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Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

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The CV-S method is a high-efficiency random sampling method to estimate statistical moments of random variables, and it uses an approximated target parameter which are linearly dependent on input as a mockup parameter. In order to enhance the applicability of the CV-S method, we propose to use a mockup parameter which is different from but similar to a target parameter and whose sensitivity coefficients are available. In the present work, nuclear fuel burnup problems are concerned, and standard deviation of k_∞ and nuclide number densities at certain fuel burnup are estimated by the CV-S method. Through numerical tests, it is clearly demonstorated that even if sensitivity coefficients of non burnup-related parameters in a simple system like a fuel pin-cell are used as the mockup, the CV-S method has a potential to efficiently estimate statistical moments of burnup-related parameters in a complicated system like a fuel assembly.

Keywords: uncertainty quantification, random sampling, sensitivity, control variates method

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1. Introduction

Numerical simulation has often been carried out in the field of reactor physics, and uncertainty quantification for results obtained through the numerical simulation has been an important research subject. There are several sources of uncertainties in the numerical simulation for reactor physics problems, and one of the most dominant sources is nuclear data. Accordingly, quantification of uncertainty induced by nuclear data is important. There are mainly two methods to quantify the uncertainty induced by nuclear data: the sensitivity method and the (random) sampling method. The latter has several advantages over the former such as no assumption of linear relation between input nuclear data and output reactor physics parameters, no requirement to prepare sensitivity coefficients of the output with respect to the input, and ease of actual implementation. However, entire computation time can be long in the sampling method when one wants to estimate nuclear data-induced uncertainties of parameters with sufficiently small standard errors since a large number of samples is required. This becomes problematic when concerned parameters require long computation time to be calculated, such as nuclear fuel burnup-related parameters in a complicated system. Deterministic sampling methods, which have a potential to significantly reduce the required number of samples, have been proposed[1], but there still be some difficulties in the application to reactor physics problems due to the large number of concerned nuclear data, especially in cases where nuclear fuel burnup-related parameters are concerned. We have developed a new method combining the sensitivity method and the random sampling method to estimate statistical moments (mean value and variance) of random variables with small standard errors from the small number of samples, and it is named as the CV-S method[2] since it utilizes the first-order sensitivity coefficients in random sampling calculations based on the control variates (CV) method[3]. When one wants to estimate statistical moments of a target parameter with the CV method, he/she needs to prepare a mockup parameter which has high correlation

with the target parameter and whose statistical moments are known or easily calculated. The original CV-S method uses a fictitious mockup parameter: an approximated target parameter which is linearly dependent on the input data, and dependence of this target parameter on the input is represented by the sensitivity coefficients of the target with respect to the input. When random samples are generated for the input data, corresponding samples of this mockup parameter can be obtained very easily without any significant computational burdens using the sensitivity coefficients. Furthermore, statistical moments of this mockup parameter can be theoretically calculated if the normal distribution is assumed to the input data. Although the high efficiency of the original CV-S method has been well demonstrated in the previous paper, the original method has a limitation that preparations of sensitivity coefficients of a target parameter are mandatory. In the present work, we attempt to adopt the CV-S method to estimate statistical moments of a target parameter whose sensitivity coefficients are unavailable. To accomplish this, we use a mockup parameter which is a linear approximation of another parameter which is different from a target parameter. It is important that this mockup parameter should have similar dependence on the input data with the target parameter, and that sensitivity coefficients for this mockup are available. If this procedure works efficiently, we can adopt the CV-S method to estimate statistical moments of a target parameter for which sensitivity coefficients cannot be calculated. For example, in reactor physics problems, even if sensitivity coefficients of burnup-related parameters cannot be obtained, we can adopt the CV-S method to this burnup-related parameters by using alternative sensitivity coefficients of non burnup-related (stationary) parameters. Another example is that statistical moments of a target parameter for a complicated system like a whole reactor core can be efficiently estimated by the CV-S method with a mockup parameter for a simple system like a fuel pin-cell.

Sections 2 and 3 describe the theory and procedure of the CV-S method, respectively.

Section 4 presents numerical calculation results, and Section 5 is devoted to conclude the present work and to give a future perspective.

2. Theory of the CV-S method

2.1. The control variates (CV) method

The CV method is a variance reduction method in Monte Carlo calculations[3], and it allows us to estimate statistical moments with smaller standard errors than the normal sampling method from the same number of samples. In the CV method, we consider two random variables: a target parameter and a mockup parameter. The target parameter is the one for which we want to estimate its statistical moments. The mockup parameter is highly correlated with the target, and whose statistical moments are known or can be calculated precisely and easily. In this method, when the correlation between these two parameters is higher, estimated statistical moments are expected more precise, and thus the correlation is important.

