[Journal of the Faculty of Science, Hokkaido University, Ser. VII (Geophysics), Vol. IV, No. 1, 1972]

Aftershocks and Earthquake Statistics (IV)

—— Analyses of the Distribution of Earthquakes in Magnitude, Time, and Space with Special Consideration to Clustering Characteristics of Earthquake Occurrence (2) ——

Tokuji UTSU

(Received Aug. 28, 1972)

Abstract

The first step in the analysis of the time distribution of earthquakes is to test the hypothesis that a given series of earthquake data are samples from a Poisson process. There are many independent methods for this test, e.g., the methods based on i) the time interval between events, ii) the number of events in a unit time interval, iii) the ratio of variance to mean of the number of events, iv) the autocorrelation, v) the spectrum, and many others. These tests have been applied to series of shallow and deep earthquakes in and near Japan and shallow earthquakes in the world. The results show that for series of shallow earthquakes from which aftershocks had been removed and for a series of deep earthquakes, the Poisson hypothesis can not be rejected by most of the methods. For series of shallow earthquakes including aftershocks, the Poisson hypothesis is rejected at very small significance levels.

Instead of the Poisson process, the branching Poisson process (first used in seismology by Vere-Jones and Davies in 1966) has been adopted. Comparisons of the data with the theoretical curves for the distribution of time intervals, the variance/mean ratio, and the spectrum indicate that this model is a suitable approximation. An important parameter for this model L_∞ can be evaluated from the variance/mean ratio and the spectrum. The values from the both methods agree well. L_∞ is related to the mean A and the variance V of the total number of aftershocks triggered by a main shock by the equation

$$L_\infty = 1+\bar{A}+\frac{V}{1+\bar{A}} \geq 1 .$$

$L_\infty=1$ for Poisson processes only. If the spectrum is defined by

$$\Phi(\omega) = \left| \sum_{k=1}^{N} e^{i\omega t_k} \right|^2 \Big/ N$$

where t_k is the origin time of the kth earthquake, $\Phi(\omega)$ tends to L_∞ and 1 when $\omega \to 0$ and $\omega \to \infty$ respectively.

Schuster's criterion for significance of spectral amplitudes is inadequate, if the data contain aftershocks, as pointed out by Jeffreys. Fisher's test for

significance of the maximum spectral amplitude has a property of compensating the effect of aftershocks. A significance test used by Matuzawa and others is closely related to the test based on the present model. The data examined here show no significant periodicities. Reccurrence of large earthquakes from the same source region at intervals of several tens to hundreds of years is recognized in some island arc areas. A simple model for this is proposed. The branching Poisson process may be considered as the superposition of such simple reccurrence of mainshock-aftershock sequence system in each source region.

14. Distribution of earthquakes in respect to time

14.1 *Statistical tests for stationary random occurrence of earthquakes in time*

The most simple and fundamental model for a series of events occurring in time is the Poisson process, in which all points representing the events are distributed completely at random along the time axis. In most cases investigated hitherto the occurrence of earthquakes does not fit a simple Poisson process. The most common reason for this may be the clustering of events due to the existence of aftershocks. In some earthquake series, however, the Poisson model seems to be adequate as a first approximation. These series usually contain few aftershock sequences or swarms. In many papers dealing with the statistical properties of earthquake occurrence in time, goodness-of-fit tests to the Poisson process have been performed. There are many independent methods for these tests, some of which have been applied to aftershock sequences in Chapter 8. In the present section, we consider several methods and apply them to the following sets of earthquake data.

(I) All shallow earthquakes (depth $\leq$60 km) of magnitude 6.0 and larger which occurred in and near Japan (the region defined in Figure 1 of the author's paper[1]) during 1926–1969 (44 years).

(II) All shallow earthquakes (depth $\leq$60 km) of magnitude 5.5 and larger which occurred in and near Japan during 1959–1970 (12 years) (Tables 1 and 23).

(III) All deep earthquakes (depth $\geq$140 km) in and near Japan of magnitude 5.0 and larger listed in Katsumata's table[368] (with additions by Katsumata) during 1951–1969 (19 years).

(IV) All shallow earthquakes (depth $\leq$100 km) in the whole world with magnitude 7.0 and larger listed in Duda's table[348] during 1915–1964 (50 years).

Hereafter these data will be called data I, data II, etc. Data I′ and II′

refer to those obtained from data I and II respectively by excluding all foreshocks, aftershocks, and shocks in swarms (except the largest one in each swarm). Thus data I′ and II′ represent the sreies of main shocks (including single shocks) only.

Table 23. Continued from Table 1.

No.	Origin Time (GMT) d h m	Epicenter °N	Epicenter °E	h km	M_0	$D_1 = M_0 - M_1$	$D_2 = M_0 - M_1$	T_1 d h m	Mark
262	1969 Feb. 21 03 05	40.3	144.1	30	5.6	a			
263	Apr. 15 17 31	39.8	143.9	20	5.9	a			
*	17 04 56	39.6	143.8	70	5.6	a			
264	21 07 19	32.1	132.1	10	6.5	4.7	4.6	03 40	
265	22 06 11	39.8	143.4	60	5.9	a			
266	June 12 05 41	40.3	144.0	40	5.6	a			
267	20 15 38	40.7	142.4	40	5.6	a			
268	July 12 19 16	39.7	143.9	10	5.6	a			
269	23 13 14	37.2	141.7	40	5.5				
270	Aug. 11 21 28	42.7	147.6	30	7.8	6.2	5.9	2 16 51	
271	12 03 34	42.9	147.7	60	5.5	a			
272	09 26	42.9	147.2	10	5.5	a			
273	13 08 32	43.5	147.9	50	5.7	a			
274	14 14 19	42.9	147.2	0	6.2	a			
275	15 04 32	42.8	147.4	10	5.6	a			
276	16 15 15	42.9	147.4	60	5.9	a			
277	17 14	52.9	147.6	60	5.5	a			
278	Sept. 3 16 20	30.7	140.5	60	6.2				
279	4 21 13	43.5	147.1	10	5.6	a			
280	9 05 15	35.8	137.1	0	6.6	4.9	4.8	1 12 47	
*	13 11 52	43.1	147.7	70	5.6	a			
281	17 18 41	30.9	131.7	0	5.9	5.5		10	
282	18 51	31.2	131.1	0	5.5	a?			
283	Oct. 31 07 00	37.0	142.5	60	5.5				
284	1970 Jan. 20 17 33	42.4	143.1	50	6.7	4.8	4.5	19 26	
285	Mar. 9 00 50	39.5	143.7	40	5.5	a			
286	May 27 19 05	40.1	143.2	30	6.2	a			
287	22 36	40.2	143.2	30	6.0	a			
288	23 56	40.3	143.1	20	5.8	a			
289	June 22 21 33	43.1	147.5	0	5.8	a			
290	July 25 22 41	32.1	132.0	10	6.7	6.1	4.8	8 29	
291	26 07 10	32.1	132.1	10	6.1	a			
292	Sept. 14 09 45	38.7	142.3	40	6.2	4.7	4.6	13 01 13	
293	Oct. 8 23 36	42.3	147.6	60	5.6	a			
294	14 21 14	43.1	146.9	40	5.7	a			
295	16 05 26	39.2	140.8	0	6.2	4.9	4.0	5 23	
296	Nov. 20 13 48	43.1	146.9	40	5.6	a			
297	Dec. 7 05 21	41.7	143.8	50	6.1				

i) *Time interval between successive events*

A Poisson process is characterized by only one parameter, the rate of occurrence of events ν. For a Poisson process the time interval τ between successive events has an exponential distribution of parameter ν (Chapter 8)

$$\phi(\tau) = \nu e^{-\nu\tau} . \tag{40}$$

Since Terada's paper[454] in 1918, several tens of papers which discuss the distribution of time intervals between earthquakes have been published. For some earthquake series, the data fit the exponential distribution reasonably well. Some authors have concluded in these cases that the earthquakes occur randomly and independently in time, but it is not logically correct, since the exponential distribution of τ is a necessary but not a sufficient condition for a Poisson process. Similar comments may be made for other tests for Poisson processes described later.

Frequency distributions of time intervals between successive earthquakes are shown in Figure 140–145 for data I, I′, II, II′, III, and IV using semi-logarithmic scales. Open circles represent frequencies of τ in the interval of length $\Delta\tau$ shown in each figure, and solid circles represent the cumulative

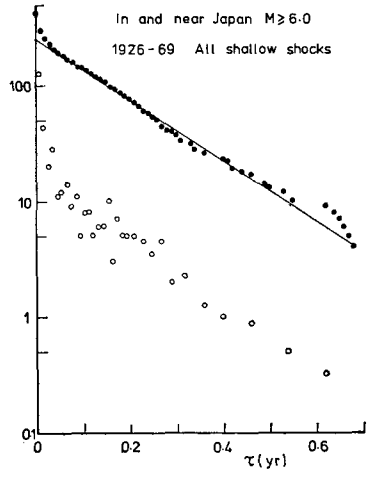

Fig. 140.

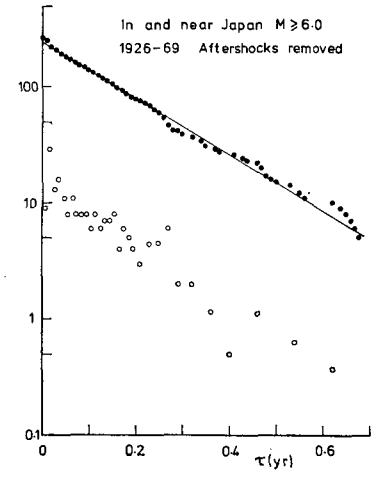

Fig. 141.

Fig. 140. Distribution of time intervals between successive events for data I ($\Delta\tau=0.01$ year).

Fig. 141. Distribution of time intervals between successive events for data I′ ($\Delta\tau=0.01$ year).

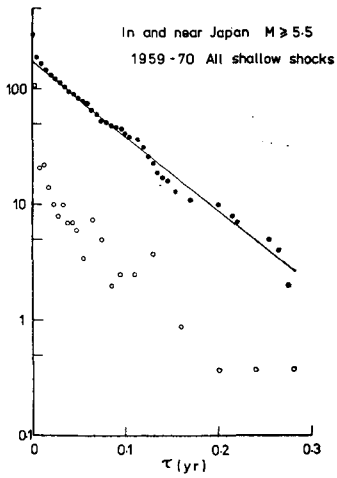

Fig. 142.

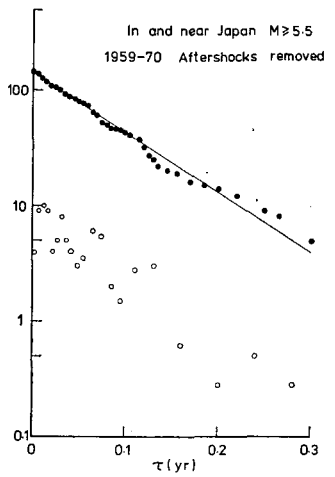

Fig. 143.

Fig. 142. Distribution of time intervals between successive events for data II ($\Delta\tau=0.005$ year).

Fig. 143. Distribution of time intervals between successive events for data II′ ($\Delta\tau=0.005$ year).

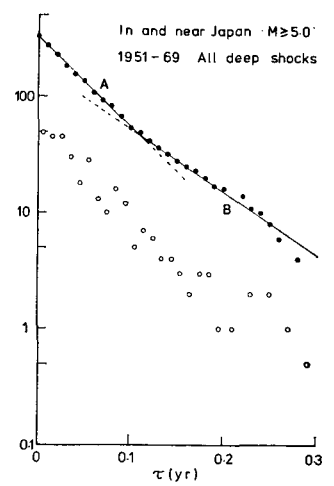

Fig. 144.

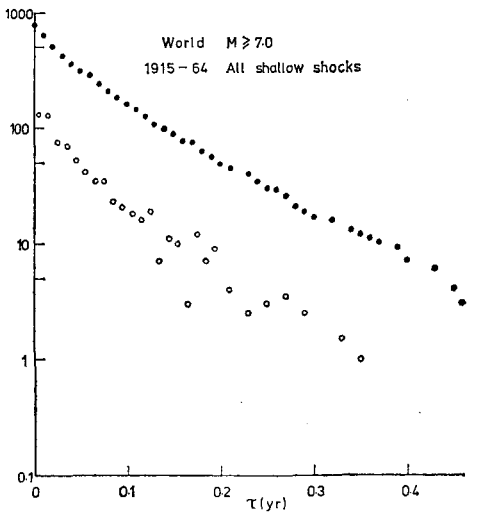

Fig. 145.

