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A Note on the Statistical Nature of Energy and Strain Release in Earthquake Sequences

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Abstract

The sum of the γ th power of energy E^{γ} for earthquakes having a magnitude distribution function $\log n = a - bM$ is discussed. When $\gamma = 1$, the sum means the total energy. Some seismologists regard the sum of E^{γ} as a representation of the total strain release (or deformation, or slip) due to the earthquakes, taking a value for γ between 1/2 and 1. The nature of this sum differs largely according to whether the value of $\kappa(=b/\beta\gamma)$ is larger than or smaller than 1, where β is the coefficient in the energy-magnitude relationship $\log E = \alpha + \beta M$. Practically the summation is done only for earthquakes larger than a certain magnitude M_s. When $\kappa < 1$, such a summation is justified, since the sum of E^{γ} for the numerous earthquakes smaller than M_s is negligible, if the lower limit of magntidue M_s is properly fixed. However, when $\kappa > 1$, the sum is critically dependent on M_s. On the average it is proportional to the number of earthquakes with magnitude larger than M_s. Some illustrations of energy or strain release characteristics are given using data on numerically generated earthquakes and a natural earthquake sequence.

1. Distribution function of energy and strain release

It is now generally accepted that the frequency of earthquakes increases with decreasing magnitude according to Gutenberg-Richter's formula[1)]

$$\log n\,(M) = a - b\,M \tag{1}$$

where $n(M)dM$ is the number of earthquakes having magnitude between M and $M+dM$ and a and b are constants depending on the seismic region and the period of time concerned. Spacial and temporal variations in the value of b in this formula have been discussed in many papers. Utsu[2)] made a review of b-value determinations publsihed before 1967 by using his method of b-value determination[3)]. The b-value for ordinary earthquakes falls in the range between 0.4 and 1.5, and values near 1.0 are most frequent.

However statistical studies of earthquake occurrence in terms of energy are in a sense more fundamental than the statistics on the magnitude, since the energy is a well-defined quantity in physics, and the sum and the ratio of

two or more energy values have definite physical meanings.

The formula (1) can be converted into a formula for frequency distribution of earthquakes in terms of energy, if a relation between magnitude and energy is provided. The generally used expression for this relation takes the form

$$\log E = \alpha + \beta M . \tag{2}$$

According to Gutenberg and Richter[4] the values for α and β are 11.8 and 1.5 respectively, when E is measured in ergs. Other values for α and β have also been published.

Combining (1) and (2), we obtain

$$n(E) = c\, E^{-(b/\beta)-1} , \tag{3}$$

and

$$c = 10^{a+(\alpha b/\beta)}/\beta \ln 10 , \tag{4}$$

where $n(E)dE$ is the number of earthquakes with energy between E and $E+dE$.

In many seismological studies the square-root of energy of an earthquake has been used as a measure of the strain released by the earthquake, and so-called strain release curves and strain release maps have been constructed for various earthquake sequences and for various regions by summing up the square-roots of energy values. However, this idea, which was first introduced by Benioff[5)-7)] in 1949, encounters some criticisms from physical and statistical points of view[8)-11)]. Anyway since the strain release J by an earthquake with magnitude M and energy E is considered to be proportional to $E^{1/2}$,

$$\log J = \frac{1}{2}(\alpha + \beta M) + \text{const.}, \tag{5}$$

and the total strain released by a series of earthquakes is proportional to $\Sigma E^{1/2}$, where the summation is performed usually for all earthquakes in the series with magnitude above a certain fixed level, assuming that such a simple addition is permissible.

Recently the seismic moment of an earthquake has used for representing the amount of slip along a fault by the earthquake[12-13)]. The total slip for a fault zone is proportional to the sum of the moments for all earthquakes occurring in the zone. The relation of moment K to the magnitude M for an earthquake is somewhat uncertain. If the linear relation between $\log K$ and M is assumed, i.e.,

$$\log K = u + v M , \tag{6}$$

K is proportional to $E^{v/\beta}$. Brune's graph[13] of $\log K$ vs M plots shows that the coefficient v is approximately 1. If $v=1$, the total slip is proportional to $\Sigma E^{1/\beta}$.

On the other hand, Båth and Duda[14] showed that the deformation D in a seismic zone by an earthquake is related to the magnitude M of the earthquake by

$$\log D = q + r M \tag{7}$$

where q and r are constants. According to them, the value of $r(=1.46)$ is almost equal to the value of $\beta(=1.44)$ by Båth.[15] Therefore the total strain or deformation in the seismic zone is approximately proportional to ΣE.

