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Aftershocks and Earthquake Statistics (III)

—— Analyses of the Distribution of Earthquakes in Magnitude, Time, and Space with Special Consideration to Clustering Characteristics of Earthquake Occurrence (1) ——

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Abstract

According to Gutenberg-Richter's law, the number $N(M)$ of earthquakes having magnitude M or larger can be expressed by the equation

$$\log N(M) = b(M_1{}^* - M)$$

where b and $M_1{}^*$ are constants. $M_1{}^*$ is the most probable magnitude of the largest earthquake in the group of earthquakes concerned. b is considered to be an important quantity that characterizes the group. The values for b have been obtained for earthquakes in various regions, time intervals, and magnitude ranges. The median of about 500 determinations of b-values found in many seismological papers is about 0.89. However, many of these determinations do not seem to be sufficiently accurate for discussing such problems as the spatial or temporal variations in b-value.

For 113 cases in which tables showing frequencies of earthquakes at each magnitude level or lists of magnitudes of individual earthquakes are available, the maximum likelihood estimates of b have been calculated by using the equation

$$b = \frac{s \log e}{\sum M_i - sM_s} \eta$$

where $\sum M_i$ is the sum of the magnitudes of all s earthquakes with magnitude M_s and larger, and η is a factor for correcting the effect of the length of magnitude interval ΔM. About 16% of the recalculated b-values differ by more than 0.2 from the values found in the original papers. The method of least squares adopted by many authors gives too heavy weight to a small number of large-magnitude earthquakes. In some cases the recalculation leads to a different conclusion from that of the original author. It appears that inaccurate b-values have most often been resulted from the use of an incomplete set of data. The cases that convincingly show the existence of regional or temporal variations in b-value are rather few, though this does not always offer support to the constant b hypothesis.

In some cases it is more adequate to consider that there is an upper limit of magnitude M_l below which the magnitude-frequency relation is expressed by the Gutenberg-Richter equation. The following equation has been used by Okada for calculating the maximum likelihood estimate of b.

$$\frac{\bar{M}-M_s}{M_l-M_s}=\frac{1}{b(\ln 10)(M_l-M_s)}-\frac{1}{\exp\{b(\ln 10)(M_l-M_s)\}-1}$$

where $\bar{M}$ is the mean magnitude for all earthquakes with magnitude between M_s and M_l. The values of $b(M_l-M_s)$ corresponding to various values of $(\bar{M}-M_s)/(M_l-M_s)$ are tabulated.

A model for the magnitude distribution of earthquakes is proposed, in which the magnitude distribution of main shocks and that of each aftershock sequence satisfy the Gutenberg-Richter equations with the coefficient of b_0 and b_a respectively. The magnitude distribution for the whole of earthquakes (main shocks and aftershocks) is dependent on the two b-values and the degree of aftershock activity u. In addition to these quantities, the upper limit of magnitude M_l and the activity of main shocks (indicated by M_{01}*) affect the magnitude distribution pattern. It is proved that the number of aftershocks A of magnitude larger than a fixed level in an aftershock sequence has an inverse power type distribution. Under some assumptions, the upper limit of magnitude M_l and the degree of aftershock activity u can be determined from the mean and the variance of A, which are estimated from the time distribution of earthquakes on the basis of a model discussed in the next part.

The discussions in previous chapters have been concerned mainly with the problems of aftershocks. In the following chapters, the statistical properties of earthquake occurrence in general will be described. Since it is often the case that considerable part of the earthquakes treated in a statistical investigation are classificable as aftershocks, and the aftershocks themselves have distinct statistical properties, the effect of aftershocks should be considered in a general discussion of earthquake occurrence. Some results from statistical studies along this line have been published by the author in several papers written in Japanese during 1964–1970.[23),175),176),192),224),225),312),313)] Most of these results will be included in the present paper together with some new results from the subsequent studies.

13. Distribution of earthquakes in respect to magnitude

13.1 *Introductory remarks*

The formula most widely used for representing the frequency of occurrence of earthquakes as a function of magnitude is Gutenberg-Richter's formula (Chapter 2)

$$\log n(M)=a-bM\,. \tag{1}$$

In this equation the number of earthquakes with magnitude between M and $M+dM$ is denoted by $n(M)dM$, and a and b are constants. Equation (1) can be written in the form

$$n(M) = n_0 \, 10^{-bM} = n_0 \, e^{b' M} \tag{121}$$

where

$$n_0 = n(0) = 10^a \, , \tag{122}$$

$$b' = b \ln 10 \, . \tag{123}$$

Equation (1) was first used by Gutenberg and Richter[314] in 1944 for southern California earthquakes, though the exponential decrease of earthquake frequency with increasing magnitude had been suggested in an earlier paper.[315]

Kawasumi[316),317),318] in 1943 independently defined a magnitude scale and presented a frequency distribution law in respect to his magnitude M_K in the following form.

$$\log n(M_K) = a_K - b_K M_K \, . \tag{124}$$

According to Kawasumi[318], M_K is connected with magnitude M used by Gutenberg and Richter[319] by a linear equation

$$M = \kappa M_K + \rho \tag{125}$$

where $\kappa = 0.5$ and $\rho = 4.85$. From this equation, we obtain the following relation between b in equation (1) and b_K in equation (124).

$$b_K = \kappa b \, . \tag{126}$$

Gutenberg-Richter's formula is closely related to Ishimoto-Iida's formula of 1939[320] expressing the frequency distribution of earthquakes in respect to maximum trace amplitude a recorded at a certain station (Chapter 5)

$$n(a) = k a^{-m} \tag{35}$$

where k and m are constants. Under some reasonable assumptions,[321),322),323] the exponent m in this equation is connected with the coefficient b in equation (1) by the equation

$$m = b + 1 \, . \tag{36}$$

Nakamura[36] showed in 1925 that the frequencies of aftershocks of the Nobi and the Kwanto earthquakes decreased exponentially with increasing seismic intensity registered at certain places. This has been supported by later studies[324),325),326] which showed that the frequency of earthquakes having seismic intensity I at a certain place is distributed as

$$\log n(I) = a_I - b_I I \, , \tag{127}$$

provided that the intensity is based on the Japanese scale. Since Kawasumi's magnitude M_K is defined as the seismic intensity at the epicentral distance

of 100 km, it is verified that the coefficient b_I in equation (127) is actually equal to the coefficient b_K in equation (124), i.e.,

$$b_I = b_K . \tag{128}$$

Therefore

$$m = (b_I/\kappa) + 1 . \tag{129}$$

The seismic intensity I is connected to the acceleration of ground a_m by Kawasumi's equation[317]

$$I = \varphi \log a_m + \psi \tag{130}$$

where $\varphi=2$ and $\psi=0.7$. Combining equations (127), (35), and (130), we obtain

$$a_m = ha^{\eta} \tag{131}$$

where h is a constant and

$$\eta = (m-1)/(\varphi b_I) . \tag{132}$$

Asada[327] first derived this relation and found $\eta=0.66$ by putting $m=1.74$,[320] and $b_I=0.56$[324] from observations in Tokyo. The fact that η is less than 1.0 was explained by the decrease of the predominant frequency of earthquake motion with increasing earthquake magnitude. On the other hand, from equations (126), (128), (36), and (132), we obtain $\eta=1/(\varphi\chi)=1.0$.

If the relation between magnitude M and energy E for an earthquake

$$\log E = \alpha + \beta M \tag{82}$$

is accepted, the distribution of earthquakes in respect to an energy index

$$K = \log E \tag{133}$$

is represented by

$$\log n(K) = c - \gamma K \tag{134}$$

in which the coefficient γ is given by

$$\gamma = b/\beta . \tag{135}$$

The distribution of earthquakes in respect to energy E is represented by

$$n(E) = CE^{-\gamma-1} \tag{136}$$

where

$$C = 10^{\alpha+(\alpha b/\beta)}/(\beta \ln 10) . \tag{137}$$

Expressions in the form of (134) and (136) are frequently used in the USSR. The inverse power type distribution of earthquake energy in the form of equation (136) was first mentioned by Wadati[328] in 1932. Enya[31),329),330] discussed

the frequency distributions of earthquakes in respect to magnitude (he used the radius of felt area as a measure of magnitude) and maximum velocity-amplitude as early as in 1907.

There are several quantities for an earthquake which are empirically related to its magnitude M by equations of the form

$$\log X = \alpha_X + \beta_X M \tag{138}$$

where X represents one of such quantities and α_X and β_X are constants. Among such quantities are the linear dimension L, area A and volume V of an aftershock region, the length l of an earthquake fault, the linear dimension L_T and area A_T of a tsunami source, and the linear dimension r of the crustal deformation accompanying an earthquake (see Chapter 4). The distribution of earthquakes in respect to one of these quantities X is expressed by

$$n(X) = C_X X^{-(b/\beta_X)-1} \tag{139}$$

where C_X is a constant. For example, if we adopt equation (15) for the size L of the aftershock region of an earthquake of magnitude $M$and assume that $b=1$, the size distribution of aftershock regions is given by

$$n(L) = C_L L^{-3}\,. \tag{140}$$

This can be regarded as the size distribution of source regions (earthquake volumes). Takeuchi and Mizutani[331] has pointed out that this distribution is the same as the size distribution of fragments in a fractured brittle solid, the size distribution of materials on the luner surface, and the mass distribution of meteoritic bodies. These distributions were reported separately by various investigators. They considered that these distributions including the size distribution of earthquakes are resulted from the same physical process of brittle fracturing. The size distributions of luner and Martian craters and cracks in walls are approximated by equations similar to (140).

13.2 *Some properties of Gutenberg-Richter's law*

In this section, formulas derived from Gutenberg-Richter's law (equation (1)) are summarized from previous papers of the author.[1),175),176),177),192),332)]

For groups of earthquakes whose magnitude distribution follows Gutenberg-Richter's law, the number of earthquakes with magnitude M and larger is given by

$$N(M) = N_0\, 10^{-bM} = N_0\, e^{-b'M} \tag{141}$$

where

$$N_0 = N(0) = 10^a/b' . \tag{142}$$

It follows that

$$\log N(M) = a - \log b' - bM , \tag{143}$$

and

$$\log N(M) = b(M_1^* - M) \tag{144}$$

where M_1^* is defined by $N(M_1^*)=1$, or in general

$$N(M_i^*) = i , \qquad i = 1, 2, \cdots \tag{145}$$

or

$$bM_i^* = a - \log b' - \log i \tag{146}$$

and

$$M_i^* = M_1^* - (\log i)/b . \tag{147}$$

Since

$$M_1^* = \frac{a}{b} - \frac{\log b'}{b} \tag{148}$$

and $(\log b')/b$ is equal to about 0.3 for values of b of 0.7 to 5, a/b can be considered as an index of the seismic activity as well as M_1^* (in this relation see Chouhan et al.[333] and Chouhan[334]).

In this section we assume that an earthquake is a sample from a population in which magnitudes are distributed in accordance with equation (1) or (141). The number of earthquakes with magnitude between M_q and M_r has a Poisson distribution with a mean of $N_0(10^{-bM_q} - 10^{-bM_r})$.

The probability that the ith largest earthquake in a group has a magnitude between M_i and $M_i + dM_i$ is given by

$$g_i(M_i)\,dM_i = \frac{b'}{(i-1)!}\lambda^i e^{-\lambda}\,dM_i = \frac{\lambda^{i-1} e^{-\lambda}}{(i-1)!}\,d\lambda \tag{149}$$

where

$$\lambda = i\,10^{-b(M_i - M_i^*)} \tag{150}$$

The cumulative distribution function of M_i is given by

$$G_i(M_i) = \int_{-\infty}^{M_i} g_i(M_i)\,dM_i = \Gamma(i, \lambda)/\Gamma(i) \tag{151}$$

where $\Gamma(i, \lambda)$ is an incomplete gamma function of the second kind. It can be easily shown that $g_i(M_i)$ reaches its maximum at $M_i = M_i^*$ (i.e., M_i^* is the most probable magnitude of the ith largest earthquake), but the expectency of M_i is not equal to M_i^*. For the magnitude of the largest earthquake M_1, we have

$$G_1(M_1) = e^{-\lambda} = \exp\{-10^{-b(M_1-M_1^*)}\} \tag{152}$$

or

$$\log\{-\log G_1(M_1)\} = -b(M_1-M_1^*) - \log(\ln 10). \tag{153}$$

This expression is the same as that given by Epstein and Lomnitz.[335] Figure 127 shows a graph of $G_1(M_1)$ plotted against M_1 for several values of b. It is to be noted that $G_1(M_1^*)=1/e=0.368$. Therefore the probability that the magnitude of the largest earthquake exceeds M_1^* is 0.632.

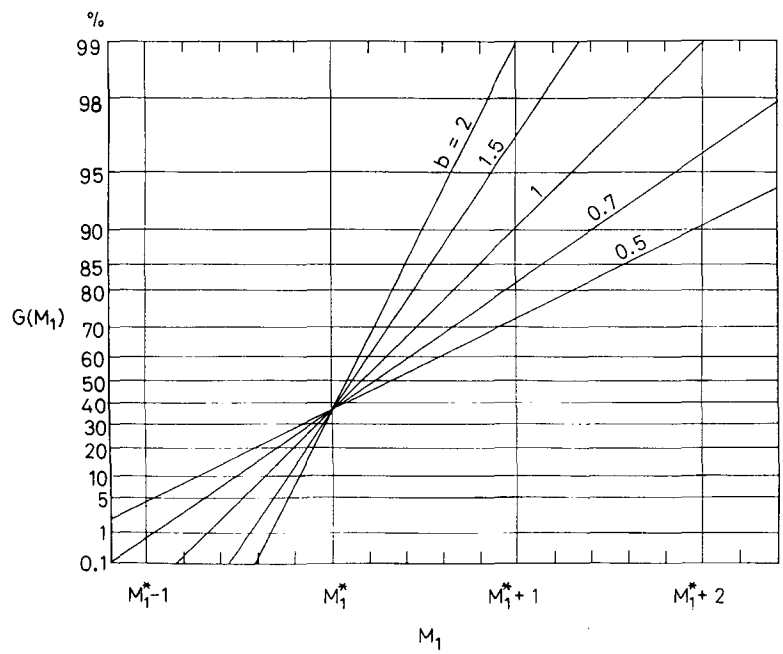


Fig. 127. Graph of $G_1(M_1)$, the probability that the magnitude of the largest earthquake is equal to or less than M_1.

Equation (152) or (153) is a case that has been treated in Gumbel's theory of extreme values.[336] These equations, or similar equations for maximum acceleration or seismic intensity at a certain site, have been used for estimating earthquake risk[337)-342] usually under the assumption of stationary random occurrence of earthquakes in time.

The probability that the ith largest earthquake has a magnitude between M_i and M_i+dM_i and the jth largest earthquake $(i<j)$ has a magnitude between M_j and M_j+dM_j is given by

$$g_{ij}(M_i, M_j)\,dM_i\,dM_j = \frac{b'^2 i^j}{(i-1)!\,(j-i-1)!}\,10^{-jb(M_i-M_i^*)}10^{(j-i)b(M_i-M_j)}$$

$$\times\,\{1-10^{-b(M_i-M_j)}\}^{j-i-1}\exp\{-i10^{-(M_j-M_i^*)}\}\,dM_i\,dM_j\,. \qquad (154)$$

The probability that M_i-M_j has a value between x and $x+dx$ is given by

$$q_{ij}(x)\,dx = \frac{b'(j-1)!}{(i-1)!\,(j-i-1)!}\,10^{-ibx}(1-10^{-bx})^{j-i-1}\,dx\,. \qquad (155)$$

This is a beta distribution as to

$$z = 1-10^{-bx}\,. \qquad (156)$$

The probability that M_i-M_j is smaller than x is given by

$$Q_{ij} = \int_0^x q_{ij}(x)\,dx = B_z(j-i, i)/B(j-i, i) \qquad (157)$$

where B_z and B denote an incomplete beta function and an ordinary beta function respectively. Equation (157) was used in estimating the accuracy of b-value determination.[175] It was found from this equation that a previous discussion on the accuracy of b-value (Utsu,[1] p. 594) was incorrect.

13.3 *Estimation of b-value and its accuracy*

The value of b in equation (1) or (144) for a given group of earthquakes was determined usually from the slope of a straight line fitted to the plotted points on a log $n(M)$ vs M diagram by the method of least squares. However, such determination of b-value may be subjected to the following criticisms.[175]

(1) The assumption the method of least squares is based on is not valid for this kind of distribution. Moreover, the ordinary least squares method gives too heavy weight to the points for large magnitude. Utsu,[175),176] using Monte Carlo technique, showed that the ordinary least squares method yields systematically small b-values when the total number of earthquakes is small (less than about 400). Deming's method of least squares[343] with proper weighting gives more accurate and unbiased b-values.

(2) Different b-values are obtained according to the choice of the length of interval ΔM of magnitude in classifying earthquakes[184),344] and also the treatment of the data in the magnitude intervals for which observed frequency of shocks is zero. The points representing $n(M)\Delta M=0$ can not be plotted on the semi-log diagram, but if these points are neglected, unreasonably low

b-values are sometimes obtained, especially when the total number of earthquakes is small. Such determination may lead to a misleading conclusion that the b-value increases with increasing number of earthquakes.

