[Journal of the Faculty of Science, Hokkaido University, Ser. VII (Geophysics), Vol. VI, No. 1, 1979]

# Size Distribution of Snowflakes and Graupel Particles Observed in Nagaoka, Niigata Prefecture

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(Received Oct. 9, 1979)

**Abstract**

Observations of the size distribution for snowflakes and graupel particles were made in a prefecture facing the Sea of Japan in two winters. Different size distributions were obtained with respect to different meteorological situations. In snowfalls related to a strong winter monsoon the existence of numerous small particles was very remarkable even for high precipitation rates, and this was in contrast to the size distributions for extratropical cyclone. Based on differences in the size distributions, the precipitation rate $R$ in heavy snowfalls under winter monsoon conditions was larger than that for snowfalls in cyclonic conditions under the same radar reflectivity factor $Z$.

## 1. Introduction

Weather radar would be a powerful means to obtain information on the snowfall area and rate on a real time basis. Such information from radar is anticipated to bring improvements to the snow removal system of highway networks in the snowy districts. However it should be noted that there are several difficulties to be overcome before practical use of radar in such local detections of snowfall can be realized. One point among them is ambiguity in the relationship of the radar reflectivity factor $Z$ to precipitation rate $R$ in the case of snow, which is necessary for the measurement of the snowfall intensity by means of radar.

For rain, a great number of measurements of the $Z$-$R$ relationship have been reported[1], and compiled with regard to the rainfall types and radar echo characteristics[2]. On the other hand, only a few reprots on the $Z$-$R$ relationships for snow are available[3)-8]. These observational reports may suggest that the $Z$-$R$ relationship for snow is beyond comprehension with respect to weather conditions. This is partly due to the fact that the scattered relationships were observed with different configurations, densities, water contents and other

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factors of snow particles. In other words, the $Z$-$R$ relationships experience a locality and/or difference in weather conditions. It might be stated, therefore, that the $Z$-$R$ relationship for snow should be operationally treated with restricted weather conditions in a specific region covered by a radar for an observation of snowfall.

The direct purpose of the present research project is to obtain the $Z$-$R$ relationships for snow in the region of Niigata under some weather conditions. The observations have been made in 1978 and 1979. In the former year heavy snowfalls were predominant under the winter monsoon, whereas in the next year, snowfalls were mainly brought on by cyclones. This preliminary paper reports the different characteristics of size distribution of snow particles observed in the two years and the ensuing discrepancy in the $Z$-$R$ relationships between both years.

## 2. Methods of observation and analysis

For calculation of the $Z$-$R$ relationship of snow particles, a space size distribution must be known in terms of the diameter of the water drop formed on melting of the snow particles, assuming any break-up does not occur. In order to provide this, the filter paper method was introduced as follows: A horizontal sheet of brushed velvet (26 cm$\times$36 cm) was exposed to snowfall on the ground. After exposure, the velvet was taken inside a shelter, then the snow particles on the velvet were photographed and melted into water drops in a heated and humidified box. The melted drop did not wet the velvet. A filter paper dusted with water-blue dye was laid on the velvet and the drops were absorbed.

From the previous calibration of the filter paper, the diameter $D$ of the drop to which the snow particles melted could be readily measured. If a large snowflake melts into several drops, all spots arising from the original snowflakes were totalized with the aid of the photograph of the snowflake and the diameter of the snowflake was taken as the diameter of the circular area according to the total number and area of spots. The diameters of melted snow particles were measured every 0.2 mm. Thus the horizontal distribution was obtained. The conversion of the horizontal distribution to the corresponding distribution in space $N_D$ were made by using the measurements of the terminal speed $V$ of snowflakes[9] and graupel particles[10].

The radar reflectivity factor $Z$ ($\mathrm{mm}^6\,\mathrm{m}^{-3}$) is given by

$$Z = \int_0^{D_{\max}} N_D D^6 \, dD \,, \tag{1}$$

and the precipitation rate $R$ (mm hr$^{-1}$) by

$$R = \frac{\pi}{6} \int_0^{D_{\max}} V N_D D^3 \, dD \,, \tag{2}$$

where $D_{\max}$ is the maximum diameter of the melted snow particles. Using $Z$ and $R$ defined above, the $Z$-$R$ relationship was determined empirically as

$$Z = BR^{\beta} \,, \tag{3}$$

where $B$ and $\beta$ are constants and depend on precipitation types.