Fig. 144. Distribution of time interval between successive events for data III ($\Delta\tau=0.01$ year).

Fig. 145. Distribution of time intervals between successive events for data IV ($\Delta\tau=0.01$ year).

frequencies, i.e. the number of time intervals τ and larger. In Figures 141 and 143 the plotted data are well represented by a straight line, whereas in other figures concave curves like curve A in Figure 146 fit the data rather closely.

These results together with the results from many previous investigations are summarized as follows:

(1) The distribution of time intervals deviates from the exponential distribution usually in the sense shown in Figure 146 (A).[15),151),274),455)~459)]

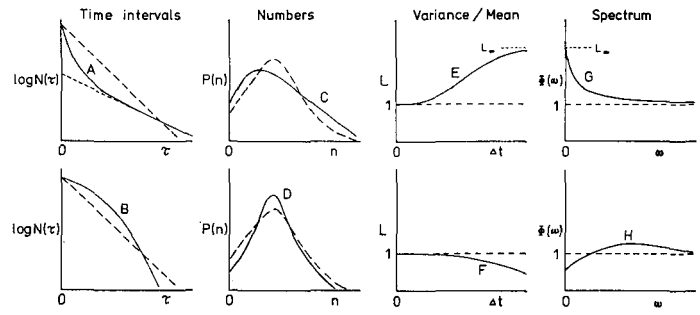

Fig. 146. Schematic representation of the deviation from the Poisson process (indicated by broken lines). The upper diagrams represent the case for the trigger model.

(2) It is most likely that such a deviation is caused by either the existence of aftershock sequences or other clusters of earthquakes in the data or the variation of the rate of occurrence with time. The latter effect, pointed out by Terada[454)] and later workers is clearly observed in the statistics of τ for aftershock sequences and swarms as discussed in Chapter 8. The former effect can be evidenced by the fact that the removal of aftershocks from the original series makes the distribution nearly exponential[455),458),460)] (Figures 141 and 143).

(3) The distribution is nearly exponential for deep earthquakes which are generally followed by no aftershocks.[17),461),462)] In data III (Figure 144), however, the plotted points fit two straight lines of different slope. This may suggests weak clustering different in nature from aftershock sequences.

(4) In a few cases deviations from exponential distribution in the opposite sense as shown in Figure 146 (B) have been reported.[454),461)] A decrease in probability of occurrence of the next shock after the occurrence of a shock may cause such an effect, but in some cases this effect may be

attributed to the missing of closely separated events in time by overlapping of seismograms. Both causes have been pointed out by Terada[454] and later workers.

(5) For some relatively dense series of small earthquakes such as data shown in Figure 147 (the Matsushiro earthquakes), the deviation from the exponential distribution is rather small. This is probably due to the effect illustrated in Figure 148, which is reproduced from the author's paper in 1962.[228] Indeed, almost all series of randomly distributed events may be considered as a result of the superposition of non-random series of events in each elementary region (S_i in Figure 148).

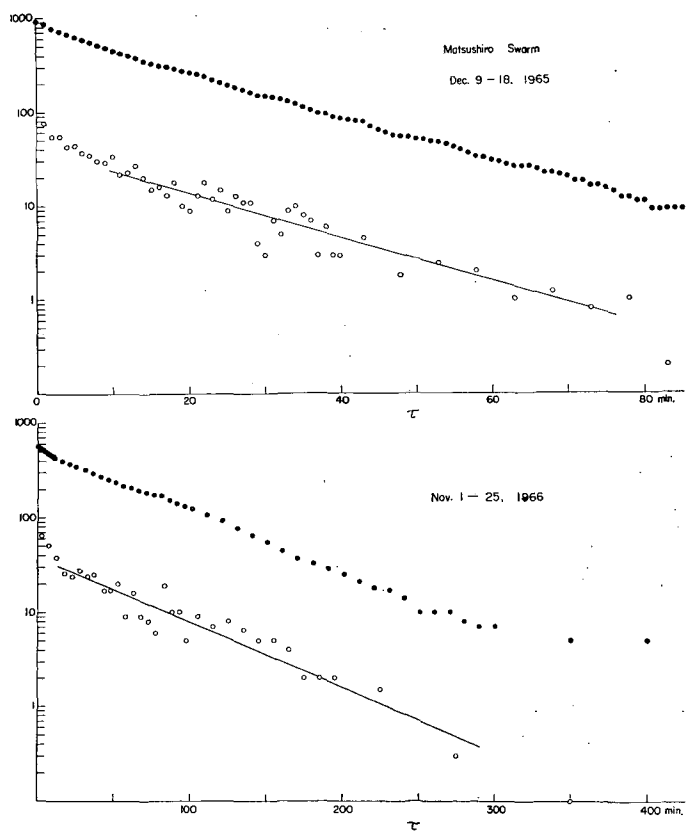

Fig. 147. Distributions of time intervals between successive events for two periods of the Matsushiro earthquake swarm.

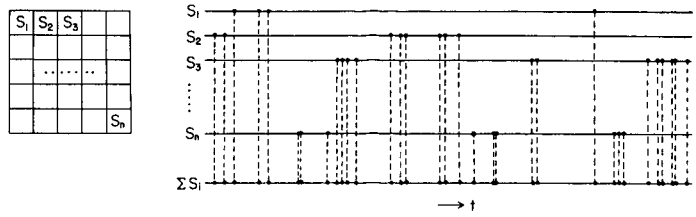

Fig. 148. Superposition of series of events yields more random distribution of events.

ii) *Number of events in a unit time interval*

For a Poisson process the number of events n occurring in a time interval of length Δt has a Poisson distribution of parameter $\nu\Delta t$ (Chapter 8).

$$p(n) = (\nu\Delta t)\, e^{-\nu\Delta t}/n! \tag{39}$$

At least 30 papers have been written dealing with the distribution of number of events. The first paper known to the author is the one by Nakamura[463] in 1920.

For data I, I′, II, II′, III, and IV, frequency distribution of number of events per specified interval of time are shown in Figures 149–154 respect-

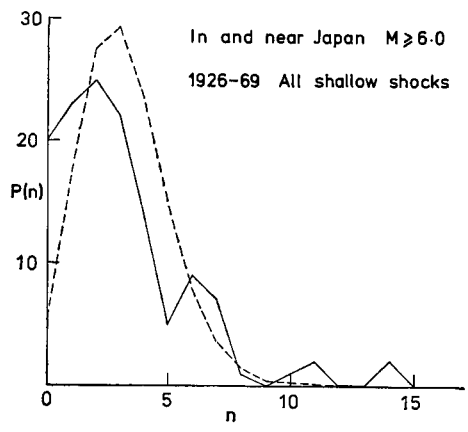

Fig. 149.

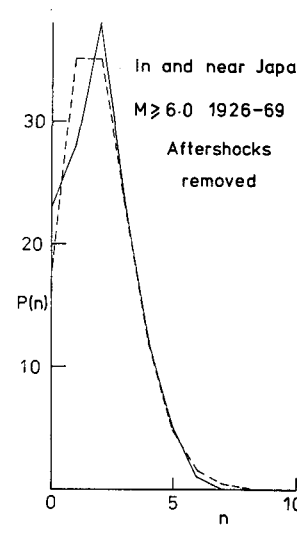

Fig. 150.

Fig. 149. Distribution of numbers of erthquakes per 1/3 year for data I. The broken line represents the corresponding Poisson distribution.

Fig. 150. Distribution of numbers of earthquakes per 1/3 year for data I′.

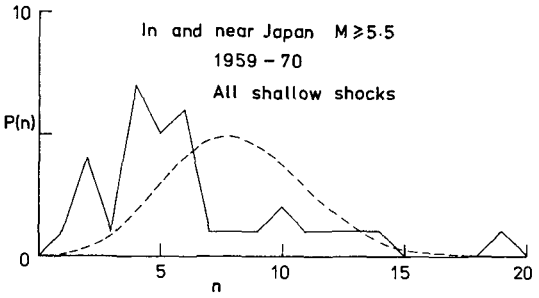

Fig. 151.

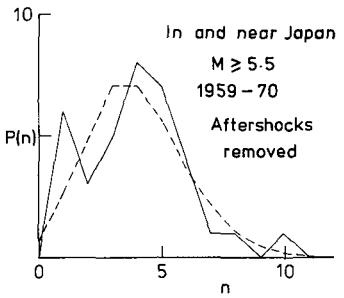

Fig. 152.

Fig. 151. Distribution of numbers of earthquakes per 1/3 year for data II.
Fig. 152. Distribution of numbers of earthquakes per 1/3 year for data II′.

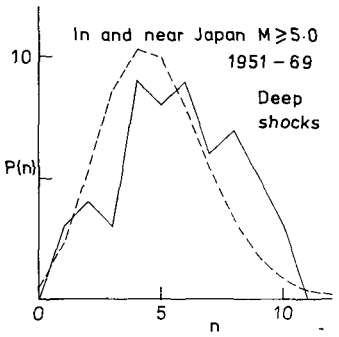

Fig. 153.

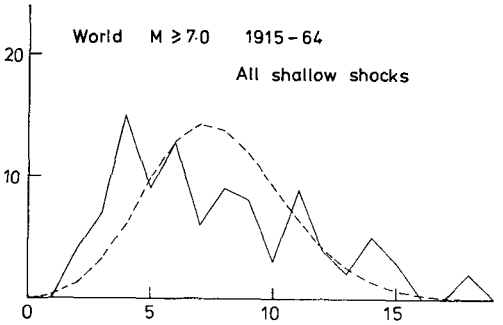

Fig. 154.

Fig. 153. Distribution of numbers of earthquakes per 1/3 year for data III.
Fig. 154. Distribution of numbers of earthquakes per 0.5 year for data IV.

ively. In Figures 150, 152, and 153, the plotted points fit the corresponding Poisson distributions indicated by broken lines fairly well, whereas in other figures systematic deviations are appreciable.

These results as well as the results reported in many previous investigations show similar characteristics to the results for the distribution of time intervals. In series of earthquakes from which aftershocks have been removed and in series of deep earthquakes, the number of events are approximately Poisson-distributed.[17),460),464),465)] Deviations from Poisson distributions in the sense as illustrated in Figure 146 (C) are generally observed for relatively small shallow earthquakes.[232),395),456),457),466)–469)] These are caused by the

temporal variation of the rate of occurrence or the existence of aftershocks in the data.

iii) *Variance-to-mean ratios*

Theoretically, the variance $V(n)$ of the number of events n for a Poisson process is equal to its mean $E(n)=\nu\Delta t$, independently of the interval length Δt. For random samples from a Poisson process the ratio

$$L = V(n)/E(n) \tag{224}$$

has a certain distribution around 1. It is known that if L is obtained from the counts of events in N non-overlapping intervals of length Δt, χ_0^2 $(=NL)$ has a χ^2-distribution with $N-1$ degrees of feedom (Chapter 8). For large values of N ($N>$about 30), $\sqrt{L}$ is approximately normally distributed with a mean of $E(\sqrt{L})=\sqrt{1-3/(2N)}$ and a varance of $V(\sqrt{L})=1/(2N)$. L is called the Poisson index of dispersion or Lexis' ratio. Since the distribution of NL is known, the hypothesis of Poisson process can be tested using this distribution, too. Actually this test is the same as the χ^2-test for a uniform distribution of the number of events n. For most non-Poisson processes, the value of L differs significantly from 1, and usually depends on the length of Δt as illustrated in Figure 146 (E, F). The L vs Δt curve represents a statistical property of the process.

The variation of the number of earthquakes with time are shown in Figures 155–160 for data I, I′, II, II′, III, and IV using appropriate lengths of Δt. Figures 161–164 show graphs of L plotted against Δt for these data.

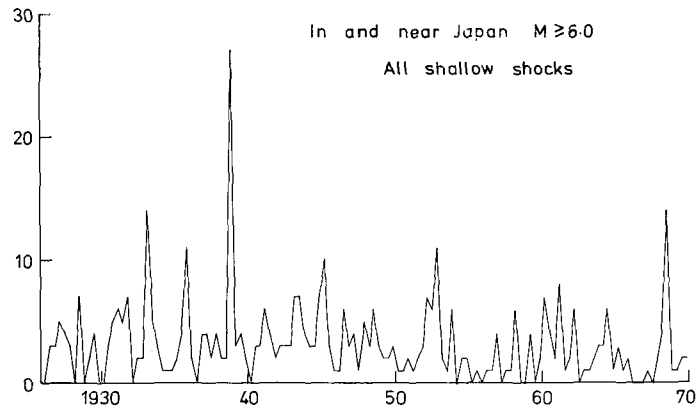

Fig. 155. Variation of frequency with time for data I ($\Delta t=1/3$ year).