In the following, we describe the procedure for estimating statistical moments with the CV method. First, we consider a random variable X as a target and another random variable Y as a mockup. Mean value and variance for a random variable Z are represented as $E[Z]$ and $V[Z]$, respectively. Here $E[X]$ and $V[X]$ are unknown and those we want to estimate while $E[Y]$ and $V[Y]$ are assumed known. In the estimation of $E[X]$, we first define a parameter H_1 as $H_1 = X - \alpha Y$ where α is a constant, and thus $E[X]$ can be written as

$$E[X] = E[H_1] + \alpha E[Y]. \tag{1}$$

Since the variance of $E[H_1]$ is proportional to $V[H_1]$, if $V[H_1]$ is adequately small, the variance of $E[H_1]$ is also small, and we can estimate $E[X]$ more precisely with Eq. (1) than the direct estimation. $V[H_1]$ can be written with $V[X]$, $V[Y]$, and the covariance

between X and Y, $\mathrm{Cov}[X,Y]$, as

$$V[H_1] = V[X - \alpha Y] = V[X] + \alpha^2 V[Y] - 2\alpha \mathrm{Cov}[X,Y], \tag{2}$$

and α is determined so as to minimize $V[H_1]$ from

$$\frac{\partial V[H_1]}{\partial \alpha} = 0. \tag{3}$$

From Eqs. (2) and (3), α can be determined as

$$\alpha = \frac{\mathrm{Cov}[X,Y]}{V[Y]}, \tag{4}$$

and $V[H_1]$ can be written with this α as

$$V[H_1] = V[X] - \frac{(\mathrm{Cov}[X,Y])^2}{V[Y]} = \left(1 - (\mathrm{Corr}[X,Y])^2\right) V[X], \tag{5}$$

where $\mathrm{Corr}[X,Y]$ represents the correlation coefficient between X and Y. Since the square of $\mathrm{Corr}[X,Y]$ ranges from 0 to 1, $V[H_1]$ is smaller than $V[X]$ whenever $\mathrm{Corr}[X,Y]$ is not 0. This means that the standard error of $E[H_1]$ is also smaller than that of $E[X]$, and thus we can estimate $E[X]$ more precisely from $E[H_1]$ using Eq. (1) than estimating $E[X]$ directly. The procedure for estimating the mean value of X by the CV method is as follows:

(1) Consider a mockup parameter Y, where $E[Y]$ and $V[Y]$ are known.

(2) Generate a finite number of samples of X and Y from a common set of randomly-sampled input data.

(3) Calculate the covariance between X and Y from these samples and obtain α from Eq. (4), where $V[Y]$ estimated from the samples is used to assure consistency between the numerator and the denominator.

(4) Prepare the samples of H_1 from the samples of X and Y, and calculate $E[H_1]$ from these samples.

(5) Estimate $E[X]$ from Eq. (1).

Next, we describe the procedure for estimating $V[X]$, which can be written using $E[X]$

and $E[X^2]$ as

$$V[X] = E[X^2] - (E[X])^2 . \tag{6}$$

Now, we can estimate $E[X]$ precisely by the CV method, and thus we need to consider how to estimate $E[X^2]$ by the CV method. In the estimation of $E[X^2]$, we first consider H_2 defined as $H_2 = X^2 - \beta Y^2$, and thus $E[X^2]$ can be written as

$$E[X^2] = E[H_2] + \beta E[Y^2]. \tag{7}$$

The constant β in Eq. (7) is determined from the minimization for $V[H_2]$ as

$$\beta = \frac{\mathrm{Cov}[X^2, Y^2]}{V[Y^2]}. \tag{8}$$

Thus, we can estimate $E[X^2]$ precisely using Eq. (7) in the same way as $E[X]$, and finally we can estimate $V[X]$ using Eq. (6). The procedure for estimating the variance of X by the CV method is as follows. The steps (1) to (5) are same as the estimation of $E[X]$.

(6) Calculate the covariance between X^2 and Y^2 from the samples and obtain β from Eq. (8), where $V[Y^2]$ estimated from the samples is used because $V[Y^2]$ is assumed unknown.

(7) Prepare the samples of H_2 and calculate $E[H_2]$ from these samples. Here, $E[Y^2]$ is obtained from $E[Y^2] = V[Y] + (E[Y])^2$ where $V[Y]$ and $E[Y]$ are known.

(8) Estimate $E[X^2]$ from Eq. (7) and obtain $V[X]$ from Eq. (6).

The above procedure has been described in the previous paper[2]. In addition to mean value and variance, we can estimate high-order statistical moments using the CV method with the similar procedure.

2.2. The CV-S method

The CV-S method uses a target parameter itself as a mockup parameter, but this mockup parameter is fictitious one which is linearly dependent on input. Hence samples and statistical moments of this mockup parameter can be easily obtained if sensitivity

coefficients of the target parameter with respect to the input are available. Also, high correlation between the target and the mockup is required. In the application of the CV-S method to reactor physics problems, we use sensitivity coefficients of target reactor physics parameters to nuclear data. Sensitivity coefficients of stationary parameters can be easily calculated with the (generalized) perturbation theory in reactor physics problems, and those of burnup-related parameters are also possible to be calculated with the depletion perturbation theory[4, 5]. However, there still be several limitations in sensitivity coefficient calculations with the perturbation theory for burnup-related parameters at present, and thus it cannot be said that sensitivity coefficients can be calculated with reasonable computational costs for any reactor physics parameters. The CV-S method cannot be applied to uncertainty quantification calculations for reactor physics parameters for which sensitivity coefficients are unavailable. To overcome this drawback, we propose to use a mockup parameter which is different from the target parameter but whose sensitivity coefficients are available. If this procedure is efficient, we can adopt the CV-S method to reactor physics parameters for which sensitivity coefficients cannot be obtained, and the applicability of the CV-S method can be enhanced.

