Time and Space Distribution of Deep Earthquakes in Japan

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Abstract

Earthquakes with focal depth larger than 140 km which occurred in and near Japan from 1961 through 1967 have been investigated for a tendency of clustering in time and space. It is shown that the occurrence of the deep earthquakes is not represented by a stationary random process in time as well as in space. It is also shown that the deep earthquakes have a tendency to cluster in time and space from the following simple test. The number of pairs of earthquakes with epicenter interval less than \( r \) and time interval less that \( t \) is counted for all combinations of \( r = 100 \) km, 150 km, and 200 km, and \( t = 15 \) days, 30 days, and 90 days. The number is then compared with the frequency distribution of the numbers of pairs counted for 150 series of events in each of which the total number, epicenter coordinates, and origin times remain the same as those for the originally observed series of deep earthquakes but the combinations of the epicenters and origin times are randomly changed. The result indicates the occurrence of some clusters which is not attributable to chance with high probability.

1. Introduction

It is generally recognized that shallow earthquakes have a tendency to cluster in space and time. Aftershock sequences and earthquake swarms are typical of such clusters. However, this tendency is not observed for most deep earthquakes. Absence of aftershock sequences and earthquake swarms has been considered to be a character of deep earthquakes. Most deep earthquakes in Japan appear to be mutually independent events, though some cases of sequential occurrence of deep earthquakes, or of deep and shallow earthquakes, or of deep earthquakes and volcanic eruptions within relatively short intervals of time and space have been pointed out in literature.\(^1\)\textsuperscript{--}\(^4\)

Recently, Isacks et al.\(^5\) studied the clustering of deep and shallow earthquakes in the Fiji-Tonga-Kermadec region, and found that for some deep earthquakes the clustering occurs in the form of multiplets, i.e., a small number of events closely grouped in space and time. Utsu\(^6\), in his study
of the seismicity of Hokkaidó, mentioned that deep earthquakes in the Hokkaidó region do not occur completely at random but they show a slight tendency to occur within certain intervals of space and time, say, 150 km and three months. This is somewhat different kind of clustering from that found by Isacks et al. in the Fiji region because of large separations in space and time (especially in time), but this is somewhat similar to the clustering of deep earthquakes in the New Zealand region reported by Vere-Jones and others. In the present paper, the clustering of deep earthquakes in and near Japan is studied in more detail.

2. Data

In this paper earthquakes deeper than 140 km are temporally classified as deep earthquakes. 198 deep earthquakes have been located by the Japan Meteorological Agency using an electronic computer in the area shown in Figure 1 during the seven year interval from 1961 through 1967. The lower limit of magnitude of these earthquakes is around 4\(\frac{1}{2}\), but it may somewhat varies with the location of hypocenter. Since the non-uniformity of the lower limit of magnitude does not affect the results of this paper seriously, all these earthquakes except three near Siberia and one far off the Pacific coast of eastern Honshú are used in this study.

Figure 1 shows the location of these earthquakes. Numerals attached to each epicenter (except the four omitted) are the serial numbers of earthquakes in order of the time of occurrence. Systematic errors may exist in the absolute location of these epicenters, resulting from the anomalous upper mantle structure under the island arcs and other causes. However the relative location between near-by epicenters is believed to be fairly accurate. As seen from Figure 1, there is a gap in the distribution of epicenters in Tōhoku district near 39°N parallel. Therefore the 194 earthquakes can conveniently divided into two groups, the N-group (72 earthquakes) and the S-group (122 earthquakes).

3. Distribution in time

First, the distributions of these earthquakes in time and space are studied separately. Three kinds of graphs (Figures 2–4) are widely used to indicate the characteristics of the time distribution of earthquakes. The variations of semianual frequencies are shown in Figure 2. Increased activities are
Fig. 1. Distribution of deep earthquakes ($h>140 \text{ km}$) in the region of Japan during 1961-1967. The numerals indicate the order of occurrence. Arrows indicate pairs of earthquakes which occurred within intervals of $r=150 \text{ km}$ and $t=30 \text{ days}$. Broken lines indicate additional pairs of earthquakes when $r$ is extended to 200 km.
observed in 1964-65 for the N-group and in 1962-64 for the S group. This is not due to the random fluctuations, since a $x^2$-test rejects the hypothesis of the uniformity of the frequencies at the 5% and 0.5% levels of significance for the N- and S-groups respectively.

