the third among three groups have smaller periods than that of the original one. The interval of them is kept constant on every trace. Therefore it has been recognized in the present experiment that the first and the third wave groups would be excited selfishly by arrivals of top and rear of the sinusoidal source of a finite duration.

The wave group having a period near 20 μs grows most in the present model. On the other hand, the wave for a period as large as 50 μs can be hardly developed. Therefore the signal for a large period is apt to be masked by undesirable "free-oscillations" whose periods are near 20 μs. Additional noises, for instance a circuit noise, are inevitably contained in traces for large periods, because the gain of the amplifier was extraordinarily raised for detecting the small signal.

The bounds occupied by the true wave group must be decided with great care against the period as well as the wave numbers, otherwise the previous misunderstanding will be repeated.

3.3 The arrival time of onset of the predominant wave group, instead of determing group velocity at once, has been decided on each trace with cautions noted above. These times are plotted by black circles on Fig. 12 in which each straight line indicates the inclination corresponding to the theoretical group velocity $d\omega/d\xi$ defined by (1.11). It is seen that the lower limit of the epicentral distance should exist where definition (1.11) might be satisfied. It was already noticed in Fig. 2 dealing a pulse, but it has now become clear.

The group velocity x/t determined from a plot on a large distance will certainly coincide with theoretical one $d\omega/d\xi$. In Fig. 13, the former at a large distance is plotted by black circles and the latter is drawn with a full line. Contrary to Fig. 10 in the previous paper, this figure shows good coincidence between $d\omega/d\xi$ calculated and x/t observed throughout the range of periods, though considerable errors might enter in the observation on the part of large periods.

3.4 The lower limit of the epicentral distance pointed out on Fig. 13 seems to change from a period to another. To tell the truth, the limit is difficult to be decided and any arbitrary range must be allowed in it. This range is illustrated with a bar in the middle of Fig. 14 in which a dotted line means nothing but a curve smoothed.

The highest among Fig. 14 illustrates the lower limit of time when the

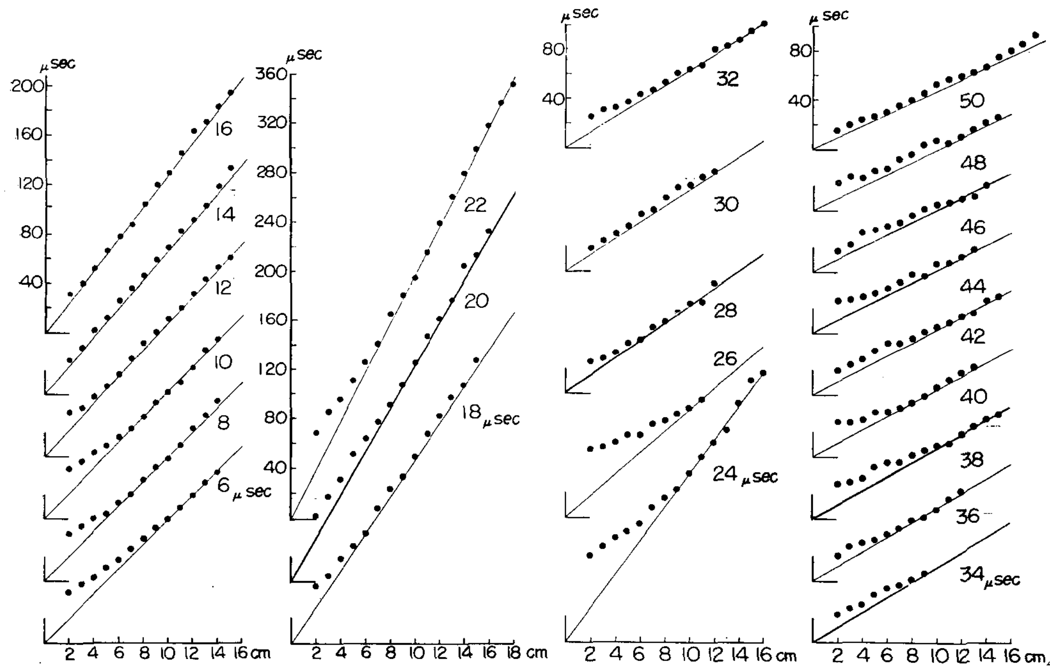

Fig. 12. Time-distance plots of the onset of the wave group, being parameter periods.