3. Numerical procedure of estimating standard deviations of reactor physics parameters by the CV-S method

In the present work, we will focus on estimations of standard deviations (SDs) of reactor physics parameters, and these will be carried out with the normal random sampling method and the CV-S method. Reactor physics parameters (quantities) defined in nuclear fuel burnup problems for the following three systems are concerned: a UO2 fuel pin-cell, a multi-cell consisting of 3×3 UO2 and MOX fuel pins, and a BWR fuel assembly. Perfect reflective boundary conditions are adopted in all these systems. Concerned parameters are the infinite neutron multiplication factor (k_∞) and nuclide number densities during

burnup, and SDs of these parameters originated from the nuclear data uncertainty are estimated.

Nuclear fuel burnup calculations are carried out with a reactor physics code system CBZ being under development at Hokkaido University[7]. Details of numerical methods of the burnup calculations with CBZ can be found elsewhere[8]. Reaction cross section data are taken from JENDL-4.0[9], and fission yields and decay data of fission products (FPs) are taken from JENDL/FPY-2011 and JENDL/FPD-2011[10], in which evaluated nuclear data for 1,400 FPs are stored. Considered nuclear data uncertainties include reaction cross sections of actinoids, decay half-lives, decay branching ratios, and independent fission yields of FPs. Regarding the reaction cross section uncertainties, we consider (n,n), (n,f), (n,γ), (n,2n), and (n,n'), and other two types of nuclear data, $\bar{\nu}$ and χ, are also considered based on the JENDL-4.0 evaluations. Regarding the uncertainties of decay half-lives, fission yields, and decay branching ratios, we use the covariance data defined in the above nuclear data files. Correlations in the fission yields among FPs belonging to the same mass chain are considered by the method presented in the reference[10]. One hundred percent standard deviation is assumed to the decay half-lives, fission yields, and decay branching ratios if variance data are not provided.

In all the calculations with the CV-S method, we use parameters defined for the UO2 pin-cell as the mockup. The sensitivity coefficients of k_∞ and nuclide number densities at certain fuel burnup with respect to nuclear data are calculated by the depletion perturbation theory capability of CBZ[6].

The above mentioned procedure is summarized as follows:

(1) Sensitivity coefficients of a mockup parameter are calculated for the single UO2 pin-cell, and the variance of this mockup parameter is calculated from the sensitivity coefficients and the covariance data by the well-known sandwich rule.

(2) Generate a set of nuclear data samples according to their covariance data and carry

out burnup calculations to obtain samples of a target parameter. In this burnup calculation, we use a simplified burnup chain consisting of 137 FPs to reduce computational time. This simplified burnup chain is generated from each set of the sampled nuclear data. At the same time, samples of the mockup parameter are calculated from the sampled nuclear data and the sensitivity coefficients without any burnup calculations.

(3) Estimate SD of the target parameter by the CV method from the sets of the samples of the target and mockup parameters.

The maximum numbers of generated samples are ten thousands in the cases of the UO2 pin-cell and the multi-cell, and one thousand in the case of the BWR assembly.

Standard errors of the estimated SD are important quantities in the present work since these are indices showing the efficiency of the sampling method. These are estimated by the bootstrap method[11], which is one of the resampling methods and has been recently introduced to the field of reactor physics[12]. In the calculations with the bootstrap method, we prepare 1,000 sets of samples which are resampled from the original set of samples (or the empirical distribution function), and calculate statistical moments for each set. Finally, the standard errors of our estimates on SD of the target parameter can be obtained. The applicability of the bootstrap method to the standard error estimation in the CV-S method has been demonstrated in the previous paper[2].

4. Numerical results

Firstly, performance of the CV-S method using mockup parameters at different fuel burnup from the target is tested with the UO2 pin-cell. Secondly, applicability of the CV-S method to estimate statistical moments of parameters defined for the multi-cell using mockup parameters defined for the pin-cell is tested. Finally, the CV-S method is applied to parameters defined for the BWR assembly using mockup parameters defined for the

pin-cell comprising the assembly.

4.1. UO2 pin-cell

We here concern SDs of k_∞ and number densities of I-129 and Eu-151 at certain fuel burnup of the UO2 fuel pin-cell whose U-235 enrichment is 4.1 wt%. These nuclides are concerned since non-linear effects of these number densities on nuclear data have been observed in the previous work[13]. First, we estimate the SD of k_∞ at 15 and 30 GWd/t with the CV-S method using the sensitivity coefficients of k_∞ of the same pin-cell at 45 GWd/t. **Figure 1** shows the results where error bars indicate 1σ standard errors estimated by the bootstrap method. Next, the SDs of the nuclide number densities at 20 GWd/t are estimated with the CV-S method using the sensitivity coefficients of the number densities of the corresponding nuclides at 45 GWd/t. **Figure 2** shows the SDs of these number densities. These results indicate that, even if we use the sensitivity coefficients at different fuel burnup from the target in the CV-S method, the SDs of k_∞ and the nuclide number densities can be estimated without any significant bias.

[Figure 1 about here.]