Open circles in Figure 3 represent the frequencies of the time interval $\tau$ between two successive earthquakes in each group plotted against $\tau$ with a logarithmic ordinate. If the earthquake occurrence is stationary and random with respect to time, the probability density function of $\tau$ takes the form

$$\phi(\tau) = \nu e^{-\nu \tau}$$  \hspace{1cm} (1)$$

where $\nu$ is the average number of earthquakes per unit time interval (five days in the present case). The straight lines in Figure 3 represent $N\phi(\tau)$ with $\nu = 5N/2556$ and $N = 72$ (N-group) or 122 (S-group). Significant departure of the open circles from the straight lines is not observed for the present case. A $x^2$-test can not reject the hypothesis that the time interval is exponentially distributed at the 5% significance level for both N- and S-groups. The cumulative frequencies, i.e., the frequencies of the time intervals larger than $\tau$ are also plotted in the same figure as solid circles, which must fit a straight line with slope $-\nu$ if the distribution of $\tau$ follows the exponential law (1).

In Figure 4 the frequencies of the monthly number $n$ of earthquakes

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Variations of semi-annual number of deep earthquakes in each group.}
\end{figure}
Fig. 3. Frequency distribution of time interval \( \tau \) between two successive deep earthquakes. Open circles: Frequency per five day interval of \( \tau \). Solid circles: Cumulative frequency of \( \tau \).

Fig. 4. Frequency distribution of monthly number of deep earthquakes (solid circles) as compared with the corresponding Poisson distribution (open circles).

are plotted against \( n \) as solid circles. For stationary and random events, \( n \) should have a Poisson distribution

\[
p(n) = \nu^n e^{-\nu}/n!
\]

where \( \nu \) is the average number of events per unit time interval (one month in the present case). Variations of \( Np(n) \) with \( n \) are shown by open circles. Significant differences between the two distributions are not seen from this...
A $x^2$-test can not reject the hypothesis that the observed frequency $n$ fits a Poisson distribution at the 5% level of significance for both N- and S-groups.

There are many other independent methods of testing the stationarity and randomness of the earthquake occurrence. The one which is based on the theory of runs is applied to the present data. The series of origin times $a_1, a_2, \ldots, a_N$ and its reversed series $b_1, b_2, \ldots, b_N$ ($b_i = T - a_{N-i+1}$, $T$ is the length of the whole period) are mixed and arranged in the ascending order of magnitude, and the number of runs $R$ is counted.* If the $N$ origin times are randomly distributed in the whole period, the number of runs is approximately normally distributed for large $N$, and its expectancy and variance are given by

$$E(R) = N + 1,$$  \hspace{1cm} (3)

and

$$V(R) = N.$$  \hspace{1cm} (4)

For the N-group, $R = 62$, $E(R) = 73$, and $V(R) = 72$, and for the S-group, $R = 106$, $E(R) = 123$, and $V(R) = 122$. The ratio $r_0 = (E(R) - R)/\sqrt{V(R)}$ becomes 1.30 and 1.54 respectively. For stationary and random events the probability that the ratio $r_0$ exceeds 1.30 and 1.54 is 0.097 and 0.062 respectively, which is rather small to accept the hypothesis of the stationary and random occurrence of the deep earthquakes.

Although the material used here is very limited, the time distribution of the deep earthquakes may not represented by a stationary random process, but its character is somewhat different from shallow earthquakes for which marked grouping is generally observed over short periods of time, say, less than ten days.\(^9\)\(^{-10}\)

4. Distribution in space

It is evident from Figure 1 that the deep earthquakes are not randomly distributed in space. Open circles in Figure 5 show the frequency distribution of the space interval $s$ between two neighboring earthquakes plotted against $s$. Solid circles represent its cumulative frequency distribution. Space intervals are measured following Tomoda's method.\(^11\) The deep earthquake zones (two areas N and S enclosed by dotted lines in Figure 1) are divided into strips having a width of 100 km by several straight lines parallel to the extension of the seismic zones. One-dimensional space coordinates are fixed along these strips and the space interval is taken as the difference in coordinate