Utsu[176),192)] recommended the following simple formula, because it gives unique, unbiased, accurate estimate of b-values.

$$b = \frac{s \log e}{\sum M_i - sM_s} \tag{158}$$

where $\sum M_i$ is the sum of magnitude of all earthquakes having magnitude equal to or larger than M_s and s is the total number of these earthquakes. (Note that $M_s = M_s' - (\Delta M/2)$, if magnitude values are given at intervals of ΔM, and M_s' is the central value of the lowest magnitude class). This equation has been derived by equating the first-order moment for the samples $\sum(M_i - M_s)$ to that for the population $\int_{M_s}^{\infty} (M-M_s)n(M)dM = (s \log e)/b$. Aki[345)] pointed out that this is the same as the maximum likelihood estimate of b and tabulated the confidence limits of b-value for large values of s ($s \geq 50$). The value of M_1^* can be estimated from

$$M_1^* = (\log s)/b - M_s\,. \tag{159}$$

Utsu[177)] has shown that the probability density function of b-value determined from equation (158) is given by

$$f(b)db = \frac{s^s}{\Gamma(s)} \left(\frac{b_0}{b}\right)^{s+1} \exp\left(-\frac{sb_0}{b}\right) \frac{db}{b_0} \tag{160}$$

where b_0 is the value of b in the population. Cumulative distributions of b/b_0 and b_0/b have been plotted for various values of s from 7 to 3000. From these graphs we can estimate the accuracy of b-value determined by this method. The statistical test for the difference in b-value between two groups of earthquakes A and B can be made by use of the F-distribution, since b_B/b_A has an F-distribution with $2s_A$ and $2s_B$ degrees of freedom (suffixes A and B indicate groups A and B respectively and it is assumed that $b_A < b_B$), if there is no difference between the two groups.

The value of magnitude is usually given to the one-tenth of the magnitude unit, i.e., it is given at intervals of $\Delta M = 0.1$. In some cases, however, the interval is 1/4 or 1/2. It has been pointed out that the estimate of b-value is dependent on this interval. Utsu[176)] showed that the maximum likelihood estimate of b is systematically small, when the interval is large. The value

calculated from equation (158) should be corrected for this bias, if $b\Delta M$ is larger than a certain value. The correction η has been obtained as follows.

The probability that the magnitude M of an earthquake falls in the range between $j\Delta M+M_s$ and $(j+1)\Delta M+M_s$ $(j=0, 1, 2, \ldots)$ is given by

$$p_j=(1-10^{-b\Delta M})\,10^{-bj\Delta M}\,. \tag{161}$$

Therefore, the mean and the variance of $M-M_s$ are given by

$$\mu_1=\sum_{j=0}^{\infty} p_j\left(j+\frac{1}{2}\right)\Delta M=\left(\frac{10^{-b\Delta M}}{1-10^{-b\Delta M}}+\frac{1}{2}\right)\Delta M\,, \tag{162}$$

and

$$\mu_2=\sum_{j=0}^{\infty} p_j\left(j+\frac{1}{2}\right)^2(\Delta M)^2-\mu_1^2=\frac{10^{-b\Delta M}(\Delta M)^2}{(1-10^{-b\Delta M})^2} \tag{163}$$

respectively. Then the mean and the variance of $\sum_{i=1}^{s}(M_i-M_s)$ are $s\mu_1$ and $s\mu_2$ respectively. Consequently, the b value corresponding to the mean of $\sum_{i=1}^{s}(M_i-M_s)$ becomes

$$\bar{b}=s(\log e)/(s\mu_1)=b/\eta \tag{164}$$

where

$$\eta=\left(\frac{10^{-b\Delta M}}{1-10^{-b\Delta M}}+\frac{1}{2}\right)b\Delta M/\log e\,. \tag{165}$$

Thus, the b value calculated from equation (158) should be multiplied by η to obtain an unbiased estimate. η is tabulated in Table 18 as a function of $b\Delta M$. It is seen from this table that no correction is necessary when $b\Delta M$ is less than about 0.2. The value of b in the expression of η should be the value for the population which is unknown to us, and we inevitably use its estimated value. This substitution causes a decrease in accuracy with increasing $b\Delta M$, though the variance of the b value from equation (158) decreases with increasing $b\Delta M$ as given by

$$v=\frac{s\mu_2}{(s\mu_1)^2}=\frac{4(10^{-b\Delta M})}{s(1+10^{-b\Delta M})^2}\,. \tag{166}$$

Table 18. Correction η as a function of $b\Delta M$.

$b\Delta M$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
η	1.000	1.004	1.017	1.039	1.070	1.108	1.154	1.208	1.268	1.344	1.407

13.4 *b-values for various seismic zones and their redeterminations*

More than 250 papers are known to the author which include descriptions of b, m, or γ values for earthquakes occurring in some regions of the world. Figures 128 and 129 show the frequency distributions of b and m values for shallow earthquakes reported in these papers. Some determinations using apparently inadequate data are not included. When many b-values have been determined for the same region for different time intervals in the same paper, only one representative value for the entire period is adopted. Due to the wide variety of the quality of data and the biased sampling of the regions investigated, these graphs are only a rough indication of the scatter of b and m values.

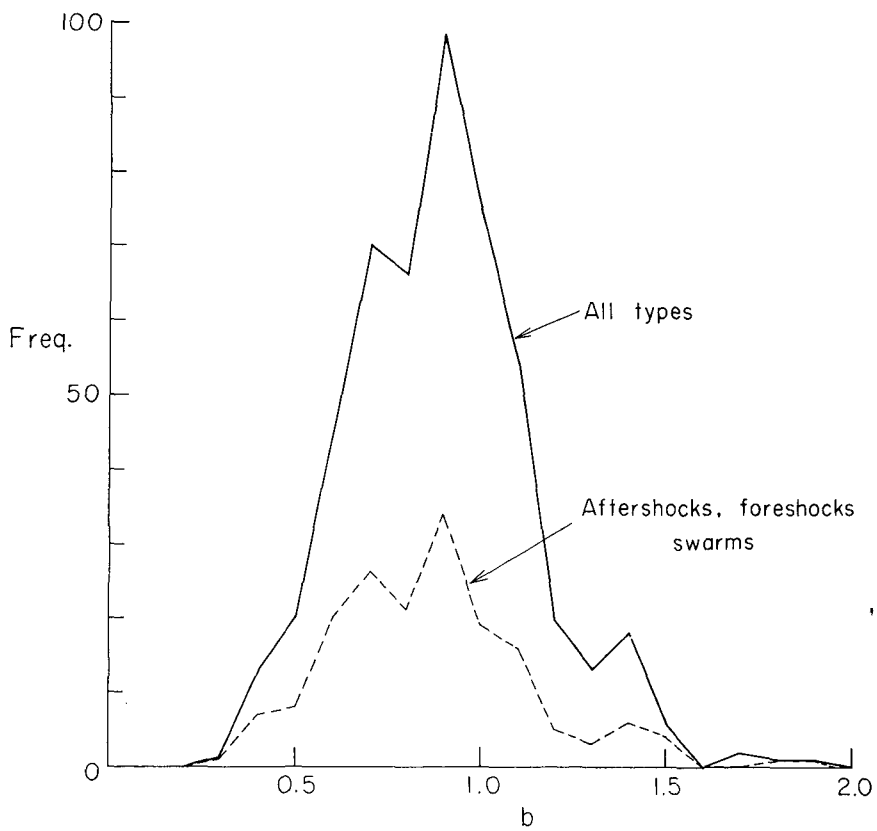


Fig. 128. Frequency distribution of b-values published in seismological literature.

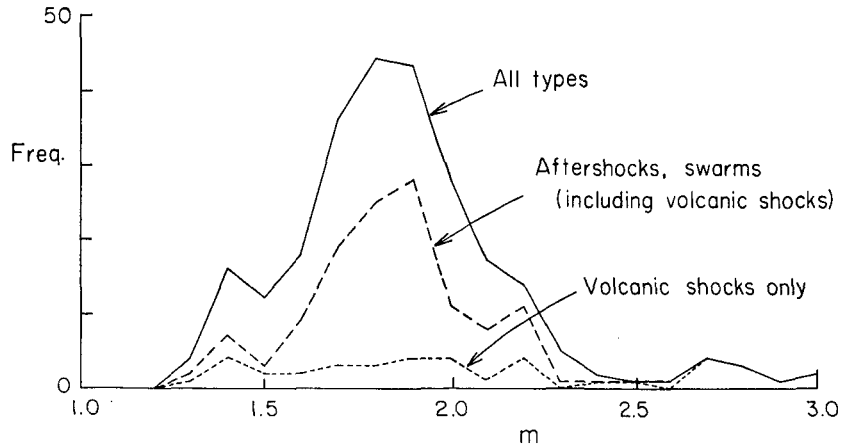


Fig. 129. Frequency distribution of m-values published in seismological literature.

In Figures 128 and 129, solid lines indicate frequencies of b and m-values for all types of earthquakes including the values for aftershock sequences, swarms, volcanic earthquakes, etc. Broken lines indicate those for aftershock sequences, foreshocks sequences, and swarms including volcanic earthquakes. Dotted lines in Figure 129 represents volcanic earthquakes only. The medians of b-values for general earthquakes (excluding the values for aftershock sequences, etc.) and for aftershock sequences, foreshock sequences, and swarms are 0.96 and 0.85 respectively. The former value is nearly equal to the b-value for Japanese shallow earthquakes and the latter value is equal to the median of b-value for Japanese aftershock sequences (Chapter 5). The median of b-values for all types of earthquakes is 0.89. The medians of m-values for all types of earthquakes and for aftershock sequences, foreshock sequences, and swarms are both 1.85. Most of the m-values have been determined generally for earthquakes of smaller magnitude than earthquakes for which the b-values have been obtained.

The main reasons for the variability of b-values are attributable to the following causes.

(1) Actual variation in the magnitude-frequency relation among various regions, time intervals, and magnitude ranges.

(2) Choice of the magnitude scale or the seismographs on which the magnitude or amplitude values are dependent.

(3) Errors in the determination of b or m-values due to i) incompleteness of data, ii) inadequacy of the method used in the determination, and iii)

random sampling fluctuations of purely statistical nature. Some probable cause of the incompleteness of data will be described in Section 13.5 (p. 416).

Some of the papers reporting b-value determinations contain tables listing the frequency of earthquakes of each magnitude interval or the magnitudes of individual earthquakes. For such groups of earthquakes b-values have been recalculated by using equation (158). Most of the recalculated values do not differ much from the values given in the original papers, but in some cases the recalculated values differ to such an extent that the conclusions of the original papers have been modified. In this section these recalculated b-values are presented together with some comments on the data and the conclusions found in respective papers. The recalculated values and the total number of earthquakes used in the calculation are denoted by $\hat{b}$ and s respectively, while the values found in the original papers is denoted by b.

i) *Worldwide studies—shallow earthquakes*

It should be noted that a comparison of b-values among various seismic regions is meaningful only when the data are based on the same magnitude system.

In their book "Seismicity of the Earth", Gutenberg and Richter[319] reported b-values for various regions of the world. The time intervals from which the data have been taken differ with magnitude ranges. The intervals are 1904–1945 (42 years) for $M \geq 7\,{}^3/_4$, 1922–1945 (24 years) for $7 \leq M \leq 7.7$, and 1932–June 1935 ($3\,{}^1/_2$ years) for $6 \leq M \leq 6.9$. Some earthquakes outside of these intervals of time and magnitude are listed in their book, but the complete listing is confined to the above intervals. For southern California and New Zealand, however, b-values have been determined for earthquakes with $M_L \geq 4$ occurring during 1934–May 1943 and October 1940–January 1944, respectively.

$\hat{b}=0.78$ ($s=54$),	Regions 1, 2, Aleutian, Alaska, Brit. Col.,	$b=1.1\pm0.1$,
$\hat{b}=0.93$ ($s=471$),	Southern California, $M_L \geq 4$,	$b=0.88\pm0.03$,
$\hat{b}=0.92$ ($s=59$),	Regions 5, 6, Mexico, Central America,	$b=0.9\pm0.1$,
$\hat{b}=0.73$ ($s=39$),	Region 8, South America, $h<100$ km,	$b=0.45\pm0.1$,
$\hat{b}=0.93$ ($s=236$),	New Zealand, $M_L \geq 4$,	$b=0.87\pm0.04$,
$\hat{b}=0.96$ ($s=23$),	Region 12, Kermadec and Tonga Is.,	$b=1.3\pm0.2$,
$\hat{b}=0.93$ ($s=54$),	Region 15, Solomon to New Britten Is.,	$b=1.01\pm0.07$,
$\hat{b}=0.78$ ($s=101$),	Region 19, Japan to Kamchatka,	$b=0.80\pm0.08$,
$\hat{b}=0.76$ ($s=32$),	Region 24, Sunda Arc,	$b=0.9\pm0.1$,

$\hat{b}=0.81$ ($s=29$),	Regions 26, 28, Pamir-Eastern Asia,	$b=0.6\pm0.14$,
$\hat{b}=0.72$ ($s=15$),	Region 30, Asia Minor-Levant-Balkans,	$b=0.9\pm0.1$,
$\hat{b}=1.41$ ($s=23$),	Region 32, Atlantic Ocean,	$b=1.4\pm0.2$,
$\hat{b}=1.30$ ($s=19$),	Region 33, Indian Ocean,	$b=1.3\pm0.1$,
$\hat{b}=0.95$ ($s=9$),	Region 43, Southeastern Pacific,	$b=1.1\pm0.2$,
$\hat{b}=1.28$ ($s=8$),	Region 45, Indian-Antarctic Swell,	$b=1.06\pm0.03$,

and for the world's shallow earthquakes

$\hat{b}=0.84$ ($s=804$), the whole world, $b=0.90\pm0.02$.

These $\hat{b}$ values, except three oceanic regions 32, 33, and 45, do not differ largely from the value for the whole world. If these three oceanic regions are combined, we obtain

$\hat{b}=1.37$ ($s=50$), Regions 32, 33, 45.

This value is higher than that for the whole world. Two high b-values of 1.6 ± 0.2 for region 40 (Arctic Ocean) and 1.8 ± 0.2 for region 44 (East Pacific) are included in the first edition of "Seismicity of the Earth", but no data are available from this book for recalculation.

On the basis of the b-values found in this book and in some other papers, Miyamura[346),347)] emphasized the regional variations of b-value and their relation to the regional tectonics. Chouhan et al.[333)] also calculated a and b values for 37 regions from the data found in the same book. The b-values obtained by them ranges from 0.35 to 1.42 with a median of 0.79. Accuracy of these values is probably low, because of the small number of data and the incompleteness of the Gutenberg-Richter's catalog for the time intervals and magnitude ranges adopted by Chouhan et al.

Duda[348)] prepared a catalog of world large earthquakes for the period 1897–1964, and calculated b-values for seven regions of the circum-Pacific belt using the data for $M\geq7$ in the years 1918–1963. The recalculated b-values for shallow earthquakes ($h\leq65$ km) are as follows.

$\hat{b}=0.93$ ($s=50$),	Region 1, South America,	$b=0.91\pm0.09$,
$\hat{b}=1.13$ ($s=60$),	Region 2, North America,	$b=1.33\pm0.08$,
$\hat{b}=1.36$ ($s=40$),	Region 3, Aleutians, Alaska,	$b=0.73\pm0.08$,
$\hat{b}=1.03$ ($s=87$),	Region 4, Japan, Kuriles, Kamchatka,	$b=1.01\pm0.05$,
$\hat{b}=1.14$ ($s=105$),	Region 5, New Guniea, Banda Sea, Moluccas, Philippines,	$b=1.24\pm0.07$,
$\hat{b}=1.32$ ($s=112$),	Region 6, New Hebrides, Solomon,	

New Guniea, $b=1.42\pm0.10$,

$\hat{b}=1.11$ ($s=26$), Region 7, New Zealand, Tonga,

Kermadec, $b=1.00\pm0.06$.

The recalculated values are in approximate agreement with those given by the original author except for region 3. The magnitude distribution for this region is shown in Figure 130. Straight lines fitted by the least squares method and the maximum likelihood method are drawn in the figure. The least squares method seems to weight three large shocks (M: 8.7, 8.6, and 8.2) too heavily. An F-test has shown that there are no significant differences in b-value between these seven regions at a significance level of 0.02.