[Figure 2 about here.]

We estimate SDs of k_∞ at several different fuel burnup with the CV-S method using the sensitivity coefficients of k_∞ at 0 GWd/t or 45 GWd/t. To quantify the efficiency of the CV-S method, we use an index, uncertainty reduction (UR), which is defined as a ratio of standard error of the estimated SDs with the CV-S method to that estimated with the normal random sampling method. **Figure 3** shows UR in the SDs of k_∞ during the burnup estimated from the 10,000 samples. When the sensitivity coefficients at 0 GWd/t are used as the mockup, UR at the beginning of the burnup is apparently small. This result demonstrates a possibility that the CV-S method can efficiently estimate SD of a burnup-related parameter using a mockup parameter which is a non burnup-related

parameter. Oppositely, when the sensitivity coefficients at 45 GWd/t are used as the mockup, UR around the end of the burnup becomes small.

[Figure 3 about here.]

We estimate SDs of the number densities of I-129 and Eu-151 at several fuel burnup using the sensitivity coefficients of these number densities at 45 GWd/t. **Figure 4** shows UR of the SDs of these number densities. This result shows that the efficiency of the CV-S method using the sensitivity coefficients at different fuel burnup from the target vary among the nuclides.

[Figure 4 about here.]

From the above results, we find that the CV-S method using the sensitivity coefficients calculated at the different fuel burnup from the target can efficiently estimate the SDs. The efficiency degradation due to the different burnup values of the mockup parameter from the target is more significant in k_∞ than in the nuclide number densities. This can be explained as follows; k_∞ is dependent on the fuel composition at the concerned burnup and is thus dependent on the nuclear data of these nuclides. The fuel composition should be significantly changed during the burnup because of the conversion from U-238 to Pu-239 and neutron capture reactions of plutonium isotopes. Thus, correlations in k_∞ between different burnup values tend to be small as shown in our previous work[14]. On the other hand, the nuclide number density should be dependent on some specific nuclear data such as fission yields and neutron capture reaction cross sections of specific nuclides. Thus, correlations in the nuclide number densities between different burnup values tend to be large as shown in our another previous work[15].

4.2. Multi-cell

The purpose of calculations in this section is to confirm that SDs can be estimated efficiently by the CV-S method when systems for which target and mockup parameters

are defined are different from each other; a target parameter is defined in the multi-cell consisting of 3×3 fuel pins and a mockup is defined in the single pin-cell. **Figure 5** shows the geometrical configuration of the multi-cell systems. These multi-cell systems consist of the UO2 fuel pin with U-235 enrichment of 4.1 wt%, which is the same as that used in the preceding section, and MOX fuel pins with Pu enrichment of 10%. We consider four cases, in which the UO2 fuel pin is located at the center and the arrangement of fuel pins around the center is different; only the UO2 fuel pins are loaded (case 1), the MOX fuel pins are loaded neighboring to the center by the corner (case 2), the MOX fuel pins are loaded neighboring to the center by the side (case 3), and the MOX fuel pins are loaded at all the positions except the center (case 4).

[Figure 5 about here.]

For these four cases, we estimate SDs of k_∞ and number densities of I-129 and Eu-151 at the central UO2 pin, and the sensitivity coefficients used in the CV-S method are calculated with the single UO2 pin-cell. The fuel burnup of the mockup is the same as the system-averaged burnup of the target. **Figures 6** and **7** show SDs of k_∞ and number densities of I-129 and Eu-151 at 45 GWd/t in case 4. These results indicate that, even if we use the sensitivity coefficients defined in a different system from the target in the CV-S method, SDs of k_∞ and nuclide number densities can be estimated without any significant bias.

[Figure 6 about here.]

[Figure 7 about here.]

Figure 8 shows UR of the SDs of k_∞ at 0 GWd/t and 45 GWd/t in these four cases estimated from the 10,000 samples. When the target fuel burnup is 45 GWd/t, the CV-S method works efficiently in all the cases and the efficiency is different among the cases. On the other hand, when the target fuel burnup is 0 GWd/t, the CV-S method works efficiently only in case 1, and almost no uncertainty reduction is observed in the others.

These results are due to the difference of the fuel pin arrangement. Since the fresh UO2 fuel pin contains only uranium isotopes, contribution of the uranium isotopes to fission and neutron capture reactions is overwhelming at the beginning of fuel burnup, while at the end of fuel burnup, the contribution of plutonium isotopes, especially Pu-239 that is produced from U-238, to these reactions is also significant. Because of this, the UO2 pin-cell can consider contribution of the plutonium isotopes to these reactions at the end of burnup, but cannot at the beginning. Thus, a good performance of the CV-S method is not observed in the multi-cell systems including the MOX pins at the beginning of burnup when the UO2 fuel pin-cell is used as the mockup system.

[Figure 8 about here.]