Another different consideration described in Appendix suggests that the deformation of an seismic zone contributed by an earthquake with energy E is poroprtional to E. If this idea is adopted, the deformation of the the zone is proportinal to ΣE.

In the above theories, the strain release, deformation, or slip, which means almost equivalent things, is proportional to the γth power of energy E^{γ}, in which γ varies from 1/2 to 1 according to the theories. The total strain release is represented by ΣE^{γ}. Theoretically the summation should be performed for all earthquakes concerned, but practically it is done for earthquakes with magnitude larger than a certain fixed level. Therefore such summation is meaningless unless it is proved that the sum of E^{γ} for earthquakes with magnitude less than the fixed level is sufficiently small. This condition, however, is not always satisfied for any combination of the conceivable values of γ, β, and b.

The frequency distribution of strain release X is given by

$$n(X) = \frac{c}{\gamma} X^{-\kappa-1}, \tag{8}$$

and

$$\kappa = b/\gamma\beta \tag{9}$$

where $n(X)dX$ is the number of earthquakes with strain release between X and $X+dX$ and the unit of strain is so chosen that $X=E^{\gamma}$. Since b, β, and γ vary in the following ranges,

$$\left.\begin{aligned} 0.4 \leq b \leq 1.5, \\ 1.4 \leq \beta \leq 2.0, \\ 0.5 \leq \gamma \leq 1.0, \end{aligned}\right\} \tag{10}$$

κ ranges from 0.2 to 2.1. However the type of the distribution function of X is the same, regardless to the theories described above. In Table 1 the values of κ for various values of b, β, and γ are shown.

Table 1. Values of $\kappa=b/\gamma\beta$ for various values of b, γ, and β.

$\beta=1.5$					$\beta=2.0$				
b \ γ	1/2	2/3	3/4	1	b \ γ	1/2	2/3	3/4	1
0.5	$0.6\dot{6}$	0.50	$0.4\dot{4}$	$0.3\dot{3}$	0.5	0.50	0.375	$0.3\dot{3}$	0.25
$0.6\dot{6}$	$0.8\dot{8}$	$0.6\dot{6}$	0.592	$0.4\dot{4}$	$0.6\dot{6}$	$0.6\dot{6}$	0.50	$0.4\dot{4}$	$0.3\dot{3}$
0.75	1.00	0.75	$0.6\dot{6}$	0.50	0.75	0.75	0.5625	0.50	0.375
1.0	$1.3\dot{3}$	1.00	$0.8\dot{8}$	$0.6\dot{6}$	1.0	1.00	0.75	$0.6\dot{6}$	0.50
1.2	$1.6\dot{6}$	1.20	$1.1\dot{1}$	$0.8\dot{8}$	1.2	1.20	0.9375	$0.8\dot{3}$	0.625
1.5	2.00	1.50	$1.3\dot{3}$	1.00	1.5	1.50	1.125	1.00	0.75

For actual earthquake data, the observed frequency of earthquakes at each level of energy or strain release is not in perfect agreement with that given by Equation (3) or (8), since the frequency fluctuates from various causes. In this paper the fluctuation is considered to be only due to the random sampling from a population whose frequency distribution of energy or strain release is expressed by one of the above equations. According to this assumption, the statistical nature of the energy and strain release for earthquake groups has been investigated. This study will provide a basic data for understanding the statistical characteristics of the energy and the strain release in earthquake sequences, such as general earthquakes occurring in limited regions, aftershock sequences, earthquake swarms, etc.

2. Some characteristics of the distribution function of energy and strain release

The cumulative frequency $N(X)$, i.e., the frequency of earthquakes with energy or strain release equal to or larger than X is given by

$$N(X)=\int_X^\infty n(X)\,dX=C\,X^{-\kappa}, \tag{11}$$

where

$$C=c/\gamma\kappa. \tag{12}$$

If we denote such a value of X that

$$N(X) = \nu, \qquad \nu = 1, 2, 3, \cdots\cdots \tag{13}$$

by X_ν^*,

$$X_\nu^* = \nu^{-1/\kappa} X_1^*, \tag{14}$$

and

$$X_1^* = C^{1/\kappa}. \tag{15}$$

Also

$$N(X) = (X/X_1^*)^{-\kappa}. \tag{16}$$

When the earthquakes in an earthquake group are arranged in order of magnitude, and the energy or the strain release of the nth earthquake is denoted by X_n, X_n is not always equal to X_n^*, but it has a certain distribution $g_n(X_n)$. The probability that the X-value for the nth earthquake falls in the range between X_n and $X_n + dX_n$ takes the form

$$g_n(X_n)\, d X_n = \frac{\kappa}{(n-1)!} \left\{ n \left(\frac{X_n}{X_n^*} \right)^{-\kappa} \right\}^n \exp\left\{ - n \left(\frac{X_n}{X_n^*} \right)^{-\kappa} \right\} \frac{d X_n}{X_n}. \tag{17}$$

This equation is derived from Equation (47) in Utsu's paper[10] or Equation (39) in Utsu's paper[2].

