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Instantaneous Shape of Raindrop Size Distribution and Its Rainparameter Relations in the Convective Rainfall

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Abstract

This research has a purpose to make clear the relations between the representative rain parameters for the intantaneous drop samplings. Two parameters which can be comparatively easily observed are used for determining the other rain parameters. They are the rainfall intensity (R) and the maximum diameter of drop ($Dmax$). The data sources are from the convective rainfalls observed at Hitoyoshi and Tsuetate in Kyushu, the southern land of Japan. The obtained instantantaneous raindrop size distributions are narrow in the liquid water content distribution with size or flat in the space number density curve in N_D. The instantaneous rain parameters derived from those size distributions are quite different from those averaged over the rainfall, for instance, Marshall and Palmer distribution. This result may be useful to the prediction of the instantaneous rain parameters which have the intimate relations to our industrial and enviromental life.

1. Introduction

Demands for rain parameter relations for an instant of rainfall are recently raised up in not only meteorology, but civil engineering, telecommunication technology, and agricultural engineering. Instantaneous rain parameter relations are derived from the instantaneous shape of raindrop size distribution. Many studies have demonstrated that the instantaneous raindrop size distributions (accumulated during 1 min or less) are usually different from the exponential distribution as shown by Marshall and Palmer[1] generally in the direction of monodispersity. In Japan, Shiotsuki[2] found the flat shape in the convective rainfall in Kyushu and Fujiwara et al.[3] found the trapezoidal shape from the double layer structure of raincloud in Owase area, as the instantaneous shape of raindrop size distribution. Anyway, those instantaneous shapes have the narrow spectrum of liquid water content with drop size, and can be expressed by the distribution equation proposed by Shiotsuki[4]. Recently, Shiotsuki[5] showed that the maximum diameter of raindrops in

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the rainfall is one of the important rain parameters. Especially in the case of instantaneous rainfall, the maximum drop has a big effect on the rain parameter relations because M (liquid water content), R (rainfall intensity) and Z (radar reflectivity) of rainfall are much contributed by the portion due to the maximum drop.

In this report, the instantaneous size distribution are determined by the instantaneous *Dmax-R* relation from the drop sampling data of the summer convective rainfalls in Kyushu, and then the instantaneous rain parameter relations are derived from those instantaneous drop size distributions.

2. Determination of the instantaneous drop size distribution

Fig. 1 shows the *R-Dmax* plots obtained in the summer convective rainfall at Hitoyoshi (1969, 1970) and at Tsuetate area in Kyushu. The drop data are based on the instantaneous sampling (1~3 sec) by use of water blue paper. Hitoyoshi 1969 rainfall was associated with the passing of cold frontal thunderstorm[2]. Hitoyoshi 1970 and Tsuetate rainfalls were associated with the typical Baiu front. Echo top heights of their clouds were more than 10 km in case of former rainfall and about 7 km in the latter, respectively. As seen in Fig. 1, the maximum values of R in correspond to each *Dmax* are considered to change due to the rainfall type. But, in here, we will set the *R-Dmax* relation from Fig. 1 as the representative one for the summer convective rainfall in Kyushu. As shown in Fig. 1, we get two *R-Dmax* lines. One is $R=0.277\ Dmax^{3.986}$ which corresponds to the regression line of all *R-Dmax* plots, and the other is $R=2.80\ Dmax^{3.021}$ which corresponds to the envelope of maximum R against each *Dmax*. The latter shows the maximum *R-Dmax* relation which means the highest rainfall efficiency in intensity and may be useful to the prediction of the instantaneous maximum rainfall intensity. Hereafter, we call the former the "average" instantaneous *R-Dmax* relation and the latter the "maximum" instantaneous *R-Dmax* relation, respectively. Moreover, other terms on the instantaneous size distribution and rain parameters which are derived from those two *R-Dmax* relations are named "average" and "maximum", respectively.