Figure 9 shows UR of the SDs of number densities of I-129 and Eu-151 at 45 GWd/t when the number of samples is 10,000. This figure indicates that the CV-S method works efficiently, but as the case number increases, namely as the number of the MOX fuel pins increases, the efficiency of the CV-S method deteriorates; UR of case 1 is the smallest, those of case 2 and 3 are larger than that of case 1 and comparable with each other, and that of case 4 is the largest. One of the reasons causing this is the difference in the fuel burnup between the target UO2 pin in the multi-cell and that of the single mockup pin-cell. In this calculation, we focus on nuclide number densities of the central UO2 fuel pin in the multi-cell, but we define the fuel burnup as the average in the whole multi-cell, and thus the fuel burnup of each fuel pin in the multi-cell is different from the average. Fuel burnup values of the central (target) UO2 pin are 45.0, 33.9, 35.9 and 28.8 GWd/t in cases 1 to 4, respectively, when the system-averaged burnup is 45 GWd/t. Impact of the difference in fuel burnup on UR of the Eu-151 number density observed here is consistent with UR obtained in Figure 4, but that on UR of the I-129 number density here is much larger than that in Figure 4. This would be explained by the difference in neutron flux energy spectrum. In cases 2 to 4, since the MOX fuels are loaded, neutron flux energy spectrum

in the central UO2 fuel should be harder than that in case 1. The cumulative fission yields of I-129 are different between U-235 and Pu-239: 0.00540 for U-235 and 0.0132 for Pu-239 in JENDL/FPY-2011. If the neutron flux energy spectrum becomes hard, relatively large amount of Pu-239 is generated by the conversion and contribution of the Pu-239 fission to the I-129 generation becomes high. This means that sensitivity profiles of the I-129 number density in the MOX-loaded cases can become different from those in the UO2 pin-cell. Due to this mechanism, UR of the I-129 number density becomes large in cases 2 to 4.

[Figure 9 about here.]

4.3. BWR fuel assembly

In this section, we use the BWR fuel assembly model which was developed in the phase III-C of the burnup credit criticality safety benchmark[16]. **Figure 10** shows geometrical configuration of the assembly. This assembly includes five types of UO2 fuel pins with different U-235 enrichment and UO2 fuel pins bearing gadolinium. These five types of the UO2 pins are identified with the *fuel pin type*, and the fuel pin type 1 corresponds to that with the highest U-235 enrichment and the pin type 5 corresponds to that with the 2.1 wt% enrichment. The fuel pin types are presented in a circle in Figure 10. The numbers of loaded pins per one assembly are 4, 32, 12, 8, and 4, respectively, for the fuel pin types 1 to 5. In addition, each fuel pin except the Gd-bearing pins is labeled by the *fuel pin index* from (a) to (j), and the correspondence can be found also in Figure 10.

The target parameters are k_∞ of the assembly and nuclides number densities of fuel pins from (a) to (j) at 40 GWd/t. In all calculations with the CV-S method, we use the sensitivity coefficients of k_∞ or the nuclide number densities defined for the five different types of the single UO2 pin-cell at 40 GWd/t.

[Figure 10 about here.]

First we concern SDs of k_∞ of the assembly. The SDs are estimated with the CV-S method using the mockup parameters of five different pin-cells, so five UR values are obtained. URs of the SDs estimated from the 1,000 samples are 0.79, 0.73, 0.65, 0.58, and 0.63 with the mockup parameters defined for the fuel pin types 1 to 5, respectively. The efficiency of the CV-S method is dependent on the fuel pin type of the mockup, and in the present result, the fuel pin type 4 is the best among the five mockup parameters. Whereas the number of loaded pins with the type 2 is the largest among the all loaded fuel pins in this assembly, this result suggests that the neutronics property of the single pin-cell of the type 4 is the closest to that of the assembly. Generally the neutron flux energy spectrum becomes soft as the U-235 enrichment is low. The reason why the fuel pin type 4, whose U-235 enrichment is relatively low among all, is chosen as the best would be contributions of the water gaps and water channels of this assembly; these structure makes the neutron flux energy spectrum soft. This result also suggests that other systems like a single fuel pin of high-enriched uranium with a large pin-pitch value are also good candidates as mockup systems.

Next, we concern number densities of Eu-154, Pu-239, Am-241, and Cm-244 of the fuel pins from (a) to (j) at 40 GWd/t. In the CV-S method, the mockup of the pin-cell with the same type is used. **Figure 11** shows UR of the SDs of these number densities. In the CV-S calculations, the sensitivity coefficients of the number density of the corresponding nuclides for the corresponding pin-cells at 40 GWd/t are used. In most cases, UR are less than 0.4. The efficiency of the CV-S method are dependent on the target nuclides and the target fuel pins, but any trends cannot be found.

[Figure 11 about here.]

From the above results, we demonstrate that the CV-S method using the sensitivity coefficients for the pin-cell system is applicable to the uncertainty quantification calculations for the parameters of the fuel assembly system.

5. Conclusion

The CV-S method is a high-efficiency random sampling method to estimate statistical moments of random variables, and it uses an approximated target parameter which is linearly dependent on input as a mockup parameter. Although the high potential of this method has been demonstrated in the previous work, it has a drawback that it can be applied to problems where sensitivity coefficients of a target parameter with respect to input data are available. In order to enhance the applicability of the CV-S method, we have proposed to use a mockup parameter which is different from but similar to a target parameter and whose sensitivity coefficients are available. In the present work, nuclear fuel burnup problems have been considered, and SD of k_∞ and nuclide number densities at certain fuel burnup have been estimated by the CV-S method. The efficiency has been quantified with the index UR which corresponds to the improvements in the estimation precision relative to the normal sampling method. Standard errors of the estimated SDs of the target parameters have been estimated by the bootstrap method.