The mean or the expectency of X_n is given by

$$E[X_n] = \left\{ \Gamma\left(n - \frac{1}{\kappa}\right) / \Gamma(n) \right\} X_1^*. \tag{18}$$

When $\kappa \leq 1$ this equation is valid for $n \geq 2$. Since

$$\Gamma\left(n - \frac{1}{\kappa}\right) / \Gamma(n) \to n^{-1/\kappa}, \qquad (n \to \infty) \tag{19}$$

refering to Equation (14) we obtain a natural result

$$E[X_n] \to X_n^*. \qquad (n \to \infty) \tag{20}$$

The cumulative distribution function of X_n, $G_n(X_n)$, is given by

$$G_n(X_n) = \int_0^{X_n} g_n(X_n)\, d X_n = \Gamma(n, \lambda)/\Gamma(n), \tag{21}$$

where

$$\lambda = n (X_n/X_n^*)^{-\kappa}. \tag{22}$$

Putting $\log X_n = \gamma(\alpha + \beta M_n)$, and $n=1$, we obtain an expression of $G_1(M_1)$

$$\log(-\log G_1(M_1)) = -b(M_1 - M_1^*) - \log(\ln 10), \tag{23}$$

which is equivalent to Epstein-Lomnitz's equation.[16]

If we deal with earthquakes whose energy or strain release is X_s or

larger (X_s can be chosen arbitrarilly), the probability that the energy or the strain release of an earthquake falls in the range between X and $X+dX$ is given by

$$\begin{aligned} p(X)\,dX &= \kappa\left(\frac{X}{X_s}\right)^{-\kappa-1}\frac{dX}{X_s}, \qquad (X \geq X_s) \\ p(X)\,dX &= 0. \qquad (X < X_s) \end{aligned} \tag{24}$$

This probability density function has somewhat different nature from ordinary distribution functions in statistics, since the kth moment of X is not existent for any integer k equal to or larger than κ. When $\kappa<1$, even the first moment, i.e., the mean of X is not existent. The nature of the function $p(X)$ differs between three cases $\kappa<1$, $\kappa=1$, and $\kappa>1$. First, we shall treat the case of $\kappa<1$.

3. Distribution of the total energy or the total strain release when $\kappa<1$

Let us consider a population of earthquakes whose energy or strain release distribution is represented by the probability density function (24). The sum of the energy or the strain release for samples of size N from this population

$$X_T = \sum_{i=1}^{N} X_i \tag{25}$$

must have a distribution with probability density function denoted by $f_N(X_T)$. It seems difficult to obtain the analytical expression of this function in a simple form, since the ordinary method is not applicable to this case owing to the nature of the function $p(X)$ mentioned above. However the following characters of the function $f_N(X_T)$ can be deduced from Equation (10).

(1) $$f_N(X_T) = 0 \qquad \text{for} \qquad X_T < N X_s. \tag{26}$$

(2) $$\left.\begin{aligned} f_1(X_T) &= p(X_T), \\ f_2(X_T) &= f_1 * p(X_T), \\ &\cdots\cdots\cdots \\ f_N(X_T) &= f_{N-1} * p(X_T), \end{aligned}\right\} \tag{27}$$

where an asterisk means the convolution.

(3) Expectency of X_T, $E[X_T] = \int_{-\infty}^{\infty} X_T f_N(X_T)\,dX_T$ is not existent.

(4) For values of X_T far larger than its mode,

$$f_N(X_T) \doteqdot N p(X_T) = \kappa C X_T^{-\kappa-1}, \tag{28}$$

where

$$C = N X_s^{\kappa}. \tag{29}$$

This is because a very large X_T value is attained mostly when one of the N values of X is very large (we denote this X_e). In this case the sum of the other $N-1$ values of X are very small as compared with X_e. The probability of the occurrence of such a case is $Np(X_T)$. The probability that two or more earthquakes have large and comparable X-values is very small as compared with $Np(X_T)$.

(5) The function $f_N(X_T)$ has three parameters κ, X_s, and N. However for large values of N, this function is approximated by a function with two parameters κ and C. This is due to the following reason.