## 3. Results

The observation was carried out in Nagaoka on 25 January through 1 February in 1978 and 27 January through 4 February in 1979. The number of effective data obtained was 106 in 1978 and 58 in 1979. The data for each year were analyzed with respect to snowflakes and graupel particles.

### *3.1 Size distribution*

Generally, the size distribution for snow particles fits well to an exponential of the form

$$N_D = N_0 \exp(-\Lambda D) \,, \tag{4}$$

as found for rain by Marshall and Palmer[11]. Examples of the size distribution for snowflakes and graupel particles are shown in Figs. 1 and 2, respectively. The distribution for snowflakes under a precipitation rate between 0.6 mm hr$^{-1}$ and 1.0 mm hr$^{-1}$ was expressed by $N_D = 4.1 \times 10^3 \exp(-2.9D)$. In the case of graupel particles for a precipitation rate between 0.5 mm hr$^{-1}$ and 1.0 mm hr$^{-1}$ corresponding equation was $N_D = 1.7 \times 10^3 \exp(-2.6D)$.

To specify the characteristics of size distributions observed, the particular value of $N_0$ and $\Lambda$ for every value of $R$ was taken into account. So all data for the types of snow particles obtained in an individual year were classified into several groups for precipitation rate, and values of $N_0$ and $\Lambda$ were computed for each class which had an averaged precipitation rate. Then the dependences of $N_0$ and $\Lambda$ upon $R$ were obtained. Attempts for such averaging were first made by Gunn and Marshall[4] to reveal a systematic change of size

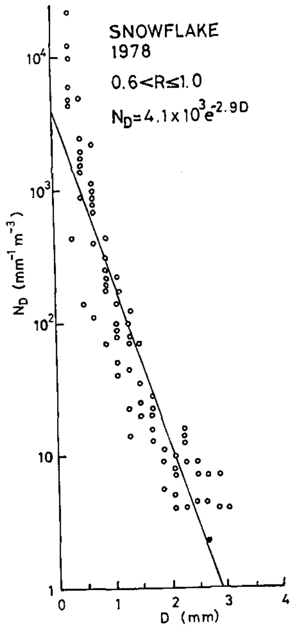


Fig. 1 Size distribution for snowflakes observed in 1978, at precipitation rates between 0.6 mm $hr^{-1}$ and 1.0 mm $hr^{-1}$.

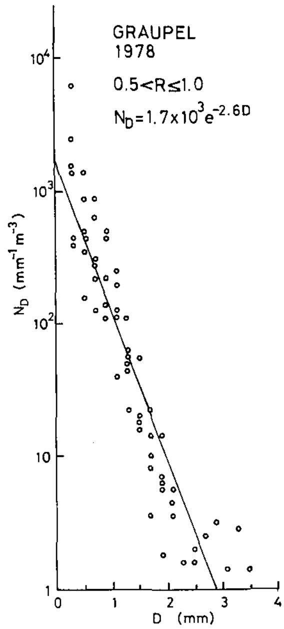


Fig. 2 Size distribution for graupel particles observed in 1978, at precipitation rates between 0.5 mm $hr^{-1}$ and 1.0 mm $hr^{-1}$.

distribution with rate of snowfall for several samples of snow obtained on different days.

*Results in 1978.* The number of data for snowflakes was 68 in 1978, and they were classified into 8 categories depending upon precipitation rate. Fig. 3 shows plots of $N_0$ and $\Lambda$ against the average precipitation rate. The solid line is a regression curve for eight points. The $N_0$ value increased ($N_0=3.7\times10^3R^{0.57}$) as the precipitation rate increased. The $\Lambda$ value decreased ($\Lambda=2.6R^{-0.10}$) as the precipitation rate increased. Namely, the characteristics for 1978 snowflakes were that the distribution became broader and the number of particles increased on the whole as the precipitation rate increased.