When we assume that the instantaneous shapes of drop size distribution in the present rainfalls are expressed by the normal distribution of liquid water content with size (see Appendix), we can determine the k value which means the width of drop spectrum, and then the instantaneous drop size distribution by the following procedure. As described in the previous paper[6], the rain

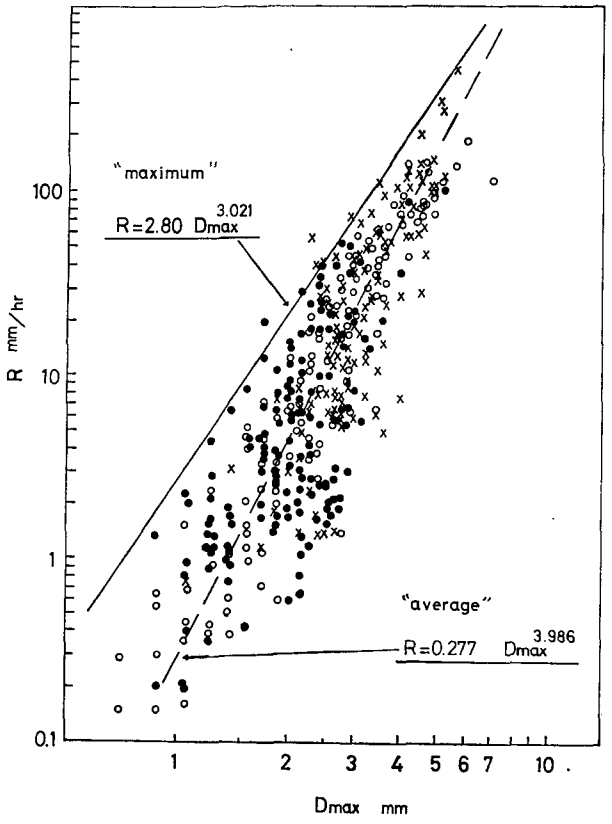

Fig. 1. R-D_{max} relation observed instantaneously in the convective rainfalls in Kyushu. o: Hitoyoshi (1969), ●: Hitoyoshi (1970), and ×: Tsuetate.

parameters R and Z are given by the following equations from the drop size equation (Eq. A. 1)

$$R = 16.0M\sqrt{\overline{D}}\left(1-\frac{k^2}{8}\right) \tag{1}$$

$$Z = 1.91\times 10^3 M\overline{D}^3(1+3k^2) \tag{2}$$

When using the above obtained *R-Dmax* relation and $\overline{D}$-*Dmax* relation (Eq. A.2) in Eq. 1, we obtain *M-Dmax* relations having the parameter k, and then *Z-Dmax* relations in Eq. 2. *M-Dmax* and *Z-Dmax* relations change according to the k value. Table 1 shows the calculation results of the relations *M-Dmax* and *Z-Dmax*, giving the observed *R-Dmax* and the k values. We can determine

Table 1.

	R-Dmax	k	M-Dmax	Z-Dmax
"average"	3.986 R=0.277 Dmax	0.1	3.462 M=0.0182 D_{max}	6.610 Z=27.0 D_{max}
		0.2	3.452 M=0.0215 D_{max}	6.659 Z=13.2 D_{max}
		0.25	3.449 M=0.0227 D_{max}	6.676 Z=10.8 D_{max}
		0.3	3.446 M=0.0238 D_{max}	6.689 Z=9.35 D_{max}
"maximum"	3.021 R=2.80 Dmax	0.1	2.497 M=0.1842 D_{max}	5.645 Z= 273 D_{max}
		0.2	2.487 M=0.2173 D_{max}	5.694 Z= 133 D_{max}
		0.25	2.484 M=0.2296 D_{max}	5.711 Z= 109 D_{max}
		0.3	2.481 M=0.2403 D_{max}	5.724 Z=94.4 D_{max}

the k value in the figures of *M-Dmax* plots and *Z-Dmax* plots obtained in the same present rainfall, by finding which *M-Dmax* line and *Z-Dmax* line with k are best fitted to the plots.

Fig. 2 and Fig. 3 show the *M-Dmax* plots and *Z-Dmax* plots in the present rainfalls, where M and Z are calculated directly by the drop data. The data sources are quite samely as for Fig. 1. As seen in the figures, both of the "maximum" and "average" relations for *M-Dmax* and *Z-Dmax* are well fitted by the $k=0.2$ or $k=0.3$ line. Thus, we find the k value in the size distribution equation as about $k=0.25$.

