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The Possibility of Measuring in-Situ Stress by Using the Dielectric Anisotropy of Rocks Caused by Aligned Cracks

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Abstract

The possibility of measuring in-situ stress by using the dielectric anisotropy of rocks is examined theoretically. Firstly, a model for the dielectric anisotropy of rocks due to aligned ellipsoidal cracks is constructed. It is found that anisotropy would appear if the cracks are filled with water.

Secondly, electric fields in an anisotropic medium are examined. It is found that arrangements of quadruple line electrodes are not adequate to measure the anisotropy. Point electrodes are better suited to obtain the anisotropy. With in-line arrangement of four point electrodes, the potential difference is

$$\Delta\phi=\frac{\rho}{\sqrt{\varepsilon^z}\, x_0(b^2-1)}\frac{1}{\sqrt{\varepsilon^y+a^2\varepsilon^x}}$$

where ρ is the charge density, $a=\tan\theta$ where θ is the direction of the line arrangement measured from the principal axis of the anisotropy, b is a constant, $2x_0$ is the distance between source electrodes, and ε^x, ε^y, and ε^z are the principal components of the average dielectric tensor.

Finally, the problems arising when conductive dielectrics is treated are examined. For wet rocks, rather high frequencies (10^6-10^{10} Hz) should be used if the above arrangement of electrodes is employed. At such frequencies, it is found that the skin depth ranges from a centimeter to a hundred meters. It means that the observed anisotropy does reflect the crack state around the observation points.

In conclusion, it is expected that in-situ stress can be measured by using the dielectric anisotropy.

1. Introduction

Several methods have been applied to measure in situ stresses for earthquake prediction in Japan. (eg. Tsukahara et al., 1978; Tanaka et al., 1980; Yoshikawa and Mogi, 1982; Yamamoto et al., 1986). In these studies, elastic responses of rocks, which are determined by the state of existing cracks, are used. Because crack states are determined by stress states, we can deduce the stress states, knowing the crack states. Maeda and Shimizu (1983) offered a new method of measuring stress states using elastic anisotropy, which also utilized the elastic response caused by aligned cracks. The method was found to be rather cumbersome for field applications. We seek another method in this report.

In general, the physical properties of rocks containing aligned cracks become anisotropic. We examine, therefore, the possibility of using the dielectric property to obtain tectonic stresses.

The dielectric constant has been used for geophysical investigations since Pascal's work (1964). Systems consisting of sand or clay mixed with water or oil have been primarily investigated (eg. Singh and Rankin, 1986). Mendelson and Cohen (1982) investigated electric properties of such a multiphase system theoretically. In these works, oil/water is considered as a matrix because of the large amount of the liquid phase and the isotropic character of the system. The anisotropy caused by anisotropic grains is only taken into account in order to interpret the diversity of a parameter in a law similar to Archie's.

In rocks, the liquid phase is confined in cracks whose total volume is very small. Many rocks can be considered nearly dielectrically isotropic if there are no cracks. The main cause of anisotropy, if it exists, may be cracks which are themself isotropic. There seems to have been no investigation of this sort of anisotropy. In this report, we theoretically investigate the dielectric anisotropy caused by aligned cracks in isotropic matrices and examined whether or not it is a measurable quantity in common rocks in the field.

2. Model

We consider rock to cosist of matrix and cracks. For simplicity, each part is considered to be isotropic and homogeneous and has a dielectric constant of ε^e or ε^i, respectively. The shape of cracks is approximated by an ellipsoid. The average electric flux density $\bar{\boldsymbol{D}}$ in the medium is