In the case of the 1978 graupel particles with 38 data and 7 categories of precipitation rate, as seen in Fig. 4, the $N_0$ value increased ($N_0=3.6\times10^3R^{0.98}$) and the $\Lambda$ value decreased only a little ($\Lambda=2.9R^{-0.002}$) as the precipitation rate increased. This resulted in almost no change in the pattern of distribution

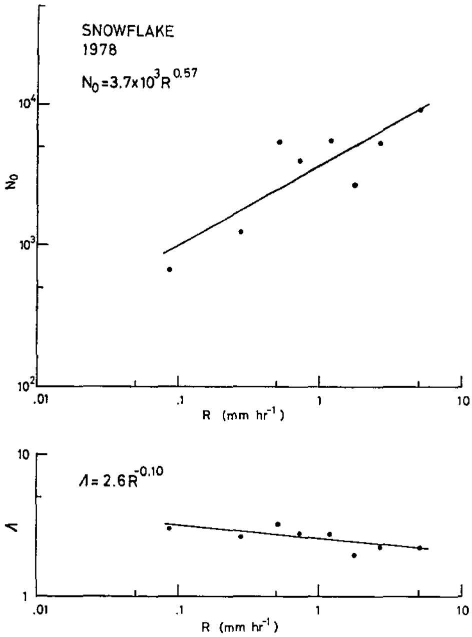


Fig. 3 Size distributions for snowflakes observed in 1978, in terms of $N_0$ and $\Lambda$ versus precipitation rate $R$ (for equation $N_D=N_0\exp(-\Lambda D)$).

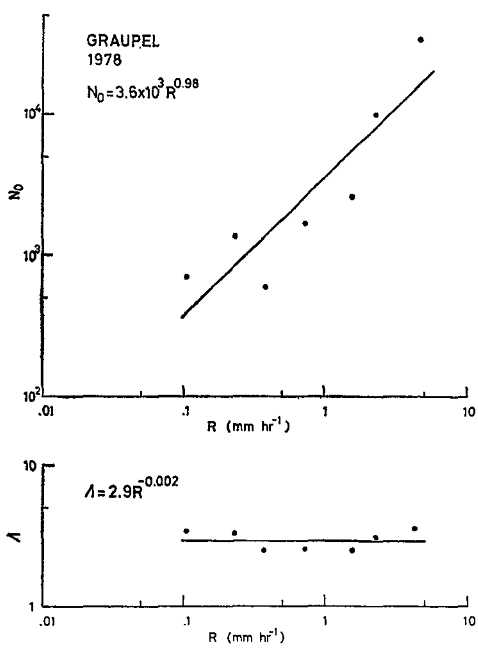


Fig. 4 Same as Fig. 3, but for graupel particles in 1978.

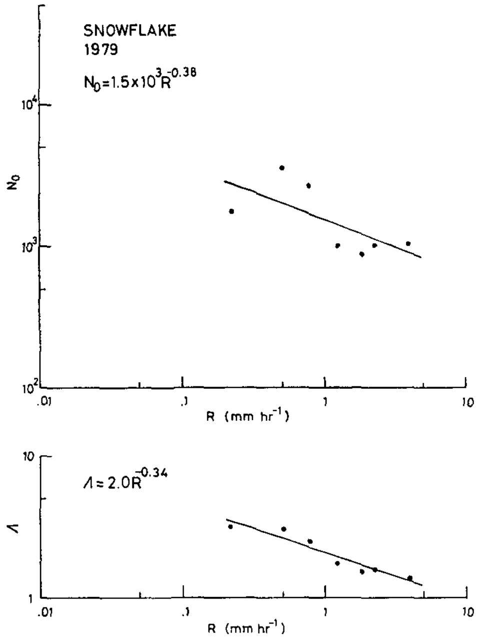


Fig. 5 Same as Fig. 3, but for snowflakes in 1979.

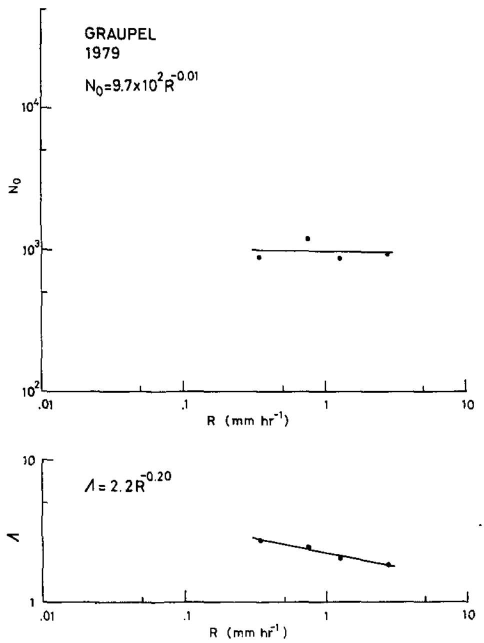


Fig. 6 Same as Fig. 3, but for graupel particles in 1979.

and the total increased number of particles is attributed to the increase of the precipitation rate.