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* For example a series $a_1, a_2, b_1, a_3, b_2, a_4, b_3, a_5$ consists of six runs $aa, b, a, b, a, bb$, then $R = 6$ and $N = 4$. Note that the runs are anti-symmetric in the present problem.
between two neighboring epicenters in each strip. If the earthquake occurrence is stationary and random with respect to space, the distribution of \( s \) must be an exponential one. The plotted points in Figure 5 roughly agree with an exponential distribution in the range of \( s < 80 \) km. The inverse power law adopted by Tomoda\(^{11}\)

\[
f(s) = k s^{-q}
\]

fits the present data less well.
The frequency distribution of the number $n$ of earthquakes per unit area is plotted in Figure 6 as open circles. The areas N and S in Figure 1 are divided into squares of 100 km×100 km and the number of epicenters falling in each square is counted. The distribution considerably differs from a Poisson distribution. An exponential distribution or a geometrical distribution proposed by Tamaki13) seems to be a better approximation rather than an inverse power distribution described by Suzuki and Suzuki13) Anyway this figure shows that the deep earthquakes in Japan do not occur at random but have a tendency of grouping in space.

5. Clustering in time and space

There are many instances that two deep earthquakes occur within relatively short intervals of time and space. In Figure 1 pairs of earthquakes which occurred within intervals of 30 days and 150 km are indicated by arrows connecting the corresponding epicenters. Table 1 shows the numbers of such pairs of earthquakes with time interval less than $t$ and space interval less than $r$ for various combinations of $t$ and $r$. However the occurrence of such pairs does not necessarily indicate that the two earthquakes are mutually dependent. Even if all earthquakes are independent events, a number of pairs may be generated only by chance. A tendency of clustering in space and time must be recognized only when the observed number of such pairs is significantly larger than that expected from the accidental occurrence of the events.

Since it has been shown that the occurrence of deep earthquakes is not completely at random in space as well as in time, it is not appropriate for the present data to compare with a model of purely random occurrence, then the following simple statistical experiment is conducted. The original series of earthquakes took place at respective epicenters in such an order as indicated in Figure 1. In the experiment this order is changed randomly 150 times.

Table 1. Numbers of pairs of deep earthquakes occurring within intervals of $r$ and $t.$

<table>
<thead>
<tr>
<th>$r$</th>
<th>N-group</th>
<th>S-group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15 days</td>
<td>30 days</td>
</tr>
<tr>
<td>100 km</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>150 km</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>200 km</td>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>
while the epicenter coordinates and origin times are kept unchanged. For 150 series of events constructed in this way, the number of pairs of events with space and time intervals less than $r$ and $t$ are counted for all combinations of $r$ and $t$ listed in Table 1.

The frequency distributions of the number of pairs for every combination of $r$ and $t$ are plotted in Figure 7 a, b in the form of cumulative probability. The vertical bars in the figure indicate the observed numbers of pairs $N_p$. If the probability corresponding to $N_p$ is $p$, the probability that $N_p$ pairs or more are generated only by chance is $1 - p$. For $r = 150$ km and $t = 30$ days, the probability that all the observed pairs are generated by chance is less than 10%. The hypothesis that all the pairs are attributed to accidental occurrence is rejected at a level of significance a little less than 10%. However it is also true that a fairly large proportion of the pairs may be produced by chance. Generally speaking, the mutual dependence of the deep earthquakes is not very strong, though the present test is rather too severe for demonstrating the clustering tendency. For example, if the similar test is performed for the combined group (N+S), the above mentioned hypothesis of accidental occurrence will be rejected more easily.

The directions of the arrows in Figure 1 seem to be distributed fairly systematically. For most of the experimentally generated series of events such systematic distributions are not observed. The comparison of Figure 1 with the distribution of the horizontal components of maximum pressure obtained in the focal mechanism studies of deep earthquakes by Honda et al.\cite{14} suggests a probable relationship between them, but the data are not sufficient to draw any definite conclusion.

It should be added that no regularities are evident in the difference in magnitude or focal depth between the first and second events in each pair of earthquakes. The first one may have a larger magnitude or a larger focal depth than the second, and vice versa.

6. Conclusion

The time and space distributions of recent deep earthquakes in the region of Japan have been studied statistically. It has been shown that the deep earthquakes have a weak tendency to cluster within certain intervals of space and time. More data must be collected to understand the nature of clustering and its relation to the physical processes occurring under the island arcs.
Fig. 7. (a)
Fig. 7. The cumulative distribution of the number of pairs occurring within intervals of $r$ and $t$ (values of $r$ and $t$ are indicated in the figure) obtained from 150 computer-generated series of events whose epicenter coordinates and origin times are the same as actually observed earthquakes but their combinations are changed randomly. Vertical bars indicate the actually observed number of pairs (cf. Table 1).
References