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ABSTRACT

Weyl semimetals (WSMs) are classified into type-I and type-II, depending on the magnitudes of the inclination of Weyl cones. It is known that these WSMs show negative longitudinal magnetoresistance originating from chiral anomaly. Moreover, we have recently revealed that type-II WSMs show positive longitudinal magnetoresistance originating from chiral anomaly. The negative longitudinal magnetoresistance in type-I WSMs can be explained utilizing the conventional formula of the chiral anomaly, which does not have the term related to the inclination of the Weyl cones. However, we cannot explain both the positive and the negative longitudinal magnetoresistance in type-II WSMs utilizing it. Therefore, in this paper, we derive the general formula including the term related to the inclination of the Weyl cones in order to explain straightforwardly the positive and the negative longitudinal magnetoresistance in type-II WSMs. Also, we consider both cases where a pair of the Weyl cones are tilted in the same direction (positive tilt chirality) and toward (or against) each other (negative tilt chirality) in order to investigate the influence of the direction to which the Weyl cones are tilted. As a result, we find that in the negative tilt chirality, the general formula is strongly affected by the inclination. These results suggest that we can estimate whether the WSMs show the positive or the negative longitudinal magnetoresistance using the general formula from the information of their tilt chirality and the magnitudes of the inclination of the Weyl cones.

Many researchers have paid much attention to topological materials[1–9] and topological phenomena[10–15] from the viewpoint of their application to spintronics devices.[16–20] Weyl semimetals (WSMs) are one kind of topological materials, which are studied extensively due to peculiar transport properties originating from the chiral anomaly. It is well known that the WSMs have linear energy dispersion, called Weyl cones, where electrons have the chirality. The WSMs have some pairs of the Weyl cones with opposite chirality. These WSMs are classified into two types, type-I and type-II, depending on the magnitudes of the inclination of the Weyl cones.[21] Type-II WSMs have electron and hole pockets due to their inclination. It is reported by experiments that TaAs, TaP, NbAs, and NbP are type-I WSMs[22–25] and WTe_2, $MoTe_2$, and $YbMnBi_2$ are type-II WSMs.[26–28] Also, it is known that these WSMs show the negative longitudinal magnetoresistance originating from the chiral anomaly.[25,29,30] In addition, recently we have revealed that type-II WSMs show not only the negative but also the positive longitudinal magnetoresistance originating from the chiral anomaly in our paper of *Journal of Applied Physics*.[31] The negative longitudinal magnetoresistance in type-I WSMs can be explained by the conventional formula of the chiral anomaly developed by Nielsen and Ninomiya,[32] which does not include the term related to the inclination of the Weyl cones. However, the positive and the negative longitudinal magnetoresistance in type-II WSMs cannot be explained by it. This is because the Weyl cones in type-II WSMs incline. Thus, in the previous paper, we have explained the positive and the negative longitudinal magnetoresistance in type-II WSMs by considering both the chiral magnetic effect and the peculiar zeroth Landau energy-levels emerging around the Weyl cones. However, it is desirable to understand the above phenomena by applying the general formula (if we can get it) to type-II WSMs. Although other authors have already derived the general formula,[33] their formula does not agree with our result and cannot describe the signs of the longitudinal magnetoresistance in type-II WSMs. Therefore, in this paper, we derive the general formula of the chiral anomaly including the term related to the inclination of the Weyl cones and explain straightforwardly the positive and the negative longitudinal magnetoresistance in type-II WSMs. Also, we consider the cases where a pair of the Weyl cones are tilted in the same direction (positive tilt chirality) and toward (or against) each other (negative tilt chirality) in order to investigate the influence of the inclination of the Weyl cones.

In this paper, we derive the formulas of the chiral anomaly in the cases of both the positive and the negative tilt chiralities utilizing the time reversal symmetry broken Hamiltonians. Since considering the time reversal symmetry broken WSMs, the minimal number of Weyl nodes is two with the opposite chirality. Thus, we consider the following two sets of Hamiltonians with two Weyl nodes corresponding to the positive tilt chiralities and the negative ones, respectively:

$$H_p^\chi = \chi\hbar v_F(\boldsymbol{k} + \chi k_0\hat{\boldsymbol{e}}_x)\cdot\boldsymbol{\sigma} + \hbar v_{\rm tilt}^\chi(k_x + \chi k_0)\sigma_0 \quad \text{(positive tilt chirality)}, \tag{1}$$