*Results in 1979.* In the case of snowflakes which had 36 data and 7 categories of precipitation rate, a remarkable decrease of the $N_0$ value with increasing $R$ value is demonstrated in Fig. 5, where $N_0=1.5\times 10^3 R^{-0.38}$ and $\Lambda=2.0R^{-0.34}$. The snowflake characteristics for 1979 were in contrast to that of 1978, namely, the number of smaller particles decreased with increasing precipitation rate and the distribution became much broader due to a notable increase of larger snowflakes also with increasing precipitation rate.

In the case of the graupel particles for 1979, 22 data and 4 categories of precipitation rate are illustrated in Fig. 6 where $N_0=9.7\times 10^2 R^{-0.01}$ and $\Lambda=2.2 R^{-0.20}$. There were almost no change in $N_0$ and a decrease in $\Lambda$, which were also in contrast to the 1978 graupel case. Namely, the increase of precipitation rate was due to the increase of the larger graupel particles in the case of 1979.

### *3.2 Z-R relationship*

Using equation 1 and 2, the radar reflectivity factor $Z$ and the precipitation rate $R$ were reduced from the size distribution data obtained. Then the relationships between $Z$ and $R$ were computed in the form of equation 3 as follows:

*Results in 1978.* The relationship between $Z$ and $R$ for snowflakes in 1978 is shown in Fig. 7. The solid line in the figure is a regression curve for 68 plots and gave $Z=1200R^{1.3}$. The range of precipitation rate in this case was 0.013 mm hr$^{-1}$ through 11 mm hr$^{-1}$.

The case for graupel particles is shown in Fig. 8. The regression curve for 38 plots was $Z=650R^{1.1}$ and the range of precipitation rate varied from 0.058 mm hr$^{-1}$ through 8.1 mm hr$^{-1}$.

*Results in 1979.* The relationship of $Z$ to $R$ for snowflakes is shown in Fig. 9. The regression curve for 36 plots was $Z=2600R^{1.8}$ and the precipitation rate ranged from 0.031 mm hr$^{-1}$ through 4.3 mm hr$^{-1}$. This result indicates greater $B$ and $\beta$ values in 1979 compared to those for 1978.

Fig. 10 shows the case for the graupel particles. The $Z$-$R$ relationship was $Z=960R^{1.6}$ with a range of 0.13 mm hr$^{-1}$ through 3.7 mm hr$^{-1}$ in precipitation rate among 22 plots. Also the greater $B$ and $\beta$ values in this case is significant when compared to the result for the 1978 graupel event.

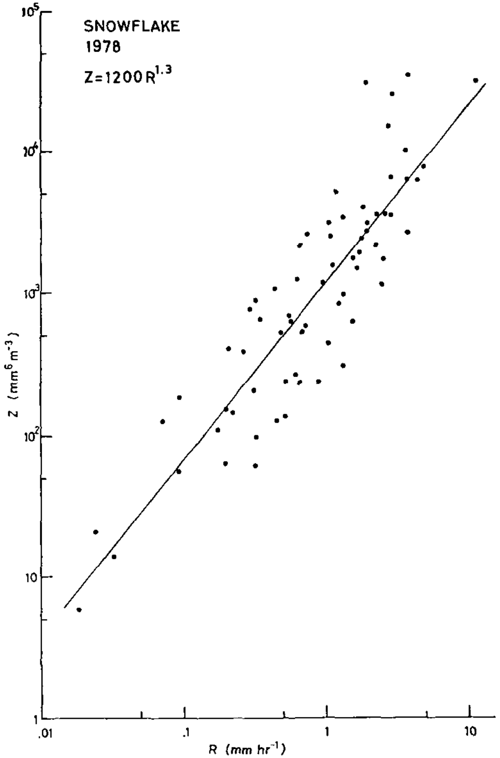


Fig. 7 *Z*-*R* relationship for snowflakes observed in 1978.