$$
\begin{aligned}
\bar{\boldsymbol{D}} &= \frac{1}{V}\int_V \boldsymbol{D}(\boldsymbol{r})d\boldsymbol{r} \\
&= \frac{1}{V}\int_{V^e} \varepsilon^e\, \boldsymbol{E}^e(\boldsymbol{r})d\boldsymbol{r} + \frac{1}{V}\int_{V^i} \varepsilon^i \boldsymbol{E}^i(\boldsymbol{r})d\boldsymbol{r} \\
&= (1-c)\frac{\varepsilon^e}{V^e}\int_{V^e} \boldsymbol{E}^e(\boldsymbol{r})d\boldsymbol{r} + \frac{c}{V^i}\,\varepsilon^i \int_{V^i} \boldsymbol{E}^i(\boldsymbol{r})d\boldsymbol{r} \\
&\equiv (1-c)\varepsilon^e \bar{\boldsymbol{E}} + c\,\frac{\varepsilon^i}{V^i}\int_{V^i} \boldsymbol{E}^i(\boldsymbol{r})d\boldsymbol{r}
\end{aligned}
$$

where $\boldsymbol{E}^e$ and $\boldsymbol{E}^i$ are the electric fields in the matrix and cracks, respectively. By assumption, ε^e and ε^i are independent of coordinates in each defined domain. It is assumed that all cracks have the same dielectric constant ε^i. V^e is the volume of matrix, V^i is the total volume of cracks and $V=V^e+V^i$. In the following, it is assumed that $V^i \ll V$, that is, that the crack porosity $c=V^i/V$ of rock is sufficiently low. This assumption seems to hold for many rocks. By this assumption, the electric interaction between cracks can be neglected. Then, the present problem is reduced to one of calculating the electric field $\boldsymbol{E}^i$ inside an ellipsoid placed in the uniform external field $\bar{\boldsymbol{E}}$. Although this problem is already solved and the solution is given (Landau and Lifshitz, 1962), we will outline the derivation in an orthodox way.

Let us take lengths of the principal axes of the ellipsoid to be a, b, and $c(a \geq b \geq c)$ and, along the principal axes, take the coordinate axes x, y, and z. The basic equatons are

$$\nabla \cdot \boldsymbol{D}^e=0, \ \nabla \cdot \boldsymbol{D}^i=0$$

and constitutive equations are

$$\boldsymbol{D}^i=\varepsilon^i \boldsymbol{E}^i, \ \boldsymbol{D}^e=\varepsilon^e \boldsymbol{E}^e$$

where suffix i and e have the same meaning as before. The basic equation becomes Laplace's equation for a potential defined by $\boldsymbol{E}=-\nabla \psi$. The potential equation is, in the ellipsoidal coordinate system,

$$\begin{aligned} \Delta\psi=&\frac{4}{(\xi-\eta)(\zeta-\xi)(\eta-\zeta)}\left[(\eta-\zeta)R_\xi \frac{\partial}{\partial\xi}\left(R_\xi \frac{\partial\psi}{\partial\xi}\right)\right. \\ &\left.+(\zeta-\xi)R_\eta \frac{\partial}{\partial\eta}\left(R_\eta \frac{\partial\psi}{\partial\eta}\right)+(\xi-\eta)R_\zeta \frac{\partial}{\partial\zeta}\left(R_\zeta \frac{\partial\psi}{\partial\zeta}\right)\right]=0, \\ R_s=&\sqrt{(s+a^2)(s+b^2)(s+c^2)} \end{aligned}$$

where ξ, η, and ζ are the solutions of following equation;

$$\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$$

Because the present problem is linear one, it is sufficient to consider only one direction, say along the x-axis. The external potential in x direction is

$$\psi_0=\bar{\boldsymbol{E}}_x \cdot x=-\frac{\bar{E}_x}{\sqrt{(b^2-a^2)(c^2-a^2)}}\sqrt{(\xi+a^2)(\eta+a^2)(\zeta+a^2)}$$

Let us assume that the true potential can be expressed as $\psi_0+\psi'$ and that ψ' has the form:

$$\psi'=\psi_0 F(\xi)$$

Substituting this expression into Laplace's equation, we obtain for F:

$$\frac{d^2F}{d\xi^2}+\frac{dF}{d\xi}\frac{d}{d\xi}\ln\left[R_\xi(\xi+a^2)\right]=0$$

Two independent solutions of this equation are

$$F_1=\text{const}\equiv B',$$
$$F_2=A\int_\xi^\infty\frac{ds}{(s+a^2)R_s}$$

Because the second solution F_2 is infinite at $\xi=-c^2$, first one F_1 must express the internal potential of the ellipsoid. Hence, the external potential must be F_2:

$$\phi^i=B\phi_0,\ B=1+B'$$
$$\phi^e=\phi_0\left[1+A\int_\xi^\infty\frac{ds}{R_s(s+a^2)}\right]$$