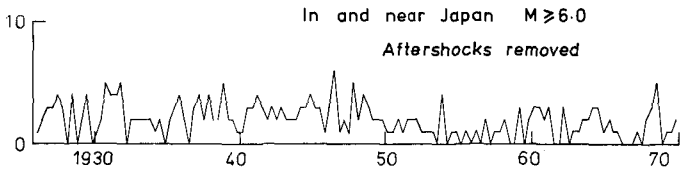

Fig. 156. Variation of frequency with time for data I' ($\Delta t = 1/3$ year).

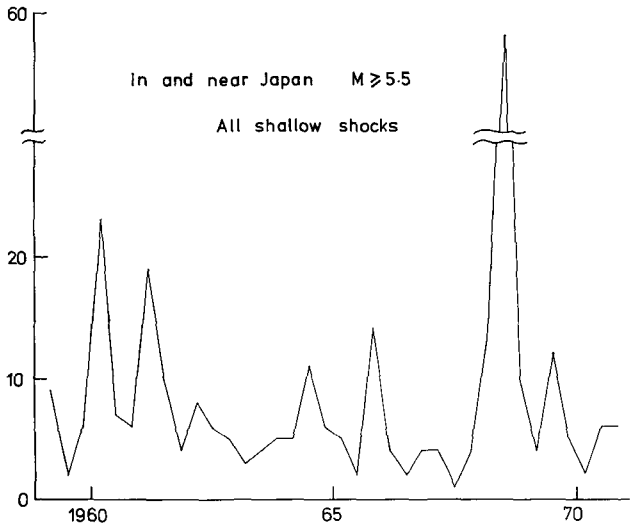

Fig. 157. Variation of frequency with time for data II ($\Delta t = 1/3$ year).

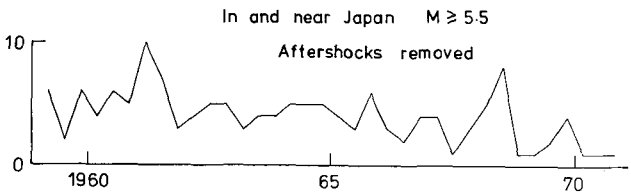

Fig. 158. Variation of frequency with time for data II' ($\Delta t = 1/3$ year).

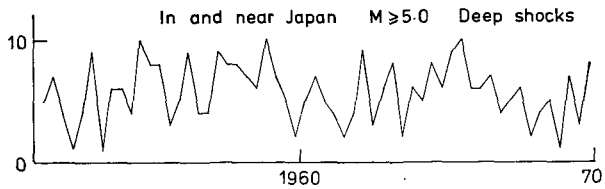

Fig. 159. Variation of frequency with time for data III ($\Delta t = 1/3$ year).

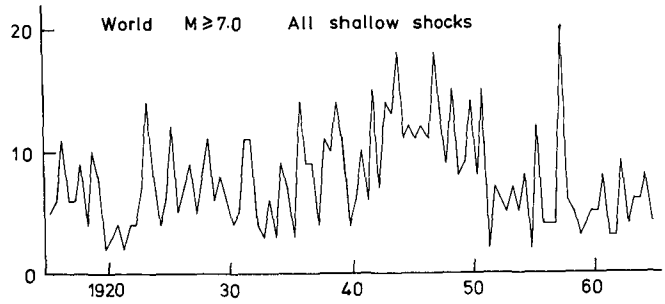

Fig. 160. Variation of frequency with time for data IV (Δt=0.5 year).

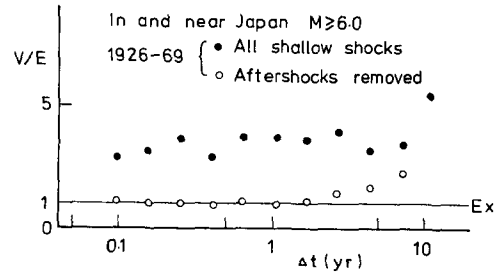

Fig. 161. Variation of $L(=V/E)$ with Δt for data I (solid circles) and for data I′ (open circles).

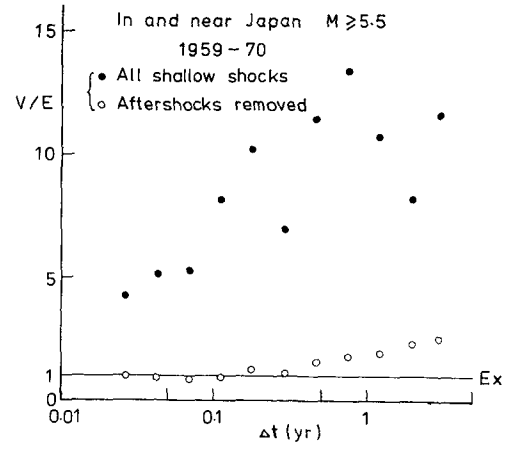

Fig. 162. Variation of $L(=V/E)$ with Δt for data II (solid circles) and for data II′ (open circles).

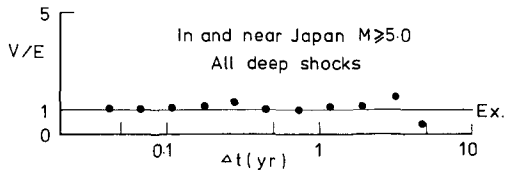

Fig. 163. Variation of $L(=V/E)$ with Δt for data III.

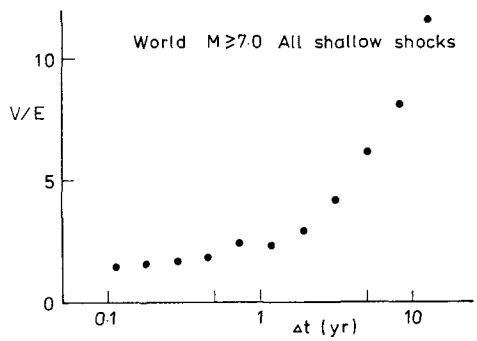

Fig. 164. Variation of $L(=V/E)$ with Δt for data IV.

For data I, II, and IV, and the data given by Takahasi[470] in 1937 and later investigators,[15),16),313),380),458)] it is seen that the value of L is usually larger than unity and has a tendency to increase with Δt. Takahasi[470] pointed out that the clustering of events or the temporal variation of the rate of occurrence caused the values of L larger than 1. For data I′, II′, and III, the value of L is close to 1. Increase of L with Δt is observed for data I′ and II′, but the hypothesis of $L=1$ is not rejected at a significance level of 0.05.

iv) *Autocorrelation*

If the period of investigation is divided into N intervals of length Δt, and the number of earthquakes in the ith interval is denoted by n_i, the autocorrelation function of the number of earthquakes is defined by

$$r_k = \frac{\sum_{i=1}^{N-k} (n_i - \bar{n})(n_{i+k} - \bar{n})}{\sum_{i=1}^{N} (n_i - \bar{n})^2} \tag{225}$$

where

$$\bar{n} = \sum_{i=1}^{N} n_i/N = \nu\Delta t\,. \tag{226}$$

In some literature somewhat different definition is given, but here we use equation (225).[471]

For a Poisson process the numbers of events in two different intervals are independent. Therefore, for $k \neq 0$,

$$E(r_k) = 0 \tag{227}$$

and for large N, r_k is approximately normally distributed with a mean of 0 and a variance of

$$V(r_k) = \frac{1}{N-k-1} \tag{228}$$

provided that n is not very small.

For the time interval τ between successive events, we can define the autocorrelation function in a similar way. For a Poisson process, or in general for a renewal process in which all the time intervals are independently and identically distributed, $E(r_k)=0$ for $k \neq 0$.

Figure 165–168 represent autocorrelation functions r_k ($k=0, 1, \ldots, 30$) for numbers of events and time intervals between events for data II and II′. Since the distribution of τ is far from the normal distribution, approximate normalization is made by putting $\tau' = \log(\tau + 0.2/\nu)$ to calculate r_ks. Similar

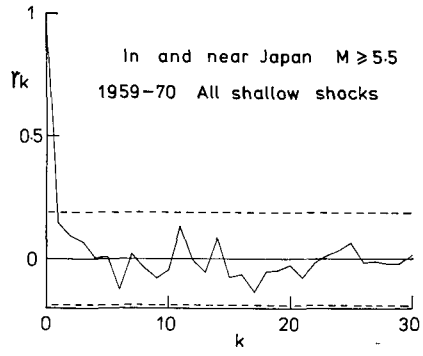

Fig. 165.

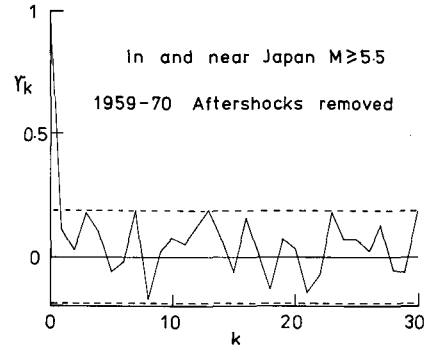

Fig. 166.

Fig. 165. Autocorrelation of numbers of counts for data II ($\Delta t = 0.110$ year).
Fig. 166. Autocorrelation of numbers of counts for data II′ ($\Delta t = 0.110$ year).

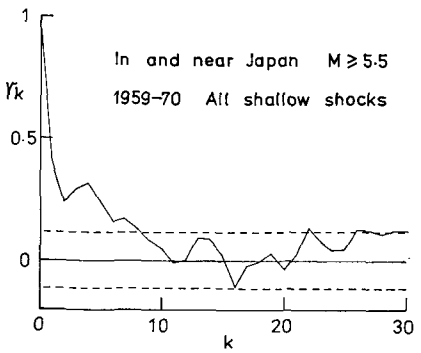

Fig. 167.

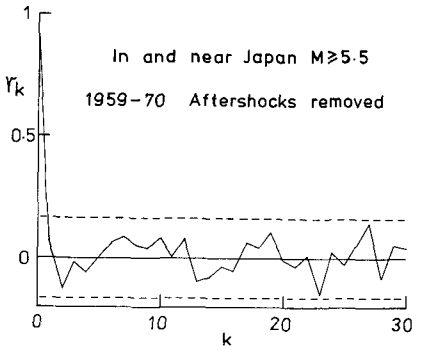

Fig. 168.

Fig. 167. Autocorrelation of time intervals between successive events for data II.
Fig. 168. Autocorrelation of time intervals between successive events for data II′.

graphs have been prepared for other data, but they are not shown here. These graphs indicate that for series of earthquakes from which aftershocks have been removed (data I′ and II′), the values of r_k for both τ and n are not significantly different from 0. For data I and II, the values of r_k for n are also nearly 0. The autocorrelation function of the number of counts does not seem to be a sensitive quantity for testing the Poisson hypothesis. On the other hand, the values of r_k for time intervals are significantly larger than zero for first several terms.

v) *Spectra*

The power spectrum for a series of earthquakes (considered as a point process) is defined here by

$$\Phi(\omega) = \left| \sum_{k=1}^{N} e^{i\omega t_k} \right|^2 / N \tag{229}$$

where ω is the angular frequency, t_k is the time of occurrence of the kth earthquake, and N is the total number of earthquakes. For a Poisson process, it is well known that

$$E[\Phi(\omega)] = 1 \tag{230}$$

and $2\Phi(\omega)$ is approximately χ^2-distributed with two degrees of freedom. Therefore the probability that $\Phi(\omega)$ exceeds a certain value φ is $e^{-\varphi}$. This property has been used for a test of the periodicity in earthquake occurrence

by Schuster[472] in 1897 and by later investigators (e.g., see Davison[473]). The problem of periodicity will be discussed in a later section.

Power spectra of the data I, I′, II, II′, III, and IV have been calculated for $\omega=2\pi/(kT)$ (T: the length of the whole period and $k=1, 2, 3, \ldots$) and plotted in Figures 169–174. In these figures, a horizontal line marked by

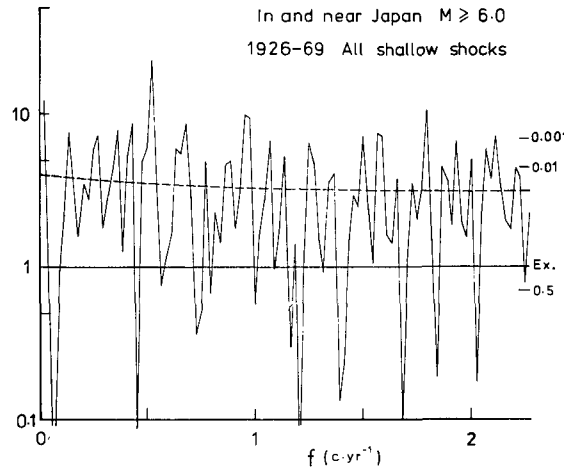

Fig. 169. Power spectrum for data I. The broken curve represents the theoretical spectrum for the trigger model with $L_\infty=4$, $p=1.3$, and $c=0.3$ day.