In the first numerical test, the newly proposed CV-S method has been tested to estimate SDs of k_∞ and the nuclide number densities of the UO2 fuel pin-cell at target fuel burnup by using the same quantities at different fuel burnup as mockup parameters. In the second test, the CV-S method has been applied to estimate SDs of k_∞ and the nuclide number densities of the multi-cell systems consisting of the UO2 and MOX fuel pins by using the same quantities of the single UO2 fuel pin-cell as mockup parameters. In both the tests, improvement in the efficiency by the CV-S method has been confirmed. This suggests that even if sensitivity coefficients are available for non burnup-related parameters in a simple system like a fuel pin-cell, the CV-S method has a potential to efficiently estimate statistical moments of burnup-related parameters at a complicated system like a fuel assembly. In the third test, the CV-S method has been applied to estimate SDs of k_∞ and the nuclide number densities of the BWR fuel assembly by using the same quantities

of the single fuel pin-cell comprising the assembly, and SDs of these target quantities have been estimated by the CV-S method more efficiently than by the normal random sampling method.

Throughout the present work, we have successfully enhanced the applicability of the CV-S method. The improvement in the efficiency is, however, highly dependent on the cases, and we should properly choose the best mockup parameter among the candidates. We are now testing to use a fictitious mockup parameter which is combining several parameters with proper weights. This result will be published in another paper in future.

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Figure Captions

Figure 1 Standard deviations of k_∞ at 15 and 30 GWd/t estimated using the sensitivity coefficients of k_∞ at 45 GWd/t

Figure 2 Standard deviations of nuclide number densities at 20 GWd/t estimated using sensitivity coefficients at 45 GWd/t (unit: [/cm/barn])

Figure 3 Uncertainty reduction of standard deviations of k_∞ using the sensitivity coefficients at 0 GWd/t or 45 GWd/t

Figure 4 Uncertainty reduction of standard deviations of the nuclide number densities estimated using the sensitivity coefficients at 45 GWd/t

Figure 5 Pin-cell arrangement of four multi-cell systems

Figure 6 Standard deviations of k_∞ at 45 GWd/t in case 4 of the multi-cell systems using the sensitivity coefficients of the single pin-cell

Figure 7 Standard deviations of nuclide number densities at 45 GWd/t in case 4 of the multi-cell systems using sensitivity coefficients of the single pin-cell (unit: [/cm/barn])

Figure 8 Uncertainty reduction of the standard deviations of k_∞ in the multi-cell systems at 0 and 45 GWd/t

Figure 9 Uncertainty reduction of the standard deviation of nuclide number densities at 45 GWd/t at 10,000 samples

Figure 10 Geometrical configuration of the BWR fuel assembly

Figure 11 Uncertainty reduction of the standard deviations of the number densities at 40 GWd/t at each fuel pin

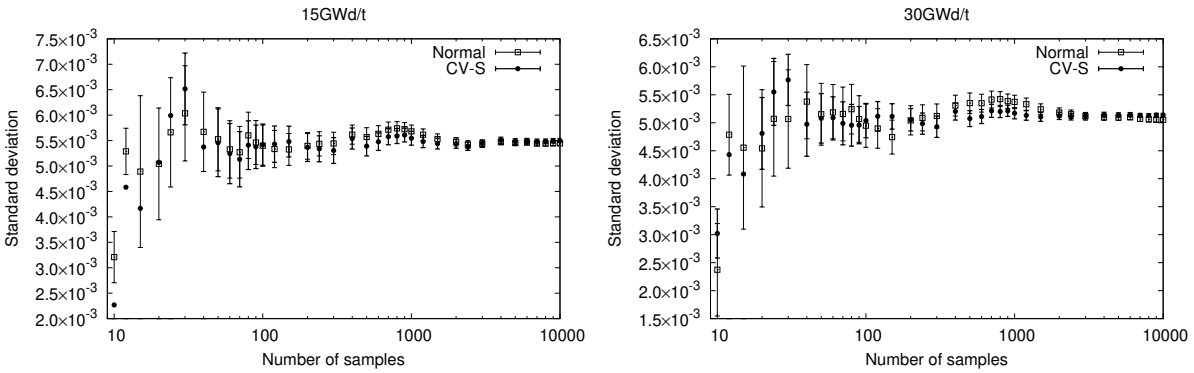

Figure 1 Standard deviations of k_∞ at 15 and 30 GWd/t estimated using the sensitivity coefficients of k_∞ at 45 GWd/t