3. Instantaneous *M-R* and *Z-R* relations

As the k value was determined in the preceding section, we can derive the representative rain parameter relations of *M-R* and *Z-R* in Eq. (1) and (2), using the relations in case of $k=0.25$ as shown in Table 1. The obtained results are as follows.

$$M\text{-}R \text{ relation} \begin{cases} \text{"average"} & M=0.0689R^{0.865} \\ \text{"maximum"} & M=0.0985R^{0.822} \end{cases}$$

$$Z\text{-}R \text{ relation} \begin{cases} \text{"average"} & Z=92.6R^{1.675} \\ \text{"maximum"} & Z=15.6R^{1.89} \end{cases}$$

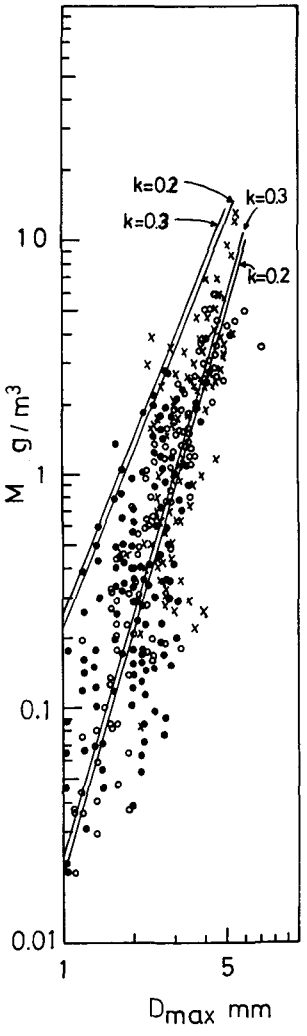

Fig. 2 M-D_{max} relation.

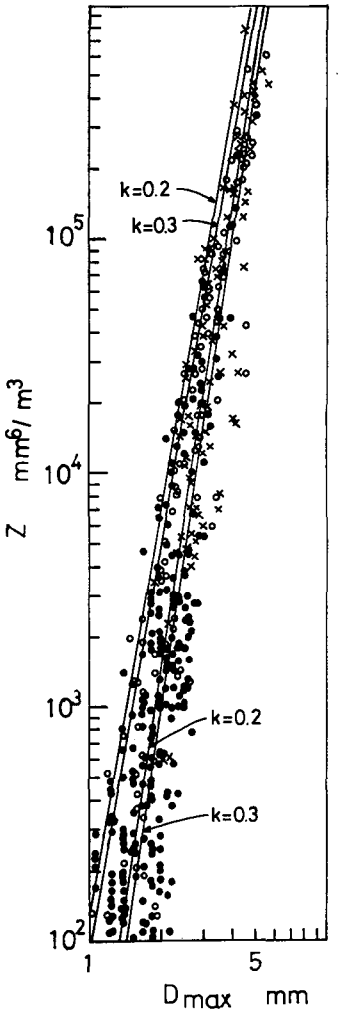

Fig. 3 Z-D_{max} relation.

Figs. 4 and 5 show the comparisons between above obtained instantaneous relations, and the relation plots obtained directly or another some representative relations. As seen in Fig. 4, the "average" instantaneous M-R relation $M=0.0689R^{0.865}$ is quite similar to $M=0.0655R^{0.867}$ which was derived from the observation results of various rainfalls, and was used as the representative M-R relation in the previous paper[5], and also similar to $M=0.058R^{0.909}$ which was