From the boundary conditions on the surface of the ellipsoid, we can determine the coefficients A and B. That is, from the continuity of the electric flux density normal to the surface at $\xi=0$, ie.

$$\varepsilon^e E_n^e=-\varepsilon^e(\nabla\phi^e)\cdot\boldsymbol{n}=-\varepsilon^e\frac{\partial\phi^e}{\partial\xi}=-\varepsilon^i\frac{\partial\phi^i}{\partial\xi},$$

we obtain

$$B\varepsilon^i=\left[1+\frac{2A}{abc}(n^x-1)\right]\varepsilon^e$$

and from the continuity of the tangential electric field on the surface $E_\eta^i=E_\eta^e$, we obtain

$$1+\frac{2An^x}{abc}=B$$

where

$$n^x=\frac{abc}{2}\int_0^\infty\frac{ds}{(s+a^2)R_s}$$

and n^y and n^z are defined by replacing a by b and c in the above equation. Finally, we obtain coefficients A and B;

$$A=\frac{abc}{2}\frac{\varepsilon^e-\varepsilon^i}{n^x(\varepsilon^i-\varepsilon^e)+\varepsilon^e}$$
$$B=\frac{\varepsilon^e}{n^x\varepsilon^i+(1-n^x)\varepsilon^e}$$

Under the assumption of dilute crack distribution, the relation between the external and internal electric fields in x-direction is

$$[n^x(\varepsilon^i-\varepsilon^e)+\varepsilon^e]E_x^i=\varepsilon^e\bar{E}_x$$

and, for y and z directions, E_x should be replaced by E_y and E_z and n^x by n^y and n^z. In a general rectangular coordinate system, the relation between the m -th component of the external field $\bar{E}_m$ and the n-th component of the internal one E_n^i is

$$\bar{E}_m=\sum_n S_{mn}E_n^i$$

where S_{mn} is a second rank tensor ;

$$S_{mn}=\delta_{mn}+n_{mn}\left(\frac{\varepsilon^i}{\varepsilon^e}-1\right)$$

n_{mn} is a tensor of which the principal components are n^x, n^y and n^z.

The m-th component of the electric flux is, using the inverse tensor of S_{mn}, S_{mn}^{-1} ;

$$\bar{D}_m=\sum_n [(1-c)\varepsilon^e\delta_{mn}+c\varepsilon^i S_{mn}^{-1}]\bar{E}_n\equiv\sum_n \bar{\varepsilon}_{mn}\bar{E}_n$$

The coefficient $\bar{\varepsilon}_{mn}$ is the average dielectric tenser of the medium containing ellipsoidal cracks.

We will estimate the degree of the anisotropy for the penny shaped cracks (elliposid). Since, for penny shaped cracks, n^x and n^y are nearly zero, and $n^z=1$, the average dielectric tensor has the form ;

$$\bar{\varepsilon}=\begin{bmatrix}(1-c)\varepsilon^e+c\varepsilon^i & 0 & 0\\ 0 & (1-c)\varepsilon^e+c\varepsilon^i & 0\\ 0 & 0 & \varepsilon^e\end{bmatrix}$$

in the principal coordinate system of the ellipsoid. The dielectric anisortropy can be defined as the ratio of the xx- or yy-component to the zz-component. The dielectric constants are listed in the table edited by Beblo (1982). The dielectric constant decreases with increasing frequency of measurement. In order to detect the dielectric anisotropy, we should use higher freqüencies (Maeda, 1988). The average dielectric constant of dried samples measured above 10^6 Hz is 7.78 in Gaussian units. This value should be used as the dielectric constant of matrix. It is easily seen that, when the cracks are empty ($\varepsilon^i=1$), no detectable anisotropy will appear. When the cracks are filled with water, whose value is 81, the anisotropy is 18.8 percent assuming a porosity of 2 percent. It is detectable.