$$H_n^\chi = \chi\left[\hbar v_F(\boldsymbol{k} + \chi k_0\hat{\boldsymbol{e}}_x)\cdot\boldsymbol{\sigma} + \hbar v_{\rm tilt}^\chi(k_x + \chi k_0)\sigma_0\right] \quad \text{(negative tilt chirality)}, \tag{2}$$

where σ are the Pauli matrices, v_F is the Fermi velocity, $\chi = \pm 1$ is the value of the chirality, and $v_{\rm tilt}^\chi$ is the tilt velocity at the Weyl node with χ. Two Hamiltonian sets (H_p^χ and H_n^χ) have two Weyl nodes at $\boldsymbol{k} = (\pm k_0, 0, 0)$, respectively, and k_0 is set to $\pi/2$ for simplicity. Note that the Weyl node at $\boldsymbol{k} = (k_0, 0, 0)$ has $\chi = -1$, and the other one at $\boldsymbol{k} = (-k_0, 0, 0)$ has $\chi = +1$. Moreover, these Hamiltonian sets can describe both type-I and type-II WSMs by tuning the tilt velocity $v_{\rm tilt}^\chi$. These become to describe type-I WSMs when $v_{\rm tilt}^\chi$ is less than v_F. On the other hand, these become to describe type-II WSMs when $v_{\rm tilt}^\chi$ is more than v_F.

As a preparation, we derive the zeroth Landau energy-levels of these Hamiltonian sets in order to derive the general formula of the chiral anomaly. We need to consider two cases where the magnetic field is applied in the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$ or is applied in the opposite direction to that. This is because the gradients of the zeroth Landau energy-levels change depending on the direction of the magnetic field. From now on, we assume that the electric field E is applied only in the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$. First, we consider the case (H_p^χ) of the positive tilt chirality. We can obtain the following zeroth Landau energy-level at the Weyl node with χ when the magnetic field B is applied in the direction from the Weyl node with $\chi = +1$ to that $\chi = -1$:[34]

$$E_0^\chi = \hbar(-\chi v_F + v_{\rm tilt}^\chi)(k_x + \chi k_0). \tag{3}$$

Here, the density of the states d is given by

$$d = \frac{1}{2\pi}\frac{eB}{h}. \tag{4}$$

Then, the rate of the change of the electron number $\dot{N}^\chi$ can be written as follows[32] utilizing Eq. (4):

$$\dot{N}^\chi = d\dot{E}_0^\chi = \frac{1}{2\pi}\frac{eB}{h}\dot{E}_0^\chi. \tag{5}$$

From Eqs. (3) and (5), we can obtain the following formula of the anomaly of the number of chiral fermions with the magnetic field applied in the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$:

$$\dot{N}^{\chi=+1} - \dot{N}^{\chi=-1} = \frac{e^2}{2\pi h}(2v_F - v_{\rm diff})EB, \tag{6}$$

where $v_{\rm diff} = v_{\rm tilt}^{\chi=+1} - v_{\rm tilt}^{\chi=-1}$. Here, we utilize the equation of the motion of an electron given by $\hbar\dot{k}_x = -eE$. On the other hand, we can obtain the following zeroth Landau energy-level at the Weyl node with χ when the magnetic field is applied in the direction from the Weyl node with $\chi = -1$ to that with $\chi = +1$:

$$E_0^\chi = \hbar(\chi v_F + v_{\rm tilt}^\chi)(k_x + \chi k_0). \tag{7}$$

From Eqs. (5) and (7), we can obtain the following formula of the anomaly of the number of chiral fermions with the magnetic field applied in the direction from the Weyl node with $\chi = -1$ to that with $\chi = +1$:

$$\dot{N}^{\chi=+1} - \dot{N}^{\chi=-1} = \frac{e^2}{2\pi h}(-2v_F - v_{\rm diff})EB. \tag{8}$$

As a result, we can write the following general formula of the chiral anomaly in the case of the positive tilt chirality utilizing Eqs. (6) and (8):

$$\dot{N}^{\chi=+1} - \dot{N}^{\chi=-1} = \frac{e^2}{2\pi h}(2v_F\boldsymbol{E}\cdot\boldsymbol{B} - v_{\rm diff}|\boldsymbol{E}\cdot\boldsymbol{B}|). \tag{9}$$