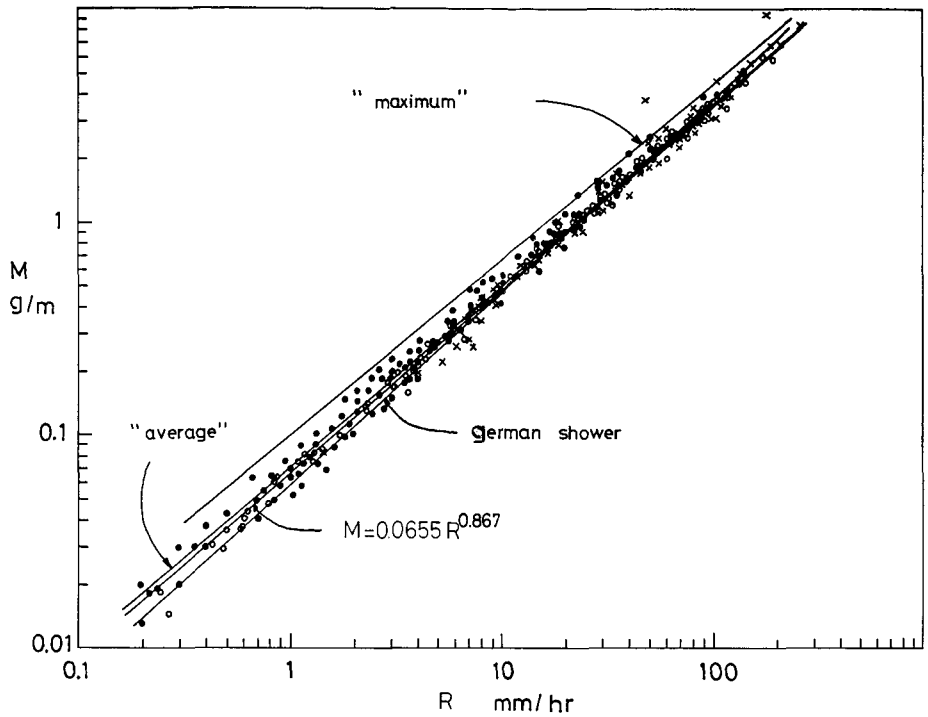

Fig. 4 M-R relation.

obtained in German shower by use of raindrop spectrometer on the time base of 5 sec (Kreuels[7]). Furthermore, the "maximum" instantaneous relation of $M=0.0985R^{0.822}$ fits well to the upper envelope of M-R plots.

On the other hand, as seen in Fig. 5, the "average" instantaneous relation $Z=92.6R^{1.675}$ is similar to the relation in case of shower ($Z=300R^{1.37}$) obtained by Fujiwara[8]. Also the above relation represents well the feature of Z-R plots. The "maximum" instantaneous relation $Z=15.6R^{1.89}$ fits well to the lower envelope of Z-R plots, while the thunderstorm relation $Z=450R^{1.46}$ by Fujiwara[8] fits well to the upper envelope of Z-R plots.

Thus, we can find the "average" instantaneous relations of M-R and Z-R well fit to those original plots and to the representative relations obtained some workers. The more liquid water content in the "maximum" instantaneous M-R relation needs to reach the same rainfall intensity in the "average", while the lower radar reflectivity in the "maximum" instantaneous Z-R

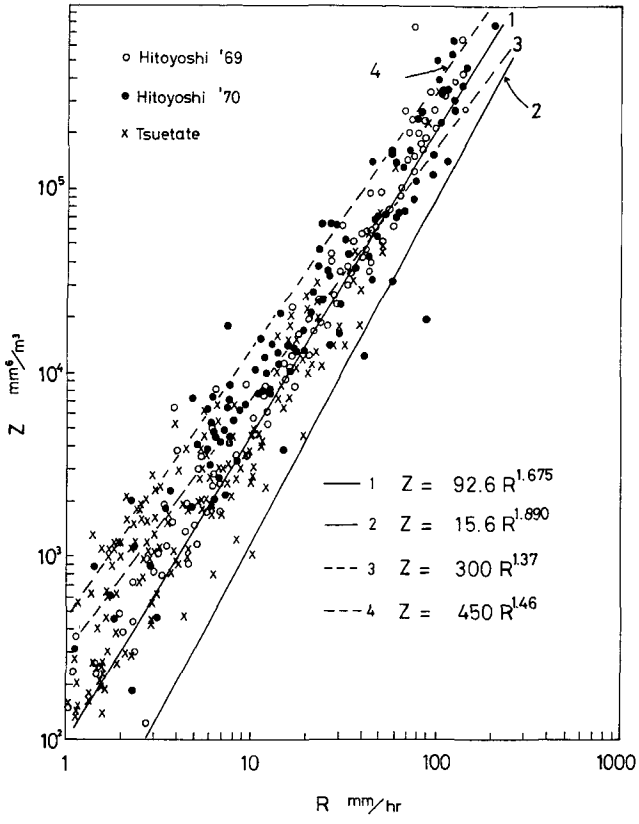

Fig. 5 Z-R relation.

relation needs to reach the same rainfall intensity in the "average". Especially, we note the latter case, because it means that the heavy rainfall is possible to occur even if the radar reflectivity is weaker than expected.