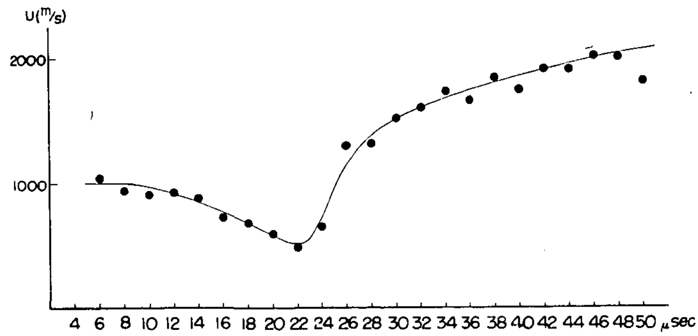

Fig. 13. Comparison of group velocity $d\omega/d\xi$ with x/t.

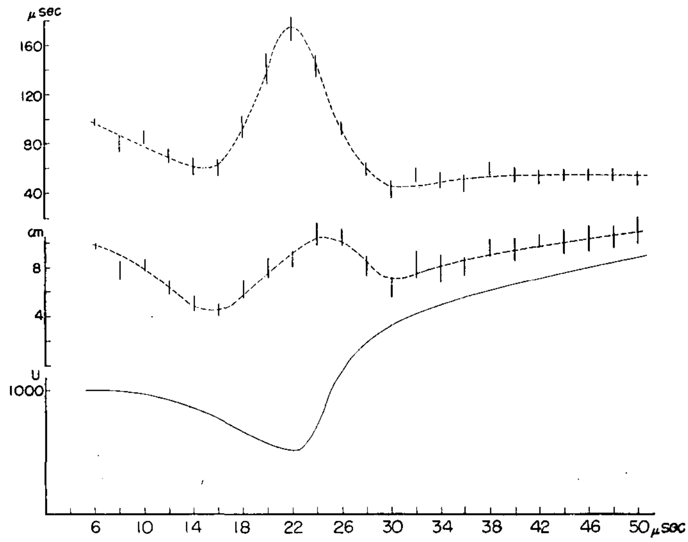

Fig. 14. Relations between the least time and period satisfying $d\omega/d\xi=x/t$, group velocity and period.

arrival of the observed wave group coincides with that deduced from the usual theory. The lowest is the same curve as that of Fig. 13 showing group velocity.

The larger the period, the longer the wave-length may be. It is somewhat strange that the upper two dotted lines do not ascend monotonously with increase in period. Any complicated phenomenon might be occured near the period for the minimum group velocity.

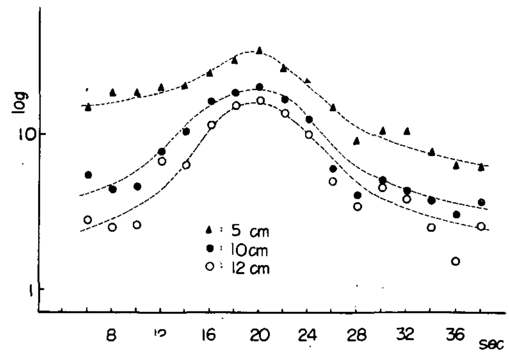

Fig. 15. The relation between amplitude and period.

The gain of the amplifier was kept constant for the same period but it was changed from a period to another. In order to compare amplitudes for different periods, the correction of amplitudes should be made on each trace which was obtained by its own gain. Corrected amplitudes of the wave group are plotted against period on Fig. 15, using three kinds of marks which distinguish epicentral distances where respective observations were carried out.