If a group of N earthquakes with $X \geq X_s$ includes N' earthquakes with $X \geq X'_s$, we easily see from Equation (8) that

$$N' X_s'^{\kappa} = N X_s^{\kappa} \ (= c/\gamma\kappa = C) \tag{30}$$

for large values of N and N'. When $\kappa < 1$, the sum of X-values of the N earthquakes is approximately equal to the sum of X-values of the N' earthquakes, i.e., $X_T \doteqdot X'_T$, since as explained later the sum of X-values of relatively small numbers of large earthquakes in a group forms the most part of the total sum of X-values of the group. Therefore the distribution function of X_T is characterized by C and κ for large values of N, and is not dependent on N.

It follows from the above statement that if the cumulative distribution function $F_N(X_T)$ $(=\int_0^{X_T} f_N(X)dX_T)$ is plotted against $\log X_T$, the shape of the curve is the same without regard to N and X_s, and only the position of the curve differs according to the value of C, which is an index of over-all seismic activity of the group. It is possible to make $C^{1/\kappa}=1$ by a proper choise of the unit of X. By using such a unit, the cumulative function for large samples is uniquely determined for each value of κ.

Figure 1 shows the curve for this function for $\kappa=2/3$ calculated by Monte Carlo method (*cf.* Chapter 5).

Strictly speaking, the total energy or the total strain release for an earthquake group should be expressed by

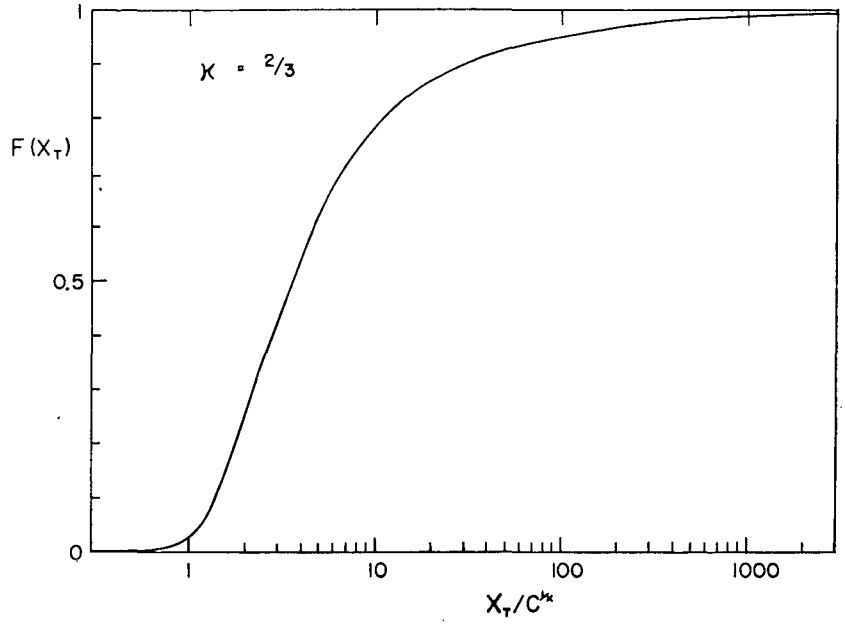

Fig. 1. Cumulative probability function $F(X_T)$ of the sum of X-values of N earthquakes plotted against $X_T/C^{1/\kappa}$ ($C^{1/\kappa}=N^{1/\kappa}X_s=X_1{}^*$), when $\kappa=2/3$ and $N\gg 1$.

$$X_T=\sum_{i=1}^{\infty} X_i\,. \tag{31}$$

However when the number N of earthquakes with $X\geq X_s$ is sufficiently large,

$$X_T \doteqdot \sum_{i=1}^{N} X_i\,. \tag{32}$$

because $\sum_{i=N+1}^{\infty} X_i$ is negligibly small as compared with X_T, as

$$\sum_{i=N+1}^{\infty} X_i \doteqdot \int_0^{X_s} X_n(X)\,dX=\frac{c}{\gamma(1-\kappa)}X_s{}^{1-\kappa}$$

$$=\frac{\kappa X_1{}^*}{(1-\kappa)N^{(1/\kappa)-1}}\to 0\,. \qquad (N\to\infty) \tag{33}$$

Therefore, when $\kappa<1$, small earthquakes, though they are numerous, do not account for a large percentage of the total energy or strain release. Only a small number of largest earthquakes in the group occupies a greater part of the total.