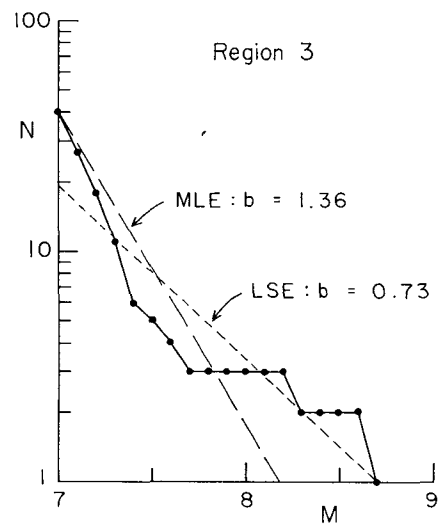


Fig. 130. An example showing the large difference between b-values estimated by the method of least squares (LSE) and by the method of maximum likelihood (MLE). Data taken from Duda.[347]

Tomita and Utsu[349] used the magnitude given by U.S. Coast and Geodetic Survey for shallow earthquakes ($h\leq100$ km) of magnitude 5 and above occurring in three years from 1964 through 1966, and obtained the maximum likelihood estimates of b for 41 regions of the world. Gutenberg-Richter's division[319] of the world into 51 regions are adopted without modification, but in ten regions the b values have not been estimated since less than ten earthquakes occurred during these years. The b-values obtained ranged

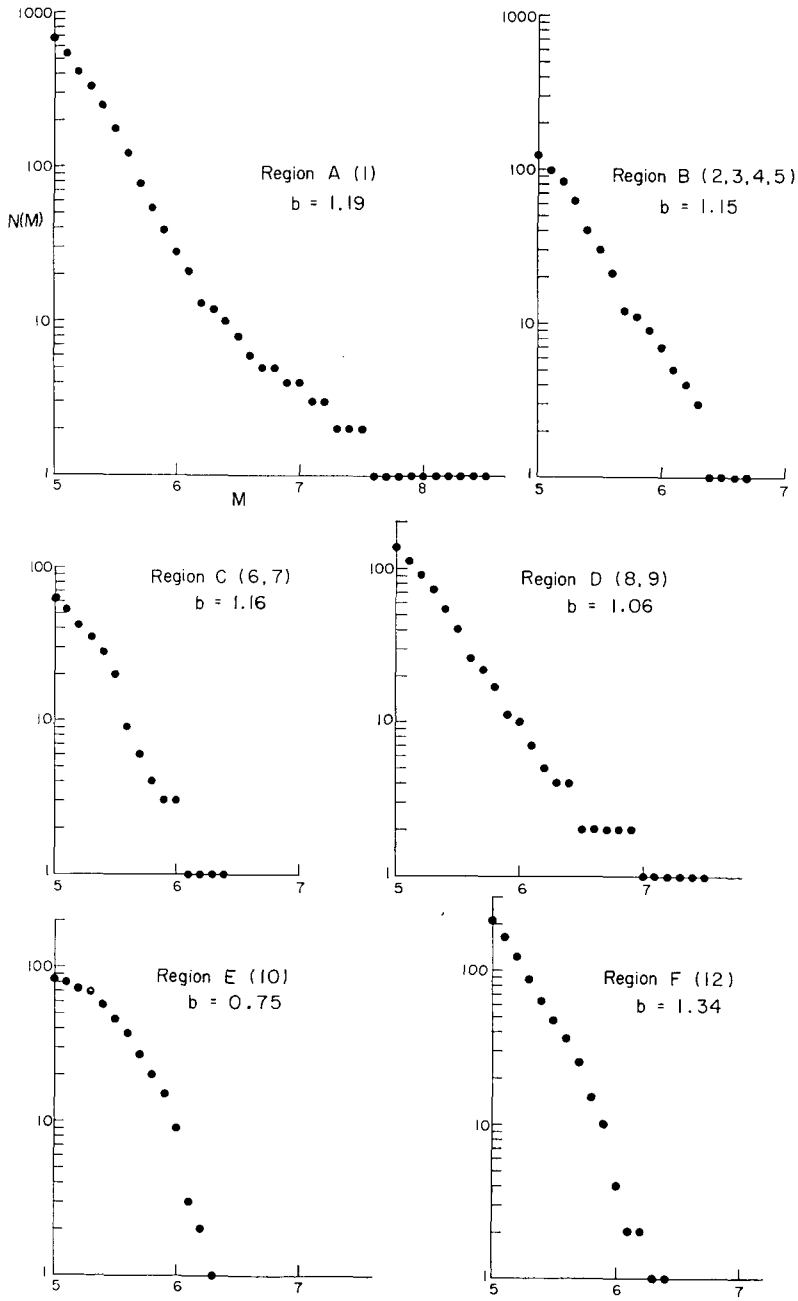
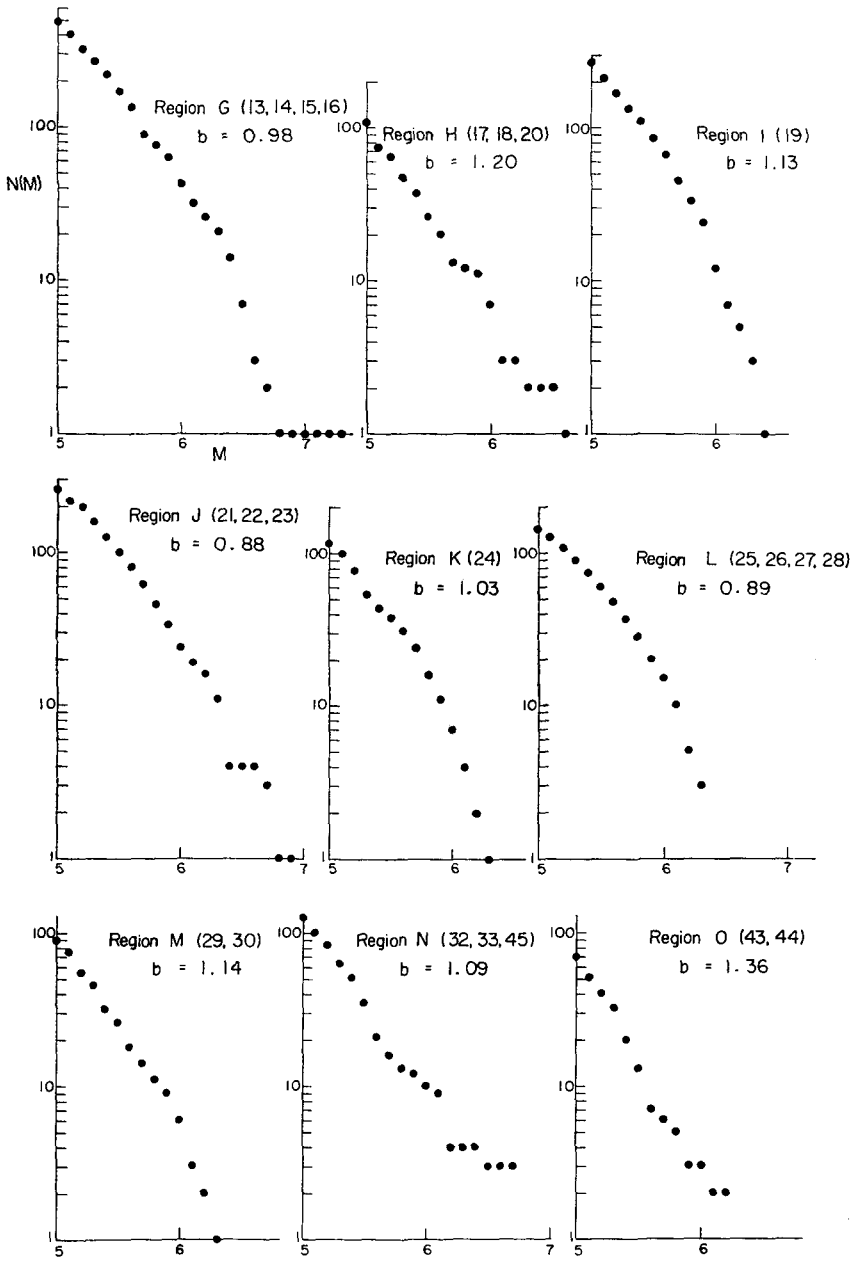


Fig. 131. $N(M)$ plotted against M for fifteen regions using CGS magnitude for the and Richter.[319] Taken from Tomita and Utsu.[348]



years 1964–1966. Numerals in parentheses indicate the regions defined by Gutenberg

from 0.60 to 1.40 with a median of 1.08. The b-value for all shallow earthquakes in the world was estimated as $\hat{b}=1.07$ ($s=3170$).

To reduce the errors due to the smallness of the data size, some of the adjoining regions with approximately equal b values have been grouped together. Figure 131 shows the plots of log $N(M)$ against M for the fifteen regions A, B,..., O thus defined. The numbers in parentheses in each graph indicate the region numbers defined in Gutenberg-Richter's book. The maximum likelihood estimates of b are also shown in each graph. These estimates scatter in the range between 0.75 and 1.36. The lowest value of 0.75 for region E (South Atlantic Ocean) might be caused by incomplete listing of events for magnitude below about 5.5. The significance of the difference in b value between these regions have been tested by using the F-distribution. The results are shown in Figure 132, in which crosses indicate that the hypothesis of equal b-values for the two regions concerned can not be rejected at a significance level of 0.1. Single and double circles indicate that this hypothesis can be rejected at significance levels of 0.1 and 0.02 respectively.

Among a total of 105 combinations of two regions from fifteen regions,

A														
×	B													
×	×	C												
×	×	×	D											
◎	○	○	○	E										
×	×	×	×	◎	F									
○	×	×	×	○	◎	G								
×	×	×	×	◎	×	×	H							
×	×	×	×	○	×	×	×	I						
◎	×	×	×	×	◎	×	○	○	J					
×	×	×	×	×	×	×	×	×	×	K				
○	×	×	×	×	◎	×	○	×	×	×	L			
×	×	×	×	○	×	×	×	×	×	×	×	M		
×	×	×	×	○	×	×	×	×	×	×	×	×	N	
×	×	×	×	◎	×	○	×	×	◎	×	○	×	×	O

Fig. 132. Significance of the difference in b-value between every pair of the regions A to O. Single and double circles indicate the difference at significance levels of 0.1 and 0.02 respectively.

23 and 9 combinations having significantly different b-values are found at the above two significance levels. However, if region E is excluded because of lack of accuracy, 12 and 5 combinations among a total of 91 combinations have significantly different b-values at the same significance levels. Considering that the ratios 12/91 and 5/91 do not differ largely from 0.1 and 0.02 respectively, it is difficult to draw a conclusion about the regional difference in b-values from these data.

Miyamura[350] reported b-values for various regions of the circum-Pacific zone on the basis of the CGS magnitude data in the year of 1965. He stated that the results showed a similar tendency to those found in his previous studies.

ii) *European regions*

Båth[351] obtained a fairly small b-value for Fennoscandia using macroseismic data during 1891–1930. The maximum likelihood estimate for shocks of magnitude 2.5 and above is

$\hat{b}=0.50$ ($s=343$), Fennoscandia, $M\geq 2.5$, $b=0.46$.

Utsu[176] considered that this small b value might be due to the magnitude scale Båth adopted. However, Miyamura[352] opposed to this view.

Miyamura,[353] using Båth's data, calculated and compared b-values for five regions of Fennoscandia. Each region has a small b-value in the range from 0.47 to 0.61.

Kàrník[10),354] estimated the magnitudes of many earthquakes in Europe from seismic intensities and obtained b-values for 39 regions on the basis of these magnitude data. The values ranged from 0.5 to 1.1 with a median of 0.83. The recalculation is performed for the following eight combined regions for which Kàrník also gave b-values in his paper.

$\hat{b}=0.85$ ($s=125$), Regions 1, 2, Iceland and vicinity, $M\geq 4.7$, $b=0.79$,
$\hat{b}=0.65$ ($s=91$), Regions 3, 10, Fennoscandia to England, $M\geq 4.1$, $b=0.96$,
$\hat{b}=0.94$ ($s=85$), Regions 5–9, 11, White Russia to France, $M\geq 4.1$, $b=0.84$,
$\hat{b}=0.75$ ($s=320$), Regions 13–16, South of Mediterranean, $M\geq 4.1$, $b=1.01$,
$\hat{b}=0.90$ ($s=1525$), Regions 17–22, East Spain to Rumania, $M\geq 4.1$, $b=1.01$,
$\hat{b}=0.72$ ($s=456$), Regions 23, 24, 27, 33, 34, NW Iran to Bulgaria, $M\geq 4.7$, $b=0.64$,

$\hat{b}=0.97$ ($s=117$), Regions 28, 32, Northeast of Black Sea, $M\geq4.7$, $b=0.97$

$\hat{b}=1.00$ ($s=94$), Regions 30, 31, 35–39, Turkey to Libya, $M\geq5.2$, $b=1.15$.

No simple relationship is evident between the b-value and the tectonic type of the region as Kàrník mentioned.

Niklovà and Kàrník[335] reported b-values for 19 regions in and near the Balkans from data on earthquakes of $M\geq4.1$ or $M\geq4.6$. The values ranged from 0.53 to 1.10 with a median of 0.76. Frequencies of earthquakes as a function of magnitude have been reported for Greece by Galanopoulos,[356] for Turkey by Öcal,[357] and for Bulgaria by Grigorova et al.[358] The maximum likelihood estimates of b for these regions are as follows.

$\hat{b}=0.65$ ($s=1613$), Greece, 1840–1959, $M\geq4.8$, $b=0.69$,
$\hat{b}=0.45$ ($s=1499$), Turkey, 1850–1960, $M\geq3.5$,
$\hat{b}=0.60$ ($s=124$), Bulgaria, 1901–1965, $M\geq4$, $b=0.58$.

In a later paper Galanopoulos[359] adopted $b=0.82$ for the region of Greece.

iii) *North America*

As already mentioned the first b-value found in seismological literature was $b=0.88$ for southern California.[314] Allen et al.[292] obtained b-values for six regions in southern California in the magnitude range 3 and greater. These values are not much different from the above value of 0.88. The lowest value is found for Kern County ($b=0.80$) and the highest value for Los Angeles Basin ($b=1.02$). Niazi[360] obtained $b=0.99$ for earthquakes with $M\geq4$ in northern California and western Nevada. Ryall et al.[361] constructed magnitude-frequency graphs for six regions in the western United States. The b-value for the period from 1932 to 1962 for magnitude 4 and greater are 0.61, 0.63, 0.68, 0.79, 0.79, and 0.90. Ryall et al. suspected that the first three low values are the results of poor detection of small shocks. Duda[362] reported the following b-values: $b=0.47$ ($2.7\leq M\leq3.6$) and $b=1.11$ ($3.7\leq M\leq4.5$) for Baja Claifornia, $b=0.46$ ($2.0\leq M\leq4.2$) and $b=1.07$ ($3.8\leq M\leq4.6$) for Imperial County, California, and $b=1.21$ ($1.5\leq M\leq2.1$) for Arizona.

Evernden[363] obtained $b=1.29$ for earthquakes in the USA using CGS magnitude data from 1961 to August 1968. His paper includes b-values for various regions of the world published by other investigators.

The following b-values are obtained from the data by Milne.[364]

$\hat{b}=0.75$ ($s=435$), Aleutians, 1898–1960, $M\geq 6$, $b=0.89$,
$\hat{b}=0.49$ ($s=181$), Western Canada, 1898–1960, $M\geq 5$, $b=0.67$,
$\hat{b}=0.80$ ($s=65$), Eastern Canada, 1898–1960, $M\geq 5$, $b=0.78$.

In addition to these determinations, b-values have been reported for several aftershock sequences and swarms as described in sub-section vii).

iv) *Japan and vicinity*

Kawasumi[318] determined the value of b_K in equation (124) for Japanese earthquakes from data for 40 years from 1904 to 1943. The maximum likelihood estimate from the same data is

$\hat{b}_K=0.52$ ($s=343$), Japan, $M_K\geq 4$, $b_K=0.537$.

From equation (126), this corresponds to the b-value of 1.04.

Tsuboi[365] in 1952 first described the regional difference in b-value in Japan using data on earthquakes with magnitude 6 and above occurring during 1931–1950. The b-values determined by the method of least squares for three regions A, B, and C differ considerably (see below). However, recalculated values using the same data do not show any appreciable differences.

$\hat{b}=0.96$ ($s=292$), Region A, Pacific Ocean side of NE Japan, $b=1.06$,
$\hat{b}=0.94$ ($s=90$), Region B, Pacific Ocean side of SW Japan, $b=0.72$,
$\hat{b}=0.96$ ($s=65$), Region C, Japan Sea side of the whole Japan, $b=0.66$.

Tsuboi also calculated b-values for the whole of Japan as a function of the length of time interval. The result indicated clear depencence of b-value on the length of time interval, i.e., on the number of earthquakes used in the calculation. The b-value increases with the length of time interval and reaches a constant value asymptotically. However, maximum likelihood estimates on the basis of the same data do not show such a tendency. The method of least squares adopted by Tsuboi yields small b-values when the size of data is small. For example, Tsuboi gave a very small value of $b=0.21$ for the year of 1950, but the recalculated value for this period is $\hat{b}=1.15$ ($s=15$).

Tsuboi[366),367] in 1957 and 1964 calculated b-values for shallow earthquakes occurring in and near Japan. The recalculated values using the same data are as follows.