Considerable errors might enter in the correction of amplitudes, because every amplitude was adjusted by taking the amplitude as the standard which was obtained at the distance 2 cm far from the origin. The present standard is not the amplitude of the source but is an amplitude influenced to some extent by frequency-characteristics of the wave.

The undesirable effect should be large at small distances. On the other hand, wave forms were seriously distorted in large periods. However, the average shape of three curves is to suggest some meaning on Fig. 15, having the maximum at $20\,\mu$s. This period is larger than that reported in the previous paper and has approached the period for the group velocity minimum.

4. A theoretical consideration on "free-oscillations".

According to the investigation by one of the author[2], displacements Ψ of Love waves which are generated from a sinusoidal source at $t=0$ are expressed by

$$\Psi = \begin{cases} 0 & \text{for} \quad t < x/v_1\,, \\ P + B & \text{for} \quad t > x/v_1\,. \end{cases}$$

In the above expression, P means the integration round a pole ν which is

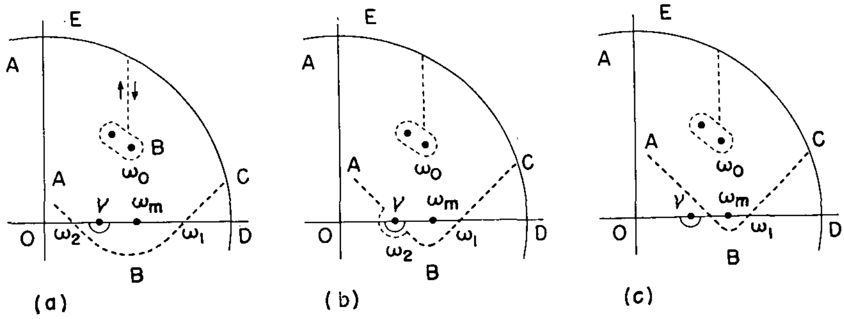

Fig. 16. The path of integration on ω-plane.

the angular frequency of the source but B does the integration round a branch point ω_0 which makes $d\omega/d\xi$ zero on ω-plane.[3].

The real part of ω_0 exists in the neighbourhood of ω_m which makes $d\omega/d\xi$ minimum. If ν is far from ω_m, therefore, B becomes considerably larger than P. However, B diminishes exponentially with increase of time and P may survive alone a little later than $t=x/v_1$.

The method of steepest descent gave paths of integration illustrated on Fig. 16 when t was between x/v_2 and x/v_1.

Taking integrations near saddle points ω_2 and ω_1 as S_2 and S_1, we have the following evaluations: (a) $\Psi=\int_{-\infty}^{\infty} d\omega=\int_{ABCD}=S_2+S_1$, (b) $\Psi=S_2+S_1=P/2+S_1$ and (c) $\Psi=P+S_2+S_1$.

Owing to CAUCHY's theorem, however, we see:

(a) $\int_{ABC}=P+B+\int_{AEC}$ that is $S_2+S_1=P+B$,

(b) $\int_{ABC}=P/2+B+\int_{AEC}$ that is $S_2+S_1=P/2+B$,

(c) $\int_{ABC}=B+\int_{AEC}$ that is $S_2+S_1=B$,

because $\int_{AEC}$ is zero for $t\geqq x/v_1$.

These relations tell again that "free-oscillations" B's are excited from $t=x/v_1$ whose frequency is near ω_m and whose velocity of propagation is equal to v_1.

It has been found by the result of the present experiment that periods of "free-oscillations" are always $22\,\mu s\sim 24\,\mu s$ which are independent of the period of the source but are near the period for the group velocity minimum. As was already pointed out in the previous section, wave groups were misunderstood on a part of Fig. 10 where all group velocities for large periods ccincided to v_1.

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3) TAZIME, K.: The Frequencies of LOVE Waves Whose Group Velocities are Zero, Zisin, **18** (1965), 82-93.