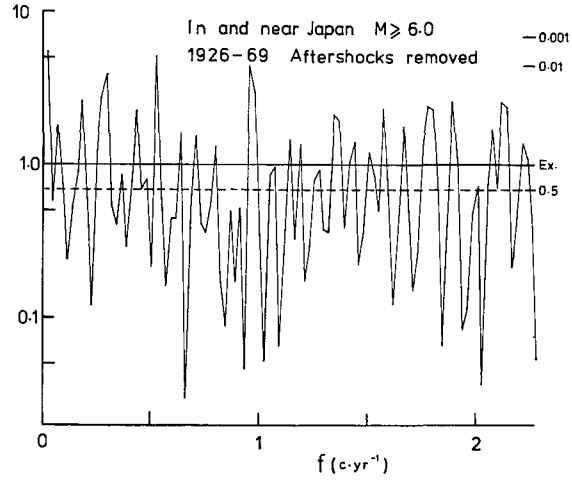

Fig. 170. Power spectrum for data I′.

Ex. indicates the expectancy and a level marked by 0.5 indicates the median of spectral values for the Poisson process. Marks 0.01 and 0.001 mean that the probability of the occurrence of spectral value larger than these levels is less than 0.01 and 0.001 respectively for the Poisson process. These graphs and the results of spectral analyses by other investigators[15),268),443),458),474)] indicate that a significant increase in spectral values towards lower

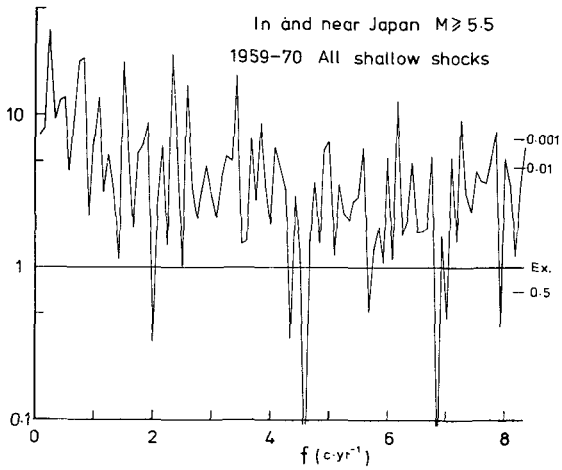

Fig. 171. Power spectrum for data II.

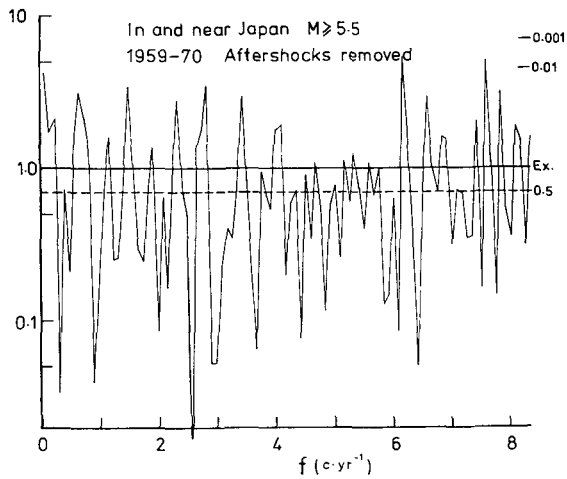

Fig. 172. Power spectrum for data II′.

frequencies (sometimes called "reddening") occurs in many cases (e.g., Figures 169, 171, and 174). For some series of deep earthquakes and of shallow earthquakes from which aftershocks have been removed, the spectral values show no systematic frequency-dependence (e.g., Figures 170, 172, and 173). Thus the main cause of the reddening seems to be the departure from the Poisson process due to the inclusion of aftershocks.

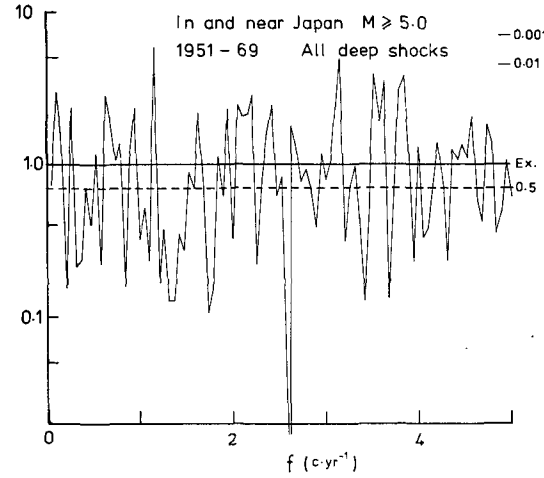

Fig. 173. Power spectrum for data III.

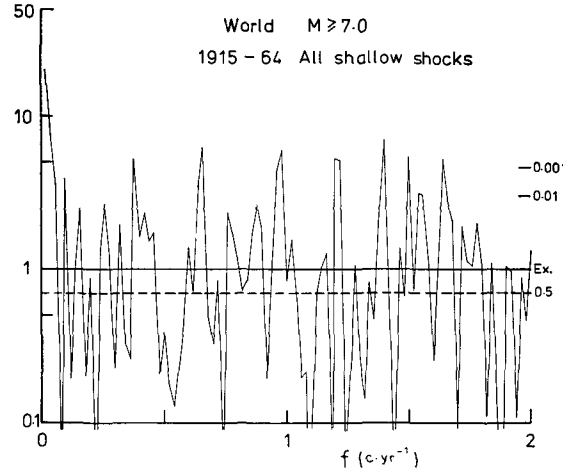

Fig. 174. Power spectrum for data IV.

vi) *Other tests for Poisson process*

(1) Time differences beween two events: Takahasi[370),374)] prepared a graph showing the frequency distribution of time difference τ_{ij} between the ith and the jth events in a series of large earthquakes in Japan since 1500 for all combinations of i and j $(i<j)$ except for τ_{ij} larger than 80 years. Based on this graph he discussed persistence and periodicity in earthquake occurrence. If the frequency of τ_{ij} falling between τ and $\tau+\Delta\tau$ is denoted by $f(\tau)\Delta\tau$, the mean and the variance of $f(\tau)$ for a Poisson process of parameter ν are approximately given by

$$E\,[f(\tau)] = V[f(\tau)] = \nu N \tag{231}$$

where N is the total number of events. In Figures 175–177 graphs of $f(\tau)$ $\Delta\tau$ are shown for data I, I′, II, II′, III, and IV. Horizontal lines indicate the expectancy given by (231). It is recognized that the frequency of τ_{ij} increases with decreasing τ for τ smaller than about 0.3 years in the case of data I, II, and IV. In the other cases plotted points scatter around their expected values in the whole range of τ studied.

(2) Use of runs: Several methods for testing the randomness in earthquake occurrence by use of the theory of runs have been described.[225)] Here the one which seems to be most sensitive is applied to the data. Let $\tau_1, \tau_2, \ldots, \tau_N$ be the series of time interval between successive events (total number of events is $N+1$). All τ_is are replaced by a + or a − sign according

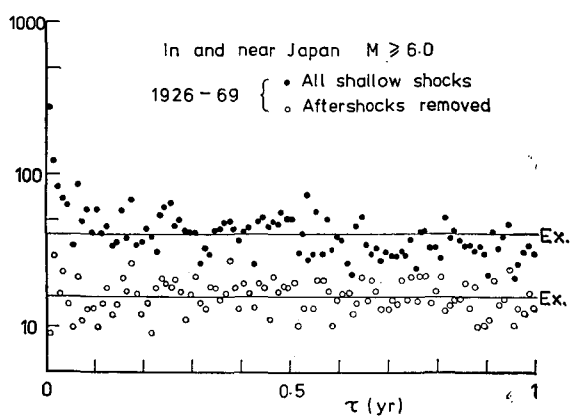

Fig. 175. Distribution of time intervals between events (all combinations of events with separations less than one year) for data I (solid circles) and for data I′ (open circles).

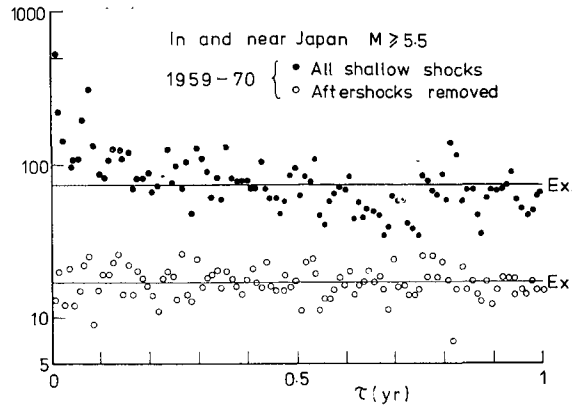

Fig. 176. Distribution of time intervals between events for data II (solid circles) and for data II′ (open circles).

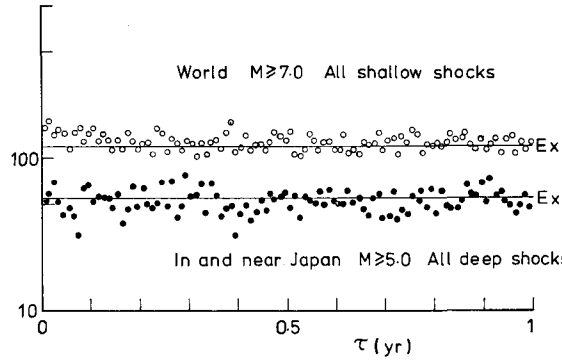

Fig. 177. Distribution of time intervals between events for data III (solid circles) and for data IV (open circles).

as they are larger than or smaller than a certain fixed value τ. The theory of runs says that the number of runs R for the series of + and – signs is approximately normally distributed with the mean and variance given by equation (60) and (61) (Chapter 8, p. 227) for the Poisson process. Table 24 contains the results of the test. $0.5/\nu$ is used as the value for τ. The hypothesis of Poisson process is rejected for data I and II.

(3) Grouping index: This index has been defined by equation (62) (Chapter 8, p. 227, hereafter we use a notation G instead of u in equation (62)). For a Poisson process G is approximately normally distributed with the expectancy and variance of

Table 24. Test for Poisson process by use of the number of runs of time intervals between events. $\sigma(R)=\sqrt{V(R)}$.

Data	n_+	n_-	R	$E(R)$	$\sigma(R)$	$\frac{E(R)-R}{\sigma(R)}$	Prob.
I	188	233	178	209.1	10.1	3.08	0.001
I′	152	110	133	128.6	7.9	−0.56	0.57*
II	137	158	96	147.8	8.5	6.07	0.0000
II′	74	67	72	70.3	5.9	−0.29	0.77*
III	182	141	154	159.9	8.8	0.67	0.50*
IV	417	355	363	384.5	13.8	1.56	0.06

(* two-sided)

$$E(G) = 1-e^{-2\eta} \tag{232}$$

$$V(G) = \frac{(1-e^{-2\eta})\,e^{-2\eta}}{N_0}\,. \tag{233}$$

if $\eta=0.5$, $E(G)=0.6321$ and $\sigma(G)=\sqrt{V(G)}=0.4822/\sqrt{N_0}$. The grouping indexes ($\eta=0.5$) calculated for the data are listed in Table 25. For data I, II, and IV remarkable grouping is recognized.

Table 25. Test for Poisson process by use of the grouping index ($\eta=0.5$).

Data	N_0	G	$\sigma(G)$	$\frac{G-E(R)}{\sigma(G)}$	Prob.
I	422	0.763	0.023	5.59	0.0000
I′	263	0.665	0.030	1.12	0.26*
II	296	0.753	0.028	4.29	0.0000
II′	142	0.697	0.040	1.62	0.053
III	324	0.673	0.027	1.52	0.064
IV	773	0.693	0.017	3.54	0.0002

(* two-sided)

(4) Uniformity: Since the Poisson process is a stationary point process, the statistical properties are uniform in time. For example, if there is a trend in the rate of occurrence, the process is not a Poisson process. The existence of a linear trend can be tested by testing the hypothesis that the regression coefficent of the number of events in unit time intervals against time is zero. Reyment[476] tested the exponential trend in volcanic eruptions by a method described in Cox and Lewis.[477] Kitagawa et al.[478] and Utsu[225] applied Pitman's test to earthquake data in Japan.