Here, we write Eq. (9) utilizing $\boldsymbol{E}\cdot\boldsymbol{B}$ since it is applicable for arbitrary orientation of $\boldsymbol{E}$ and $\boldsymbol{B}$. We can show that Eq. (9) is applicable for arbitrary orientation of $\boldsymbol{E}$ and $\boldsymbol{B}$ considering the case where both the electric field of $\boldsymbol{E} = (E_0, 0, 0)$ and the magnetic field of $\boldsymbol{B} = (B_0\cos\theta, B_0\sin\theta, 0)$ are applied. Here, we set the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$ as the positive direction of the x-axis. The Landau energy-levels are formed in the plane perpendicular to the direction of the magnetic field. Thus, electrons can move along the direction, which is inclined by the angle of θ from the x-axis. The movement of electrons can be cast into k_x and k_y axes, which means that k_x and k_y are good quantum numbers and that the Landau energy-levels formed in the plane perpendicular to the magnetic field can be also cast into k_x and k_y axes. Therefore, we can obtain discrete Landau energy-levels around origins of k_x and k_y axes. As a result, we can get the Landau energy-levels as a function of k_x or k_y except for the cases of $\theta \neq \pi/2, 3\pi/2$. The Landau energy-levels we obtain in the above way correspond to just ones under the magnetic field of $B_0\cos\theta$ along the x-axis. Thus, Eq. (9) also holds for the magnetic field inclined by the angle of θ. For the cases of $\theta = \pi/2, 3\pi/2$, the Landau energy-levels are formed in the plane parallel to the x-axis. Thus, they do not contribute to conductivity of the x-direction. As a result, the chiral anomaly does not happen, which corresponds to the case of $\boldsymbol{E}\cdot\boldsymbol{B} = 0$. Therefore, Eq. (9) is applicable for arbitrary orientation of $\boldsymbol{E}$ and $\boldsymbol{B}$. As you can see, the general formula of the chiral anomaly in the WSMs with the positive tilt chirality includes the term related to the inclination of the Weyl cones. However, it is found that this general formula does not include the term related to the inclination of the Weyl cones if and only if the magnitudes of the inclination of a pair of the Weyl cones are the same ($v_{\rm tilt}^{\chi=+1} = v_{\rm tilt}^{\chi=-1}$). Namely, the general formula of the chiral anomaly reduces to the conventional one when $v_{\rm tilt}^{\chi=+1} = v_{\rm tilt}^{\chi=-1}$. Therefore, if and only if $v_{\rm tilt}^{\chi=+1} = v_{\rm tilt}^{\chi=-1}$, we can use the conventional formula of the chiral anomaly in order to explain the longitudinal magnetoresistance of the WSMs with the positive tilt chirality. Also, we can express the chiral anomaly using the chiral current j_5 as follows:

$$\partial_\mu j_5^\mu = \frac{e^2}{2\pi h}(2v_F\boldsymbol{E}\cdot\boldsymbol{B} - v_{\rm diff}|\boldsymbol{E}\cdot\boldsymbol{B}|). \tag{10}$$

Next, we consider the case (H_n^χ) of the negative tilt chirality. We can obtain the following zeroth Landau energy-level at the Weyl node with χ when the magnetic field is applied in the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$:

$$E_0^\chi = \chi\hbar(-v_F + v_{\text{tilt}}^\chi)(k_x + \chi k_0). \tag{11}$$

From Eqs. (5) and (11), we can obtain the following formula of the anomaly of the number of chiral fermions with the magnetic field applied in the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$:

$$\dot{N}^{\chi=+1} - \dot{N}^{\chi=-1} = \frac{e^2}{2\pi h}(2v_F - v_{\text{sum}})EB, \tag{12}$$

where $v_{\text{sum}} = v_{\text{tilt}}^{\chi=+1} + v_{\text{tilt}}^{\chi=-1}$. On the other hand, we can obtain the following zeroth Landau energy-level at the Weyl node with χ when the magnetic field is applied in the direction from the Weyl node with $\chi = -1$ to that with $\chi = +1$:

$$E_0^\chi = \chi\hbar(v_F + v_{\text{tilt}}^\chi)(k_x + \chi k_0). \tag{13}$$

From Eqs. (5) and (13), we can obtain the following formula of the anomaly of the number of chiral fermions with the magnetic field applied in the direction from the Weyl node with $\chi = -1$ to that with $\chi = +1$:

$$\dot{N}^{\chi=+1} - \dot{N}^{\chi=-1} = \frac{e^2}{2\pi h}(-2v_F - v_{\text{sum}})EB. \tag{14}$$