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

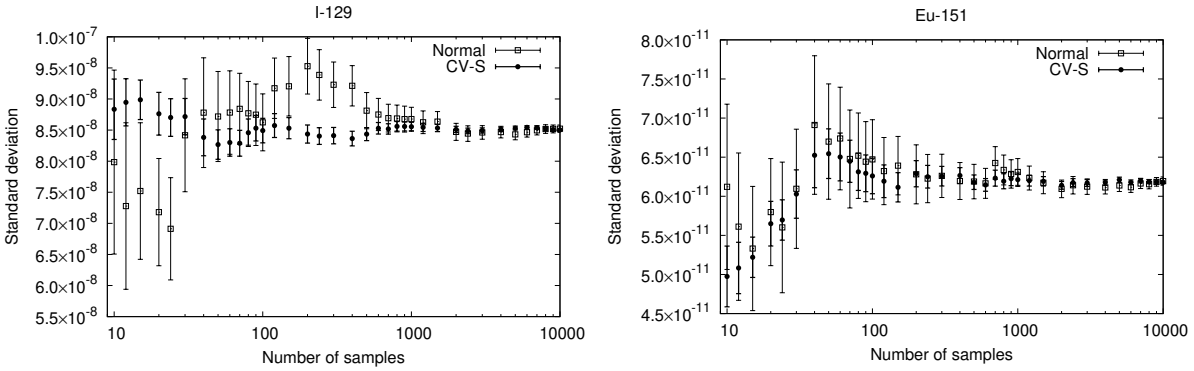

Figure 2 Standard deviations of nuclide number densities at 20 GWd/t estimated using sensitivity coefficients at 45 GWd/t (unit: [/cm/barn])

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

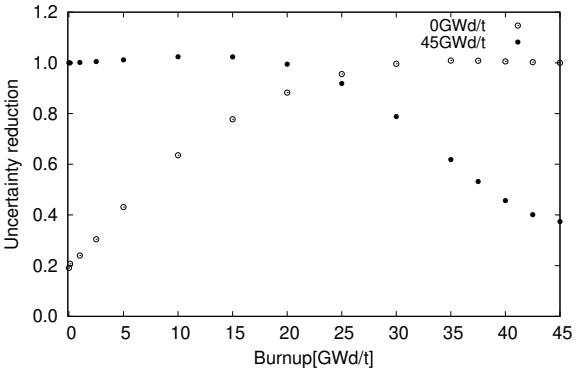

Figure 3 Uncertainty reduction of standard deviations of k_∞ using the sensitivity coefficients at 0 GWd/t or 45 GWd/t

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

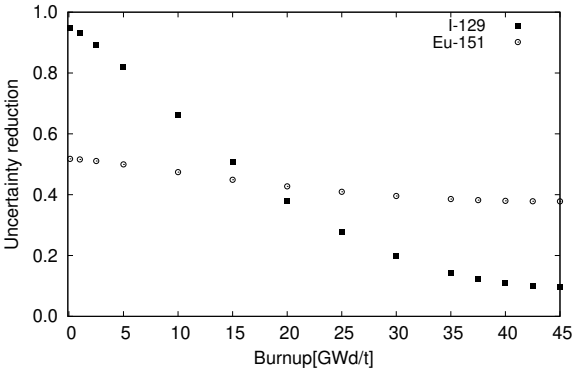

Figure 4 Uncertainty reduction of standard deviations of the nuclide number densities estimated using the sensitivity coefficients at 45 GWd/t

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

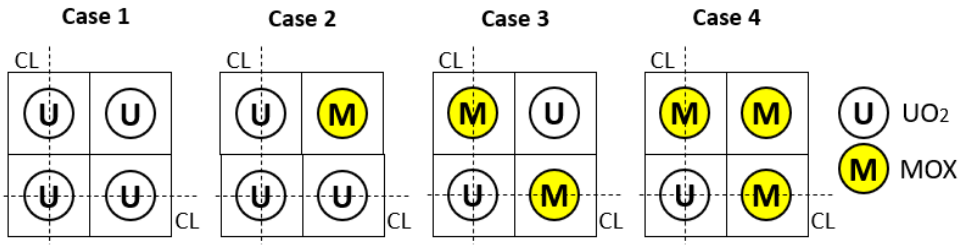

Figure 5 Pin-cell arrangement of four multi-cell systems

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

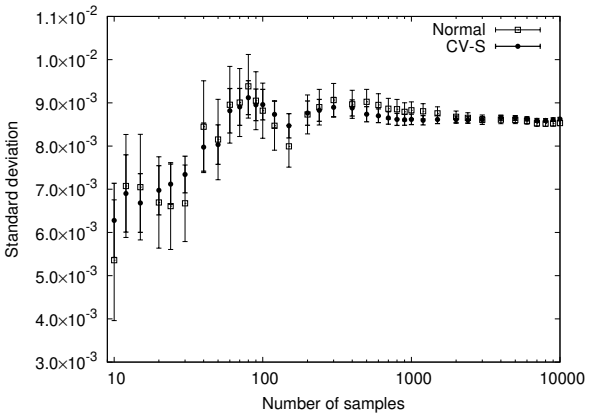

Figure 6 Standard deviations of k_∞ at 45 GWd/t in case 4 of the multi-cell systems using the sensitivity coefficients of the single pin-cell