Figure 2 is a graph of the cumulative distribution $Q_n(r)$ of the ratio r of the sum of X-values for the largest n earthquakes to the total sum of X-values for the group,

$$r = \sum_{i=1}^{n} X_i / X_T \,. \tag{34}$$

In this figure $Q_n(r)$ claculated by Monte Carlo method is plotted against r for $n=1$, 2, 3, 5, and 10. This figure shows, for example, that for a group of earthquakes with $b=1.0$ and $\beta=1.5$, the propability that the largest five earthquakes account for more than 50% of the total energy of the group is 87%, etc.

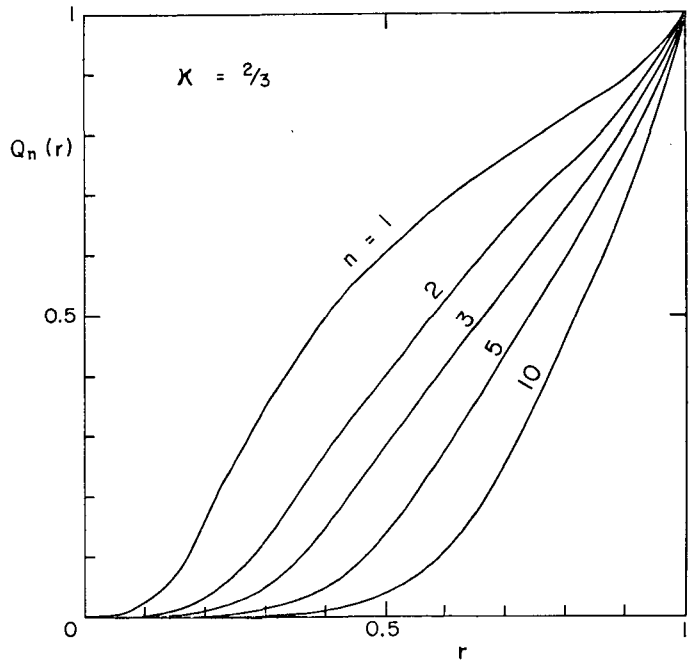

Fig. 2. Cumulative probability function $Q_n(r)$ of the ratio r of the sum of X-values of the largest n earthquakes to the total sum of X-values of all earthquakes for $\kappa=2/3$ and $n=1$, 2, 3, 5, and 10.

4. Summation of energy or strain release when $\kappa>1$ and $\kappa=1$

When $\kappa>1$ the mean or the expectency of X is expressed by

$$E[X] = \int_{X_s}^{\infty} X p(X)\, dX = \frac{\kappa}{\kappa - 1} X_s \,. \tag{35}$$

Therefore the mean or the expectency of the sum of X-values for N earthquakes is

$$E[X_T] = \frac{\kappa}{\kappa - 1} N X_s . \tag{36}$$

If a group of earthquakes contains N earthquakes with $X \geq X_s$, and N' earthquakes with $X \geq X_s'$, the expectency of the sum of X-values for the N' earthquake is

$$E[X_T'] = \frac{\kappa}{\kappa - 1} N' X_s' . \tag{37}$$

By using Equation (30) which is also valid when $\kappa > 1$, we obtain

$$\frac{E[X_T']}{E[X_T]} = \left(\frac{X_s}{X_s'}\right)^{\kappa-1}. \tag{38}$$

This equation means that the expectency of the sum of X-values increases unlimitedly with a decrease in the lower limit of X or magnitude above which all earthquakes are included in the summation. Therefore the choise of the lower limit of magnitude seriously affects the result. This makes a remarkable contrast to the equation $X_T \fallingdotseq X_T'$ when $\kappa < 1$, in that case smaller earthquakes do not constitute a large percentage of the total sum of X-values.

From the above description, it is apparent that the attempt to obtain the total strain release by summing up the X-values is not supported when $\kappa > 1$. On the average, such a sum is proportional to the number of earthquakes included in the summation.

When $\kappa = 1$, the integral

$$\int_{\varepsilon}^{\eta} X\, n(X)\, dX = \frac{c}{\gamma} \ln(\eta/\varepsilon) \tag{39}$$

does not converge in both cases of $\eta \to \infty$ and $\varepsilon \to 0$. Therefore it is difficult to interprete the meaning of the sum of X-values in this case.

5. Energy and strain release curves for earthquake sequences — Simulation and actual seismic data

In an earthquake sequence, let $n(t)dt$ denotes the frequency of earthquakes with magnitude larger than a fixed value M_s occurring in the time interval between t and $t+dt$, and $N(t)$ the cumulative frequency, i.e., $N(t) =$

$\int_0^t n(t)\,dt$. We now deal with the sum of X-values, $X_T(t)$, for earthquakes with $M \geq M_s$ occurring in the period between $t=0$ and t. The total energy or the total strain release until time t is approximated by $X_T(t)$, when $\kappa < 1$.

