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Conductive Heat Flux Through Active Layer in Tundra Plain

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Abstract

A method was developed to estimate conductive heat flux through active layer and thawing rate of ice in permafrost from ground temperature, and it was applied to the field data obtained in July 1983 at the Arara To in St. Lawrence Island, Bering Sea. The estimated heat flux and thawing rate show diurnal fluctuation in response to solar radiation and air temperature, but their daily amounts are maintained at constant rates during the observation period due to presence of a large downward temperature gradient created by annual variations in the solar radiation and air temperature.

1. Introduction

Measurements of ground temperature were made from July 10 to 14, 1983, at the Arara To about 15 km due south of Gambell in St. Lawrence Island, Bering Sea. The measurements form part of the meteorological and hydrological observations at the Arara To. Nakao et al. (1986) discuss the water budget there, and they estimate the inflow of thawing water into the lake from other components as a residual term in the water budget equation.

This paper deals with heat flux in an active layer. A method for estimating heat flux and thawing rate of ice in permafrost from a known ground temperature, which has been developed using the heat conduction theory, is applied to the field data obtained at the Arara To. This is a thermal approach to estimate the inflow of thawing water into the lake from the surrounding tundra plain.

2. Basic equation and its periodic solution

Consider a shallow active layer underlain by thick permafrost (Fig. 1). The active layer is homogeneous and uniform in thickness throughout. It is

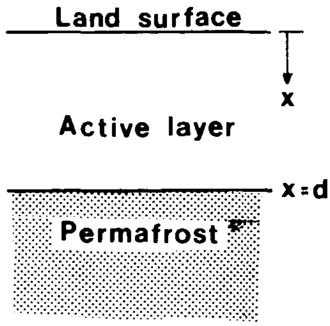

Fig. 1 A model of active layer in tundra region.

assumed that flow of heat in the active layer is conductive and that the active layer is heated at its surface by periodic heat flux, but its thickness is kept unchanged.

With these assumptions, the basic equation for heat transfer in the active layer is expressed as

$$\frac{\partial \theta}{\partial t}=\chi \frac{\partial^2 \theta}{\partial x^2} \tag{1}$$

and boundary conditions are

$$-K\frac{\partial \theta}{\partial t}=a \sin(\omega t+\pi/4+\varepsilon),\ x=0 \tag{2}$$

$$\theta=0,\ x=d \tag{3}$$

where θ is the ground temperature ; χ and K are the thermal diffusivity and the thermal conductivity of the active layer, respectively ; a and ε are the amplitude and the phase constant of the periodic heat flux with time period of $2\pi/\omega$; t is time ; and x is the depth from the surface of the active layer.

Now we introduce complex variables and seek a solution of (1) of the type,

$$\theta=ue^{i(\omega t+\varepsilon)} \tag{4}$$

where u is a function of x only. Substituting (4) in (1) it follows that u must satisfy

$$\frac{d^2 u}{dx^2}=\frac{i\omega}{\chi}u \tag{5}$$

The boundary conditions (2) and (3), treated in the same way, give

$$-K\frac{du}{dx}=ae^{i(\omega t+\pi/4+\varepsilon)},\ x=0 \tag{6}$$

$$u=0,\ x=d \tag{7}$$

The solution of (5) which satisfies (6) and (7) is

$$u=\frac{a}{\sqrt{2}\,kK}\frac{\sinh k'(d-x)}{\cosh k'(d-x)} \tag{8}$$

where

$$k=\sqrt{\omega/2\kappa},\ k'=\sqrt{i\omega/2\kappa} \tag{9}$$

Hence, taking the imaginary part of

$$\frac{a}{\sqrt{2}\,kK}\frac{\sinh k'(d-x)}{\cosh k'(d-x)}e^{i(\omega t+\varepsilon)}$$

the solution which satisfies (1), (2), and (3) becomes

$$\theta=\frac{a}{\sqrt{\rho cK\omega}}\frac{R(x)}{D}\sin\{\omega t+\varepsilon+\phi(x)\} \tag{10}$$

where ρ and c are the density and the specific heat of the active layer, respectively, and

$$\left.\begin{aligned}
P(x)&=\cosh k(d-x)\sin k(d-x)\sinh kd\sin kd\\
Q(x)&=\sinh k(d-x)\cos k(d-x)\cosh kd\cos kd\\
R(x)&=[\{P(x)+Q(x)\}^2+\{P(x)-Q(x)\}^2]^{\frac{1}{2}}\\
D&=(\cosh kd\cos kd)^2+(\sinh kd\sin kd)^2\\
\phi(x)&=\arctan\left[\frac{P(x)-Q(x)}{P(x)+Q(x)}\right]
\end{aligned}\right\} \tag{11}$$