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

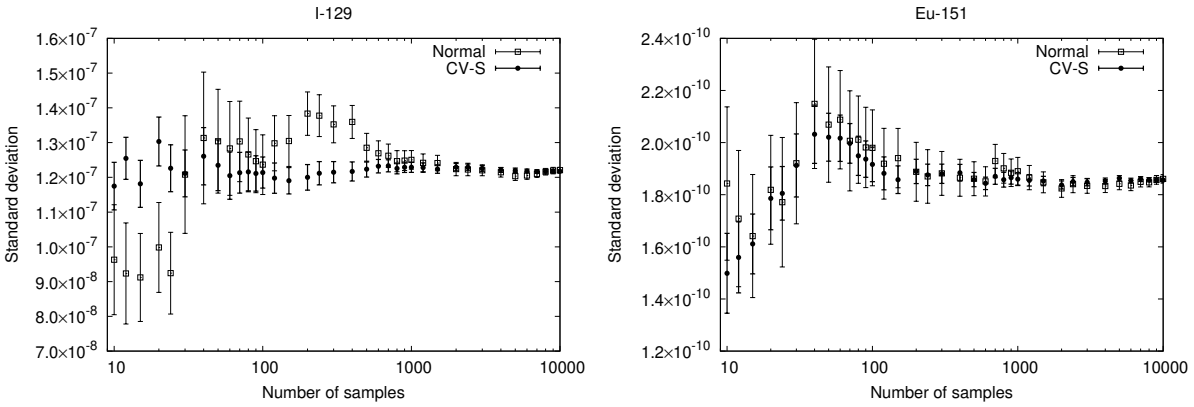

Figure 7 Standard deviations of nuclide number densities at 45 GWd/t in case 4 of the multi-cell systems using sensitivity coefficients of the single pin-cell (unit: [/cm/barn])

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

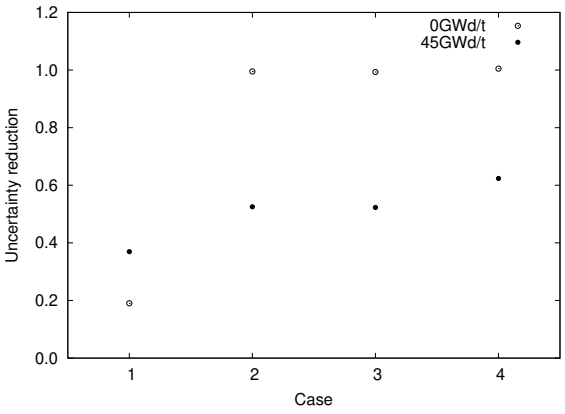

Figure 8 Uncertainty reduction of the standard deviations of k_∞ in the multi-cell systems at 0 and 45 GWd/t

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

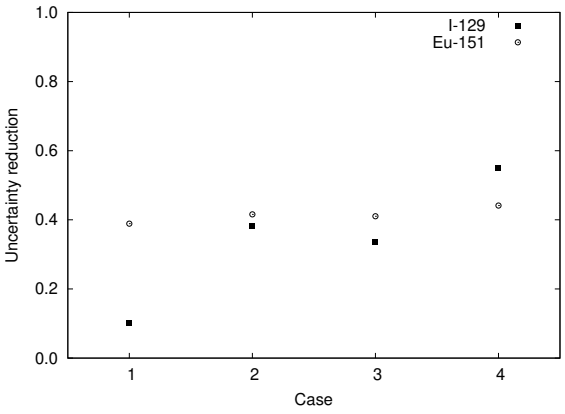

Figure 9 Uncertainty reduction of the standard deviation of nuclide number densities at 45 GWd/t at 10,000 samples

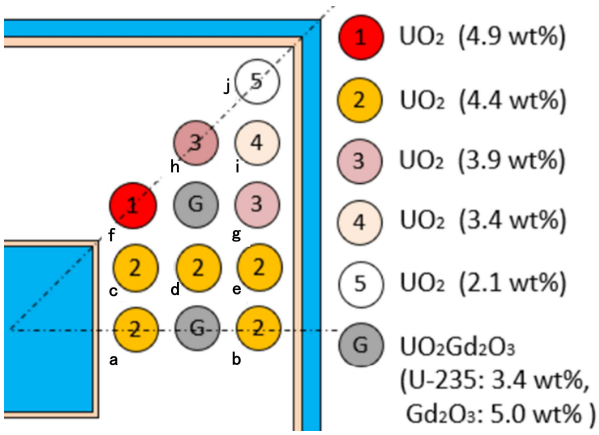

Figure 10 Geometrical configuration of the BWR fuel assembly

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients

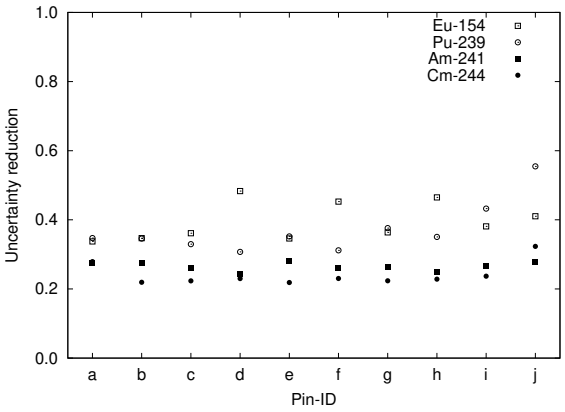

Figure 11 Uncertainty reduction of the standard deviations of the number densities at 40 GWd/t at each fuel pin

T. Kida and G. Chiba

Enhancement of applicability of high-efficiency random sampling method using control variates method and sensitivity coefficients