To demonstrate the nature of $X_T(t)$ in connection with the discussions in the previous chapters, we first use a series of experimentally generated earthquakes from random digits. These earthquakes are random samples from a population whose energy distribution is represented by Equation (3) with an index of $b/\beta = 2/3$. For the sake of simplcity, we assume that the distribution of earthquakes with respect to time is uniform, i.e., $n(t) =$ constant. This distribution can be converted into any distribution of $n(t)$ by a proper transformation of the time axis.

A series of 25,000 earthquakes with $E \geq 1$ (in arbitrary unit) is divided into 500 groups each of which contains 50 earthquakes. The unit of time is taken in such a way that 50 earthquakes occur in a unit time interval. In Figures 3 and 4 curves marked with 1 show the cumulative sum of E (i.e., X when $\kappa = 2/3$) and $\sqrt{E}$ (i.e., X when $\kappa = 4/3$) respectively for earthquakes with $E \geq 1$. The curve for E is largely deviated from a straight line, but the curve of $\sqrt{E}$ is nearly a straight line, or $X_T(t) \propto n(t)$ for $\kappa = 4/3$.

The curve marked with 10^3 in Figure 3 is constructed using earthquakes with $E \geq 10^3$. Although the number of data is reduced to 1/100, the curve remains almost the same. The number of data, 250, is enough for representing the energy release characteristics of the whole series. In Figure 4, the curves marked with $10^{3/2}$ and $10^{9/4}$ are drawn using the data of $E \geq 10^{3/2}$ and $E \geq 10^{9/4}$ respectively. The numbers of data are about 1/100 and 1/1000 of the data used for the curve 1. In this case the three cruves are quite different. To compare the shape of them, the curves $10^{3/2}$ and $10^{9/4}$ are drawn again in the same figure (broken lines) with the same average slope as that of the curve 1 using different scales in the ordinate. It is seen from this figure that the cumulative curve of X deviates widely from the staight line when the number of data is small, but it approaches a straight line or, generally speaking, the cumulative frequency curve $N(t)$, as the number of data increases.

It is noticed during an examination of many cumulative curves of $\sqrt{E}$ published hitherto that the cruves constructed of a large number of earthquakes (say a few hundreds or more) have in general smoother shape. For exmaple, curves for the aftershock sequences of the Kamtchatka earthquake of 1952 (Båth and Benioff[17]), the Aleutian earthquake of 1957 (Duda[18]), and the