As a result, we can obtain the following general formula of the chiral anomaly in the case of the negative tilt chirality utilizing Eqs. (12) and (14):

$$\dot{N}^{\chi=+1} - \dot{N}^{\chi=-1} = \frac{e^2}{2\pi h}(2v_F \boldsymbol{E}\cdot\boldsymbol{B} - v_{\text{sum}}|\boldsymbol{E}\cdot\boldsymbol{B}|). \tag{15}$$

It is found that the general formula of the chiral anomaly in the WSMs with the negative tilt chirality includes the term related to the inclination of the Weyl cones. Also, this general formula does not include the term related to the inclination of the Weyl cones if and only if a pair of the Weyl cones are not tilted ($v_{\text{tilt}}^{\chi=+1} = v_{\text{tilt}}^{\chi=-1} = 0$). Therefore, we need to utilize the general formula of the chiral anomaly when considering type-II WSMs ($v_{\text{tilt}}^\chi \neq 0$) with the negative tilt chirality. Moreover, we find that the net chirality in the system decreases when v_{sum} is larger than $2v_F$ even though the electric field and the magnetic field point in any direction. You will see that this fact results in the emergence of the positive magnetoresistance from the chiral anomaly in type-II WSMs reported in the previous paper. Also, we can express the chiral anomaly using the chiral current j_5 as follows:

$$\partial_\mu j_5^\mu = \frac{e^2}{2\pi h}(2v_F \boldsymbol{E}\cdot\boldsymbol{B} - v_{\text{sum}}|\boldsymbol{E}\cdot\boldsymbol{B}|). \tag{16}$$

Finally, we summarize the general formulas of the chiral anomaly derived in this paper in order to highlight them. The general formulas of the chiral anomaly in the cases of the positive and the negative tilt chirality can be written, respectively, as follows:

$$\partial_\mu j_5^\mu = \frac{e^2}{2\pi h}(2v_F \boldsymbol{E}\cdot\boldsymbol{B} - v_{\text{diff}}|\boldsymbol{E}\cdot\boldsymbol{B}|) \quad (\text{positive tilt chirality}), \tag{17}$$

$$\partial_\mu j_5^\mu = \frac{e^2}{2\pi h}(2v_F \boldsymbol{E}\cdot\boldsymbol{B} - v_{\text{sum}}|\boldsymbol{E}\cdot\boldsymbol{B}|) \quad (\text{negative tilt chirality}). \tag{18}$$

In this paper, we apply the above general formula to the concrete model Hamiltonians describing type-II WSMs in order to investigate the usefulness and the validity of the general formula. Namely, we confirm whether the general formulas [Eqs. (17) and (18)] can explain the positive and the negative longitudinal magnetoresistance of the model Hamiltonians describing type-II WSMs with the positive and the negative tilt chirality. For this purpose, we perform the numerical calculations of the MR ratios for the above-mentioned Hamiltonians in order to investigate directly whether they show the positive or the negative longitudinal magnetoresistance. We perform the numerical calculations of magnetoconductivities using the following formula:[35]

$$\sigma_{ij} = e^2\tau\int_{\text{BZ}}\frac{d^3\boldsymbol{k}}{(2\pi)^3}D_{\boldsymbol{k}}\left(v_i + \frac{eB_i}{\hbar}(\boldsymbol{v_k}\cdot\boldsymbol{\Omega_k})\right) \times\left(v_j + \frac{eB_j}{\hbar}(\boldsymbol{v_k}\cdot\boldsymbol{\Omega_k})\right)\left(-\frac{\partial f(\varepsilon_{\boldsymbol{k}})}{\partial\varepsilon_{\boldsymbol{k}}}\right), \tag{19}$$

where τ is the relaxation time, $\boldsymbol{v_k}$ is the group velocity of an electron, $\boldsymbol{\Omega_k}$ is the Berry curvature, $f(\varepsilon_{\boldsymbol{k}})$ is the Fermi-Dirac distribution function, and $D_{\boldsymbol{k}}$ is defined by $[1 + e(\boldsymbol{B}\cdot\boldsymbol{\Omega_k})/\hbar]^{-1}$. This formula can be obtained from the Boltzmann equation including the Berry curvature. Then, we obtain the MR ratios using the calculated results.