Differentiating (10) with respect to x and putting $x=d$ in it, we have the heat flux at the lower boundary, namely,

$$F_d=\frac{a}{\sqrt{D}}\sin(\omega t+\varepsilon+\gamma) \tag{12}$$

in which

$$\gamma=\arctan\left[\frac{\cosh kd\cos kd-\sinh kd\sin kd}{\cosh kd\cos kd+\sinh kd\sin kd}\right] \tag{13}$$

3. Method of estimating heat flux from ground temperature

If the ground temperature at $x=d_1$ is expressed as a simple form:

$$\theta_1=A\sin(\omega t+\alpha) \tag{14}$$

comparison with (10) yields

$$a=\frac{\sqrt{\rho cK\omega}}{R(d_1)}A,\ \varepsilon=\alpha-\phi(d_1) \tag{15}$$

These relations can be used in estimating heat flux from $f(t)$, the observed temperature. When $f(t)$ is a periodic function of the time of period T, we can obtain the heat flux by using the Fourier series for $f(t)$:

$$f(t)=\frac{A_0}{2}+\sum_{n=1}^{N} A_n \sin(\omega_n t+\alpha_n) \tag{16}$$

where

$$\left.\begin{aligned} &\omega_n=\frac{n\pi}{T},\ A_n=\{B_n^2+C_n^2\}^{\frac{1}{2}},\ B_n=\frac{1}{T}\int_{-T}^{T} f(t)\sin\omega_n t\,dt,\\ &C_n=\frac{1}{T}\int_{-T}^{T} f(t)\cos\omega_n t\,dt,\ \alpha_n=\arctan\{B_n/C_n\} \end{aligned}\right\} \tag{17}$$

With this value of $f(t)$ we have from (12) and (15)

$$F_d=\frac{A_0}{2(d-d_1)}+\sqrt{\rho cK}\sum_{n=1}^{N}\frac{\sqrt{\omega_n D_n}}{R_n}A_n\sin\{\omega_n t+\alpha_n+\gamma_n-\phi_n\} \tag{18}$$

where

$$\left.\begin{aligned} &k_n=\sqrt{\omega_n/2\kappa}\\ &P_n=\cosh k_n(d-d_1)\sin k_n(d-d_1)\sinh k_n d\sin k_n d\\ &Q_n=\sinh k_n(d-d_1)\cos k_n(d-d_1)\cosh k_n d\cos k_n d\\ &R_n=\{(P_n+Q_n)^2+(P_n-Q_n)^2\}^{\frac{1}{2}}\\ &D_n=(\cosh k_n d\cos k_n d)^2+(\sinh k_n d\sin k_n d)^2\\ &\phi_n=\arctan\left[\frac{P_n-Q_n}{P_n+Q_n}\right]\\ &\gamma_n=\arctan\left[\frac{\cosh k_n d\cos k_n d-\sinh k_n d\sin k_n d}{\cosh k_n d\cos k_n d+\sinh k_n d\sin k_n d}\right] \end{aligned}\right\} \tag{19}$$

The first term of (18) indicates a constant flux derived from a steady state solution. The second term is proportional to the thermal constant, $\sqrt{\rho cK}$, known as conductive capacity.

Similarly, using (2), the flux of heat into the active layer from its surface is expressed as

$$F_0=\frac{A_0K}{2(d-d_1)}+\sqrt{\rho cK}\sum_{n=1}^{N}\frac{\sqrt{\omega_n D_n}}{R_n}A_n\sin(\omega_n t+\pi/4+\alpha_n-\phi_n) \tag{20}$$

Futhermore, using (10), the temperature at the surface is

$$\theta_0=\frac{A_0K}{2(d-d_1)}+\sum_{n=1}^{N}\frac{S_n}{R_n}A_n\sin(\omega_n t+\alpha_n+\delta_n-\phi_n) \tag{21}$$

where

$$\left.\begin{aligned} &S_n=\{(\sinh k_n d\cosh k_n d)^2+(\sin k_n d\cos k_n d)^2\}^{\frac{1}{2}}\\ &\delta_n=\arctan\left[\frac{\sin k_n d\cos k_n d}{\sinh k_n d\cosh k_n d}\right] \end{aligned}\right\} \tag{22}$$

