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A Determination of Entrainment Rate and Drag Coefficient of Cumulus Clouds

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Abstract

An equation which shows difference between horizontal speed of cumulus cloud and ambient wind speed is introduced. It includes entrainment rate and drag coefficient. Since the difference between both speeds was observed, the rate and coefficient are determined.

As a result, if the rising air parcel takes a spherical form, the entrainment rate becomes approximately 0.54 which is almost the same as values which have been previously obtained. On the other hand, it seems that the drag force is too small for cumulus cloud to move horizontally.

1. Introduction

As a cumulus cloud grows through a cool, dry environment, an amount of the outside air is entrained into the rising cloud. The entraining which is usually called as entrainment dilutes the buoyancy and the liquid water by addition of cool, dry air. The entrainment rate is expressed by α/R where R is the radius of the cloud. The entrainment coefficient: α has been obtained by laboratory experiments (Scorer, 1957; Turner, 1963). The inverse radius dependence on the rate has been tested by aircraft observations of cumulus clouds, on the basis of the buoyancy and the liquid water content (Sloss, 1967; McCarthy, 1974). The rate can be obtained by the conservation of momentum flux in cumulus clouds, likewise. Malkus (1952) discussed the slope of cumulus clouds in relation to external wind shear and the entrainment rate which was obtained by the conservation of momentum flux. The difference in horizontal speed between the cloud and its environment was calculated by Austin and Houze (1973) from the conservation of horizontal momentum and the drag force exerted by the environment on the air in the updraft. They set the drag coefficient equal to 1. Kotsovinos (1978) examined the conservation law for the flux of axial momentum in a turbulent jet and noted that the pressure

field generated in the ambient fluid reduced the axial momentum flux. However the understanding of the drag force induced by the difference of horizontal speeds between cloud and environment is lacking.

Recently a wind field has been determined by the movement of clouds observed from Weather Satellites. Then the difference between cloud and wind speeds must be known. It was discussed by Fujita and Pearl (1975) and Hasler et al. (1977).

In this paper, a simple equation which expresses a horizontal speed of cloud is introduced. It contains entrainment rate and drag coefficient. An entrainment rate and a drag coefficient will be determined from the difference between cloud and wind speeds which were observed by Chiyu et al. (1973). Data used in this paper are quoted from Chiyu et al. (1973).

2. A horizontal speed of cumulus clouds under the influence of entrainment and drag force

A parcel which is warmer than the neighborhood ascends to make a cloud from the ground. The ascending parcel is horizontally carried away by the ambient wind. It is considered that a horizontal movement of the parcel is affected by mixing of surrounding air and the drag force due to the difference between parcel and wind speeds. We considered that a cloud is a visible form of the parcel.

It is assumed that a parcel has a mass: M and a horizontal speed: u_c at a height. After the parcel with the upward speed: w has ascended to an amount of dz , the mass increases to $M+dM$ and the horizontal speed u_c+du_c . If the ambient wind speed: u_e increases du_e for dz , the increase of the horizontal momentum for the parcel by the entrainment is expressed as $(u_e+du_e)dM$.

On the other hand, if the drag force due to the difference between both speeds is represented by F , the conservation of the horizontal momentum for the parcel is expressed as follows:

$$(M+dM)(u_c+du_c) = Mu_c + (u_e+du_e)dM + Fdt \quad (1)$$

where dt is a time increment for dz . Using $dt=dz/w$ and after simple arrangement, Eq. (1) becomes

$$M \frac{du_c}{dz} = (u_e - u_c) \frac{dM}{dz} + \frac{F}{w} \quad (2)$$

If the parcel takes a spherical form, the drag force is expressed by

$$F = C_d \rho (u_e - u_c)^2 \pi R^2 / 2 \quad (3)$$

Further if the density is constant, the above equation is transformed to

$$\frac{du_c}{dz} = \frac{3\Delta u}{R} \frac{dR}{dz} + \frac{3C_d \Delta u^2}{8wR} \quad (4)$$

where $u_e - u_c$ is replaced with Δu . This equation means that the horizontal speed of the parcel depends on the difference between both speeds, Δu , the radius of the parcel, R , the entrainment rate, $3dR/Rdz$, the vertical speed, w , the drag coefficient, C_d and the height, z .

3. A determination of entrainment rate and drag coefficient of a cumulus cloud

Eq. (4) is transformed into

$$\frac{R}{\Delta u^2} \frac{du_c}{dz} = \frac{3dR}{dz} \frac{1}{\Delta u} + \frac{3C_d}{8w} \quad (5)$$

to search for the value of unknown factors. If Δu , R and z are known and if u_c takes zero at the ground and linearly increases with z , we can obtain the values of dR/dz and C_d . The difference between horizontal speed of cumulus humilis cloud and ambient wind speed has been observed by Chiyu et al. (1973). The data are summarized in Table 1 of that paper. We must select the data to fit our assumptions. The following criterions are used to select them.