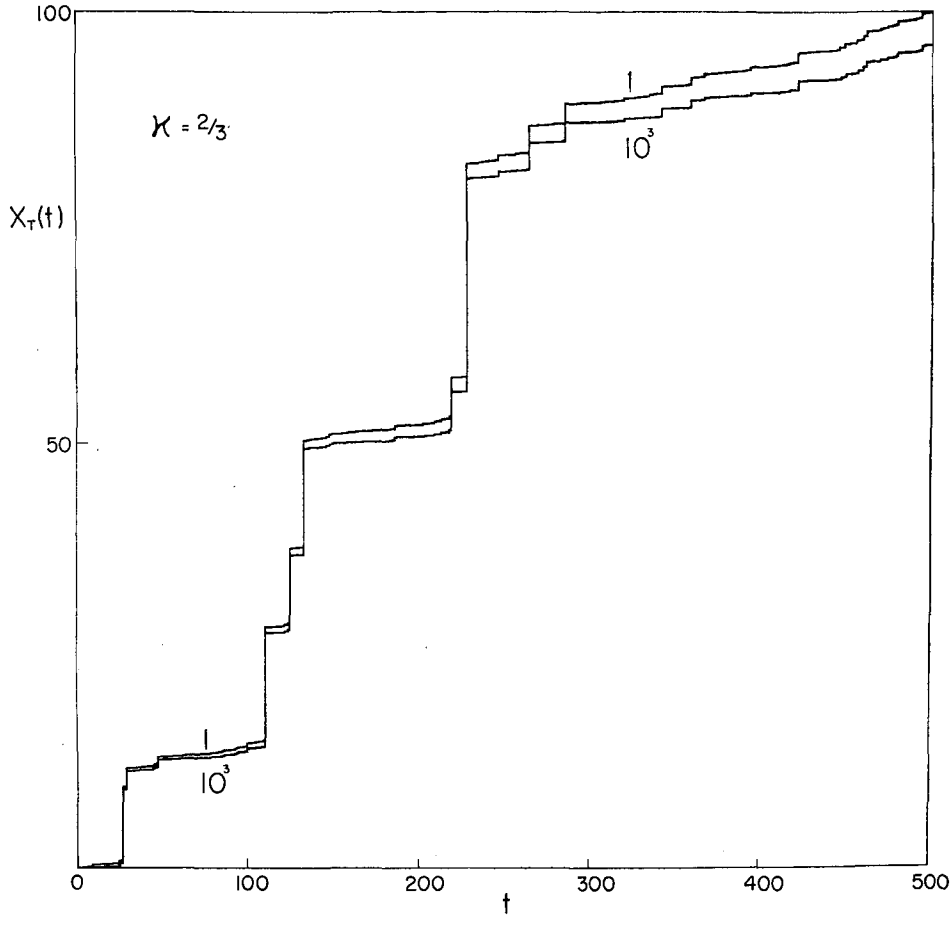

Fig. 3. Cumulative sum of X-values when $\kappa=2/3$ (or cumulative sum of energy release when $b/\beta=2/3$) for numerically generated earthquakes. A unit of t corresponds to 50 earthquakes. Curve 1: earthquakes with X or $E\geq 1$, curve 10^3: earthquakes with X or $E\geq 10^3$. Scale of the ordinate, 100: 1.14×10^7.

Southern Kurile Island earthquake of 1963 (Santo[19]) are fairly smooth and resemble the curves for cumulative distribution of aftershock frequency usually observed for aftershock sequences.[10]

The curves constructed of fewer, data for example those, for earthqaukes near Hawaii (Furumoto[20]), earthquakes in the Indian Ocean (Stover[21]), some aftershock sequences studied by Benioff[6], etc., are not so smooth, but