4. Concluding remarks

As described in the preceding sections, $k=0.25$ drop size distributions based on the normal distribution of liquid water content are considered as for the instantaneous rainfall. This coincides well with the results of size distribution estimated in case of the heavy rainfall at Tsuetate in Kyushu[6]. Fig. 6 shows the family of $k=0.25$ drop size distribution in N_D curve when the liquid water content is fixed to 1 g/m^3 and the various maximum drop dia-

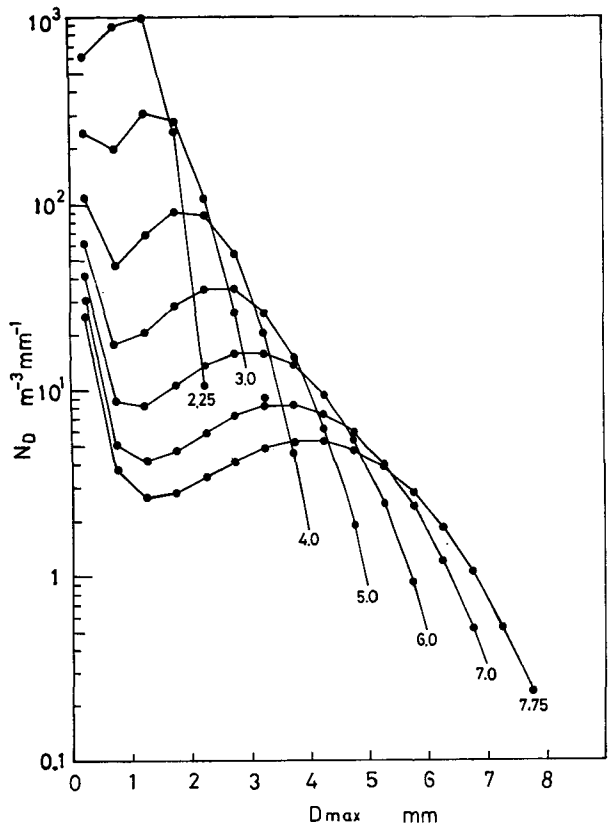

Fig. 6 Family of $k=0.25$ drop-size distributions giving $M=1$ g/m³ and each D_{max} value in Eqs. A1 and A2.

meters are given. As seen in the figure, the flat part of distribution becomes wide according to the increment of *Dmax* size. This coincides well with the results of instantaneous flat drop size distributions observed in the convective rainfall[2].

The rain parameter relations such as *M-R* and *Z-R* were derived from the instantaneous drop size distribution. The "average" instantaneous relations represent well the original plots of each relation, and the "maximum" instantaneous relations fit well to each envelope of the original plots. The latter "maximum" instantaneous relations and size distributions may be useful to the prediction of the instantaneous rain parameters, such as instantaneous rainfall intensity, visibility, microwave attenuation, and so on,

related to our industial and environmental life.

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Appendix

The equation of raindrop size distribution that was proposed in the previous paper[4] is

$$N_D = 10^3 \frac{6M}{\rho\pi} D^{-3} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{(D-\bar{D})^2}{2\sigma^2}\right\} \tag{A.1}$$

where N_D: number density of drops, $m^{-3}\ mm^{-1}$
M : liquid water content of drops, g/m^3
ρ : density of water, g/cm^3
D : diameter of drop, mm
$\bar{D}$: mean diameter of drops, mm
σ : standard deviation from $\bar{D}$, mm

When we set the density function

$$F(D) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{(D-\bar{D})^2}{2\sigma^2}\right\}$$

and consider that D becomes large, the distribution function $F(D)$ is shown from the theory of mathematical statistics by

$$1-F(D) = \frac{1}{\sqrt{2\pi}\,D} \exp\left\{-\frac{(D-\bar{D})^2}{2\sigma^2}\right\}$$

Giving D_{max} in the above equation and setting $F(D_{max})=0.99$ (99%), we obtain

$$\sigma = 0.7071 \times \frac{D_{max}-\bar{D}}{\sqrt{3.6862-\ln D_{max}}}$$

Using $k=\sigma/\bar{D}$

$$\bar{D} = D_{max}/(1+1.414\,k\sqrt{3.6862-\ln D_{max}})$$
$$\fallingdotseq 0.3113 k^{-0.466} D_{max}{}^{1.117k^{0.0272}} \tag{A.2}$$

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