$\hat{b}=0.81$ ($s=382$), In and near Japan, 1931–1955, $M\geq 6$, $b=0.72$,[366]
$\hat{b}=1.03$ ($s=433$), In and near Japan, 1926–1963, $M\geq 6$, $b=0.87$,[367]

$\hat{b}=0.96$ ($s=1231$), In and near Japan, 1885–1963, $M\geq 6$, $b=1.03$.[367]

Usami et al.[258] obtained $b=1.18$ for earthquakes of $M\geq 6$ at all depth ranges in and near Japan during 1926–1956. The recalculation has been made for earthquakes of focal depths 60 km and less, since the magnitudes of earthquakes deeper than 60 km have been provided from different sources.

$\hat{b}=1.00$ ($s=340$), In and near Japan, $h\leq 60$ km, $M\geq 6$.

In their paper, b-values are also shown for two focal depth ranges 0–30 km and 30–60 km. Recalculated values are

$\hat{b}=0.90$ ($s=185$), In and near Japan, $0\leq h\leq 30$ km, $M\geq 6$, $b=0.92$,
$\hat{b}=1.15$ ($s=155$), In and near Japan, $30<h\leq 60$ km, $M\geq 6$, $b=1.25$.

These values are nearly equal to those obtained by Hamamatsu,[186] since the materials are mostly the same.

Hamamatsu,[186] giving due consideration to the limitation of the materials, calculated b-values for Japanese regions for two depth ranges. Recalculated values are as follows.

$\hat{b}=1.02$ ($s=363$), The whole Japan, $0\leq h\leq 60$ km, $M\geq 6$, $b=1.07$,
$\hat{b}=0.93$ ($s=204$), The whole Japan, $0\leq h\leq 30$ km, $M\geq 6$, $b=1.00$,
$\hat{b}=1.17$ ($s=159$), The whole Japan, $30<h\leq 60$ km, $M\geq 6$, $b=1.13$,
$\hat{b}=1.04$ ($s=278$), NE Japan, $0\leq h\leq 60$ km, $M\geq 6$, $b=1.10$,
$\hat{b}=0.89$ ($s=139$), NE Japan, $0\leq h\leq 30$ km, $M\geq 6$, $b=1.05$,
$\hat{b}=1.16$ ($s=139$), NE Japan $30<h\leq 60$ km, $M\geq 6$, $b=1.10$,
$\hat{b}=0.95$ ($s=84$), SW Japan, $0\leq h\leq 60$ km, $M\geq 6$, $b=0.89$,
$\hat{b}=0.95$ ($s=64$), SW Japan, $0\leq h\leq 30$ km, $M\geq 6$, $b=0.83$,
$\hat{b}=1.32$ ($s=20$), SW Japan, $30<h\leq 60$ km, $M\geq 6$, $b=1.21$.

The two values $\hat{b}=0.93$ and $\hat{b}=1.17$ for the two depth ranges for the whole Japan differ significantly at a significance level of 0.1.

Katsumata[187] determined b-values for several selected regions of high seismic activity in and near Japan. He prepared additional materials to the JMA catalog of earthquakes (supplementary volumes of the Seismological Bulletin of JMA) in order to avoid the inaccuracy of b-values due to the use of incomplete data. Erroneous b-values had been obtained in several papers of different authors who used the JMA catalog without proper consideration to its limitations. Katsumata[368] later recalculated the b-values for the same region using equation (158). Some of the recalculated values are listed below, together with the values from the old paper.

$\hat{b}=1.03$ ($s=213$), Off the Pacific coast of N Honshu, $M\geq 6$, $b=1.07$,
$\hat{b}=0.88$ ($s=146$), S Hokkaido and off the coast of S Hokkaido, $M\geq 5$, $b=0.89$,
$\hat{b}=1.03$ ($s=238$), Kwanto district including nearby sea areas, $M\geq 4.5$, $b=0.99$,
$\hat{b}=0.95$ ($s=73$), Kinki district and part of Shikoku including sea areas, $4.5\leq M\leq 5.5$, $b=0.95$,
$\hat{b}=0.85$ ($s=78$), Kyushu and off Pacific coast of Kyushu, $M\geq 5$, $b=0.91$.

Katsumata concluded that there are no remarkable differences in b-value between these regions.

Ichikawa[369] reported b-values calculated from equation (158) for the following four regions. He tried three cases $M_s=5.0$, 4.8, and 4.6, but these M_s values should be modified to 4.9, 4.7, and 4.5 respectively, since the frequencies were given at intervals of 0.2 unit of magnitude. The recalculated values are as follows.

$\hat{b}=0.82$ ($s=234$), Chubu district, $M\geq 5$, $b=1.17$,
$\hat{b}=0.81$ ($s=134$), Western Japan, $M\geq 5$, $b=1.20$,
$\hat{b}=1.21$ ($s=62$), SW Ibaragi Prefecture, $M\geq 5$, $b=1.67$,
$\hat{b}=0.84$ ($s=37$), Northern Chiba Prefecture, $M\geq 5$, $b=1.04$.

It is concluded that earthquakes in SW Ibaragi Prefecture have a significantly higher b-value than those in Chubu district and western Japan. Ichikawa[232] discussed the b-values also for four regions in and near the Izu Peninsula.

Some of the b-values reported by Miyamura,[247] by Maki,[370] and by Welkner[371] seem too small. These authors used the JMA catalog only. Welkner determined the b-values by the least squares method and the maximum likelihood method for earthquakes of magnitude 5.5 and above in nine regions covering Japan and surrounding areas. The b-value ranged from 0.63 to 0.90. However, the JMA catalog is far from being complete for earthquakes of magnitude about 6 or less occurrring in the regions concerned. The similar comment may be made on a value of $b=0.72$ for earthqaukes with $M\geq 5$ in the Hokkaido region published by the Sapporo District Meterorological Observatory.[372]

Ikegami[325] determined the values of b_I in equation (127) using mean annual frequencies of felt shocks at each seismic intensity averaged for stations in each of the three regions A, B, and C in Japan. The three regions are the

same as those defined by Tsuboi.[365] In calculating the mean annual frequencies, Ikegami neglected the stations at which no felt shocks have been registered at a given seismic intensity. For example, in the C region which includes 27 stations, the seismic intensity 6 has been recorded only once at one station during the 30 years investigated. He regarded the mean annual frequency at intensity 6 in region C as 1/30, but this should be modified to $1/(30\times27)$. The recalculated values using the modified mean annual frequencies do not show large regional variations in contrast with Ikegami's conclusion.

$\hat{b}_I=0.50$,	Region A, Pacific Ocean side of NE Japan,	$b_I=0.58$,
$\hat{b}_I=0.52$,	Region B, Pacific Ocean side of SW Japan,	$b_I=0.46$,
$\hat{b}_I=0.44$,	Region C, Japan Sea side of the whole of Japan,	$b_I=0.39$,
$\hat{b}_I=0.49$,	Regions A, B, and C, the whole of Japan,	$b_I=0.49$.

Katsumata and Tokunaga[326] obtained values of b_I for 64 stations in Japan. The value varies from 0.44 at Toyooka to 0.93 at Wakayama with a median of 0.61. No large-scale regional variations are evident.

v) *Oceanic seismic belts*

Sykes[373] obtained a b-value of 0.91 for 267 earthquakes of $M\geq4.0$ occurring in the ridge system which extends from the the north Atlantic to the Arctic during the years 1955 through 1964. The recalculation has been done for $M\geq4.5$.

$\hat{b}=0.90$ ($s=101$), Arctic region, $M\geq4.5$, ($b=0.91$).

From Stover's data for the Indian Ocean[294] and the South Atlantic Ocean,[374] the following values are obtained in the recalculation.

$\hat{b}=0.97$ ($s=76$),	Indian Ocean, 1922–1951, $M\geq6$,	$b=0.91$,
$\hat{b}=0.96$ ($s=83$),	South Atlantic Ocean, 1964–1966, $M\geq4.7$,	$b=1.04$.

Francis[375] studied the magnitude-frequency relation for earthquakes in the mid-Atlantic ridge using magnitudes determined by CGS from 1963 to May 1967, and found that the b-value for earthquakes in the fracture zone is normal ($b=1.03$) whereas the b-value for earthquakes at median rift is high ($b=1.73$). He mentioned four possible interpretations for this difference: (1) Focal mechanism, (2) temperature gradient, (3) volcanism, (4) systematic error in the measurement of magnitude (see also Sykes[376])

For Hawaii, the recalculation yields the same value as found in the

original paper by Furumoto.[293]

$\hat{b}=0.97$ ($s=976$), Hawaii, 1950–Sept. 1959, $M\geq 2.5$, $b=0.97\pm 0.064$.

vi) *USSR*

In the USSR, the frequency distribution of earthquake magnitude is usually represented by the γ-value in equation (134). Most of the papers dealing with γ-values do not include tables showing the frequency of earthquakes in each energy class. The reported γ-values (e.g., Buné,[377] Fedotov et al.,[134),378] Gzovsky et al.,[379] Riznichenko,[380–382] Riznichenko and Nersesov[383] Tskhakaya,[384]) fall in the range from 0.35 to 0.60, and the values from 0.45 to 0.50 are most frequent. Kantorovich et al.[385] obtained a maximum likelihood estimate of γ, $\hat{\gamma}=0.45$ ($s=707$) for central Asia.

Some Soviet authors[384),386),387] use b-values, too. Recalculated values of γ and b for earthquakes in the Caucasus[376] are given below.

$\hat{\gamma}=0.54$ ($s=1633$), Caucasus, 1924–1960, $K\geq 10$,
$\hat{b}=0.90$ ($s=1435$), Caucasus, 1912–1960, $M\geq 3.5$, $b=0.89$.

vii) *Other regions*

Sutton and Berg[290] obtained a b-value for earthquakes in Western Rift Valley of Africa from three-year observation. The recalculated value is

$\hat{b}=0.65$ ($s=234$), Western Rift Valley, Africa, $b=0.6$,

In Sutton and Berg's paper, we find the following explanation for the low b-value: "The observational radius of the African station decreases with decreasing magnitude and the larger area is monitored for the large shocks than the smaller shocks."

In Niazi and Basford's paper[388] on seismicity of Iranian Plateau and Hindu Kush region, a magnitude-frequency table is found for region C (the belt surrounding the central desert of Iran).

$\hat{b}=0.65$ ($s=476$), Belt surrounding the central desert of Iran, $M\geq 4$.

Ram and Rathor[389] reported b-values for three regions of India, $b=0.59$ (Delhi and Himalayan region), $b=0.74$ (Assam region), and $b=0.81$ (Koyna region), but the accuracy of these values can not be inferred from their paper (see also Ram and Singh,[390] and Ram and Sriniwas[391]).

Mei[392] studied the seismicity of China and obtained fairly low b-values (0.4 to 0.6) for six regions in China. For the whole of China excluding

Taiwan, the recalculated value from the data for fifty years is

$\hat{b}=0.59$ ($s=311$), China, $M\geq 5.5$, $b=0.57$.

Keimatsu[393] studied earthquakes of China in Ming (1368–1644) and Sing (1644–1911) dynasties. Using his catalog of earthquakes in which magnitudes have been estimated from macroseismic effects, the b-value has beeen calculated.

$\hat{b}=0.53$ ($s=87$), China, 1368–1911, $M\geq 7.0$,
$\hat{b}=1.00$ ($s=62$), China, 1368–1911, $M\geq 7.6$.

For the New Guinea-Solomon region, the following b-value is obtained from data in Denham's paper.[394]

$\hat{b}=1.04$ ($s=549$), New Guinea-Solomon, 1963–1966, $M\geq 5$, $b=1.16$.

Welkner[395] obtained maximum likelihood estimates of b for each area of $1°\times 1°$ in northern Chile from the CGS epicenter and magnitude data for 1963–1966, and attempted to compare the b-value with tectonics. The estimated values scatter very widely from 0.46 to 3.95 with a median of 0.85. The smallness of the data size may account for such a wide scatter.

vii) *Aftershock sequences and earthquake swarms*

b-values for aftershock sequences and earthquake swarms are listed in Tables 4 and 5 (Chapter 3). The b-values for Japanese aftershock sequences and swarms listed in Table 4 have been determined on the basis of magnitudes calculated from Tsuboi's formula.[396] The median of these values is 0.85, which is somewhat smaller than the value of about 1.0 for general earthquakes (including aftershocks, etc.) of magnitude 6 and larger in the Japanese region obtained by several investigators[1),186),188),367),368] who used the JMA magnitudes calculated from Tsuboi's formula.

The values for m in Ishimoto-Iida's formula for aftershocks, especially micro-aftershocks have been determined in the case of some Japanese earthquakes (see Table 7 in Chapter 5). The median of the m-values is about 1.85.

Some of the b-values reported for aftershock sequences and swarms in California are fairly small. The maximum likelihood estimates of b have been obtained for several sequences[176] using data given by Richter et al.[122] (aftershocks of the Desert Hot Springs earthquake of December 4, 1948; earthquake swarm near Kitching Peak, June 10, 1944), Richter[123] (aftershocks of the Kern County earthquake of July 21, 1952), Tocher[127] (aftershocks of the San

Francisco earthquake of March 22, 1958), Richter[178] (earthquakes near China Lake, January 7–16, 1959), Udias[133),397] (aftershocks of the Salinas-Watsonville earthquakes of August 31 and September 14, 1963), and McEvilly et al.[64] (foreshocks and aftershocks of the Parkfield earthquake of June 28, 1966) are as follows.

$\hat{b}=1.00$ ($s=89$),	Aftershocks, Desert Hot Springs, $M\geq 3.0$,		
$\hat{b}=0.68$ ($s=44$),	Kitching Peak Swarm, $M\geq 3.0$,		
$\hat{b}=0.84$ ($s=200$),	Aftershocks, Kern County, $M\geq 4.0$,		
$\hat{b}=0.74$ ($s=182$),	Aftershocks, San Francisco, $M\geq 2.0$,		
$\hat{b}=0.36$ ($s=81$),	China Lake swarm, $M\geq 0.0$,		
$\hat{b}=0.58$ ($s=54$),	〃 $M\geq 0.8$,		
$\hat{b}=0.46$ ($s=35$),	Aftershocks, Watsonville (Aug. 31),[133]	$M\geq 1.0$,	$b=0.49$,
$\hat{b}=0.47$ ($s=45$),	〃 (Sept., 14),[133]	$M\geq 1.0$,	$b=0.41$,
$\hat{b}=0.93$ ($s=18$),	〃 (Aug. 31),[397]	$M\geq 2.0$,	
$\hat{b}=0.85$ ($s=22$),	〃 (Sept. 14),[397]	$M\geq 2.0$,	$b=0.84$,
$\hat{b}=0.75$ ($s=224$),	Parkfield sequence, $M\geq 2.0$		$b=0.63$.

The values of $b=0.53$ for the aftershock sequence of the earthquake near Corralitos on November 16, 1964 reported by McEvilly,[137] $b=0.78$ for the foreshock-aftershock sequence of the earthquake near Antioch on September 10, 1965 by McEvilly and Casaday,[140] and $b=1.40$ for the microaftershocks of the Truckee earthquake of September 12, 1966 by Greensfelder,[140] are listed in Table 5. In addition, Ryall et al.[65] reported a value of $b=0.81$ for the Truckee sequence and Fitch[398] reported $b=0.59$ for microaftershocks of the Parkfield earthquake.

Eaton et al.[399] made a detailed study of the aftershocks of the Parkfield earthquake and obtained b-values for various ranges of focal depth in two sections of the hypocentral zone. From the list of aftershocks for July 1 to September 15, 1966 in their paper, we obtain the following maximum likelihood estimate of b.

$\hat{b}=0.88$ ($s=67$), Aftershocks, Parkfield, $M\geq 2.0$, $b=0.85$.

Ranalli[400] calculated the maximum likelihood estimates of b for fifteen aftershock sequences using data published by various authors.[9),62),64),66),122),126),127),133),282] These sequences include six Californian sequences, five of which have been treated above. For the aftershocks of the Long Beach earthquake of March 10, 1933, he gives $\hat{b}=0.79$ ($s=78$).

Fairly large b-values of 1.5 and 1.45 have been reported for the aftershock

sequences of the Kamchatka earthquake of 1952 and of the Aleutian earthquake of 1957 by Båth and Benioff[125] and by Duda[126] respectively. These values are based on the so-called body wave magnitudes. Renalli[400] recalculated the b-value for the Aleutian sequence and obtained $\hat{b}=1.277$ ($s=205$). Utsu[57] obtained $b=0.70$ for the same sequence on the basis of surface wave magnitude converted from body wave magnitude by using Gutenberg-Richter's formula. Utsu[57] also obtained $b=0.93$ and $b=0.88$ for the aftershock sequences of the central Alaska earthquake of 1958 and of the southeast Alaska earthquake of 1958, respectively. Page[62] obtained the maximum likelihood estimate of b, $\hat{b}=0.88$ ($s=293$) for the aftershocks of the Alaska earthquake of March 28, 1964 on the basis of body wave magnitude ($4.5\leq m\leq 5.7$). Ranalli[400] reported $\hat{b}=0.987$ ($s=294$) for the same sequence on the basis of Page's paper. Press and Jackson[180] obtained $b=1.1$ for the same sequence using amplitudes recorded by Wood-Anderson seismographs.