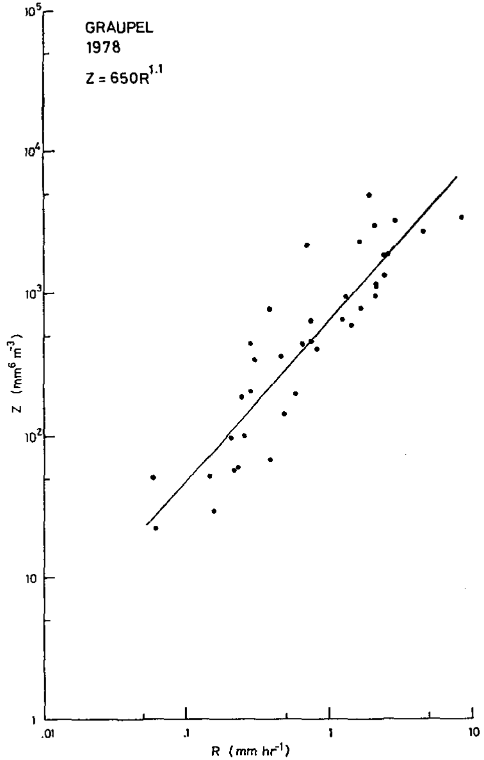


Fig. 8 Same as Fig. 7, but for graupel particles in 1978.

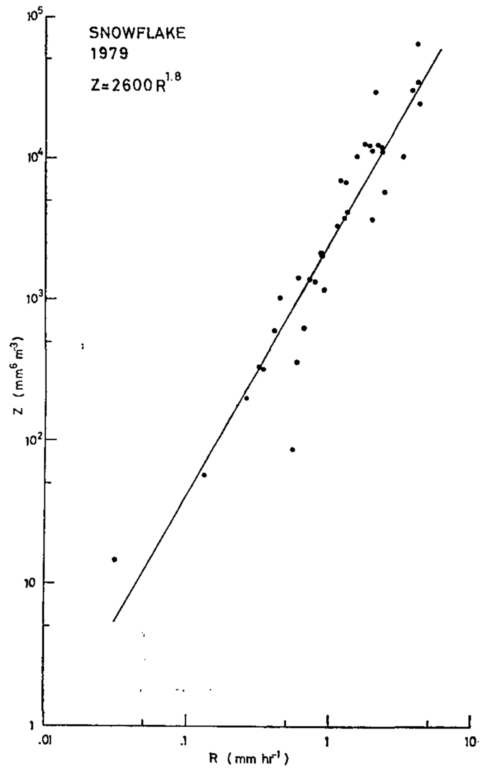


Fig. 9 Same as Fig. 7, but for snowflakes in 1979.

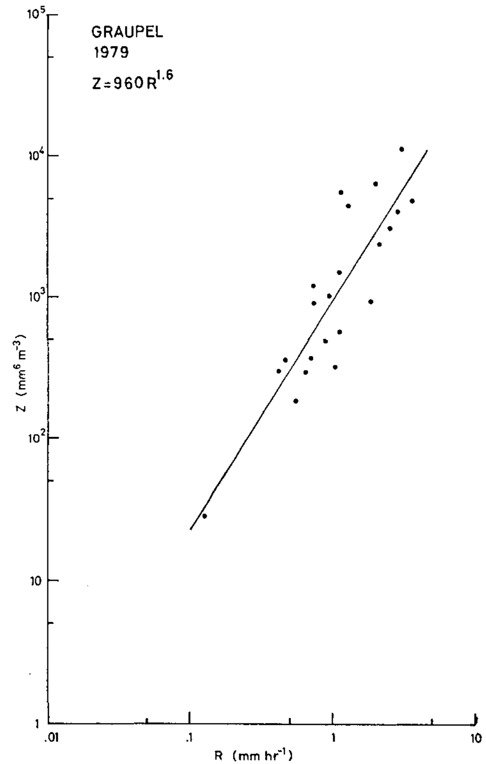


Fig. 10 Same as Fig. 7, but for graupel particles in 1979.