14.2 *Effects of aftershocks — the trigger model*

The previous examples of tests for Poisson processes and a review of earlier investigations suggest that the occurrence of earthquakes in time has two main statistical properties: randomness and clustering. Which property is more prominent depends on the selection of data. In some series of earthquakes the Poisson hypothesis is not rejected by several statistical tests. This is of course not a proof that the earthquakes occur as a Poisson process, but it may be natural to consider that the Poisson process is an adequate model for such series. This by no means indicates that the earthquakes are essentially independent events. It is quite possible that the randomness in time is resulted from the superposition of many non-random processes (Figure 148). Actually there is evidence for large earthquakes in limited regions to occur intermittently at intervals of a few tens to hundreds of years or more.

We consider here some stochastic models for earthquake occurrence.

(1) Poisson process: As mentioned above this model may be an adequate approximation in some cases (e.g., data I′, II′, and III), but it is apparently inapplicable to other cases (e.g., data I, II, IV).

(2) Time-dependent Poisson process: This is the case in which the parameter ν of the Poisson process is a function of time. In chapter 8 this model is discussed in relation to the temporal distribution of shocks in aftershock sequences (the rate of occurrence of aftershocks was denoted by $n(t)$).

(3) Branching Poisson process (or trigger model): In this model there is a series of primary events (main shocks) distributed completely at random in time. Each of these primary events may generate a secondary series of events (aftershocks) as shown in Figure 178. It is assumed that the temporal distribution of aftershocks (of magnitude above a certain level) triggered by a main shock at time t_0 is represented by

$$\begin{aligned} n(t) &= A\lambda(t-t_0)\,, \qquad t \geq t_0 \\ n(t) &= 0\,, \qquad t < t_0 \end{aligned} \tag{234}$$

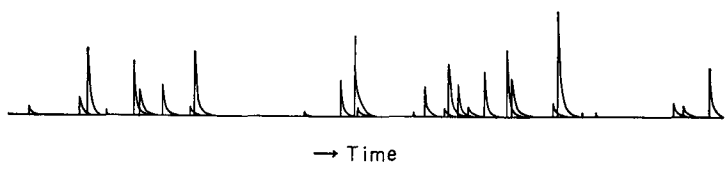

Fig. 178. Schematic representation of the trigger model.

where $\lambda(t)$ is a normarized function, i.e.,

$$\int_0^\infty \lambda(t)\, dt = 1 . \tag{235}$$

This model of earthquake occurrence has first been discussed by Vere-Jones and Davies[15] in 1966 using earthquake data from New Zealand. This model is compatible with a model for the distribution of magnitude described in Section 13.7, where A is the total number of shocks triggered by a main shock. In this model for magnitude distribution, A has an inverse power type distribution (equation (194)). In the present section, however, the functional form of $f(A)$ will not be specified. The mean and the variance of A are denoted by $\bar{A}$ and V respectively, as in Section 13.7.

A generalized Poisson model discussed by Shlien and Toksöz[458] is a special case of the trigger model, in which $\lambda(t)$ is a delta function centered at $t=t_0$. This means that more than one shock occur at an instant of time.

(4) Renewal process: This process is defined as a series of events in which the time intervals between successive events are independently and identically distributed. The Poisson process is a special renewal process in which time intervals have an exponential distribution. A renewal process with a non-exponential distribution of intervals will be discussed in a later section as a model for recurrence of large earthquakes in the same source region.

More complciated models, such as superposition of branching renewal processes, can be constructed, but models with too many parameters may be of little practical use. We first discuss some properties of a branching Poisson process (trigger model introduced by Vere-Jones and Davies[15]) in some detail.

i) *Time intervals between events in the trigger model*

For the branching Poisson process (trigger model), the number of time intervals between successive events τ and larger plotted in the semi-logarithmic coordinates has a form shown in Figure 146 (A). For large τ the curve tends to a straight line assymptotically. Since this line represents roughly the cumulative distribution of time intervals between primary events, the total number of the primary events is approximately equal to N_m+1, where N_m is the ordinate of the line at $\tau=0$. Thus, if a straight line can be fitted to the right side part of the cumulative frequency plots of time intervals, we can find the approximate value for

$$H_\infty = N/(N_m+1) = 1+\overline{A} \tag{236}$$

without counting the number of primary events. However, if primary events occur very frequently, the cumultative frequency curve will be concave throughout and the staight line will be difficult to find. Equation (236) is also inapplicable, if the process is not stationary.

Rough estimates of H_∞-values for data I, I′, II, II′, III, and IV are listed in Table 26. For data III (Figure 144), two straight lines can be fitted. If line B is adopted, $H_\infty \fallingdotseq 1.82$. This value seems too high. Two staight lines may be resulted from non-stationarity of the series. For data I, II, and IV, $H_\infty \fallingdotseq 1.7$, i.e., $\overline{A} \fallingdotseq 0.7$. We note that $\overline{A}$ rarely exceeds 1.0 (see Figure 134).

Table 26. Rough estimates of H_∞ and L_∞ values.

Data	I	I′	II	II′	III	IV
H_∞	1.68	1.15	1.69	1.0	1.0*	1.7
L_∞ (from V/E plots)	4	1	13	1	1	10?
L_∞ (from spectra)	4	1.5?	15?	1	1	5?—15?

* From line A in Fig. 134

ii) *Variance to mean ratios for the trigger model*

For the branching Poisson process (trigger model), if the rate of occurrence of primary events is denoted by μ, the rate of occurrence of all events is $\mu(1+\overline{A})$. The mean and the variance of the number of events n in the time interval of length Δt is given by

$$E(n) = \mu(1+\overline{A})\,\Delta t\,, \tag{237}$$

and

$$V(n) = \mu(1+\overline{A})\,\Delta t + 2\int_0^{\Delta t} (\Delta t - u)\,C(u)\,du \tag{238}$$

where $C(u)$ is the autocovariance function of the process expresssed by

$$C(u) = \mu\overline{A}\lambda(u) + \mu\int_0^\infty\int_0^\infty A\lambda(t)\cdot(A-1)\,\lambda(t+u)\cdot f(A)\,dA\,dt$$

$$= \mu\overline{A}\lambda(u) + \mu(\overline{A^2}+V-\overline{A})\int_0^\infty \lambda(t)\,\lambda(t+u)\,dt \tag{239}$$

for $u>0$. Thus $V(n)$ depends on the functional form of $\lambda(t)$.

It is verified that for $\Delta t \to \infty$, the variance-to-mean ratio $L(=V(n)/E(n))$ tends to a certain value L_∞ given by

$$L_\infty = 1 + \overline{A} + \frac{V}{1+\overline{A}} . \tag{240}$$

Equation (240) can be derived from equations (237), (238), and (239)[15], but here a different proof[313] will be given.

If the interval length Δt is very large, each interval includes many primary events, and almost all secondary events triggered by them occur in the same interval. In this case the number of events in the time interval of Δt can be approximated by a compound Poisson distribution. The probability generating function of this distribution is given by

$$h(k) = \exp(-\mu + \mu g(k)) \tag{241}$$

where $g(k)$ is the probability generation function of the number of secondary events triggered by each primary event plus one (see, e.g., Feller[479]). It follows that

$$g'(1) = 1 + \overline{A} , \tag{242}$$

$$g''(1) = V - g'(1) + (g'(1))^2 = V + \overline{A^2} + \overline{A} , \tag{243}$$

$$h'(1) = \mu h(1)\, g'(1) = \mu(1+\overline{A}) , \tag{244}$$

$$h''(1) = \mu\{h'(1)\, g'(1) + h(1)\, g''(1)\} , \tag{245}$$

$$E(n) = h'(1) = \mu(1+\overline{A}) , \tag{246}$$

$$V(n) = h''(1) + h'(1) - (h'(1))^2 = \mu(1 + 2\overline{A} + \overline{A^2} + V) . \tag{247}$$

The ratio of $V(n)$ and $E(n)$ given above leads to equation (240).

On the other hand, if Δt is very small, most intervals contain no events at all, and other intervals contain only very small number of events. This is almost similar to a Poisson process having a mean of nearly zero. Therefore, it is evident that for $\Delta t \to 0$, $L \to 1$.

In Table 26 rough estimates of L_∞ from L vs Δt plots for data I to IV shown in Figures 161–164 are listed.

The expression of L as a function of Δt is not simple, if $\lambda(t)$ takes the form

$$\lambda(t) = (p-1)c^{p-1}/(t+c)^p \qquad (t > 0) \tag{248}$$

satisfying the modified Omori formula (11). However, for

$$\lambda(t) = \rho e^{-\rho t} , \qquad (t > 0) \tag{249}$$

L is given by

$$L = 1 + \left(\overline{A} + \frac{V}{1+\overline{A}}\right)\left(1 - \frac{1-e^{-\rho\Delta t}}{\rho\Delta t}\right). \tag{250}$$

It is easy to find that for $\Delta t \to \infty$, $L \to L_\infty$, and for $\Delta t \to 0$, $L \to 1$.

iii) *Spectra for the trigger model*

For the branching Poisson process (trigger model), the spectrum given by equation (229) is a decreasing function of ω as shown in Figure 146 (G). If we put $\Phi(\omega) \to \Phi_0$ $(\omega \to 0)$, and $\Phi(\omega) \to \Phi_\infty$ $(\omega \to \infty)$, it is shown[15] that

$$\Phi_0 = 1 + \overline{A} + \frac{V}{1+\overline{A}} = L_\infty\,, \tag{251}$$

$$\Phi_\infty = 1\,. \tag{252}$$

These equations can be derived directly from an expression of the spectrum

$$\Phi(\omega) = \int_{-\infty}^{\infty} C(u)\, e^{i\omega u}\, du/\nu\,, \tag{253}$$

but another proof will be given below.

In a two-dimensional random walk with steps of variable length, the distance R from the origin reached after W steps is approximately distributed as

$$p(R) = \frac{2R}{W\overline{a^2}} \exp\left(-\frac{R}{W\overline{a^2}}\right) \tag{254}$$

for large W, where $\overline{a^2}$ is the mean squared step-length. Then

$$E(R^2) = W\overline{a^2}\,. \tag{255}$$

If $\omega \to 0$, the period $2\pi/\omega$ becomes far larger than the time spread of secondary events generated by each primary event. In this case $|\Sigma e^{i\omega t_k}|^2$ can be regarded as the square of the distance from the origin reached by random walk of μT steps (T is the length of the whole period) whose mean squared length is equal to $E[(1+A)^2]$, since a group of a primary event and its subsidiary events can be regarded as a step of length $1+A$. Therefore

$$\begin{aligned}\Phi_0 &= E(|\Sigma\, e^{i\omega t_k}|^2)_{\omega\to 0}/N = \mu T E\,[(1+A)^2]/N \\ &= \mu(1+2\overline{A}+\overline{A^2}+V)/\mu(1+\overline{A}) = L_\infty\,.\end{aligned} \tag{256}$$

On the other hand, for $\omega \to \infty$, each events can be regarded as a unit step of random walk, then

$$\Phi_\infty = E(|\Sigma\, e^{i\omega t_k}|^2)/N = \mu T(1+\bar{A})/N = 1\,. \tag{257}$$

If $\lambda(t)$ has a form given by equation (249), the spectrum is expressed by

$$\Phi(\omega) = 1 + \left(\bar{A} + \frac{V}{1+\bar{A}}\right)\frac{\rho^2}{\rho^2+\omega^2}\,. \tag{258}$$

It is obvious that $\Phi(\omega) \to L_\infty$ $(\omega \to 0)$ and $\Phi(\omega) \to 1$ $(\omega \to \infty)$. If $\lambda(t)$ is an inverse-power type given by equation (248), the spectrum can not be expressed by a simple form. In this case we can calculate the spectral values numerically by the following equation.

$$\Phi(\omega) = 1 + \frac{L_\infty - 1}{\Gamma(p-1)} \int_0^\infty \frac{\xi^p e^{-\xi}}{\xi^2 + (c\omega)^2}\, d\xi \tag{259}$$

Figure 179 shows spectral cruves for the trigger model with equation (248) for various p and L_∞ values. If the standard aftershock sequence (cf. Chapter 8) is adopted, the abscissa at the bottom of the figure (c=0.3 day) must be used with curves for p=1.3. It is seen that for the standard aftershock sequence the spectral values are more than 50% higher than those expected from the Poisson process at frequencies 365 c/yr (=1 c/day) for $L_\infty \geq 10$.

Comparison of the observed spectral curves such as shown in Figure 169, 171, 174 etc. with Figure 179 indicates that the remarkable increase in spectral values can be explained by the use of the trigger model. A broken line in Figure 169 represents the theoretical spectral curve for the trigger model with p=1.3, c=0.3 day, and L_∞=4. This L_∞ value is equal to that estimated from the L vs Δt curve (Table 26). For Figures 171 and 174 the theoretical curve for p=1.3 and c=0.3 day fits the data less well. Theoretical curves with larger c value or smaller p value fit the data better.