3. Potentials in an anisotropic medium

In order to measure the dielectric anisotropy in a field efficiently, we should know optimum arrangement of electrodes. Here we will calculate potentials for quadruple electrode arrangement.

The basic equations are

$$\nabla\cdot\boldsymbol{D}=4\pi\rho,\ \boldsymbol{E}=-\nabla\psi,\ D_i=\sum_i\varepsilon_{ij}E_j$$

In the principal coordinate system of the anisotropic medium, the consitutive equation above becomes

$$D_i=\varepsilon_i E_i$$

At first we consider the case of a line source placed at the origin. We take the plane $z=0$ is vertical to the line. Then the potential equation becomes

$$\varepsilon^x\frac{\partial^2\psi}{\partial x^2}+\varepsilon^y\frac{\partial\psi}{\partial y^2}=4\pi\rho\delta(\boldsymbol{r})$$

Setting $x=\sqrt{\varepsilon^x}\,x'$ and $y=\sqrt{\varepsilon^y}\,y'$, we obtain a two-dimensional Poisson's equation;

$$\left(\frac{\partial^2}{\partial x'^2}+\frac{\partial^2}{\partial y'^2}\right)\psi=\frac{4\pi\rho}{\sqrt{\varepsilon^x\varepsilon^y}}\,\delta(x')\delta(y')\equiv4\pi\rho'\delta(\boldsymbol{r})$$

The solution is

$$\psi(\boldsymbol{r})=\rho'\ln r'=\frac{\rho}{2\sqrt{\varepsilon^x\varepsilon^y}}\ln\left(\frac{x^2}{\varepsilon^x}+\frac{y^2}{\varepsilon^y}\right)$$

Let the coordinates of a positive and negative source be (x_0, y_0) and $(-x_0, -y_0)$, respectively. The potential at an arbitary point (x, y) is

$$\psi(x, y)=\frac{\rho}{2\sqrt{\varepsilon^x\varepsilon^y}}\ln\frac{\varepsilon^y(x-x_0)^2+\varepsilon^x(y-y_0)^2}{\varepsilon^y(x+x_0)^2+\varepsilon^x(y+y_0)^2}$$

Then, the potential difference between two measurement points (x_1, y_1) and (x_2, y_2) is

$$\begin{aligned}\Delta\psi&=\psi(x_1, y_1)-\psi(x_2, y_2)\\&=\frac{\rho}{2\sqrt{\varepsilon^x\varepsilon^y}}\ln\left[\frac{\varepsilon^y(x_1-x_0)^2+\varepsilon^x(y_1-y_0)^2}{\varepsilon^y(x_1+x_0)^2+\varepsilon^x(y_1+y_0)^2}\cdot\frac{\varepsilon^y(x_2+x_0)^2+\varepsilon^x(y_2+y_0)^2}{\varepsilon^y(x_2-x_0)^2+\varepsilon^x(y_2-y_0)^2}\right]\end{aligned}$$

Let us examine the line arrangement case, that is, four electrodes are arranged in a line;

$$
\begin{aligned}
&\text{source 1} && S_1=(x_0,\ y_0)=(x_0,\ ax_0)\\
&\text{source 2} && S_2=(-x_0,\ -ax_0)\\
&\text{measurement point 1} && O_1=(x_1,\ y_1)=(bx_0,\ abx_0)\\
&\text{measurement point 2} && O_2=(x_2,\ y_2)=(-bx_0,\ -abx_0)
\end{aligned}
$$

where a is tangent of the angle between the line and the principal axis x and b is a simple constant. The potential difference is

$$
\begin{aligned}
\Delta\phi &= \frac{\rho}{2\sqrt{\varepsilon^x\varepsilon^y}}\ln\left[\frac{\varepsilon^y(b-1)^2+\varepsilon^x(b-1)^2a^2}{\varepsilon^y(b+1)^2+\varepsilon^x(b+1)^2a^2}\cdot\right.\\
&\left.\frac{\varepsilon^y(-b+1)^2+\varepsilon^x(-b+1)^2a^2}{\varepsilon^y(-b-1)^2+\varepsilon^x(-b-1)^2a^2}\right]\\
&= \frac{2\rho}{\sqrt{\varepsilon^x\varepsilon^y}}\ln\frac{b-1}{b+1}
\end{aligned}
$$

The last expression does not depend on a, that is, it is independent of the direction of measurement line. It can be seen that we cannt detect any anisotropy by this arrangement of electrodes.