First, we investigate whether they show the positive or the negative longitudinal magnetoresistance by performing the numerical calculations of the MR ratios of the model Hamiltonians describing type-II WSMs [Eqs. (1) and (2)]. In most cases, the magnitudes of the inclination of a pair of the Weyl cones are the same. Therefore, the numerical calculations of the MR ratio are performed under the condition of $v_{\text{tilt}}^{\chi=+1} = v_{\text{tilt}}^{\chi=-1} = 2v_F$. Figures 1(a) and 1(b) show the B-dependences of the MR ratios of type-II WSMs with the positive and the negative tilt chirality, respectively. In the above figures, we set the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$ as the positive direction of the magnetic field. Henceforth, we call direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$ (from the Weyl node with $\chi = -1$ to that with $\chi = +1$) as the positive (negative) direction for simplicity. We find that type-II WSMs with the positive tilt chirality show the negative longitudinal magnetoresistance even though the magnetic field points in any direction. On the other hand, those with the negative tilt chirality show the positive (the negative) longitudinal magnetoresistance when the magnetic field is applied in the positive (the negative) direction.

Next, we explain the positive and the negative longitudinal magnetoresistance of type-II WSMs using Eqs. (17) and (18). First, we investigate the changes of the net chirality in the system in order to explain the negative longitudinal magnetoresistance in the case of the positive tilt chirality. Note that the electric field is applied in the positive direction. From Eq. (17), we find that the net chirality in the system increases when the magnetic field is applied in the positive direction. On the other hand, the net chirality in the system decreases when the magnetic field is applied in the negative direction. These changes of the net chirality in the system show that the differences of the chemical potentials $\Delta\varepsilon$ between the Wely cone with $\chi = +1$ and that with $\chi = -1$ occur. Therefore, $\Delta\varepsilon$ is positive (negative) when the magnetic field is applied in the positive (the negative) direction. Moreover, these differences of the chemical potentials between the

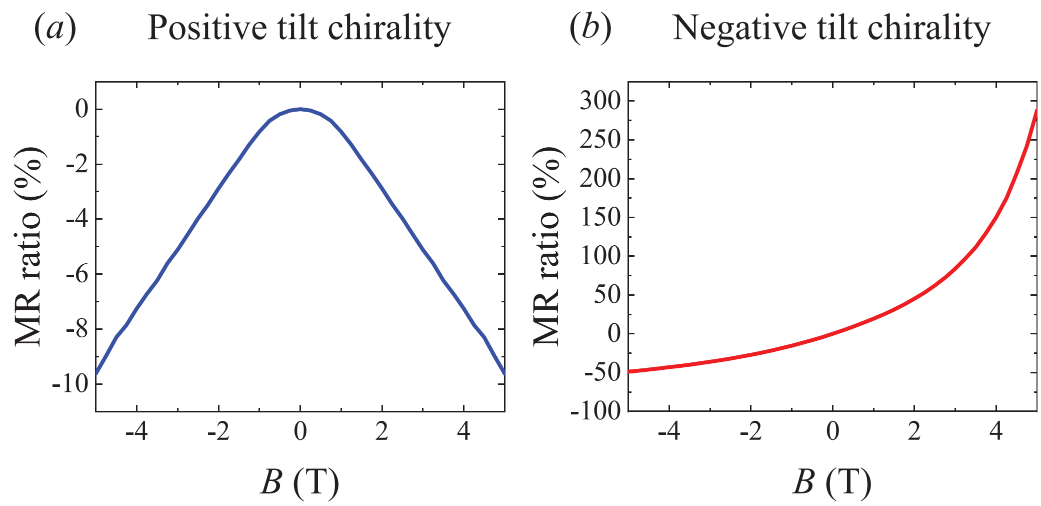

FIG. 1. The B-dependences of the MR ratios of type-II WSMs with (a) the positive and (b) the negative tilt chirality. We set the direction from the Weyl node with $\chi = +1$ to that with $\chi = -1$ as the positive direction of the magnetic field.