1. Wind speed onesidedly increases from ground height to cloud height, that is to say, a light wind does not participate in the profile of the wind speed between both heights.

2. Wind direction does not change beyond 90 degrees.

3. The difference between both speeds is larger than 1 m/s.

Suitable data are shown in Table 1. z is the ground height of a cloud base, u_c is a horizontal speed of a cloud and Δu is a difference between cloud and wind speeds. R is a radius which is calculated from the size: S shown by them as follows:

$$R = \sqrt{\frac{2S}{\pi}} \quad (6)$$

That is, it is assumed that the parcel consists of a hemispherical cumulus cloud and a hemispherical dry air mass which corresponds to the root of the cloud.

Table 1. Data used in quest of entrainment rate and drag coefficient.

date	Δu	z	u_c	R
1970				
27 June	1.6	500	2.4	160
	1.8		2.2	170
2 July	7.4	650	10.6	250
	11.2		6.8	310
7 Sept	3.7	1500	8.3	550
11	6.5	1300	7.5	620
	4.0		10.0	450
	4.7		9.3	480
	4.2		8.8	480
13	2.0	800	5.0	470
27	7.5	1400	9.5	560
	3.8		13.2	290
11 Oct	1.2	800	4.8	140
1971				
6 Sept	3.3	1000	6.7	330
	2.5	1400	7.5	190
15	2.9	900	7.1	330
	4.6		5.4	450
	3.3		6.7	310
Mean	4.4			360

Δu : difference between wind and cloud speed, z : cloud height, u_c : cloud movement speed and R : radius of cloud.

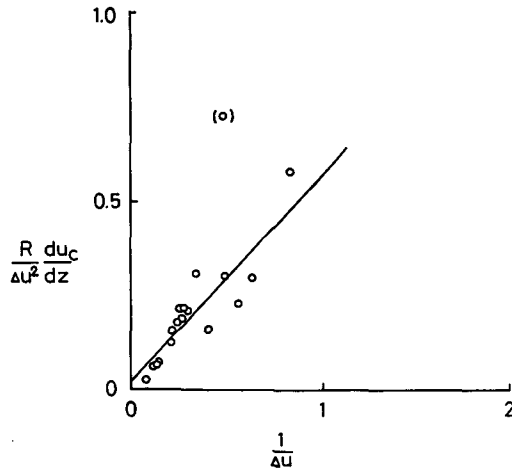


Fig. 1. A relationship between $1/\Delta u$ and $R/\Delta u^2 \cdot du_c/dz$ in Eq. (5). A point with parenthesis is excepted from statistics.

$1/\Delta u$ for $R/\Delta u^2 \cdot du_c/dz$ is plotted for each case in Fig. 1 except for a case in Sept. 13 which varies largely the tendency of statistics. du_c/dz is calculated by u_c/z because it is considered that u_c is zero at the ground and linearly increases. The solid line in the figure shows the best fit line. The equation of this line is expressed as follows:

$$\frac{R}{\Delta u^2} \frac{du_c}{dz} = 0.57 \frac{1}{\Delta u} + 0.017 \quad (7)$$

Accordingly, from a comparison of Eqs. (5) and (7), it is seen that

$$\frac{dR}{dz} = 0.19$$

and

$$\frac{C_d}{w} = 0.045. \quad (8)$$

The number of dR/dz is almost the same as the values obtained by laboratory experiments of thermals (e.g. Fig. 4 of Kon et al. 1976). The number of C_d becomes 0.045 to 0.23 according to the vertical speed of 1 to 5 m/s which are reasonable values for small cumulus clouds (Steiner, 1973; Warner, 1977).

4. Consideration and conclusion

An equation which gives the difference between the horizontal speed of cumulus humilis cloud and ambient wind speed is introduced. It contains the entrainment rate and drag coefficient. Since the difference between both speeds has been observed, the rate and the coefficient can be determined. As a result, the entrainment rate becomes about 0.57 which is almost the same as the value which was previously obtained by laboratory experiments. On the other hand, the drag coefficient is estimated to be from 0.045 to 0.23.

The mean radius of cumulus clouds and the mean difference between both speeds shown in Table 1 are used to estimate the magnitude of two terms in the right side of Eq. (4). The first term becomes

$$\frac{0.57 \Delta u}{R} = 7.0 \times 10^{-3}$$

and the second term becomes

$$\frac{0.017 \Delta u^2}{R} = 9.1 \times 10^{-4}$$

The second term is smaller by one order than the first one. Therefore it is concluded that cumulus clouds are almost carried away by the entrainment.

Case Sept. 13 is excepted from the statistics. If the case is included, the drag coefficient becomes negative. It is considered that if the cloud grows newly to the rear, the coefficient becomes negative because the movement speed of cloud is different from an air particle. However the author could not determine the magnitude of the effect in the present paper.

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