For the aftershocks of the southern Kurile Islands earthquake of October 13, 1963, the b-value is calculated using CGS magnitudes listed in a table by Santo[135] for the period from 09 h, October 13 through December 31.

$\hat{b}=1.15$ ($s=256$), Aftershocks, southern Kurile Islands, $m\geq 4.5$.

Two $\hat{b}$-values were reported by Motoya[218] for the aftershocks of the magnitude 7.8 earthquake on August 12, 1969 off Hokkaido, $\hat{b}=1.07$ for $m\geq 4.6$ using CGS magnitudes and $\hat{b}=1.44$ for $M\geq 5.1$ using JMA magnitudes.

Papazachos et al.[9] studied many aftershock sequences in the region of Greece, and obtained b-values for 29 sequences. The value ranged from 0.33 to 1.45 with a median of 0.73. Ranalli[400] recalculated b-values for five of these sequences. The values are $\hat{b}=0.557$ (s=85) for the Chalkidike sequence of 1932 ($b=0.52$), $\hat{b}=0.639$ ($s=299$) for the west Thessaly sequence of 1954 ($b=1.00$), $\hat{b}=0.926$ ($s=400$) for the Amorgos sequence of 1956 ($b=0.92$), $\hat{b}=0.905$ ($s=291$) for the Magnesia sequence of 1957 ($b=0.73$), and $\hat{b}=1.357$ ($s=139$) for the Zante sequence of 1962 ($b=1.07$). He also obtained $\hat{b}=1.086$ ($s=103$) for the Cremasta sequence of 1966 ($b=1.12$) studied by Comminakis et al.[66] Drakopoulos and Srivastava[401] obtained $\hat{b}=1.25$ ($s=308$) for the aftershock sequence of the earthquake in Epidavros, Greece on July 4, 1968.

Gupta et al.[402] obtained $\hat{b}=0.56$ ($s=52$) for the aftershock sequence of the Godavari Valley, India, earthquake of April 13, 1969. Utsu[175] obtained $b=1.22$ for aftershocks of the great Outer Mongolian earthquake of December 4, 1957 from records of the Benioff short-period seismographs at College-Outpost, Alaska.

From Duda's table[129] for the Chilean earthquake sequence of 1960, the b-value is recalculated as follows.

$\hat{b}=0.75$ ($s=110$), Chile, May–December 1960, $M\geq 5.8$, $b=0.7$.

b-values have been obtained for some earthquake sequences in New Zealand. Ranalli[400] obtained $\hat{b}=0.557$ ($s=71$) for the aftershocks of the Hawke's Bay earthquake of February 3, 1931. For the aftershocks of the West Port earthquake of May 10, 1962, the following value is obtained from the talbe of Adams and Le Fort.[59]

$\hat{b}=0.64$ ($s=71$), Aftershocks, West Port, $M\geq 3.0$.

Hamilton[58] reported a value of $b=1.05$ for the foreshock-aftershock sequence of the Fiordland earthquake of May 24, 1960.

viii) *Variation of b-value with depth*

According to Gutenberg and Richter[319] b-values for the world's intermediate and deep earthquakes are both $b=1.2\pm 0.2$, which are somewhat higher than the value for the world's shallow earthquakes ($b=0.90$). Gutenberg[403] developed the "unified magnitude scale". The b-values determined on the basis of this scale for the world's shallow, intermediate, and deep earthquakes of magnitude above 7 are all 1.7, while the values for earthquakes of magnitude between 6 and 7 are 0.92 (shallow), 0.54 (intermediate), and 0.51 (deep).

Katsumata[404] also determined and compared b-values for the world's shallow, intermediate, and deep earthquakes. The recalculated values by using equation (158) are

$\hat{b}=1.20$ ($s=804$), World's shallow earthquakes, 1950–1965, $M\geq 6.5$, $b=1.23$,

$\hat{b}=1.09$ ($s=195$), World's intermediate earthquakes, 1950–1965, $M\geq 6.5$, $b=1.14$,

$\hat{b}=1.06$ ($s=73$), World's deep earthquakes, 1950–1965, $M\geq 6.5$, $b=1.16$.

No significant differences in b-value are found between these three groups of earthquakes.

Tomita and Utsu[349] described that the maximum likelihood estimate of b for the world's shallow earthquakes of $h\leq 100$ km ($\hat{b}=1.07$, $s=3170$) is approximately equal to that for the world's intermediate and deep earthquakes

($\hat{b}=1.03$, $s=802$) on the basis of CGS magnitude of $m\geq5$ in the years 1964–1966.

Brazee and Stover[405] calculated b-values for the world's shallow, intermediate, and deep earthquakes on the basis of CGS magnitudes. The result of recalculation using data on shocks of magnitude 5.4 and above is as follows.

$\hat{b}=1.33$ ($s=861$),	World, 1964–1965,	$h<70$ km, $M\geq5.4$,	$b=1.361$,
$\hat{b}=1.27$ ($s=264$),	World, 1964–1965,	$70\leq h\leq300$ km, $M\geq5.4$,	$b=1.124$,
$\hat{b}=1.38$ ($s=68$),	World, 1964–1965,	$h>300$ km, $M\geq5.4$,	$b=1.138$.

The date reported by Rothé[452] for the world's earthquakes yield the following b-values.

$\hat{b}=1.11$ ($s=2060$),	World, 1953–1965,	$h<70$ km, $M\geq6$,	$b=1.22$,
$\hat{b}=0.98$ ($s=517$),	〃	$70\leq h<300$ km, $M\geq6$,	
$\hat{b}=1.04$ ($s=173$),	〃	$h\leq300$ km, $M\geq6$.	

No significant differences are evident between the three depth ranges.

Chohan and Srivastava[406] investigated the variation of b with depth for the world's earthquakes of magnitude 6 and larger based on the catalog in Gutenberg and Richter's book.[319] They found b-values of about 0.67 for four depth reanges covering 0 to 400 km, and $b=1.09$ for a range between 400 and 500 km, etc. These b-values, however, seem to be resulted from inadequate use of the catalog.

In the Japanese region, Katsumata[199] obtained $b=1.2$ for earthquakes deeper than 150 km and $b=1.1$ for those deeper than 300 km. Later Katsumata[404] reported b-values of around 1.0 for shallow, intermediate, and deep earthquakes in and near Japan. The recalcualted values are as follows.

$\hat{b}=0.99$ ($s=356$),	In and near Japan,	$h<70$ km, $M\geq6.0$,	$b=1.00$,
$\hat{b}=1.02$ ($s=80$),	〃	$70\leq h<300$ km, $M\geq6.0$,	$b=1.07$,
$\hat{b}=0.98$ ($s=78$),	〃	$h\geq300$ km, $M\geq6.0$,	$b=0.99$.

Suyehiro,[407] from a careful observation of small deep earthquakes at Matsushiro, central Japan, showed that deep earthquakes of small magnitude, say magnitude around 3, are not so frequent as expected from the frequency of large deep earthquake of magnitude 6 or more. Thus the magnitude-frequency relation is not represented by a linear equation of $\log n(M)$ and M.

Suyehiro[408),409)] obtained similar results for deep earthquakes in the Fiji region and South America.

The b-values for intermediate earthquakes in the Hindu Kush region have been described in several papers. The first b-value for this region is $b=0.6\pm0.1$ given by Gutenberg and Richter.[319)] From Santô's paper[410)] on Hindu Kush seismicity, we obtain the following estimate of b.

$\hat{b}=1.41$ ($s=84$), Hindu Kush, $h<100$ km, $m\geq5.1$,
$\hat{b}=1.25$ ($s=43$), 〃 $h>100$ km, $m\geq5.1$.

Ram and Sriniwas[391)] showed a graph of logarithm of frequency versus magnitude for earthquakes in the Hindu Kush and Delhi region. The plotted points lie on a straight line of slope 0.59 in the magnitude range from 2 to more than 6 with unbelievably small scatter. It is also unbelievable that such small earthquakes as of magnitude 2 or 3 have been observed without omission in this region. Lukk[20)] obtained a high γ-value for aftershocks of the Dzhurm earthquake of 1965 ($h=210$ km) in the Hindu Kush region. Recalculation yields the following value.

$\hat{\gamma}=0.66$ ($s=171$), Aftershocks, Hindu Kush, $K\geq10$, $\gamma=0.87$.

If shocks with $K\geq9$ are used, $\hat{\gamma}=0.58$ ($s=358$).

Another region of high seismic activity at intermediate depths is the Vrancea region in Rumania. Enescu and Jinau[411)] obtained $b=0.70$–0.75 for this activity. Constantinescu and Enescu[412)] reported that $\gamma=0.49$ ($K\geq10$) and $b=0.73$ ($M=3.5$) for the Vrancea region. The recalculated γ-value is as follows (shocks of $K=10$ are excluded).

$\hat{\gamma}=0.60$ ($s=248$), Vrancea, $K\geq11$.

Radu and Tobias reported that the frequency of small earthquakes in the Vrancea region with $K<9$ is fewer than that expected from the larger earthquakes.[381)] Purcaru[21)] obtained the b-value for the aftershock sequence of the intermediate earthquake on November 10, 1940 in the Vrancea region. On the basis of his paper we obtain

$\hat{b}=0.75$ ($s=35$), Aftershocks, Vrancea, $M\geq3.9$, $b=0.71$.

Galanopoulos[413)] reported b-values of 0.82 and 0.42 for shallow and intermediate earthquakes in Greece, respectively. Tryggavason and Lawson[414)] obtained $b=0.57$ for intermediate earthquakes near Bucaramanga, Colombia, with body wave magnitude between 3.9 and 4.8 during 1962–February 1969.

The earthquake frequency decreases more rapidly with increasing magnitude in the range between 4.8 and 6.0.

As noted in a previous section, the b-value for Japanese earthquakes of focal depths 0 to 30 km is somewhat lower than that of focal depths 30 to 60 km. Miyamura[347] stated that in Japan $b=0.7$ ca. for crustal earthquakes and $b=0.9$ ca. for subcrustal earthquakes. Parazachos et al.[94] concluded that the b-value for an aftershock sequence in Greece decreases with increasing depth of the main shock as

$$b = 1.26 - 0.0135\,h. \qquad 14\ \text{km} \leq h \leq 70\ \text{km} \tag{167}$$

It is well known that b-values are very high for some volcanic earthquakes (for reviews of volcanic earthquakes, see e.g., Sakuma and Nagata,[415] Minakami,[416] Kubodera[417]). These earthquakes are very shallow, less than a few kilometers below the surface.

ix) *Variation of b-value with time*

In addition to the spatial variation of b-value, temporal variation of b or m value has been discussed by several seismologists. Riznichenko[382] stated that the temporal variations of γ exceed considerably the spatial variations of time average of γ.

Ikegami[418] studied the variation of b-value with time during the recent forty years for earthquakes in two sea regions A and B off the Pacific coast of northeastern Japan. He used $\log \Delta$ as a measure of magnitude, where Δ is the epicentral distance to the farthest station where the shock was felt. On the basis of an empirical relation between $\log \Delta$ and M, i.e., $M=\gamma' \log \Delta - \delta'$, he assumed a relation

$$\log n(\Delta) = \alpha' - \beta' \log \Delta \tag{168}$$

where $n(\Delta)$ is the number of earthquakes with maximum felt distance between Δ–50 km and Δ+50 km, and estimated β'-values for each of the five-year intervals starting from 1924, 1925,.., 1960. The values of β' vary considerably with time with periods of 10 to 20 years in the both regions. The average β'-values are 2.67 for region A, 3.70 for region B, and 3.01 for region A+B. Ikegami described that these values correspond to b-values of 0.99, 1.37, and 1.11 respectively, since $\beta'=b\gamma'$ and $\gamma'=2.7$. However, the correct relation is $\beta'=b\gamma'+1$. Thus the b-values should be 0.62, 1.00, and 0.74 respectively.

The main reason for the small b-value for region A may be that many earthquakes in this region occurred at places located more than 150 km from

the coast, while the statistics was taken for earthquakes with Δ 150 km and over. Many earthquakes having such mangitude that corresponds to Δ of more than 150 km have not been included in the statistics as they were not felt in Japan. Such earthquakes are more frequent in region A than in region B, because the intense seismic activity spreads farther off the coast in region A. Ikegami[419),420)] later studied the same problem by the same method for several other regions and reached the conclusion that the b-value varies with time in each region and the period of low b-value corresponds to the period of high seismic energy release. This negative correlation between b-value and energy release is naturally expected, even if earthquakes are random samples from a population in which magnitudes are distributed in accordance with Gutenberg-Richter's law with a time invariant b-value. For a group of samples which includes large magnitude events by chance, the b-value calculated by the ordinary least squares method is usually lower than the value in population, since the method gives very heavy weight to a small number of large events.

As described in Chapter 5, no significant variations of b-value with time have been found in most aftershock sequences. The Matsushiro earthquake swarm of 1965–1969 has been investigated in detail by many investigators. b or m values have been reported for various stages of the activity and in various magnitude ranges. The magnitude-frequency relation is normal for the magnitude range between 4.0 and 5.1 on the basis of magnitude values given by JMA, but earthquakes having magnitude larger than about 5.3 are too few.[421)]. Okada[422)] obtained $\hat{b}=0.89$ ($s=230$) for $4.0 \leq M \leq 5.5$ for the period from August 1965 through 1967. In this calculation equation (177) was used together with an estimate of upper limit of the range $M_l=5.5$ from the same equation. Okada also obtained $\hat{b}=0.43$, $\hat{b}=1.13$ and $\hat{b}=0.75$ for three periods, August 1965 to March 1966, April and May 1966, and June 1966 to February 1967, respectively. Katsumata[368)] obtained $\hat{b}=1.27$ for $M \geq 4.0$ for the period from August 1965 through December 1966 without setting the upper limit.

The Party for Seismographic Observation of Matsushiro Earthquakes and the Seismometrical Section, ERI[149)] obtained m-values for every month from October 1965 to February 1966 and for every ten-day intervals from March 1966 to February 1967 from observations by acceleration seismographs at three stations in the swarm area. The m-values at the three stations show similar variations. The values were high ($m=2.2$–2.5) until February

1966 and normal ($m=1.7$–2.0) after March 1966. In the second decade of January 1967, m-values were considerably low at the three stations. Hamada and Hagiwara,[350] and Hamada[151] determined m-values for every ten-day intervals for microearthquakes recorded at Hoshina in the swarm area. Between February 1966 and May 1967 the m-value fluctuated in the range from 1.4 to 2.6. In the periods of increased activity (April to June 1966, and late August and early September 1966, and middle of January 1967), the m-values were low, around 1.5, and in the other periods the values were about 2 or more. m and b values determined from the observations at the Matsushiro Seismological Observatory did not exhibit such a tendency, though some temporal variations were observed.[152]

Drakopoulos et al.[423] obtained b-values from observations of microearthquakes at Suzaka and Matsushiro during three different intervals of the swarm. They estimated the values from the slope of the $\log N(M)$ vs M curves, but these curves were not correct since the data were lacking above certain magnitude levels. The recalculation has been performed by using equation (177).

$\hat{b}=0.66$ ($s=830$), Matsushiro swarm, Suzaka, Dec. 20–26, 1965, $-0.4 \leq M \leq 1.1$, $b=1.05$,
$\hat{b}=0.69$ ($s=515$), 〃 〃 May 13–14, 1966, $-0.5 \leq M \leq 0.8$, $b=1.19$,
$\hat{b}=0.50$ ($s=550$), 〃 Matsushiro, May 21, 1966, $-3.2 \leq M \leq -1.6$, $b=0.84$.

Nishida[424] made a seismic observation at Sanada near Matsushiro for 82 days in the summer of 1966 with a four channel recorder covering a dyanmic range of about 60 db. He obtained $m=1.267 \pm 0.04$, $m=1.877 \pm 0.153$, and $m=1.901 \pm 0.067$ for the first, second, and third channels respectively. For the fourth channel, which is of the lowest sensitivity, the number of recoded shocks was too small to determine an accurate m-value. For the first channel (the highest sensitivity) the variation of m with time is not remarkable, but for the second and third channel, m values show appreciable temporal variations in the range between about 1.6 and about 2.4. In late July and early August m-values for these two channels are the lowest, while the value for the first channel is the highest. In the first channel the average frequency of recoded shocks was about 100 per hour. Such a high density of shocks may cause considerable losses of small shocks by overlapping on seismograms.

The investigations of the Matsushiro earthquake swarm reported in several papers introduced above show appreciable temporal variations in m or b value, but the times of high or low m-values do not agree among different observations, though the patterns of temporal variation in earthquake frequency show good agreement among these observations.