14.3 *Periodicity in earthquake occurrence*

i) *Definition*

The problems connected with periodicities in earthquake occurrence have been discussed by many seismologists since the last century. However, critical review of these studies suggests that if there is any periodicity, it is usually so weak that it may be detected only by careful statistical analysis. In such discussions the term "periodicity" must first be defined clearly.

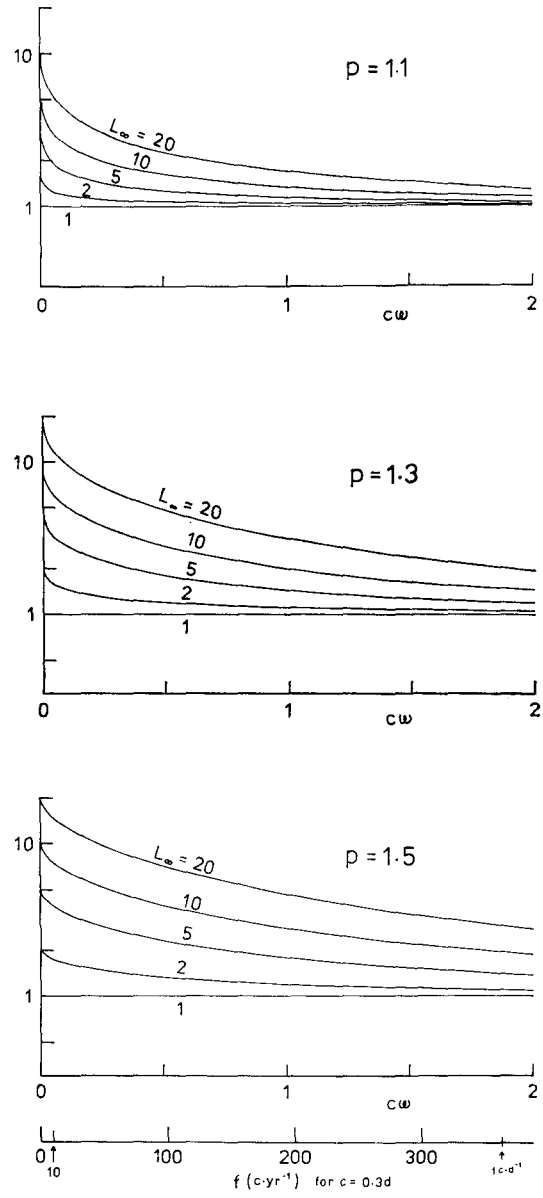

Fig. 179. Theoretical spectra for the trigger model with various L_∞ and p values plotted against $c\omega$. The scale at the bottom indicates the frequency scale for $c=0.3$ day.

Many authors have considered that the periodicity is established if a peak amplitude of the spectral curve calculated for a series of earthquake data exceeds a certain threshold value determined by assuming a Poisson process. In this case an implicit definition of periodicity is given by using a criterion for spectral amplitudes. However, the argument against such a definition is that the Poisson process is not the only process that has no periodic structure. For some non-Poisson processes without periodic structure (e.g., the trigger model), the above-mentioned threshold values may be higher than those for the corresponding Poisson processes. The spectral amplitude is of coruse a sensitive quantity to the periodicity, but the rejection of the Poisson process on the basis of the spectral amplitude does not provide a proof for the existence of the periodicity.

Generally speaking, it is possible to know whether a stochastic process defined mathematically has some periodic structure or not. Examples of non-periodic point processes are the Poisson process, renewal processes in which the distribution of time intervals is a monotonically decreasing function, and branching Poisson processes in which the rate of occurrence of subsiderary events is a monotonically decreasing function. Examples of periodic point processes of period T are renewal processes in which the distribution of time intervals has a peak at $\tau=T$ and time-dependent Poisson processes in which the occurrence rate varies with time periodically such as

$$\nu(t) = \nu_0 + \nu_1 \sin \frac{2\pi}{T} t \tag{260}$$

where ν_0, and ν_1 are constants.

The objects of our discussions are not the stochastic process itself, but series of actual earthquake data of finite size. It is impossible to prove that such empirical data are samples from a certain specified stochastic process. We can test the hypothesis that the data are samples from a given stochastic process by various independent methods, but the results do not lead to the conclusion about the presence or absence of the periodicity in the data. If the hypothesis of a non-periodic process is rejected, it does not necessarily mean that the data are periodic, because there are possibilities that the data are samples from another non-periodic process. If this hypothesis is not rejected, it never mean that the hypothesis is accepted and the data are non-periodic. If the hypothesis of a periodic process is rejected, it does not necessarily mean that the data are non-periodic, because there are possiblities that the data are samples from another periodic process. If this hypothesis is not rejected, it

never means that the hypothesis is accepted and the data are periodic.

Considering such conditions, it seems to impossible to give a perfect definition of periodicity. We must seek some practical methods for discussing the periodicity. The following procedure is one of such practical methods.

ii) *Statistical tests for periodicity*

First we must realize which of the next two cases we are going to discuss.

Case 1: To test the existence of the periodicity of some suspected period T for some geophysical reasons. For example, one day or one year period due to astronomical causes.

Case 2: To discover the periodicity of some period which is unknown before the analysis.

(1) Tests for the Poisson process.

In case 1, the spectral value $\Phi(\omega)$ for $\omega=2\pi/T$ is calculated from equation (229). This value is tested against the Poisson model by Schuster's method, i.e., if $\Phi(\omega)$ is larger than φ given in Table 27, the Poisson model is rejected at a signfiicance level smaller than p. The rejection of the Poisson process does not provide a proof for the existence of the periodicity of period T.

Table 27. Schuster's criterion for the Poisson process. $p=e^{-\varphi}$.

p	φ
0.1	2.30
0.05	3.00
0.01	4.61
0.001	6.90
0.0001	9.21

In case 2, the spectral values are calculated for $\omega_k=2\pi/T_k$, $T_k=T_0/k$ ($k=1, 2, \ldots, m$) where T_0 is the length of the whole period of investigation. If the largest spectral value among the m spectral values is denoted by Φ_1, this value is tested against the Poisson hypothesis by Fisher's method. Fisher[480] obtained the probability $P_1(m, g)$ that $G_1(=\Phi_1/\sum_{k=1}^{m}\Phi(\omega_k))$ exceeds g for the Poisson process. The g values have been tabulated by e.g., Nowroozi[481] for various values of m and $P_1(m, g)$. If G_1 is larger than g, the Poisson model is rejected at a significance level smaller than $P_1(m, g)$. Similarly, for the sth

largest spectral values Φ_s ($s=2, 3, \ldots$), the probability $P_s(m, g)$ that G_s ($=\Phi_s/\sum_{k=1}^{m}\Phi(\omega_k)$) exceeds g has been obtained for the Poisson process and g values have been tabulated by Shimshoni[482] for various values of s, m, and $P_s(m, g)$.

It should be noted that there is an essential difference between case 1 and case 2. It is not adequate to use Schuster's criterion in case 2. Shimazaki[483] pointed out this in a discussion to the 69-year periodicity of destructive earthquakes in southern Kwanto.[484]

(2) Tests for the trigger model.

Since the increase of spectral values due to the existence of aftershocks is a general feature of series of shallow earthquakes, this effect must be considered in the discussion of periodicity as Jeffreys[39] first pointed out in 1938. At present, the trigger model with $\lambda(t)$ of the type given by equation (248) seems to be the most adequate one to the approximation of the occurrence of shallow earthquakes in a relatively simple form. This model has no periodic structure. If this model with appropriate parameters is not rejected on the basis of the spectral amplitudes, the existence of periodicity can not be concluded even if the spectral amplitude test rejects the Poisson process.

To test the observed spectral values for the trigger model, the theoretical spectrum $\Psi(\omega)$ for the trigger model must first been calculated using appropriate values for the parameters L_∞, p, and c. A rough estimate of the theoretical spectrum can be obtained by drawing a smooth concave curve similar to those in Figure 179 which fits the observed spectral curve. Since the ratio of the observed and the theoretical spectra $\Phi(\omega)/\Psi(\omega)$ has approximately the same distribution as the spectrum for the corresponding Poisson process, the same procedure for the tests for Poisson process described before can be applied to this spectral ratio. If the trigger model is rejected on the basis of the spectral amplitude at period T, we may say that there is evidence for the periodicity of period T, as long as no other adequate non-periodic model is proposed. Several examples are given below.

Example 1. Data I (Figure 169). Spectral values have been calculated for 100 frequencies or 100 periods from $T_1=44$ years to $T_{100}=0.44$ year. Spectral values for 81 frequencies exceed 1 (the expectancy for the Poisson process). This is very unusual if the Poisson process is assumed. Furthermore, for 14 frequencies the values exceed 6.9 (probability level of 0.001). The maximum spectral value is 22.2 at $T_k=1.913$ years ($k=23$). If this value is tested by Schuster's method, the probability of the occurrence of this value is extremely low, $e^{-22.2}=10^{-9.6}$. However, if this value is tested by Fisher's

method, $G_1=\Phi_1/\sum_{k=1}^{100}\Phi(\omega_k)=22.2/356.3=0.062$. From the table of Nowroozi it is found that P_1 (100, 0.0674)=0.1. Thus the probability of the occurrence of $\Phi_1=22.2$ is larger than 0.1, and the periodicity of $T_{23}=1.913$ years is not accepted at all. The theoretical spectral value for this period according to the trigger model is $\Psi(\omega_{23})\fallingdotseq 3.5$ (see broken line in Figure 169). Then $\Phi(\omega_{23})/\Psi(\omega_{23})=6.3$. This value is also insignfiicant according to Fisher's criterion. Spectral amplitude at 2nd, 3rd, ... peaks are also insignificant according to the extended Fisher test. In conclusion, no significant periodicities are found from these data.

Example 2. Data II (Figure 171). The highest peak at $T_3=4.0$ years has the amplitude of $\Phi_1=30.60$. Then $G_1=30.6/534=0.057$. This value is smaller than G_1 for data I. For the second peak, $\Phi_2=25.22$ at $T_{23}=0.429$ year, and $G_2=0.047$. From Shimshoni's table, P_2 (100, 0.0543)=0.05. After all, no significant periodicities are found for data II. The same conclusions are obtained for data I′, II′, III, and IV except $T_1=50$ years for data IV.

Example 3. Remarkable spectral peaks at 1.000 c/day (local time) have been found by Morgan et al.[485] and Haubrich[474] for different sets of worldwide data. In Figure 3 of Haubrich's paper, the spectral amplitude at 1.000 c/day is by about 6 db higher than the average spectral level around this frequency. If this average level is assumed to represent an approximate theoretical spectral value, the difference of 6 db is not large enough to reject the random occurrence according to Schuster's test. In Figure 5 of the paper by Morgan et al., the squared amplitude at 1.000 c/day is about four times the average level around this frequency. This amplitude is significant at a significance level of about 0.03. Morgan et al. also found a more evident yearly peak in the same data. Haubrich suggested the possibility that yearly and daily periodicities were resulted from changes in the detection threshold due to the variation in the seismic noise rather than from the actual changes in the occurrence of earthquakes.

Example 4. Shimshoni[486] also found significant periodicity of one day using 15325 events reproted from NOAA for 1968–1970. The expectancy of the power spectrum for the Poisson process is $4\times 15325/24^2=106.4$ in the unit of his paper. The observed squared amplitude is 50.2^2, which is about 24 times of the expectancy. The effect of aftershocks may not be small, but the increase of expectancy due to aftershocks may be less than four times (estimated by using Figures 134 and 179). If the theoretical spectral amplitude is assumed to be 400, the observed amplitude is still six times as large as the

theoretical amplitude. Therefore Schuster's test rejects the trigger model, provided the effect of the daily variation of seismic noise is not so strong.

Since 1936 Matuzawa and his colleagues[78),487)-489)] examined periodicities of one year, half year, one day, one luner month, etc. for earthquakes occurring in various regions of the world and of Japan. In these studies they applied a special method for significance in harmonic amplitudes. Many papers on periodicities of earthquakes in Japan have been published by the later workers[221),456),490)-497)] using the same method. This method is closely connected with the test based on the trigger model as described below.