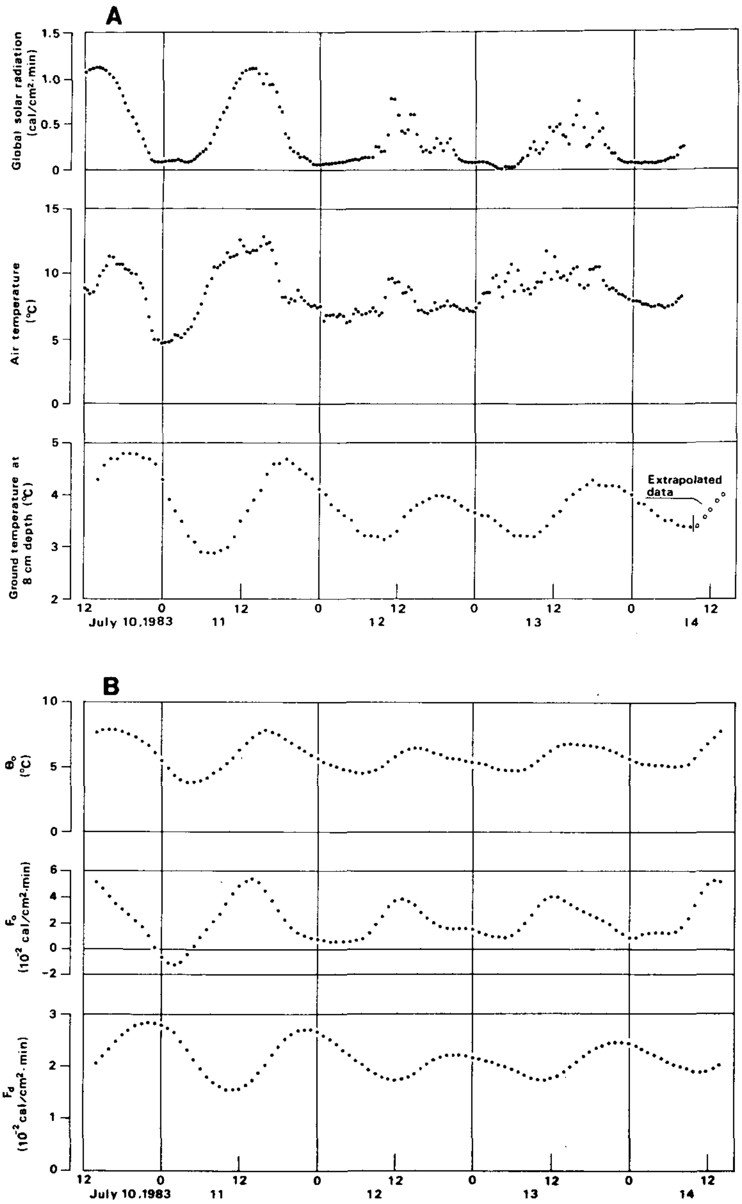

Fig. 2 **A**: Global solar radiation, air temperature, and ground temperature at 8 cm depth observed at the Arara To in St. Lawrence Island, Bering Sea.
B: Surface temperature θ_0, surface heat flux F_0, and heat flux at top of permafrost F_d estimated from the observed ground temperature.

4. Application to field data

Global solar radiation, air temperature and ground temperature at a depth of 8 cm, observed at the Arara To from July 10 to 14, 1983, are shown in Fig. 2-A. Surface temperarure θ_0, surface heat flux F_0 and heat flux at the top of permafrost F_0 are also shown in Fig. 2-B, which are estimated from the observed ground temperature using the proposed method. Thermal constants used in the estimation were $\rho c = 0.8$ cal/cm$^2\cdot$deg ; $K = 1.8\times10^{-3}$ cal/cm$\cdot$sec$\cdot$deg ; $\kappa(=K/\rho c) = 2.3\times10^{-3}$ cm^2/sec ; and $\sqrt{\rho c K} = 3.8\times10^{-2}$ cal/cm$^2\cdot$sec$^{\frac{1}{2}}\cdot$deg. Thickness of the active layer was taken as 23 cm according to the observation at the measurement site.

The estimated heat flux shows diurnal fluctuation in response to the global solar radiation and air temperature. Its amplitude decreases and its phase retards progressively with increasing depth. The phase of the heat flux at the top of permafrost lags behind those of the global solar radiation and air temperature by 8 hours. Therefore, thawing rate of ice in the permafrost has a maximum in the nighttime. But its daily amount is maintained at a constant of about 4 mm/day during the observation period (Table 1) because the constant component of the heat flux is about two times larger than that of the fluctuation. These constant components result from long term variation such as an annual variation. The daily amount of thawing rate of ice in the permafrost agrees approximately with that of thawing water estimated from the water budget at the Arara To.

Table 1 Daily amounts of heat flux and thawing rate

Date	Surface heat flux F_0 (cal/cm$^2\cdot$day)	Heat flux at top of permafrost F_d (cal/cm$^2\cdot$day)	Thawing rate of ice in permafrost (mm/day)
July 11	30.1	31.2	3.9
July 12	26.4	30.1	3.8
July 13	32.2	29.8	3.7

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