If the source electrodes and measurement electrodes are not in line but in a plane, the potential difference depends on the direction of the line connecting two source electrodes in a complex way. Therefore, on top of the difficulty of setting this arrangement in a real sutuation, this arrangement is considered to be inadequate.

In actual situations, we may use a drilled hole in the ground. The simplest way of arranging the line electrodes is to attach them to the wall parallel to the axis of the hole. Then the problem is reduced to one in which four electrodes are on a circle with a unit radius. Consider a point whose azimuth (measured from x-axis) is θ. Let us set source electrodes at

$$
\begin{aligned}
(x_{01},\ y_{01}) &= (\cos(\theta-\beta),\ \sin(\theta-\beta)),\\
(x_{02},\ y_{02}) &= (\cos(\theta+\beta),\ \sin(\theta+\beta))
\end{aligned}
$$

and measurement electrodes at

$$
\begin{aligned}
(x_1,\ y_1) &= (\cos(\theta-\beta-\alpha),\ \sin(\theta-\beta-\alpha)),\\
(x_2,\ y_2) &= (\cos(\theta+\beta+\alpha),\ \sin(\theta+\beta+\alpha))
\end{aligned}
$$

where α and β are constant angles. If we ignore the fact that, inside the hole, the medium will be isotropic, we can calculated the potential difference;

$$\Delta\psi=\frac{\rho}{2\sqrt{\varepsilon^x\varepsilon^y}}\ln\left[\frac{\varepsilon^y(x_1-x_{01})^2+\varepsilon^x(y_1-y_{01})^2}{\varepsilon^y(x_1-x_{02})^2+\varepsilon^x(y_1-y_{02})^2}\cdot\frac{\varepsilon^y(x_2-x_{02})^2+\varepsilon^x(y_2-y_{02})^2}{\varepsilon^y(x_2-x_{01})^2+\varepsilon^x(y_2-y_{01})^2}\right]$$

$$=\frac{\rho}{2\sqrt{\varepsilon^x\varepsilon^y}}\ln\left[\frac{\sin^4\frac{\alpha}{2}}{\sin^4(\beta+\frac{\alpha}{2})}\cdot\frac{\varepsilon^y\sin^2(\theta-\beta-\frac{\alpha}{2})+\varepsilon^x\cos^2(\theta-\beta-\frac{\alpha}{2})}{\varepsilon^y\sin^2(\theta-\frac{\alpha}{2})+\varepsilon^x\cos^2(\theta-\frac{\alpha}{2})}\cdot\frac{\varepsilon^y\sin^2(\theta+\beta+\frac{\alpha}{2})+\varepsilon^x\cos^2(\theta+\beta+\frac{\alpha}{2})}{\varepsilon^y\sin^2(\theta+\frac{\alpha}{2})+\varepsilon^x\cos^2(\theta+\frac{\alpha}{2})}\right]$$

It can be seen from the expression that the potential difference depends on the azimuth in a rather complex way. We can conclude that the arrangements of line electrodes are not adequate to measure the anisotropy.

Next, we consider point electrodes. Employing the same procedure used in the two dimensional case, we oatain a potential produced by a point source placed at the origin;

$$\psi=\frac{\rho'}{r'}=\frac{\rho}{\sqrt{\varepsilon^x\varepsilon^y\varepsilon^z}}\frac{1}{\sqrt{\frac{x^2}{\varepsilon^x}+\frac{y^2}{\varepsilon^y}+\frac{z^2}{\varepsilon^z}}}$$

We will consider only the case in which the electrodes are arranged in a line as in the two dimensional case discussed first: The electrodes are in a plane of $z=0$. Then the above potential becomes

$$\psi(x,\ y,\ o)=\frac{\rho}{\sqrt{\varepsilon^z}}\frac{1}{\sqrt{\varepsilon^y x^2+\varepsilon^x y^2}}$$