## 4. Discussion

All results described in the former section are outlined in Table 1, including the water content $M$ (g m$^{-3}$) and the median volume diameter $D_0$ (mm) which were reduced for snowflakes and graupel particles in each year. In this section, a locality and/or difference between meteorological conditions of snowfall is discussed with respect to the size distribution and $Z$-$R$ relationship.

Table 1 Precipitation factors for snowflakes and graupel particles observed in 1978 and 1979; precipitation rate $R$ in mm hr$^{-1}$, $B$ and $\beta$ values in $Z$-$R$ relationship $Z=BR^{\beta}$, $N_0$ and $\Lambda$ values in size distribution $N_D=N_0\exp(-\Lambda D)$, water content $M$ in g m$^{-3}$ and median volume diameter $D_0$ in mm.

| year | data | R | B | $\beta$ | $N_0$ | $\Lambda$ | M | $D_0$ |
|---|---|---|---|---|---|---|---|---|
| Snowflakes | | | | | | | | |
| 1978 | 68 | 0.013-11 | 1200 | 1.3 | $3.7\times10^3R^{0.57}$ | $2.6R^{-0.10}$ | $0.23R^{0.95}$ | $0.9R^{0.11}$ |
| 1979 | 36 | 0.031-4.3 | 2600 | 1.8 | $1.5\times10^3R^{-0.38}$ | $2.0R^{-0.34}$ | $0.20R^{0.82}$ | $1.3R^{0.41}$ |
| Graupel particles | | | | | | | | |
| 1978 | 38 | 0.058-8.1 | 650 | 1.1 | $3.6\times10^3R^{0.98}$ | $2.9R^{-0.002}$ | $0.15R^{0.89}$ | $1.0R^{0.02}$ |
| 1979 | 22 | 0.13-3.7 | 960 | 1.6 | $9.7\times10^2R^{-0.01}$ | $2.2R^{-0.20}$ | $0.11R^{0.84}$ | $1.3R^{0.24}$ |

First, in the case of snowflakes, the observed results show that the $Z$-$R$ relationship in 1978 was $Z=1200R^{1.3}$ and that in 1979 $Z=2600R^{1.8}$ A discrepancy in the relationship between the results for both years could be attributed to a difference in the size distributions, as a result of the meteorological situation peculiar to the observation period of each year. The data in 1978 was mainly obtained under the so-called heavy snowfalls during a strong winter monsoon. On the other hand the majority of the data obtained in 1979 was for snowfalls related to extratropical cyclones which passed over the observation point. In the 1979 winter the strong monsoon as usual did not occur; it was a warm winter in 1979. This difference in weather conditions between both winters would appear in the form of size distribution as follows:

For 1979 snowflakes where cyclonic conditions prevailed, the value of $N_0$ in equation 4 decreased with increasing precipitation rate and the value of $\Lambda$ also showed a distinct decrease. An explanation for such a trend could be that the number of small snowflakes or snow crystals decreased relative to that of the larger snowflakes which increased due to processes of accretion and coagulation as the precipitation rate increased. These characteristics of size distribution were the same as the results of Gunn and Marshall[4] obtained from observations for snowflakes from stratiform clouds. And such similarity in size distribution would result in a relatively good coincidence in the $Z$-$R$ relationship, that is, $Z=2600R^{1.8}$ by the present authors and $Z=2000R^{2.0}$ by Gunn and Marshall.

For the snowflakes in 1978 formed under strong monsoon conditions, the $N_0$ value showed a remarkable increase and the $\Lambda$ value gently decreased with increasing precipitation rate. This may indicate that the number of larger snowflakes increased, but still smaller particles existed and grew abundant in number as the precipitation rate increased. Namely, a very distinctive feature in size distribution for snowflakes in 1978 was that numerous small particles fell with larger-size snowflakes at the higher precipitation rate. Due to the existence of numerous small particles in 1978 the median volume diameter in 1978 was small in comparison with that in 1979, and also the water content in 1978 was large in comparison with that in 1979 because of a lower fall velocity of small particles. These features would be reflected in the 1978 $Z$-$R$ relationship, $Z=1200R^{1.3}$, i.e. smaller $B$ and $\beta$ values than those in 1979. It is also considered that the features in size distribution observed under heavy snowfalls was related to a constant supply of water vapour from the warm Sea of Japan to the cold atmosphere during the strong winter mon soon season.