Now we are going to test the periodicity of period T for events distributed in the time interval of length mT (m: interger). If the rth interval of length T ($r=1, 2, \ldots, m$) is divided into n sub-intervals of length $\Delta t=T/n$, and the number of events in the sth sub-interval is denoted by x_{sr}, the harmonic analysis gives the values for the coefficients a_{0r}, a_{1r}, b_{1r}, ... in the equation

$$x_{sr} = a_{0r} + a_{1r} \cos \frac{2\pi}{T} s\Delta t + b_{1r} \sin \frac{2\pi}{T} s\Delta t + \cdots \qquad (s = 1, 2, \cdots, n) \tag{261}$$

If

$$c_r = (a_{1r}/a_{0r}) + (b_{1r}/a_{0r})\, i\,, \qquad (i = \sqrt{-1}) \tag{262}$$

$$l_m{}^2 = \sum_{r=1}^{m} |c_r{}^2|/m\,, \tag{263}$$

and

$$c_m{}^2 = |\sum_{r=1}^{m} c_r{}^2|/m\,, \tag{264}$$

the criterion used by Matuzawa is based on the condition that the probability that

$$c_m{}^2/l_m{}^2 = |\Sigma c_r|^2/\Sigma |c_r{}^2| \tag{265}$$

exceeds a value φ is $w=e^{-\varphi}$ for non-periodic processes.

The spectrum in the complex form (the power spectrum is the square of its absolute value) for the rth interval of length T is given by

$$\varphi_r(\omega) = \sum_{(r-1)T \leq t_k < rT} e^{i\omega t_k}/\sqrt{N_r} = \sqrt{N_r}\, c_r/2\,, \tag{266}$$

where N_r is the total number of events in the rth interval. The spectrum for the whole interval of length mT becomes

$$\varphi(\omega)=\sum_{0\leq t_k<mT} e^{i\omega t_k}/\sqrt{N}=\sum_{r=1}^{m}\varphi_r(\omega)\sqrt{m}=\sqrt{N}\sum_{r=1}^{m}c_r/(2m)\,, \qquad (267)$$

provided that $N_r=N/m$ for $r=1, 2, \ldots, m$. If we consider that the theoretical power spectrum $\Psi(\omega)$ for the trigger model is approximated by the average of the power spectra $|\varphi_r(\omega)|^2$ $(r=1, 2, \ldots, m)$, i.e.,

$$\Psi(\omega)\fallingdotseq\sum_{r=1}^{m}|\varphi_r(\omega)|^2/m=N|c_r|^2/(4m^2)\,, \qquad (268)$$

the probability that $\Phi(\omega)/\Psi(\omega)$ exceeds a value φ is $e^{-\varphi}$ from Schuster's criterion for the trigger model. Since

$$\Phi(\omega)/\Psi(\omega)=|\varphi(\omega)|^2/\Psi(\omega)=|\sum_{r=1}^{m}c_r|^2/\sum_{r=1}^{m}|c_r|^2=c_m{}^2/l_m{}^2 \qquad (269)$$

the both approach is the same under the assumption expressed by equations (267) and (268). Both assumptions seem to be reasonable for stationary processes.

The results of analyses by Matuzawa et al. and later investigators indicate that in most cases no periodicities are confirmed, but in a few cases the probability w is very small suggesting the existence of periodicity. For example, Matuzawa et al.[489]. reported $w=0.018$ for the half luner-month period in the world earthquakes of 1921–1930. Matuzawa et al.[488] examined the yearly periodicities for earthquakes occurring each of 69 regions in and near Japan. Of 69 regions only eight regions have the probability w for yearly periodicity of less than 0.1. The smallest one is 0.00044 for a region near Amami-Oshima, but recalculation yields $w=0.08$. The next smallest one is $\omega=0.009$. It is natural that a probability of about 0.01 is obtained by chance in 69 trials, if there is no periodicity at all.

Many other papers have been published dealing with periodicities of earthquakes, the results of which will not be discussed here. For reviews of some of these studies, see, for example, Conrad,[498] Davison,[473] Aki[272] and Lomnitz.[499]

The correlations between earthquake occurrence and some periodic phenomena, such as the position of the sun or the moon, the ocean tides, the earth tides, etc. have been reported for various regions of the world (e.g., references [65),239),472),500)–520)]). Some authors consider such phenomena as secondary causes of earthquakes. The author has not checked the statistical significance of these reports, but it should be mentioned that the consideration to the

effects of aftershocks is needed in some of these discussions. Tests against the hypothesis of Poisson process only often leads to a misleading conclusion.

There is another type of misleading conclusions in statistical seismology. Burr[521] criticized a paper[522] which contained this type of error. Tamrazyan wrote more than twenty papers dealing with the relations between earthquakes and the astronomical positions of the sun and the moon. In a paper[523] on the synodic ages of Japanese destructive earthquakes he says "14 destructive earthquakes in Japan since 1700 accompanying the deaths of 1000 persons or more occurred in the half month from the 20th to the 5th day of the synodic month. Only two of such earthquakes occurred in the other half month." If the earthquakes occur randomly in time, the probability that the 14 earthquakes out of 16 ones fall in a half month specified beforehand (e.g., 20th to 5th) is very low, about 0.002. However, the probability that 14 events out of 16 events fall in any unspecified half month is not very low, about 0.04. Moreover, six earthquakes in Japan with deaths of more than 1000 should be added to his list. (Imamura's list to which he referred was incomplete.) Including these, 16 events out of 22 events fall in the above mentioned half month. The probability for this is about 0.3, if the half month is not specified. Thus in this case no relation is established between destructive earthquakes and the moon, though the existence of such a relation is not improbable.

iii) *Reccurrence of large earthquakes in the same source region*

It has been pointed out by several seismologists[524)–531)] that great earthquakes (e.g., $M \gtrsim 8$) originate repeatedly from the same source region at intervals of several tens to hundreds of years in some island arc areas (Japan, Kurile-Kamchatka, Aleutian-Alaska, South America, etc.). This seismic process may simply explained by gradual accumulation of strain energy and sudden release of it by an earthquake. If the rate of energy supply is constant for a long time, repetition will occur, but the interval length between earthquakes may fluctuate owing to the probabilistic nature of the fracture.

The most simple mechanical model for this process is a system of a spring and a slider connected in series (Figure 126, center). If the probability of slip μ is related to the stress σ in the spring by

$$\mu = Ae^{\beta\sigma} \tag{99}$$

and if σ increases linearly with time

$$\sigma = kt\,, \tag{270}$$

the rate of slip (hazard function) is given by

$$\mu(t) = Ae^{Bt}\,, \qquad B = \beta k\,. \tag{271}$$

In this case the probability that a slip takes place between t and $t+dt$ (t is measured from the time of the last slip) is expressed by

$$q(t) = \mu(t)\exp\left(-\int_0^t \mu(t)\,dt\right) \tag{94}$$

$$= Ae^{Bt}\exp\left\{\frac{A}{B}(1-e^{Bt})\right\}, \tag{272}$$

and the probabilty that the slip takes place at a time later than t (reliability function) becomes

$$p(t) = q(t)/\mu(t) = \exp\left\{\frac{A}{B}(1-e^{Bt})\right\}. \tag{273}$$

The mean, the median, and the mode of times to slip are given by

$$\bar{t} = \int_0^\infty p(t)\,dt = -\frac{1}{B}\left\{e^{A/B}\,Ei\left(-\frac{A}{B}\right)\right\}, \tag{274}$$

$$\tilde{t} = \frac{1}{B}\ln\left(1+\frac{B}{A}\ln 2\right), \tag{275}$$

$$\hat{t} = \frac{1}{B}\ln\left(\frac{A}{B}\right) \tag{276}$$

respectively, where $Ei(-x)$ represents the exponential integral.

Figure 180 is a plot of $P(t)$ $(=1-p(t))$ against $t/\bar{t}$ for various values of $B'(=B\bar{t})$. Sufficient historical data are not available for determining the values of parameters for this model. The data on large earthquakes in the Hokkaido-Southern Kurile region give a rough estimate of B' of 3 to 5.[530] This model is quite different from the trigger model discussed before. However, it is most probable that the trigger model is resulted from superposition of such reccurrence processes of main shock-aftershock sequence systems in many source regions.

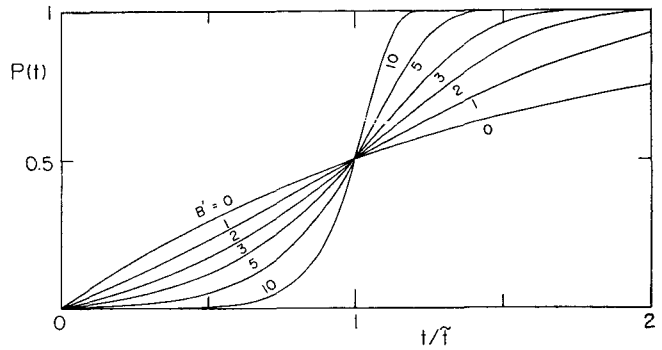

Fig. 180. Curves for $P(t)$ plotted against $t/\bar{t}$ for several values of B'. The curve of $B'=0$ corresponds to the Poisson process.

15. Distribution of earthquakes in respect to space

This chapter had been scheduled to be published in the next number of this journal. However, the schedule has changed as the author has transferred from Hokkaido University. It will be published elsewhere as an independent paper.

16. Summary

Aftershock sequences are one of the most remarkable phenomena connected with the occurrence of earthquakes. They have unique statistical properties and the physical explanation of these is of great importance in understanding the processes of earthquake generation. In statistical studies of earthquake occurrence in general, the effect of aftershock sequences and other clusters must be considered properly. Statistical significance tests under the assumption that all earthquakes are mutually independent events sometimes yield misleading results.

In Part I of the present study, some results from investigations of the statistical properties of aftershock sequences have been presented. Several parameters characterizing an aftershock sequence have been evaluated for many Japanese aftershock sequences, and the interrelations between these parameters have been investigated. There are slight correlations between some parameters, such as p (Utsu, 1957), c (Omori, 1894), D_1 $(=M_0-M_1)$ (Utsu, 1957), b (Gutenberg-Richter, 1944), etc. In Part II, on the basis of abundant examples of the multiple occurrence of simple sequences in relatively short intervals of space and time, a new classification of earthquake sequences

(an extention of that of Mogi, 1963) has been proposed. As a result of this calssification, earthquake swarms are classified into two types. This calssification is helpful in the interpretation of some statistical properties of earthquake swarms. A model for aftershock occurrence has been proposed on the basis of the known statistical properties and the new classification. Ordinary aftershocks are caused by delayed fracture in some parts of the source region of the main shock where stress redistribution occur at the time of the main shock.

Part III and IV discuss the distributions of general earthquakes in magnitude and time, the effects of aftershocks being considered. Discussions on the space and space-time distributions will be published elsewhere. In Part III, b-values have been redetermined for more than 100 groups of earthquakes using the maximum likelihood method (Utsu, 1965), and the difference in b-value between some groups has been tested. The spatial or temporal variation of b-values has been found in only a few cases. A model for the magnitude distribution of earthquakes has been proposed. In Part IV, the temporal distribution of earthquakes has been investigated by testing the hypothesis of Poisson process. The Poisson process seems to be a good approximation only for series of earthquakes from which all aftershocks have been removed or for series of deep earthquakes followed by few aftershocks. Applying a branching Poisson process (Vere-Jones and Davies, 1966), the parameters for the process have been estimated from the distribution of time intervals between events, the variance/mean curve, and the spectra. It is found that this model explains the data reasonably well. The periodicities in the occurrence of earthquakes have been tested on the basis of this model. No significant periodicities are found in most of the data analysed.

Acknowledgements: I with to express my thans to staff members of the Department of Geophysics, Hokkaido University for their helpfulness throughout this study. Most of the figures were drafted by Miss. R. Yashiro and some by Miss. I. Sanjô. Computations were performed on the NEAC 2201 and the FACOM 230–60 computers at the Computing Center of Hokkaido University.

References*

454) TERADA, T.: On the frequency of earthquake and allied phenomena. Proc. Tokyo Math.-Phys. Soc. (ii), **9** (1918), 515–522.

455) SHLANGER, A.B.M.: Some consequences of earthquake statistics for the years 1918–1955. Gerl. Beitr. Geophys., **69** (1960), 68–72.

456) IIDA, K.: Observations sur les séismes enregistrés par le microsismographe construit dernièrment (2). Bull. Earthq. Res. Inst., **17** (1939), 741–782 (in Japanese with French summary).

457) ICHIKAWA, M.: Tremblements de terre du district du Kwanto (1933–1952). Quart. J. Seism., **21** (1956), 113–123 (in Japanese with French summary).

458) SHLIEN, S. and TOKSÖZ, M.N.: A clustering model for earthquake occurrence. Bull. Seism. Soc. Am., **60** (1970), 1765–1787.

459) FRANCIS, T.J.G. and PORTER, I.T.: A statistical study of mid-Atlantic ridge earthquakes. Geophys. J., **24** (1971), 31–50.