Weyl cones cause the chiral magnetic effect, by which the current flows. The current by the chiral magnetic effect $\boldsymbol{j}_c$ is given by[5]

$$\boldsymbol{j}_c = \frac{e^2}{h^2}\Delta\varepsilon\boldsymbol{B}. \tag{20}$$

From Eq. (20), the current flows in the positive direction when the magnetic field is applied in the positive direction. As a result, the negative longitudinal magnetoresistance occurs when the magnetic field is applied in the positive direction. Also, the current flows in the positive direction when the magnetic field is applied in the negative direction since the signs of both $\Delta\varepsilon$ and B cancel each other in Eq. (20). As a result, the negative longitudinal magnetoresistance occurs again when the magnetic field is applied in the negative direction. Therefore, we find that the general formula of the chiral anomaly can explain the negative longitudinal magnetoresistance of type-II WSMs in the case of the positive tilt chirality.

Next, we investigate the change of the net chirality in the system in order to explain both the positive and the negative longitudinal magnetoresistance in the case of the negative tilt chirality. From Eq. (18), we find that the net chirality in the system decreases even though the magnetic field points in any direction. The current flows in the opposite direction to the magnetic field utilizing Eq. (20) since $\Delta\varepsilon$ is negative. Therefore, the positive (the negative) longitudinal magnetoresistance occurs when the magnetic field is applied in the positive (the negative) direction. As a result, we find that the general formula of the chiral anomaly can explain both the positive and the negative longitudinal magnetoresistance of type-II WSMs in the case of the negative tilt chirality. Moreover, it is found that the positive longitudinal magnetoresistance occurs only in type-II WSMs with the negative tilt chirality ($v_{\mathrm{sum}} > 2v_F$) from Eqs. (18) and (20). According to the above results, we find that the general formulas [Eqs. (17) and (18)] can explain the positive and the negative longitudinal magnetoresistance of type-II WSMs in both the cases of the positive and the negative tilt chirality.

Finally, we show that our results are compatible with those obtained from the semiclassical Boltzmann model. The term of $\boldsymbol{v}_{\boldsymbol{k}} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}}$ included in Eq. (4) in Ref. 35 and the term of $\boldsymbol{E} \cdot \boldsymbol{B}$ included in Eq. (5) in Ref. 35 strongly relate to the chiral magnetic effect and the chiral anomaly, respectively. Furthermore, the term of $D_{\boldsymbol{k}}$ included in Eqs. (4) and (5) in Ref. 35 corresponds to considering the zeroth Landau energy-levels in this paper. Namely, the use of $D_{\boldsymbol{k}}$ can implicitly pickup the information of the tilt and the tilt chirality inheriting from the Berry curvature and the inclinations of the zeroth Landau energy-levels. We can explain it by utilizing the energy bands and the Berry curvature. The large $D_{\boldsymbol{k}}$ contributes largely to the magnetoconductivity as you can see from Eq. (10) in Ref. 35. The value of $D_{\boldsymbol{k}}$ is determined by $e(\boldsymbol{B} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}})/\hbar$ since $D_{\boldsymbol{k}}$ is $[1 + e(\boldsymbol{B} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}})/\hbar]^{-1}$. The value of $e(\boldsymbol{B} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}})/\hbar$ is smaller than unity. Therefore, when the inner dot $\boldsymbol{B} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}}$ is positive, the smaller the value of the Berry curvature becomes, the larger the value of $D_{\boldsymbol{k}}$ becomes. On the other hand, when the inner dot $\boldsymbol{B} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}}$ is negative, the larger the value of the Berry curvature becomes, the larger the value of $D_{\boldsymbol{k}}$ becomes. Since the Berry curvature is calculated using the energy bands, the above things mean that the magnetoconductivity incorporates mainly the contribution from the energy bands having the wavenumber where the value of $D_{\boldsymbol{k}}$ becomes large. Namely, the magnitude of the value of $D_{\boldsymbol{k}}$ determines which band to pickup. As an example, we consider the case of the positive tilt chirality. Here, we set $E_F = 0.1$ eV and $\boldsymbol{B} = (3, 0, 0)$ T. Figure 2(a) shows the energy bands and the x-components of the Berry curvatures in the case of the positive tilt chirality. There exist the Weyl nodes at $k_x = \pm\pi/2$, and the Weyl cones are formed around each Weyl nodes. The values of $D_{\boldsymbol{k}}$ at the wavenumbers where the Fermi level crosses the energy bands of (I), (II), (III), and (IV) in Fig. 2(a) are equal to 0.53, 1.11, 0.91, and 9.71, respectively. Namely, the above results mean that the energy band (IV) contributes to the magnetoconductivity most. When the magnetic field of $\boldsymbol{B} = (3, 0, 0)$ T is applied, the zeroth Landau energy-levels are formed as shown in Fig. 2(b). As that time, the energy bands (II) and (IV) become the zeroth Landau energy-levels themselves. This fact is very crucial. Therefore, the fact that the energy band (IV) contributes to the magnetoconductivity most is to consider the zeroth Landau energy-levels. Comparing the values of $D_{\boldsymbol{k}}$,