Next, in the case of graupel particles, the observed results show that $Z=650R^{1.1}$ in 1978 and $Z=960R^{1.6}$ in 1979. The characteristics in size distribution were an increase of $N_0$ and constancy of $\Lambda$ in 1978, and the constancy of $N_0$ and a decrease of $\Lambda$ in 1979, respectively with increasing precipitation rate. In 1978, numerous powder snow particles fell with graupel particles, then such size distributions as mentioned above were resulted. So the $B$ and $\beta$ values in 1978 were smaller than those in 1979, similar to the case of snowflakes. This follows that the precipitation rate $R$ in heavy snowfalls under winter monsoon conditions is larger than that in snowfalls related to a cyclone under the same radar reflectivity factor $Z$.

Finally, a comparison between the present results and other observers' for the $Z$-$R$ relationship is attempted. Harimaya[8] drew two domains for snowflakes and graupel particles on the $B$-$\beta$ plane, based on his measurements and those of several workers[3),4),5),6)], as seen in Fig. 11. In this figure the $Z$-$R$ relationship for snowflakes is indicated by the borken line and those of graupel particles by the solid line. As the present authors' results are superimposed on the same figure, then it can be said that they are included in respective domains. This is good in a way, because Harimaya's compilation was designed to include, possibly, the observational results for various conditions of weather. However, it must be noted that the observed data available now are still insufficient to classify the $Z$-$R$ relationship with respect to different meteorological situations of snowy bad weather. This point is left to be settled in future.

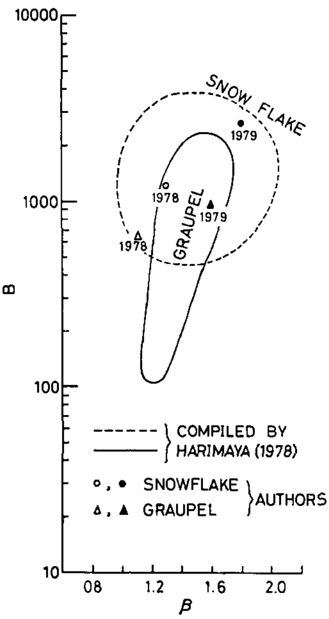


Fig. 11 The domains of $B$ and $\beta$ values in the equation $Z=BR^{\beta}$ for snowflakes and graupel particles compiled by Harimaya[8] and the present authors' results superimposed on the same $B$-$\beta$ plane.

## 5. Conclusion

Observation of size distribution for snowflakes and graupel particles were made in Nagaoka, Niigata Prefecture in January to February, in 1978 and 1979. The data in 1978 were obtained mainly in heavy snowfalls when winter monsoon conditions were prevalent, whereas those in 1979 were obtained under extratropical-cyclonic conditions. Such a difference in meteorological situations gave discrepancy between the size distributions of snowflakes and graupel particles for two years.

In the case of snowflakes, in 1978 the size distribution grew slightly broader with a distinct increase of smaller particles as the precipitation rate increased, and in 1979 the size distribution became remarkably broad with a distinct decrease in the smaller particles as the precipitation rate increased.

In the case of graupel particles, the significant feature in size distributions was the same as in the case of snowflakes; the existence of numerous smaller particles was very remarkable in 1978 under the higher precipitation rate.

Based on such differences in size distribution, it can be stated that the precipitation rate $R$ in heavy snowfalls under winter monsoon conditions are larger than that in snowfalls related to a cyclone, under same radar reflectivity factor $Z$.

*Acknowledgements*: The authors wish to express their hearty thanks to Dr. M. Kajikawa, Akita University, Mr. T. Yoshida, Science and Technology Agency and Dr. T. Harimaya, Hokkaido University for their useful suggestions regarding the planning of the observations, and to Mr. T. Yonetani, National Research Center for Disaster Prevention for his help in the 1978 observation. The authors also appreciate the asssistance afforded by the Institute of Snow and Ice Studies during observations.

This work was partially supported by the Special Fund for the Promoting of Multiministerial Research Project under the jurisdiction of the Science and Technology Agency, entitled," Studies on watch-system of snow coverage for snow removal of highway networks and precipitation process in heavy snowfalls".

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