460) WANNER, E.: Zur Statistik der Erdbeben, I. Gerl. Beitr. Geophys., **50** (1937), 85–99; II. ibid, 223–228.

461) GAISKIĬ, V.N.: On the similarity of earthquake series, the relationship between them, and the "tendency" toward periodicity. Bull. Acad. Sci. USSR, Phys. Solid Earth, (1967), 432–437 (English translation).

462) YAMAGUTI, S.: World distribution of deep earthquakes. Bull. Earthq. Res. Inst., **15** (1937), 170–178.

463) NAKAMURA, S.: On the frequency of earthquakes in Tokyo. Kishoshushi, **39** (1920), 79–82 (in Japanese).

464) GAISKIĬ, V.N.: Some features of the seismic process from a study of earthquakes in Tadzhikistan. Bull. Acad. Sci. USSR, Ser. Geophys., (1961), 368–371 (English translation).

465) KNOPOFF, L.: The statistics of earthquakes in southern California. Bull. Seism. Soc. Am., **54**, (1964), 1871–1873.

466) INOUYE, W.: Statistical investigation on earthquake numbers. Bull. Earthq. Res. Inst., **10**, (1932), 43–54 (in Japanese with English summary).

467) ISHIMOTO, M.: Etudes statistiques sur les seismes observés à Tôkyô depuis 1876. Bull. Earthq. Res. Inst., **14** (1936), 610–615 (in Japanese with French summary).

468) TAKEHANA, M.: Statistics of felt earthquakes in Japan in the last 30 years. Quart. J. Seism., **10** (1940), 95–146 (in Japanese).

469) FERRAS, S.G.: Test of Poisson process for earthquakes in Mexico City. J. Geophys. Res., **72** (1967), 3741–3742.

470) TAKAHASI, K.: On the statistical properties of frequency of earthquakes. Kishoshushi (ii), **15** (1937), 7–16 (in Japanese).

471) JENKINS, G.M. and WATTS, D.G.: Spectral Analysis and Its Application, (1968), Holden-Day, San Francisco.

472) SCHUSTER, A.: On luner and solar periodicities of earthquakes. Proc. Roy. Soc., **61** (1897), 455–465.

473) DAVISON, C.: Studies in the Periodicity of Earthquakes, (1938), T. Murby, London.

---

* See Part I, II, and III of this series for references 1)–193), 194)–311), and 312)–453) respectively.

474) HAUBRICH, R.A.: Spectra of earthquake time series. EOS, **50** (1969), 409–410.

475) TAKAHASI, K.: On analysis of random fluctuation, persistence, and periodicities and some application to meteorological and geophysical phenomena. Geophys. Mag., **11** (1937), 237–268.

476) REYMENT, R.A.: Statistical analysis of some volcanologic data regarded as series of point events. Pure Appl. Geophys., **74** (1969), 57–77.

477) COX, D.R. and LEWIS, P.A.: The Statistical Analysis of Series of Events, (1966), Methuen & Co, London.

478) KITAGAWA, T., HURUYA, S., and YAMADA, T.: The probabilistic analysis of the time series of rare events, I. Mem. Fac. Sci. Kyushu Imp. Univ. Ser. A, **2** (1941), 152–204.

479) FELLER, W.: An Introduction to Probability Theory and Its Applications, Vol. 1, 2nd Ed., (1957), John Wiley & Sons, New York.

480) FISHER, R.A.: Test of significance in harmonic analysis. Proc. Roy. Soc., A **125** (1929), 54–59.

481) NOWROOZI, A.A.: Table for Fisher's test of significance in harmonic analysis. Geophys, J., **12** (1967), 517–520.

482) SHIMSHONI, M.: On Fisher's test of significance in harmonic analysis. Geophys. J., **23** (1971), 373–377.

483) SHIMAZAKI, K.: Hidden periodicities of destructive earthquakes at Tokyo. Zisin (J. Seism. Soc. Jap.), ii, **25** (1972), 24–32, (in Japanese with English summary).

484) KAWASUMI, H.: Proofs of 69 years periodicity and imminence of destructive earthquake in southern Kwanto district and problems in the countermeasures thereof. Chigaku-Zasshi, **79** (1970), 115–138, (in Japanese with English summary).

485) MORGAN, W.J., STOVER, J.O., and DICKE, R.H.: Periodicity of earthquakes and the invariance of the gravitational constant. J. Geophys. Res., **66** (1961), 3831–3843.

486) SHIMSHONI, M.: Evidence for higher seismic activity during the night. Geophys. J., **24** (1971), 97–99.

487) MATUZAWA, T.: Remarks on the statistical study of earthquake frequencies. Zisin (J. Seism. Soc. Jap.) i, **8** (1936), 16–23, (in Japanese).

488) MATUZAWA, T., NAKAMATI, H., NISHIKAWA, Y., and YOSIMURA, Y.: Über die Jahresschwankung der Erdbebenhäufigkeit in Japan. Bull. Earthq. Res. Inst., **15** (1937), 711–784.

489) MATUZAWA, T., HAYAKAWA, M., HATTORI, Y., KANEKO, T., and MIYAMURA, S.: Erdbebenhaufigkeit in Bezug auf die Stellung der Sonne und des Mondes. Bull. Earthq. Res. Inst., **18** (1940), 265–280.

490) KISHINOUYE, F.: On the diurnal periodicity of felt earthquakes. Bull. Earthq. Res. Inst., **14** (1936), 604–609.

491) SHINOHARA, S.: On the diurnal variation in earthquake frequencies. Kenshin-jiho (Quart. J. Seism.), **9** (1937), 272–277, (in Japanese).

492) KINOSHITA, J.: Statistical study of earthquakes occurring in the west side of the Kii Peninsula. Zisin (J. Seism. Soc. Jap.), i, **10** (1938), 129–150, (in Japanese).

493) KISHINOUYE, F.: Statistical investigation of monthly numbers of earthquakes felt at Tokyo. Bull. Earthq. Res. Inst., **26** (1948), 73–79.

494) IIDA, K.: Observations sur les séismes enregistrés par microsismographe (3). Bull. Earthq. Res. Inst., **18** (1940), 575–674, (in Japanese with French summary).

495) HOMMA, S. and KOMIYA, T.: Investigations of the diurnal variation of earthquake occurrence in Japan, Part 1, Ibaragi Prefecture and vicinity. Kenshinjiho (Quart. J. Seism.), **12** (1942), 56–64, (in Japanese).

496) NAGAMUNE, T.: Investigation of the diurnal variation of earthquake occurrence in Japan, Part 2, the Nansei Islands. Kenshinjiho (Quart. J. Seism.), **14** (No. 3–4), 18–23 (in Japanese).

497) FURUMOTO, A.S.: Some statistical investigations of Hawaiian volcanic eruptions and earthquakes I. Kazan (J. Volc. Soc. Jap.) ii, **2** (1957), 26–36, (in Japanese with English summary).

498) CONRAD, V.: Die zeitlich Folge der Erdbeben und bebenauslösende Ursachen. Handbuch der Geophysik, **4** (1932), 1007–1185,

499) LOMNITZ, C.: Statistical prediction of earthquakes. Rev. Geophys., **4** (1966), 377–393.

500) IMAMURA, A.: Synodic-monthly variation of seismic frequency in Japan. Pub. Earthq. Inv. Comm., **18** (1904), 41–71.

501) OMORI, F.: Note on the luner-daily distribution of earthquakes. Pub. Earthq. Inv. Comm., **18** (1904), 27–40.

502) NAKAMURA, S.: On the effect of tides on the occurrence of earthquakes in Kwanto district. Jap. J. Astro. Geophys., **4** (1927), 139–165.

503) YAMAGUTI, S.: Relation between tidal phases and the earthquakes. Bull. Earthq. Res. Inst., **8** (1930), 393–403.

504) TAKAYAMA, T. and SUZUKI, T.: On the relation between the sunspot number and the destructive earthquakes in Japan. Bull. Earthq. Res. Inst., **8** (1930), 364–374, (in Japanese with English summary).

505) FUKUTOMI, T. and KAWASE, J.: Relation of earthquake occurrence and volcanic eruption to tides. Zisin (J. Seism. Soc. Jap.) i, **3** (1931), 484–498 (in Japanese).

506) STETSON, H.T.: The correlation of deep focus earthquakes with lunar hour angle and declination. Science, **82** (1935), 523–524.

507) TAGUCHI, T.: On the recent seismic activities near Kii. Kenshinjiho (Quart. J. Seism.), **8** (1935), 139–145.

508) ALLEN, M.W.: The luner triggering effect on earthquakes in Southern California. Bull. Seism. Soc. Am., **26** (1936), 147–157.

509) STETSON, H.T.: Correlation of frequency of seismic disturbances with the hour angle of the moon. Proc. Am. Phil. Soc., **78** (1937), 411–424.

510) MORITA, M.: Relation between aftershocks of the Karenko-oki earthquake of June 14, 1925 and the tide. Kenshinjiho (Quart. J. Seism.), **9** (1937), 265–271, (in Japanese).

511) MCMURRY, H.: Periodicity of deep-focus earthquakes. Bull. Seism. Soc. Am., **31** (1941), 33–82.

512) TANAKA, Y.: Volcanic earthquakes triggered by tides. Kenshinjiho (Quart. J. Seism.), **26** (1961), 7–15, (in Japanese with English summary).

513) DIX, C.H.: Triggering of some earthquakes. Proc. Jap. Acad., **40** (1964), 410–415.

514) KNOPOFF, L.: Earth tides as a triggering mechanism for earthquakes. Bull. Seism. Soc. Am., **54** (1964), 1865–1870.

515) BERG, E.: Triggering of the Alaskan earthquake of March 28, 1964 and major aftershocks by low ocean tide loads. Nature **210** (1966), 983–896.
516) SIMPSON, J.F.: Earth tides as a triggering mechanism for earthquakes. Earth Planet. Sci. Lett., **2** (1967), 473–478.
517) SIMPSON, J.F.: Solar activity as a triggering mechanism for earthquakes. Earth Planet. Sci. Lett., **3** (1967), 417–425.
518) TAMRAZYAN, G.P.: Earthquakes of Nevada (USA) and the tidal forces. J. Geophys. Res., **73** (1968), 6013–6018.
519) TAMRAZYAN, G.P.: Principal regularities in the distribution of major earthquakes relative to solar and luner tides and other cosmic forces. Icarus, **9** (1968), 574–592.
520) MOGI, K.: Monthly distribution of large earthquakes in Japan. Bull. Earthq. Res. Inst., **47** (1969), 419–427.
521) BURR, E.J.: Earthquakes and Uranus: Misuse of a statistical test of significance. Nature, **186** (1960), 336–337.
522) TOMASCHEK, R.: Great earthquakes and the astronomical positions of Uranus. Nature, **184** (1959), 177–178.
523) TAMRAZYAN, G.P.: On the synodic age of destructive earthquakes in Japan. Zisin (J. Seism. Soc. Jap.) ii, **24** (1971), 67–68 (in Japanese).
524) IMAMURA, A.: Reccurrence of great earthquakes in Kwanto and Kinki districts and phenomena forerunning great earthquakes. Zisin (J. Seism. Soc. Jap.) i, **1** (1929), 4–16, (in Japanese).
525) FEDOTOV, S.A.: Regularities of the distribution of strong earthquakes of Kamchatka, the Kurile Islands, and northeastern Japan. Trudy Inst. Phys. Earth, Acad. Sci. USSR, **36** (1965), 66–93, (in Russian).
526) FEDOTOV, S.A., DOLBILKINA, N.A., MOROZOV, V.N., MYACHIKIN, V.I., PREOBRAZENSKY, V.B., and SOBOLEV, G.A.: Investigation on earthquake prediction in Kamchatka. Tectonophys., **9** (1970), 249–258.
527) MATSUDA, T.: Active faults and large earthquakes. Kagaku, **39** (1969), 398–407, (in Japanese).
528) KELLEHER, J.: Space-time seismicity of the Alaska-Aleutian seismic zone. J. Geophys. Res., **75** (1970), 5745–5756.
529) SYKES, L.R.: Aftershock zones of major earthquakes, seismicity gaps, and earthquake prediction for Alaska and the Aleutians. J. Geophys. Res., **76** (1971), 8021–8041.
530) UTSU, T.: Large earthquakes near Hokkaido and the expectancy of occurrence of a large earthquake off Nemuro. Rep. Coordinating Comm. Earthq. Predic., **7** (1972), 7–13, (in Japanese).
531) KELLEHER, J.A.: Rupture zones of large south American earthquakes and some predictions. J. Geophys. Res., **77** (1972), 2087–2103.