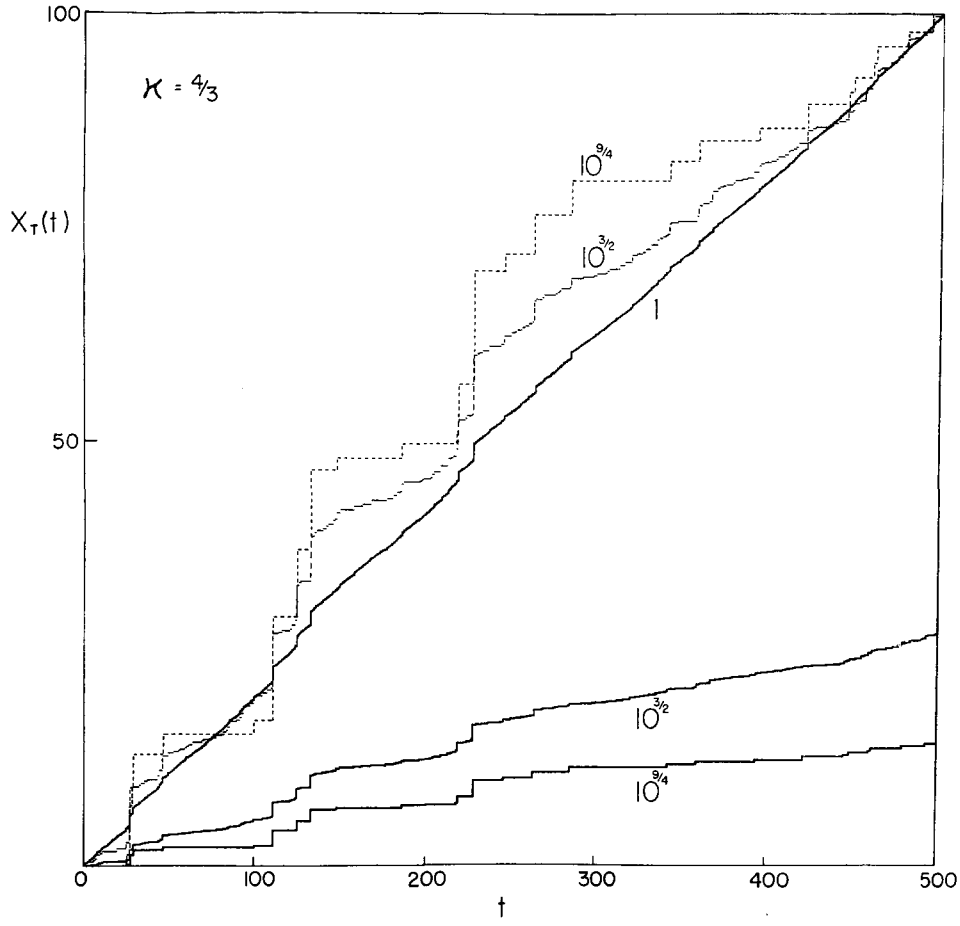

Fig. 4. Cumulative sum of X-values when $\kappa=4/3$ (or cumulative sum of square-roots of energy when $b/\beta=2/3$) for the same earthquake group as in Figure 3. Curve 1: earthquakes with X or $\sqrt{E}\geq 1$, curve $10^{3/2}$: earthquakes with X or $\sqrt{E}\geq 10^{3/2}$, curve $10^{9/4}$: earthquakes with X or $\sqrt{E}\geq 10^{9/4}$. Scale of the ordinate, 100: 8.74×10^4, for the thick curves. The broken curves are normarized ones.

exhibit remarkable jumps as usually seen in curves of cumulative energy release, like Figure 3.

A cumulative energy release curve for Japanese earthquakes during 1885–1963 presented by Tsuboi[22] (Figure 5) shows a considerable uniformity in energy release with respect to time. The energy of the largest shock constitutes about 11% of the total energy release in the period. The probability that

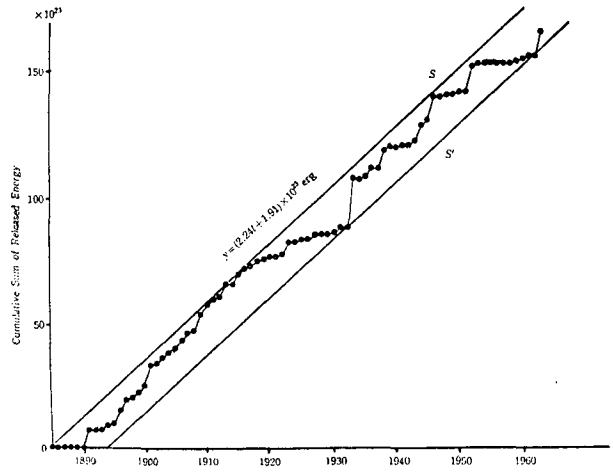

Fig. 5. Cumulative sum of energy released by earthquakes in and near Japan (1885–1963), after C. Tsuboi[22].

the largest shock accounts for 11% or less of the total energy is about 3% from Figure 2 as $b=1.03$ for these earthquakes and $\beta=1.5$ has been adopted. This means that such an energy release curve with small abrupt increments is seldom realized. The existence of an upper limit of magnitude of the earthquakes may partly explain the uniformity.

6. Conclusion

In this paper the statistical nature of the sum of the γth power of energy E^{γ} of earthquakes has been studied, assuming the magnitude-frequency relationship $\log n(M)=a-bM$ and the energy-magnitude relationship $\log E=\alpha+\beta M$. When $\kappa(=b/\beta\gamma)<1$, relatively small number of largest earthquakes occupies a greater part of the total sum of E^{γ} of the earthquakes. When $\kappa>1$, however, the sum depends largely on the lower limit of magnitude above which all earthquakes are included in the summation. Special care is required in the interpretation of so-called strain release curves (usually values of $\beta=1.5$ or 1.8 and $\gamma=1/2$ are adopted and $b=0.8\sim1.0$, then $\kappa>1$) which appear in many seismological investigations.

Appendix

Here we shall consider a relation between the deformation of a seismic region caused by an earthquake in the region and the energy of the earthquake under the following assumptions.

(1) The energy E released by an earthquake is proportional to the source volume V of the earthquake, i.e.,

$$E \propto V . \tag{40}$$

(2) The source volume is proportinal to the cube of the linear dimension L of the source, i.e.,

$$V \propto L^3 . \tag{41}$$

(3) The deformation takes place in one direction only at least in the region under consideration.

The deformation dL of a source volume in this direction is proportional to the linear dimension L of the source, since the strain energy release per unit volume associated with the linear deformation dL is proportinal to $(dL/L)^2$,and is proportional to E/V, a constant. Accordingly

$$d\,L \propto L \propto V^{1/3} \propto E^{1/3} . \tag{42}$$

If the linear dimension of the whole seismic region is A, a set of $(A/L)^2$ earthquakes with deformation dL causes a deformation dL of the whole region. Therefore the contribution from each earthquake to the deformation of the whole region is

$$S = d\,L/(A/L)^2 \propto E . \tag{43}$$

This means that the deformation of a seismic region due to an earthquake with energy E is proportional to E itself on the average.

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