The potential difference is calculated to be

$$\Delta\rho=\frac{\rho}{\sqrt{\varepsilon^z}}\left[\frac{1}{\sqrt{\varepsilon^y(bx_0-x_0)^2+\varepsilon^x(abx_0-ax_0)^2}}-\frac{1}{\sqrt{\varepsilon^y(bx_0+x_0)^2+\varepsilon^x(abx_0+ax_0)^2}}-\frac{1}{\sqrt{\varepsilon^y(bx_0+x_0)^2+\varepsilon^x(abx_0+ax_0)^2}}+\frac{1}{\sqrt{\varepsilon^y(bx_0-x_0)^2+\varepsilon^x(abx_0-ax_0)^2}}\right]$$

$$=\frac{4\rho}{\sqrt{\varepsilon^z}\ x_0(b^2-1)}\frac{1}{\sqrt{\varepsilon^y+a^2\varepsilon^x}}$$

where $(x_0,\ y_0)$, a, and b respectively have the same meanings as before. The dependence of the potential difference on the line direction is simple. The ratio of the potential difference to its maximum, which is attained when the line direction coincides with the maximum principal axis of the anisotropy, varies from zero to one monotonically as the line direction varies.

Let us consider the the case in which the measurement is made inside a drilled hole as above. It can be assumed that one of the principal axis of the anisotropy coincides with the axis of the hole, which is taken as the z-axis. This assumption is usually made in the hydrofracturing tests (eg. Tsukahara et al., 1978). Attaching the electrodes horizontally (if the hole is drilled vertically) on the wall of the hole, we can measure a relative anisotropy. The word relative is used because, in the derivation of the last equation, we ignored the fact that medium inside the hole will be isotropic. If we can measure the anisotropy for core samples with the same arrangement of electrodes under various stresses, we will know the true anisotropy and thus the tectonic stress.

4. Practical problems

There arises a severe problem when we apply the above theory to in situ stress measurements: In order that the anisotropy be detectable, the cracks must be filled with water. As a result, rocks become semi-conducting materials. Then we have to take into account the conductivity of the medium as well as the dielectric property. We will consider this problem in details. For this, we follow the argument given by Landau and Lifshitz (1962).

The microscopic Maxwell's equations are

$$\nabla \cdot \boldsymbol{e} = 4\pi\rho$$
$$\nabla \cdot \boldsymbol{h} = 0$$
$$\nabla \times \boldsymbol{e} = -\frac{1}{c}\frac{\partial \boldsymbol{h}}{\partial t}$$
$$\nabla \times \boldsymbol{h} = \frac{1}{c}\frac{\partial \boldsymbol{e}}{\partial t} + \frac{4\pi}{c}\boldsymbol{i}, \quad \boldsymbol{i} = \rho \boldsymbol{v}$$

where small letters $\boldsymbol{h}$ and $\boldsymbol{e}$ are the microscopic magnetic and electric fields, respectively. ρ is charge density, $\boldsymbol{v}$ is the velocity of the charge, and c is the velocity of light in vacuum.

As usual we take the average of these equations. The first three equations take the usual form, but for the last one we have to average it in the following manner. At first we divide the true current into three parts; the part which contributes to the macroscopic current $\overline{\rho \boldsymbol{v}}_{macro} = \boldsymbol{j}$, the part which contributes to the magnetic moment $\overline{\rho \boldsymbol{v}_{\mathrm{M}}}=c\nabla\times M$, and the part which contributes to the electric polarization $\overline{\rho \boldsymbol{v}_{p}}=\frac{\partial}{\partial t}\int \rho r dV=\frac{\partial \boldsymbol{P}}{\partial t}$. Then, substituting these terms into the last equation, we obtain the basic equations for a semi-conducting materials;