Hamaguchi and Hasegawa[425] observed more than fifty thousands aftershocks of the Tokachi-oki earthquake of May 16, 1968 ($M=7.9$) at Karumai about 150 km from the center of the aftershock area with a four-channel recorder having a dynamic range of 93 db. The m-values calculated by the maximum likelihood method vary with time to some extent, but a statistical significance test indicates that the differences are not highly significant except the value of 1.53 for an interval in the middle of June when the activity increased remarkably due to the occurrence of a magnitude 7.2 earthquake (they compared m_B/m_A with F, but correctly $(m_B-1)/(m_A-1)$ should be compared with F). Later, Hasegawa and Hamaguchi[426] reported that this low value of 1.53 was a result from the very frequent occurrence of shocks (the average frequency was 57 shocks per hour), because small shocks have been missing due to overlapping of seismograms. They evaluate this effect of overlapping on the m-value based on some reasonable assumptions. On the other hand, it was found that the m-value is significantly low for aftershocks in the western part of the aftershock area ($m=1.58$, $s=1035$) than the m-values for aftershocks in the northeastern part ($m=1.79$, $s=13{,}154$) and in the southeastern part ($m=1.82$, $s=26{,}115$). Okada and Motoya[212),213] also studied the time variation of b-value for the aftershocks of magnitude 3.0 and larger in the northern part of the aftershock area.

It has been pointed out that in some cases the b-value for foreshocks is smaller than that for aftershocks (see Chapter 9). Seismological Section and Niijima Weather Station, JMA[107] reported in 1958 that b-values for foreshocks of the Niijima earthquake of 1957 were 0.53, 0.44, 0.67, and 0.69 for different periods before the occurrence of the main shock on November 11. These values are smaller than the b-value of 0.85 for the entire period of the sequence investigated. The recalculated b-values are

$\hat{b}=0.54$ ($s=27$), Foreshocks, Niijima earthquake, $M\geq 3.3$,
$\hat{b}=0.93$ ($s=119$), Aftershocks, 〃 $M\geq 3.3$.

The difference is significant at a level of 0.02.

The data of Suyehiro et al.[147] who first emphasized the difference in b-

value between foreshocks and aftershocks give the following recalculated values.

$\hat{b}=0.46$ ($s=17$), Foreshocks, earthquake in Nagano, Jan. 1964, $b=0.35\pm0.01$,
$\hat{b}=0.81$ ($s=101$), Aftershocks, 〃 $b=0.76\pm0.02$.

From Suyehiro's paper on the Chilean earthquakes, we obtain

$\hat{b}=0.75$ ($s=31$), Foreshocks, Chilean earthquake of May, 1960, $b=0.55\pm0.05$,
$\hat{b}=1.22$ ($s=122$), Aftershocks, 〃 $b=1.13\pm0.04$.

The difference in b-value between foreshocks and aftershocks is significant at the 2% level for both sequences. (The conclusion in the author's paper in 1966[177] should be corrected, because the correction for the length of the magnitude interval ΔM was not applied in that paper). Suyehiro[234] also compared b-values between foreshocks and aftershocks of an earthquake of magnitude 5.1 on September 14, 1967 in Nagano. The recalculated values do not differ significantly.

$\hat{b}=0.66$ ($s=171$), Foreshocks, earthquake in Nagano, 1967, $b=0.59\pm0.05$,
$\hat{b}=0.74$ ($s=876$), Aftershocks, 〃 $b=0.89\pm0.02$.

Fedotov[427] reported a small γ-value of 0.37 for the seismic activity during a ten-month interval before the great southern Kurile earthquake of November 6, 1958 (see also Riznichenko[381]).

Watanabe and Kuroiso[217] studied the Wachi earthquake of 1968. The b-value for the foreshock sequence from February to August 18 (when the main shock of magnitude 5.6 occurred) has been estimated as $\hat{b}=0.59$ based on the data of about 300 shocks of magnitude between −0.4 and 3.0. This value is significantly lower than the values for more numerous aftershocks, $\hat{b}=0.80$ (August 18–December 18) and $\hat{b}=0.86$ (December 18–April 29).

b-values for three foreshock sequences have been reported for Greek earthquakes studied by Papazachos et al.[9] The recalculation yields the following values for foreshocks and aftershocks.

$\hat{b}=0.89$ ($s=46$), Foreshocks, Kephallenia earthquake of 1953, $M\geq4.0$, $b=0.61$,
$\hat{b}=0.72$ ($s=361$), Aftershocks, 〃 $M\geq4.0$, $b=0.85$,
$\hat{b}=0.77$ ($s=7$), Foreshocks, Volos earthquake of 1955, $M\geq3.6$, $b=0.43$,
$\hat{b}=0.72$ ($s=45$), Aftershocks, 〃 $M\geq3.3$, $b=0.63$,

$\hat{b}=0.74$ ($s=8$), Foreshocks, NE Crete earthquake of 1956, $M \geq 4.0$, $b=0.55$,
$\hat{b}=0.58$ ($s=19$), Aftershocks, 〃 $M \geq 3.7$, $b=0.65$.

All recalculated b-values for foreshocks are larger than the values for the corresponding aftershocks, but the differences are not significant.

In the case of the Cremasta earthquake of 1966 studied by Comminakis et al.[66] the b-value for foreshock sequence of $b=1.41$ is larger than that for aftershock sequence $b=1.12$. Since the numbers of foreshocks and aftershocks used in constructing the magnitude-frequency graphs are fairly large (about 500 and 1500 as read in the graph prepared by them), the accuracy of these b-values is believed to be high.

The Party for Seismological Observation of Matsushiro earthquakes and the Seismometrical Section, ERI[149] showed that no appreciable difference is evident in m-value between foreshocks and aftershocks of the Izu earthquake of November 26, 1930. Smith et al.[148] studied the microearthquake swarm near Mould Bay in 1965. The equal b-values have been obtained for 129 shocks before the largest shock in the swarm and for 1896 shocks after the largest shock. The recalculated values are

$\hat{b}=0.61$ ($s=129$), Foreshocks, Mould Bay, $-1.1<M<2.4$, $b=0.68$,
$\hat{b}=0.70$ ($s=1896$), Aftershocks, 〃 $-1.1<M<2.9$, $b=0.68$.

The difference between the two b-values is not signfiicant. Okada and Motoya[212),213] noted that the earthquakes following the magnitude 5.3 earthquake on May 2, 1968, which may be considered to be foreshocks of the Tokachi-oki earthquake ($M=7.9$) on May 16, has an m-value of 1.84. This value is not significantly different from the m-value of 1.76 for the Tokachi-oki aftershock sequence.

Lahr and Pomeroy[428] obtained maximum likelihood estimates of b for the foreshock and aftershock sequences of the earthquake on March 20, 1966 in the Republic of Congo. The value for the aftershocks ($\hat{b}=1.05$, $s=779$) is almost equal to the value for the foreshocks ($\hat{b}=1.06$, $s=35$).

Guha et al.[453] showed changes in b-value for foreshock and aftershock sequences of the Koyna, India, earthquake of December 10, 1967. The aftershocks have considerably larger b-values (0.75–0.94) than those of the foreshocks (0.45–0.79).

Inouye[429] and Sekiya[430] have remarked that a peculiar pattern of the log $N(M)$ vs M diagram (deficiency of large earthquakes) before the occurrence

of a great earthquake. It may be difficult to prove that this is due to a cause other than the statistical fluctuation, but they seem to be of the opinion that this should not be treated as a statistical phenomenon only. Nagumo[431] also discussed the development of the magnitude-frequency relation in the aftershock sequence of the Tokachi-oki earthquake.

13.5 *Non-exponential formulas for magnitude distribution*

It is occasionally noticed that a curve, especially an upward convex one, fits the plotted points in a graph of log $n(M)$ versus M (or log $n(a)$ versus log a) rather than a straight line as expected from Gutenberg-Richter's law. Some authors (e.g., Gutenberg[403]) used two straight lines for different magnitude ranges to represent such a distribution. On the other hand, some investigators (e.g., Suzuki[184], Abe[432]) tried to explain the curvature from statistical points of view. However, it has been pointed out by Utsu[176] that the predominance of upward convex curves over downward convex ones can not be resulted from the random sampling fluctuations. The fact that $G_1(M_1^*)=0.368<0.5$ (see Section 13.2) is a support of this result.

It is generally believed that there is an upper limit of magnitude M_l for a region concerned below which the magnitude distribution follows Gutenberg-Richter's law. This limit is equal to, or somewhat smaller than, the maximum conceivable magnitude $M_{\max}$ in the region. If we treat an earthquake group for which M_1^* is far smaller than M_l, we may consider that M_l is infinitely large. In the previous sections, the discussions have been confined to this case. The discussions including the effect of M_l will be made in the following sections. The magnitude M_l or $M_{\max}$ is often considered as a parameter that characterizes the seismic activity of a region. Mogi[433] obtained the magnitude M_l for several regions of Japan and discussed the relation between M_l and the tectonic feature of the region.

Except the above point, it is possible that some non-linear log $n(M)$ versus M curves are resulted from the incompleteness of data used in constructing the curve. The probable causes of the incompleteness are as follows:

(1) Omission of small earthquakes outside of the detection capability of the recording station

(2) Omission of small earthquakes below the background noise.

(3) Omission of small earthquakes due to overlapping of seismograms when the seismic activity is high.

(4) Erroneous mesurements of amplitudes of large earthquakes due to

saturation of the recording instrument.

(5) Erroneous measurements of amplitudes of small earthquakes due to the friction in the recording instrument, due to the effect of the width of the recoding trace, and due to the effect of background noise.

(6) Relatively small trace amplitude of large or small earthquakes due to deviation of their predominant frequencies from the frequency-band of the recording instrument.

Microearthquake observations at various sites of the world (e.g., Asada and Suzuki,[181] Asada,[434] Asada et al.,[435] Buné et al.,[436] Buné[377] Sanford and Holmes[437] Muramatsu et al.,[438] Isacks and Oliver,[439] Oliver et al.,[440] Hashizume et al.,[441] Hashizume.[442])) usually show that the frequency of earthquakes increases approximately exponentially with decreasing magnitude to a magnitude as low as –1 or –2. However, some investigators believe that at least in some cases the frequency does not increase exponentially with decreasing magnitude. They proposed different forms for the frequency distribution.

A Poisson distribution was tried by Niazi[360]. Lomnitz[443] showed that a lognormal distribution for energy E represents the energy-frequency relation for the world's earthquakes of $M \geq 6.5$ as well as the inverse power type distribution. Neunhöfer[444],[445] also applied a lognormal distribution to energies of rockbursts and earthquakes. Sacuiu and Zorilescu[446],[447] concluded that the magnitude of earthquakes in the Vrancea region has a lognormal distribution or a distribution in the form

$$\log n(M) = a + b \log M - c (\log M)^2 , \qquad c > 0 . \tag{169}$$

Purcaru and Zorilescu[448] disscussed the method for determining the values for the coefficient a, b, and c.

The lognormal distribution of M is not compatible with the observed fact that the frequency of shallow earthquakes increases approximately exponentially with decreasing magnitude from great earthquakes of magnitude 8 to microearthquakes of magnitude –2. The author proposes another three-parameter equation for the magnitude distribution of shallow earthquakes (see equation (215) in Section 13.8). For deep earthquakes there is a possibility that equation (169) is a suitable approximation.

13.6 *Some properties of the truncated form of Gutenberg-Richter's law*

In this section we assume that there is an upper limit of magnitude M_l, below which the magnitude distribution is expressed by Gutenberg-Richter's

formula and above which no earthquakes take place, i.e.,

$$\left.\begin{aligned} &\log n(M) = a - bM\,, && M \leq M_l \\ \text{and}\quad &n(M) = 0\,. && M > M_l \end{aligned}\right\} \tag{170}$$

Then for $M \leq M_l$

$$N(M) = N_0 \frac{10^{-bM} - 10^{-bM_l}}{1 - 10^{-bM_l}} = N_0 \frac{e^{-b'M} - e^{-b'M_l}}{1 - e^{-b'M_l}} \tag{171}$$

where

$$N_0 = N(0) = (10^a/b')\,(1 - 10^{bM_l})\,. \tag{172}$$

If M_1^{**} is defined by $N(M_1^{**}) = 1$, we obtain

$$bM_i^{**} = bM_1^{*} - \log(i + 10^{a - bM_l}/b') \tag{173}$$

It is easily found that $M_1^{**} \to M_1^{*}$ $(M_l \to \infty)$ and $M_1^{**} \to M_l$ $(M_1^{*} \to \infty)$.

The probability that the ith largest earthquake in a group has a magnitude between M_i and $M_i + dM_i$ is given by

$$g_i(M_i)\,dM_i = \frac{b'}{(i-1)\,!}\,\lambda^i e^{-\lambda} \frac{10^{-bM_i}}{10^{-bM_i} - 10^{-bM_l}} = \frac{\lambda^{i-1} e^{-\lambda}}{(i-1)\,!}\,d\lambda \tag{174}$$

where

$$\lambda = i\,(10^{-bM_i} - 10^{-bM_l})\,/\,(10^{-bM_i^{*}} - 10^{-bM_l})\,. \tag{175}$$

Since the expression of $g_i(M_i)$ in terms of λ is exactly the same as equation (149), the cumulative destribution function of M_i, $G_i(M_i)$, has the same form as equation (151), though the definition of the parameter λ is different.

The maximum likelihood estimate of b in the distribution function expressed by equation (170) is given by the equation[62),422)]

$$\overline{M} = \frac{1}{b'} + \frac{M_l - M_s\, e^{-b'(M_l - M_s)}}{1 - e^{-b'(M_l - M_s)}} \tag{176}$$

or

$$\frac{\overline{M} - M_s}{M_l - M_s} = \frac{1}{b \ln 10\,(M_l - M_s)} - \frac{1}{e^{b \ln 10 (M_l - M_s)} - 1} \tag{177}$$

where $\overline{M}$ is the mean magnitude of earthquakes with magnitude between M_s and M_l. Equation (177) used by Okada[422)] provides the relationship between $(\overline{M} - M_s)/(M_l - M_s)$ and $b(M_l - M_s)$, which is tabulated in Table 19. The value of b can easily determined from this table, when $\overline{M}$, M_s, and M_l are known.

If the distribution of maximum amplitude recorded at a station satisfies Ishimoto-Iida's law in the range below a_l, i.e.,

Table 19. Values of $b(M_l - M_s)$ as a function of $X = (\bar{M} - M_s)/(M_l - M_s)$.

X	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.10	4.341	4.298	4.256	4.215	4.174	4.133	4.093	4.054	4.016	3.979
.11	3.943	3.907	3.872	3.838	3.804	3.771	3.738	3.706	3.674	3.643
.12	3.612	3.582	3.552	3.522	3.493	3.464	3.436	3.408	3.381	3.354
.13	3.328	3.302	3.276	3.251	3.226	3.201	3.177	3.153	3.130	3.107
.14	3.084	3.061	3.039	3.017	2.995	2.973	2.952	2.931	2.910	2.890
.15	2.869	2.849	2.830	2.810	2.791	2.772	2.753	2.735	2.717	2.699
.16	2.680	2.662	2.644	2.626	2.609	2.591	2.574	2.557	2.541	2.524
.17	2.508	2.492	2.476	2.460	2.445	2.429	2.414	2.399	2.384	2.730
.18	2.355	2.340	2.326	2.311	2.297	2.283	2.269	2.255	2.241	2.227
.19	2.214	2.201	2.188	2.775	2.162	2.149	2.136	2.123	2.110	2.098
.20	2.085	2.074	2.060	2.048	2.036	2.024	2.012	2.000	1.989	1.977
.21	1.965	1.954	1.943	1.931	1.920	1.909	1.898	1.887	1.876	1.865
.22	1.854	1.844	1.833	1.823	1.812	1.802	1.791	1.781	1.771	1.761
.23	1.751	1.741	1.731	1.721	1.711	1.701	1.692	1.682	1.672	1.663
.24	1.653	1.644	1.634	1.625	1.616	1.606	1.597	1.588	1.579	1.570
.25	1.561	1.552	1.543	1.534	1.525	1.517	1.508	1.499	1.490	1.482
.26	1.473	1.464	1.456	1.448	1.439	1.431	1.423	1.414	1.406	1.998
.27	1.390	1.382	1.374	1.366	1.358	1.350	1.342	1.334	1.326	1.318
.28	1.310	1.303	1.295	1.287	1.280	1.272	1.264	1.257	1.249	1.242
.29	1.234	1.226	1.219	1.212	1.204	1.197	1.190	1.182	1.175	1.167
.30	1.160	1.153	1.146	1.139	1.132	1.125	1.118	1.111	1.104	1.097
.31	1.090	1.083	1.076	1.069	1.062	1.055	1.048	1.041	1.035	1.028
.32	1.021	1.015	1.008	1.001	.995	.988	.981	.975	.968	.962
.33	.955	.948	.942	.936	.929	.922	.916	.910	.903	.897
.34	.891	.884	.878	.872	.865	.859	.853	.846	.840	.834
.35	.828	.822	.816	.809	.803	.797	.791	.785	.779	.773
.36	.767	.761	.755	.749	.743	.737	.731	.725	.719	.913
.37	.707	.701	.695	.689	.683	.677	.671	.666	.660	.654
.38	.648	.642	.637	.631	.625	.619	.614	.608	.602	.596
.39	.591	.585	.579	.574	.568	.562	.557	.551	.545	.540
.40	.534	.529	.523	.517	.512	.506	.501	.495	.489	.484
.41	.478	.473	.467	.462	.456	.451	.446	.440	.435	.429
.42	.423	.418	.413	.407	.402	.396	.391	.386	.380	.374
.43	.369	.364	.358	.353	.348	.342	.337	.331	.326	.321
.44	.315	.310	.305	.300	.294	.289	.284	.278	.273	.268
.45	.262	.257	.252	.246	.241	.235	.230	.225	.220	.215
.46	.209	.204	.199	.194	.189	.183	.178	.173	.167	.162
.47	.157	.152	.146	.141	.136	.131	.125	.120	.115	.109
.48	.104	.099	.094	.089	.083	.078	.073	.068	.062	.057
.49	.052	.047	.042	.036	.031	.026	.021	.016	.010	.005

$$n(a) = ka^{-m} \qquad a \leq a_l \tag{178}$$

and

$$n(a) = 0\,. \qquad a > a_l \tag{179}$$

The maximum likelihood equation is[62)]

$$\overline{\ln a} = \frac{1}{m-1} + \frac{a_l^{1-m} \ln a_l - a_s^{1-m} \ln a_s}{a_l - a_s} \tag{180}$$

where $\overline{\ln a}$ is the mean of $\ln a$ for earthquakes with amplitude between a_s and

a_l. This equation can be transformed into the form

$$\frac{\ln \bar{a}-\ln a_s}{\ln a_l-\ln a_s}=\frac{1}{\ln (a_l/a_s)^{m-1}}-\frac{1}{(a_l/a_s)^{m-1}-1}\,. \tag{181}$$

It should be noted here that M_l in equation (177) (or a_l in equation (181)) does not indicate the largest magnitude (or largest amplitude) in a group of earthquakes for which the b-value (or m-value) is to be determined. If there is no evidence indicating the existense of the upper limit of magnitude above which Gutenberg-Richter's formula is not applicable, equation (158) should be used. Equation (177) should be used when the magnitude distribution follows Gutenberg-Richter's law for the range of magnitude between M_s and M_l only, or when the data are available for this magnitude range only.