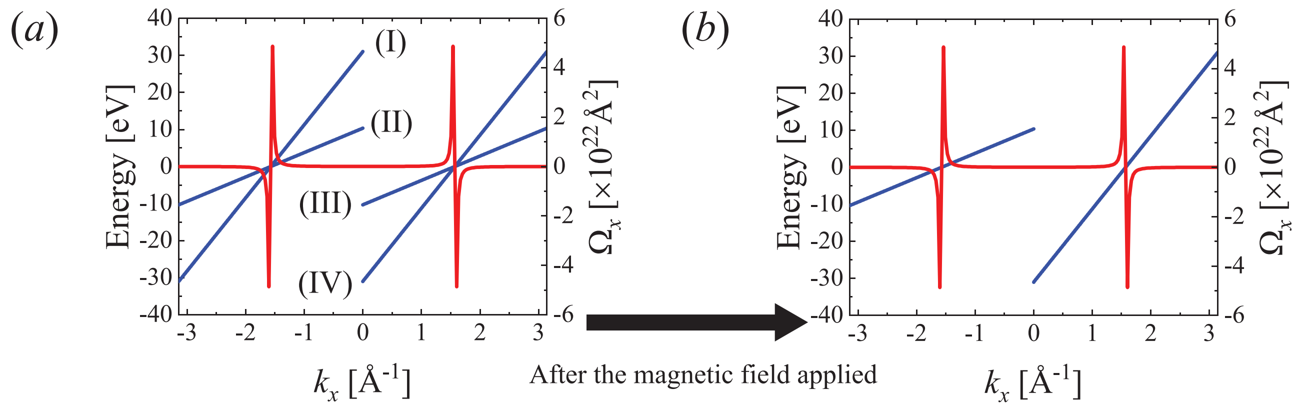

FIG. 2. (a) The energy bands and the x-components of the Berry curvatures in the case of the positive tilt chirality. The blue and the red lines show the energy bands and the x-components of the Berry curvatures, respectively. (b) The zeroth Landau energy-levels and the x-components of the Berry curvatures in the case of the positive tilt chirality under the magnetic field of $\boldsymbol{B} = (3, 0, 0)$ T. The blue and the red lines show the zeroth Landau energy-levels and the x-components of the Berry curvatures, respectively.

we find that the energy band (IV) is more important than the energy band (II). Although these two bands themselves become the zeroth Landau energy-levels, $D_{\boldsymbol{k}}$ prefers the energy band (IV) to the energy band (II). This preference is to consider the tilt chirality. As a result, the use of $D_{\boldsymbol{k}}$ corresponds to incorporating both the zeroth Landau energy-levels and the tilt chirality. The same discussion also holds even when the magnetic field of $\boldsymbol{B} = (-3, 0, 0)$ T is applied and when we consider the case of the negative tilt chirality.

In this paper, we have derived the general formulas of the chiral anomaly in the case that Weyl cones are tilting, contrary to the original case where Weyl cones do not tilt. We have also considered two kinds of the tilt-type, such as the positive tilt chirality and the negative one. Also, the usefulness and the validity of the general formulas have been shown by confirming that they can explain both the positive and the negative longitudinal magnetoresistance of type-II WSMs with the positive and the negative tilt chirality. Therefore, these results suggest that the general formula can determine straightforwardly whether the WSMs show the positive or the negative longitudinal magnetoresistance using both the information of the tilt chirality and the magnitudes of the inclination of the Weyl cones.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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