$$\nabla\cdot\boldsymbol{D}=0$$

$$\nabla\cdot\boldsymbol{B}=0$$

$$\nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\begin{aligned}\nabla\times\boldsymbol{B}+4\pi\nabla\times\boldsymbol{M}&=\frac{4\pi}{c}\boldsymbol{j}+\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}+\frac{4\pi}{c}\frac{\partial \boldsymbol{P}}{\partial t}=\nabla\times\boldsymbol{H}\\&=\frac{4\pi}{c}\sigma\boldsymbol{E}+\frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}\\&=\frac{4\pi}{c}\sigma\boldsymbol{E}+\frac{\varepsilon_0}{c}\frac{\partial \boldsymbol{E}}{\partial t}\end{aligned}$$

where $\boldsymbol{B}$ is the magnetic flux density and ε_0 is the intrinsic part of the dielectric constant. Here we used two consitutive equations for the electric field-current and for the electric field-electric flux density.

For a simple harmonic oscillation whose angular frequency is ω, the rotation of the magnetic field has the form;

$$\nabla\times\boldsymbol{H}=-\frac{i\omega}{c}\left(\varepsilon_0+\frac{4\pi\sigma}{\omega}i\right)\boldsymbol{E}=-\frac{i\omega}{c}\varepsilon(\omega)\boldsymbol{E}$$

It can be seen from this equation that the angular frequency ω determines whether a material behaves like conductor or like dielectrics; if $\omega\gg 4\pi\sigma/\varepsilon_0$, then, it behaves like dielectrics and vice versa.

Let us estimate the value ω_0 above which rocks behave like dielectrics. The resistivities of wet rocks ranges from 10^2 to 10^6 in kms units (Beblo, 1982). This means that the conductivity ranges from 10^6 to 10^{10} in Gaussian units. Therefore, for most rocks containing water, ω_0 should be higher than $10^6\sim10^{10}$ depending on the type of rock but less than the frequency of 10^{11}, at which the dispersion will occur in water (Landau and Lifshitz, 1962).

Next we will examine the skin effect, which is necessary because of the use of very high frequency obtained above. From the equations for the rotations of the electric and magnetic fields, we obtain a wave equation for the electric field;

$$\nabla\boldsymbol{E}=\frac{\mu\varepsilon(\omega)}{c}\frac{\partial^2}{\partial t^2}\boldsymbol{E},\ \boldsymbol{B}=\mu\boldsymbol{H}$$

which has a plane wave solution of the form;

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t}$$

where the wave vector $\boldsymbol{k}$ must have a complex form of $\boldsymbol{k}=\boldsymbol{k}'+i\boldsymbol{k}''$ where $\boldsymbol{k}'$ and $\boldsymbol{k}''$ are real. From the ω-dependent dielectric constant $\varepsilon(\omega)$, we can calculate the imaginary part of the wave vector, assuming the magnetic permeability μ is 1. That is, from the wave equation, we obtain

$$k^2=\left(\frac{\omega}{c}\right)^2\mu\varepsilon(\omega)$$

Then solving this equation with $\varepsilon(\omega)=\varepsilon_0+\frac{4\pi\sigma}{\omega}i$, we obtain

$$|\boldsymbol{k}''|=k''=\frac{\omega}{c}\sqrt{2\varepsilon_0+2\sqrt{\varepsilon_0^2+\left(\frac{2\pi\sigma}{\omega}\right)^2}}$$

The skin depth, d, where the amplitude of the electric field attenutes to 1/e, is

$$d=\frac{1}{k''}=\frac{c}{\omega}\frac{1}{\sqrt{2\varepsilon_0+2\sqrt{\varepsilon_0^2+\left(\frac{2\pi\sigma}{\omega}\right)^2}}}$$

Incidentally, for $\omega\ll\omega_0$, ie., the metallic case, the skin depth is reduced to the usual expression;

$$d=\sqrt{\frac{c^2}{2\pi\sigma\omega}}$$

For the ω_0 obtained above, we can neglect the second term in the double square. Then we find that the skin depth ranges from a hundred meter to a centimeter. This means that the measured dielectric anisotropy reflects the state of cracks around a drilled holl with sufficient width.

5. Conclusion

There is a strong possibility that in-situ stress can be measured by measuring the dielectric anisortopy. The anisotropy can be measured by point quadruple electrodes attached to the wall of a drilled hole with a measuring frequency of 10^6-10^{10} Hz.

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