Utsu[192] suggested the use of the following equation, when Gutenberg-Richter's law is assumed to be valid for the whole range of magnitude above M_s but for some reasons the magnitudes are not available for l earthquakes having magnitude M_l and larger.

$$b=\frac{(s-l)\log e}{\sum M_i-sM_s+lM_l} \tag{182}$$

where $\sum M_i$ is the sum of magnitudes of $s-l$ earthquakes having magnitude between M_s and M_l. This equation has been derived from equation (177) by putting

$$e^{b\ln 10(M_l-M_s)}=l/s\,. \tag{183}$$

Since equation (183) is an approximation, equation (182) is an approximate formula. However, this formula is simple and provides generally a good approximation.

13.7 *Magnitude-frequency relation and the aftershock activity*

In this section we will discuss a statistical model for magnitude-frequency distribution of earthquake occurrence. In this model all earthquakes are classified into two groups, "main shock" group and "aftershock" group. We deal with earthquakes of magnitude larger than an arbitrarily fixed level M_s.

It is assumed that the magnitude distribution for all the main shocks occurring in a certain limited space and time intervals is represented by

$$\log N(M_0)=\log N_s-b_0(M_0-M_s) \tag{184}$$

where $N(M_0)$ is the number of shocks with magnitude M_0 and larger and $N_s=$

$N(M_s)$. In a sequence of aftershocks triggered by a main shock, the magnitude-frequency relationship in the same form is assumed, i.e.,

$$\log N(M) = \log A - b_a (M - M_s) \tag{185}$$

where A is the total number of aftershocks in the sequence having magnitude M_s and larger. If no aftershocks of magnitude M_s or larger follow a main shock, the main shock is nothing but a single event as far as the shocks of $M \geq M_s$ are concerned, but it is still calssified here as a main shock. Although the b-value for each aftershock sequence, b_a, is not necessary equal to b_0, it is assumed here that all aftershock sequences have the same value for b_a.

From equation (147), the most probable magnitude of the largest shock in an aftershock sequence is given by

$$M_1{}^* = \frac{\log A}{b_a} + M_s \tag{186}$$

Referring to equation (6) (Chapter 2, p. 141), if we assume that the difference between M_0 and $M_1{}^*$ is expressed by an equation of the form

$$M_0 - M_1{}^* = u - vM_0 , \tag{187}$$

we obtain

$$\log A = b_a (1+v) M_0 - M_s - u . \tag{188}$$

Therefore the frequency distribution of A takes the form

$$f(A) = \frac{dN(M_0)}{dM_0} \frac{dM_0}{dA} = CA^{-q} \tag{189}$$

where

$$C = \frac{N_0 b_0}{(1+v) b_0} 10^{b_0 (v M_s - u)/(1+v)} \tag{190}$$

and

$$q = \frac{b_0}{(1+v) b_a} + 1 . \tag{191}$$

If $b_0 = b_a = 0.85$ and $v = 0.5$, $q = 1.67$. In Figure 133 the number of aftershock sequences whose size is A and larger is plotted against A on logarithmic scales for shallow earthquakes of magnitude 6 and above occurring in and near Japan during 1926–1968. This figure indicates that the inverse power type distribution of A with q of about 2 is roughly applicable to this case.

If there is an upper limit of magnitude M_l as discussed in the last section,

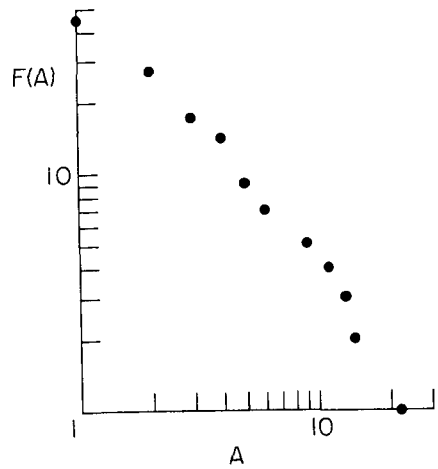


Fig. 133. Cumulative frequency distribution of the number of aftershocks with $M \geq 6$ accompanying the Japanese shallow earthquakes of $M \geq 6$ during 1926–1968.

equation (184) can be written in the form

$$N(M_0) = N_s \frac{10^{-b_0 M_0} - 10^{-b_0 M_l}}{10^{-b_0 M_s} - 10^{-b_0 M_l}} \qquad M_s \leq M_0 \leq M_l \tag{192}$$

and

$$n(M_0) = 0\,. \qquad M_0 \leq M_s \quad \text{and} \quad M_0 \geq M_l \tag{193}$$

In this case, A is distributed as

$$f(A) = C_l A^{-q} \tag{194}$$

in the range between A_s and A_l, where

$$A_s = 10^{b_a(vM_s - u)} \tag{195}$$

$$A_l = 10^{b_a\{(1+v)M_l - M_s - u\}} \tag{196}$$

and

$$C_l = C/(1 - 10^{-b_a(M_l - M_s)}) \tag{197}$$

The mean and the variance of A can be written in the form

$$\bar{A} = \left(\frac{q-1}{2-q}\right)\left(\frac{\alpha^{2-q}-1}{1-\alpha^{1-q}}\right) A_s \qquad (q \neq 2) \tag{198}$$

and

$$V=\left(\frac{q-1}{3-q}\right)\left(\frac{\alpha^{3-q}-1}{1-\alpha^{1-q}}\right)A_s^2-\bar{A}^2 \qquad (q\neq 3) \tag{199}$$

respectively, where

$$\alpha=A_l/A_s=10^{b_a(1+\cdot)(M_l-M_s)}\,. \tag{200}$$

For $q=2$,

$$\bar{A}=\left\{\ln\alpha\Big/\left(1-\frac{1}{\alpha}\right)\right\}A_s\,, \tag{201}$$

and

$$V=\alpha A_s^2-\bar{A}^2\,. \tag{202}$$

Equations (198) and (199) can be solved for q and A_s, if we know both $\bar{A}$ and V. Then we can calculate the values of u and v from equations (191) and (195), if b_0 and b_a are known. In some cases the values for $\bar{A}$ and V can be estimated on the basis of a model for the time distribution of earthquakes as discussed in Chapter 14 without classifying every earthquake into the two groups, "main shocks" and "aftershocks".

In the simple case in which $b_0=b_a$ and $v=0$, we obtain the following approximation for $M_l-M_s\simeq 1.5$.

$$\bar{A}=\ln 10\, b_a(M_l-M_s)\,10^{-b_a u} \tag{203}$$

and

$$\log V=b_a(M_l-M_s-2u)\,. \tag{204}$$

Therefore. we can calculate the value for $b_a(M_l-M_s)$ and $b_a u$ from $\bar{A}$ and V. Figure 134 is a graph showing the relation between these quantities. If b_a is given, we can obtain the values of u and M_l. Actually v may not be equal to zero, but the values of u and M_l calculated under the above assumptions may represent rough approximations for the magnitude of maximum earthquake and the degree of aftershock activity for the earthquake group concerned.

Strictly speaking, the magnitude distribution for all earthquakes (main shocks plus aftershocks) can not be represented by Gutenberg-Richter's formula, if the magnitude distributions for main shocks and for each aftershock sequence obey Gutenberg-Richter's law as assumed in this section. Nevertheless, the magnitude distribution for all earthquakes is approximately represented by Gutenberg-Richter's formula with a b-value generally larger than b_0 and b_a. It should also be mentioned that the magnitude distribution for all aftershocks (shocks in all aftershock sequences put together) roughly

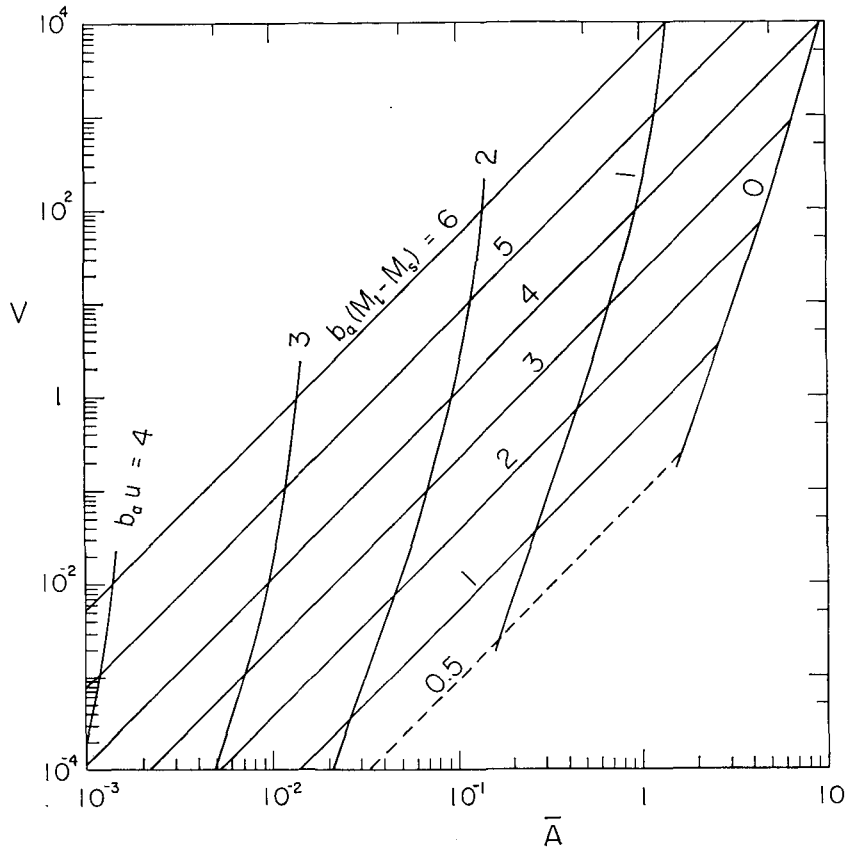


Fig. 134. Graph showing the relation between (A, V) and $(b_a u, b_a(M_l - M_s))$.

follows Gutenberg-Richter's law with a b-value (denoted here by $b_a{}^+$) somewhat higher than b_a for individual aftershock sequences.

Figure 135 shows an example. In this figure the number of earthquakes having magnitude M and larger are plotted against M for all aftershocks, for all main shocks, and for all earthquakes, using data on shallow earthquakes occurring in and near Japan during 1926–1968. The relatively small number of foreshocks and swarm earthquakes (except the largest one in each swarm) have been included in the "aftershock" group, but this does not seriously influence the discussions and conclusions in this section. In this example, b-values are calculated by using equation (158) to be $\hat{b}=0.97$ for all earthquakes, $\hat{b}_0=0.86$ for main shocks, and $\hat{b}_a{}^+=1.24$ for all aftershocks. As

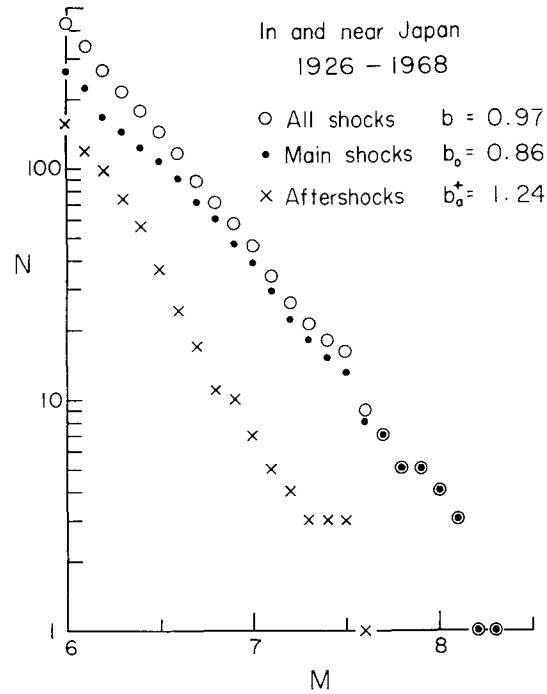


Fig. 135. Cumulative distribution of magnitudes for shallow earthquakes in and near Japan.

indicated previously, the median of b-value for aftershock sequences in Japan is about 0.85. Thus, the value of $b_a{}^+$ is about 50% larger than this value.

The value of $b_a{}^+$ of 1.24 is comparable to that reported by Mogi.[188] However, this high b-value does not mean that the average b-value for individual aftershock sequences is larger than that for general earthquakes.

In a model shown in Figure 136, magnitudes for main shocks are distributed according to Gutenberg-Richter's law with a b_0 value of 0.85. Each main shock accompanies an aftershock sequence, in which the ith largest shock has a magnitude given by $M_i{}^*=M_1{}^*-(\log i)/b_0$, where $M_1{}^*$ is given by equation (185), and $b_0=0.85$. A in equation (185) is given by equation (188) with $u=1.0$ and $v=0$. The points representing all aftershocks and the points representing all earthquakes fit approximately straight lines of different slope. The maximum likelihood estimates of b for the two groups

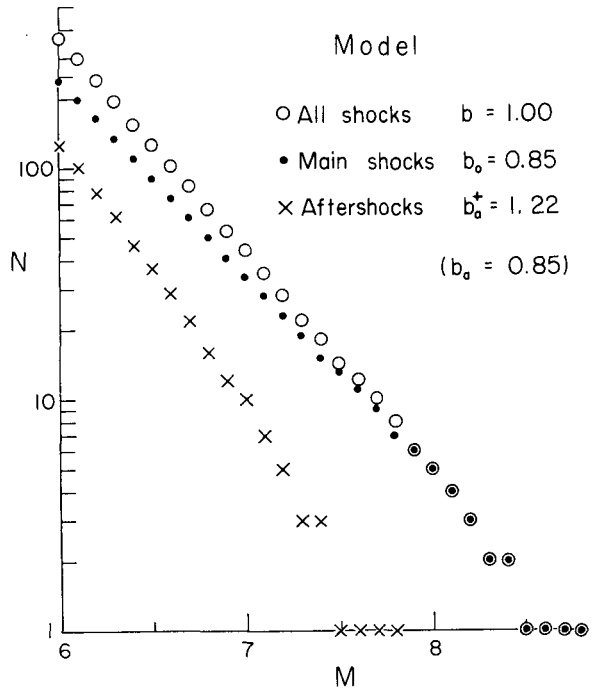


Fig. 136. A model for the magnitude distribution of earthquakes that attempts to explain the observational data shown in Figure 135.

are $b_a{}^+=1.22$ and $b=1.00$ respectively. It is seen that the values of b, b_0, b_a, and $b_a{}^+$ are nearly equal to the respective values obtained for the actual data for Japan shown in Figure 135.

It is now evident that the b-value for general earthquakes depends on the b-value for the main shocks (b_0), that for the aftershock sequences (b_a), and the degree of aftershock activity u. According to this view, it seems possible that the regional variation of aftershock activity is partly responsible for the observed regional variation of b-value.

The expression for the distribution of magnitude for all aftershocks (shocks in all aftershock sequences put together), $n^+(M)$, can be derived as follows. The magnitude distribution for an aftershock sequence accompanying a main shock of magnitude M_0 can be written in the form

$$n(M) = b_a{}^+ \, 10^{-b_a\{M-(1+v)M_0+u\}} \, . \tag{205}$$

If the magnitude distribution for main shocks is expressed by

$$\left.\begin{aligned} \log n_0(M_0) &= a_0 - b_0 M_0 , & M \leq M_l \\ n_0(M_0) &= 0 , & M > M_l \end{aligned}\right\} \tag{206}$$

we have for $n^+(M)$

$$n^+(M) = \int_M^{M_l} n(M)\, n_0(M_0)\, dM_0 . \tag{207}$$

If $b_a = b_0$ and $v = 0$,

$$n^+(M) = C_a\, 10^{-b_a M} (M_l - M) \qquad M \leq M_l \tag{208}$$

where

$$C_a = b_a{}^+ 10^{a_0 - b_a u} \tag{209}$$

or

$$\log n^+(M) = a_a - b_a M + \log(M_l - M) , \qquad M \leq M_l \tag{210}$$

where

$$a_a = \log C_a . \tag{211}$$

Equations (207), (208), and (210) are based on the assumption that the magnitude of the largest aftershock does not exceed M_0. If the upper limit of the magnitude of the largest aftershock is fixed at $M_0 - w$, M_l in equations (208) and (210) should be replaced by $M_l - w$. Some remarks on these equations will be added in the next section.

13.8 *Discussions*

As described in Section 13.5, 113 b-values has been recalculated by the maximum likelihood method. As shown in Figure 137, about 16% of the recalculated values differ from the values found in the original papers by more than 0.2. It is seen that the use of different methods for the same data sometimes causes considerable differences in the b-value determinations. The b-values for the same region determined from different sources of data shows more remarkable variability, and the conclusions of different investigators are sometimes contradictory. Since the b-values are subjected to errors of several kinds as mentioned previously (p. 390 and p. 416), the comparison should be made only between values calculated from carefully collected and processed data.

There are several magnitude scales currently used in seismological investigations; the local magnitude M_L (originally defined by Richter), the surface wave magnitude M_S, the body wave magnitude M_B (or m), etc. The magnitude used in Gutenberg-Richter's book[319] is the weighted mean of surface wave magnitudes determined from surface wave observations and

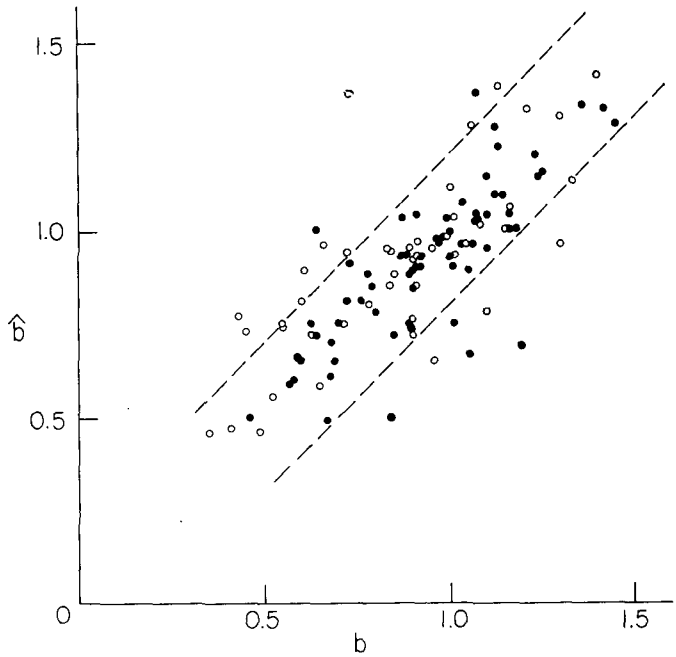


Fig. 137. Comparison between two b-values, $\hat{b}$: the maximum likelihood estimate, and b: the value found in the original paper. Solid and open circles indicate that the data size is larger than 100 and smaller than 100 respectively.

surface wave magnitudes converted from body wave magnitudes. The conversion formula used by them is $M_S - m = (M_s - 7)/4$,[449] but later the following equation[403] has been used widely.

$$m = 0.63\, M_S + 2.5\,. \tag{212}$$

However, there are several other formulas connecting m and M_s, some of which are quite different from the above equation. For example, Ichikawa and Basham[450] obtained

$$M_S = 0.76\, m + 1.58\,. \tag{213}$$

The magnitude used in JMA for earthquakes in Japan is calculated from Tsuboi's formula,[396] which was developed with the intention of obtaining the same magnitude as found in Gutenberg-Richter's book.[319] Thus the JMA magnitude is a kind of surface wave magnitude. From a comparison between the JMA magnitude and the magnitude given by CGS for many Japanese earthquakes, Ichiakwa[444] showed that the realtion (213) is also applicable

between these two magnitude scales (see also Hori[451]). Generally speaking, two b-values determined from data on different magnitude scales should not be compared unless the accurate conversion formula is available.

Although the b-value is frequently used as an index of the properties of earthquake occurrence, a review in Section 13.4 indicates that many of the published b-values are not sufficiently accurate for discussing such problems as regional or temporal variations of the magnitude-frequency relation of earthqaukes in connection with the state of materials and stresses within the earth. Table 20 shows the range and the median of the recalculated b-values for general shallow earthquakes for the three types of regions. About half of the b-values for island arcs are those for regions in and near Japan. If these values are excluded, the median will be 1.03. There is no systematic difference in b-value between oceanic ridges and island arcs.

Table 20. The recalculated b-values for general shallow earthquakes (the values for aftershock sequences, foreshock sequences, and swarms are excluded).

Region	Number of determinations	Range	Median
Oceanic ridge	11	0.75—1.41	0.97
Island arc	45	0.75—1.36	0.96
Eurasian continent	20	0.50—1.14	0.78

b-values for earthquakes in the Eurasian continent seem to be somewhat smaller. However, most of these determinations have been based on the magnitudes estimated from seismic intensity data. To estimate the magnitude from the seismic intensity I_0 at the epicenter, an equation in the form of

$$M = \alpha_1 I_0 + \beta_1 \log h - \gamma_1 \tag{214}$$

is usually used. More than ten formulas with different α_1, β_1 and γ_1 values have been proposed (see e.g., Karnik[353]). The value of α_1, which directly influences the value of b, ranges from 0.48 to 0.93 on the basis of the modified Mercalli scale. The uncertainty of α_1 value means the uncertainty of b-value.

b-values for aftershock sequences differ to some extent from sequence to sequence. Table 21 shows the range and the median of the b-values for aftershock sequences in three regions. No significant systematic regional variations are recognized.

The b-value for foreshocks is smaller than that for aftershocks in six

Table 21. The maximum likelihood estimates of b for aftershock sequences.

Region	Number of determinations	Range	Median
Japan	13	0.51—1.33	0.83
Southern California	10	0.46—1.00	0.82
Greece	10	0.56—1.36	0.82

cases out of eleven cases in which the maximum likelihood estimates are available. The significant difference is found in four cases. It has been reported[401),423)] that Drakopoulos found the relation $b_f=(0.11\pm0.13)+(0.66\pm0.11)b_a$ between the values for foreshocks b_f and aftershocks b_a, but the present results do not agree well to this relation.

To a more accurate representation of the magnitude-frequency relationship of earthquakes, the upper limit of magnitude M_l has been introduced. It is assumed that the magnitude distribution satisfies Gutenberg-Richter's formula only in the range below M_l. In this case the plot of log $N(M)$ against M is an upward convex curve as shown in Figure 138. (When M_l is far larger than M_1* for the earthquake group concerned, no effect of M_l

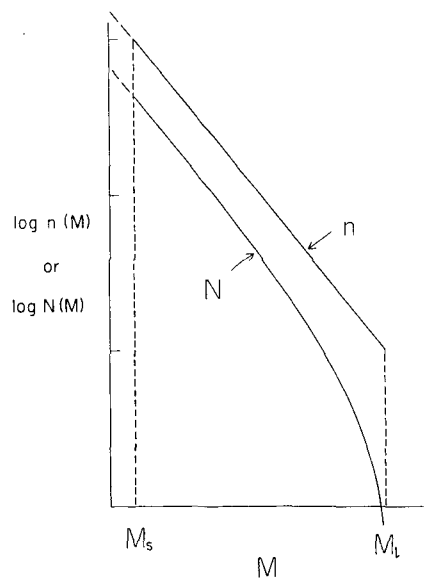


Fig. 138. A truncated distribution of magnitude (marked with n) and its cumulative form (marked with N).

will be observed.) The value of M_l seems to vary regionally and with depth. If a large region consists of small regions with equal b but different M_l, the curve of log $N(M)$ plotted against M is upward convex in a wider range of magnitude than that shown in Figure 138. The b-value determined for the large region is usually higher than the b-value of each small constituent regions. It is very likely that the regional variations of M_l and the seismic activity (i.e., M_1*) cause an apparent variation of regional b-value.

If the distribution of constituent regions in respect to M_l is given by Gutenberg-Richter's equation with the same b-value as that for the magnitude distribution in each constituent region for $M_l \leq c$, and no region with $M_l > c$ exists, the magnitude-frequency relation for earthquakes in the whole region is expressed by

$$\log n(M) = a - bM + \log(c - M). \qquad M_l < c \tag{215}$$

This equation can be obtained in the same manner as the derivation of equation (210). The slope of the curve of log $n(M)$ plotted against M is

$$\frac{d \log n(M)}{dM} = -b - \frac{1}{(c-M)\ln 10}. \tag{216}$$

The slope is nearly equal to $-b$ for $M \leq c$. For $c-M$ of 2 and 4, it is equal to $-(b+0.22)$ and $-(b+0.11)$ respectively.

If two groups of earthquakes with different a and c values (a_1, c_1, and a_2, c_2, $c_1 > c_2$) but with equal b-values are treated in one group, the magnitude distribution becomes

$$\log n(M) = a_3 - bM + \log(c_3 - M), \qquad M < c_2 \tag{217}$$

where

$$c_3 = (10^{a_1} c_1 + 10^{a_2} c_2) / (10^{a_1} + 10^{a_2}). \tag{218}$$

For the distribution of energy, we have

$$n(E) = (C/\beta) E^{-\gamma-1} (\log E_c - \log E) \tag{219}$$

where

$$\log E_c = \alpha + \beta c \tag{220}$$

and α, β, γ, and C are given by equations (82), (135), and (137). If $\gamma < 1$, the total energy of earthquakes with energy E and larger is given by

$$\varepsilon_E = \int_E^{E_c} E n(E)\, dE = \frac{C}{\beta(1-\gamma)} \left\{ \frac{E_c^{1-\gamma}}{(1-\gamma)\ln 10} - \left(\log E_c - \log E + \frac{1}{(1-\gamma)\ln 10} \right) E^{1-\gamma} \right\}. \tag{221}$$

The total energy of earthquakes of all sizes is obtained by putting $E=0$.

$$\mathcal{E}_0 = C_E E_c^{1-\gamma} \tag{222}$$

where

$$C_E = 10^{a+\alpha\gamma}/\{\beta(1-\gamma)\ln 10\}^2 . \tag{223}$$

Another factor that may affect the magnitude-frequency relation is the degree of aftershock activity. If we adopt a model discussed above that an earthquake group under consideration is composed of a number of "main shock-aftershock sequences", and also assume the existence of an upper limit of magnitude for the main shocks, the magnitude distribution for the group is controlled by the following quantities: b_0 (b-value for the main shocks), b_a (b-value for each aftershock sequence), u (average value of the difference in magnitude between a main shock and the largest aftershock: an index of the aftershock activity), M_l (upper limit of magnitude below which the magnitude distribution of the main shocks is expressed by $\log N(M_0)=b_0(M_{01}{}^*-M_0)$), and $M_{01}{}^*$ (an index of the activity of the main shocks defined by the above equation).

Figure 139 shows schematically $\log N(M)$ versus M diagrams in several cases. The curve marked with m indicates the magnitude distribution for the main shocks only. The thick line marked with m+a indicates the magnitude distribution for the whole earthquake group. In the cases of A and B, b is large, while in the cases of C and D, b is small. It is not confirmed that the curve m keeps straight to a very small magnitude. It is possible, especially for deep earthquakes, the frequency of main shocks decreases with decreasing magnitude below a certain magnitude level, thus the $\log N(M)$ curve become horizontal in a range of small magnitude as shown in E and F of Figure 139. It is known that the aftershock activity of deep earthquakes is very low, i.e., u is very large. In this case, the magnitude distribution is represented by a curve shown in Figure 139-F.

The author considers that some low b-values for earhquakes in the continental regions may be related to the low aftershock activity. Table 22 lists the difference in magnitude between a main shock of magnitude 8 or more and the largest aftershock. The average difference for earthquakes in the island arc regions from Japan to Aleutians is 1.15, while that for earthquakes in the Asiatic continent is more than 1.8 (probably around 2). This difference may cause the apparent difference of b-value in a magnitude range from about 6 to 8.

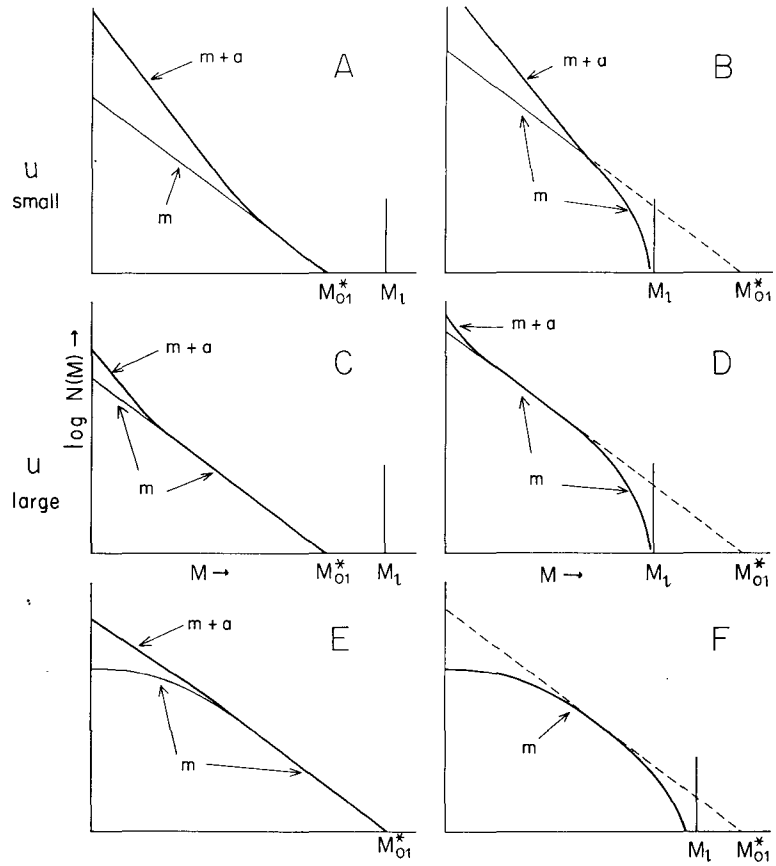


Fig. 139. Schematic diagrams for cumulative frequency distribution of magnitudes for main shocks (marked with m) and for all earthquakes (main shocks plus aftershocks, marked with m+a).

Mogi[433] discussed regional variations of the magnitude-frequency relation of Japanese shallow earthquakes, dividing the Japanese region into eight sub-regions, A, B,.., H. Roughly speaking, the magnitude-frequency relations in these regions correspond to the types shown in Figure 139 as follows: region A: type A, regions B, E, F: type B or D, region C: type C, region D: type A or C, regions G, H: type D.

(to be continued)

Table 22. Magnitude difference $u(=M_0-M_1)$ between the main shock and the largest aftershock for the main shocks of magnitude 8 or larger in the two regions (1) and (2).

(1) Japan-Kuriles Kamchatka-Aleutians

Date	Region	M_0	M_1	u
1918 Sept. 7	Kuriles	8.3	7.9	0.4
1923 Feb. 3	Kamchatka	8.4	7.4	1.0
1923 Sept. 1	Japan	8.3	7.7	0.6
1929 Mar. 7	Aleutians	8.6	7.3	1.3
1933 Mar. 2	Japan	8.9	7.3	1.6
1944 Dec. 7	Japan	8.3	7.1	1.2
1946 Dec. 20	Japan	8.4	7.3	1.1
1952 Mar. 4	Japan	8.6	7.1	1.5
1952 Nov. 4	Kamchatka	8.4	7.0	1.4
1957 Mar. 9	Aleutians	8.25	7.3	0.95
1958 Nov. 6	Kuriles	8.7	7.3	1.4
1959 May 4	Kamchatka	8.25	7.0	1.25
1963 Oct. 13	Kuriles	8.25	7.0	1 25
			Mean:	$u=1.15$

(2) Asiatic Continent

Date	Region	M_0	M_1	u
1920 Dec. 16	China	8.6	<7	≥1.7
1927 May 22	China	8.3	<7	≥1.4
1934 Jan. 15	Nepal	8.4	(<6)	≥2.5
1945 Nov. 27	Pakistan	8.3	<7	≥1.4
1950 Aug. 15	India	8.7	(6.5)	2.2
1957 Dec. 4	Mongolia	8.3	(6.5)	1.8
			Mean:	$u\geq1.83$

Magnitudes are taken from Duda.[348] M_1-values in parentheses are taken from other sources.

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