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Dielectric Anisotropy of Rocks due to Aligned Cracks

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Abstract

Dielectric anisotropy due to aligned cracks is examined. A theoretical model is constructed, assuming dilute distribution of the ellipsoidal cracks embedded in an isotropic matrix. In the principal coordinate system of the ellipsoid, the average dielectric tensor for penny shaped cracks and the matrix is found to have the form:

$$\varepsilon_{xx}=\varepsilon_{yy}=(1-c)\varepsilon^{e}+c\varepsilon^{i},\ \varepsilon_{zz}=\varepsilon^{e}$$

while the other components are zero. In these equations, ε^e and ε^i are the dielectric constants of the matrix and the material filling the cracks, respectively, and c is the porosity. The dielectric anisotropy is defined as the ratio of the xx- or yy-component to the zz-component and depends linearly on the porosity. Using published data of dielectric constants of dry rocks, we find that many kinds of rocks will show the dielectric anisotropy of eight percent to more than thirty percent when their cracks are filled with water at the assumed porosity of 2 percent. On the other hand, the anisotropy is negligibly small in the case of empty cracks.

1. Introduction

In general, the physical properties of rocks containing aligned cracks become anisotropic. In the case of elastic anisotropy, Shimizu (1984) deduced distributions of orientation and the aspect ratio of cracks by analyzing data on elastic wave velocities. Maeda and Shimizu (1983) showed that applied stress (or tectonic stress in the crust) can be deduced by measuring the anisotropy of elastic wave velocity. It is expected by measuring other kinds of anisotropy that we can deduce the stress state which determines crack state. The dielectric property is one such possibility. The dielectric constant has been used for geophysical investigations since Pascal's work (1964). Systems consisting of sand or clay mixed with oil or water have been frequently investigated (eg. Singh and Rankin, 1986). Mendelson and Cohen (1982) investigated the electric properties of such a multiphase system theoretically. In these works, oil or water is considered as a matrix because of the large amount of the liquid phase and the isotropic character of the systems. The anisotropy caused by anisotropic grains is only taken into account in order to interpret the diversity of a parameter in a law similar to Archie's. In rocks, the liquid phase is confined in cracks whose total volume is very small. Many rocks can be considered nearly dielectrically isotropic if there are no cracks. The main cause of the anisotropy, if it exists, may be cracks which are themselves isotropic. There seems to have been no investigation of this sort of anisotropy. In this report, we theoretically investigate the dielectric anisotropy caused by aligned cracks in an isotropic matrix and examine whether or not it is a measurable quantity in common rocks.

2. Model

We consider that rock cosists of matrix and cracks. For simplicity, each part is considered to be isotropic and homogeneous and has a dielectric constant of ε^e or ε^i, respectively. The shape of cracks is approximated by an ellipsoid. The average electric flux density $\bar{\boldsymbol{D}}$ in the medium is

$$\begin{aligned}\bar{\boldsymbol{D}} &= \frac{1}{V}\int_V \boldsymbol{D}(\boldsymbol{r})d\boldsymbol{r} \\ &= \frac{1}{V}\int_{Ve}\varepsilon^e \boldsymbol{E}^e(\boldsymbol{r})d\boldsymbol{r} + \frac{1}{V}\int_{Vi}\varepsilon^i \boldsymbol{E}^i(\boldsymbol{r})d\boldsymbol{r} \\ &= (1-c)\frac{\varepsilon^e}{V^e}\int_{Ve}\boldsymbol{E}^e(\boldsymbol{r})d\boldsymbol{r} + c\frac{\varepsilon^e}{V^i}\int_{Vi}\boldsymbol{E}^i(\boldsymbol{r})d\boldsymbol{r} \\ &\equiv (1-c)\varepsilon^e\bar{\boldsymbol{E}} + c\frac{\varepsilon^i}{V^i}\int_{Vi}\boldsymbol{E}^i(\boldsymbol{r})d\boldsymbol{r}\end{aligned}$$

where $\boldsymbol{E}^e$ and $\boldsymbol{E}^i$ are the electric fields in the matrix and cracks, respectively. By assumption, ε^e and ε^i are independent of the coordinates in each defined domain. It is assumed that all cracks have the same dielectric constant ε^i. V^e is the volume of matrix, V^i is the total volume of cracks and $V = V^i + V^e$. In the following, it is assumed that $V^i \ll V$, that is, that the crack porosity $c = V^i/V$ of rock is sufficiently low. This assumption seems to hold for many rocks. By this assumption, the electric interaction between cracks can be neglected. Then, the present problem is reduced to the one of calculating the electric field $\boldsymbol{E}^i$ inside an ellipsoid placed in the uniform external field $\bar{\boldsymbol{E}}$.

This problem has already been solved (Landau and Lifshitz, 1962). The relation between the m-th component of the external field $\bar{\boldsymbol{E}}$ and the n-th component of the internal one $\boldsymbol{E}^i$ is

$$\bar{\boldsymbol{E}}_m=\sum_n S_{mn}\boldsymbol{E}_n^j$$

where S_{mn} is a second rank tensor ;

$$S_{mn}=\delta_{mn}+n_{mn}\left(\frac{\varepsilon^i}{\varepsilon^e}-1\right).$$

n_{mn} is a tessor of which the principal components are the depolarization factors, given as

$$n^x=\frac{abc}{2}\int_0^\infty\frac{ds}{(s+a^2)R_s},\quad n^y=\frac{abc}{2}\int_0^\infty\frac{ds}{(s+b^2)R_s},$$
$$n^z=\frac{abc}{2}\int_0^\infty\frac{ds}{(s+c^2)R_s},$$
$$R_s=\sqrt{(s+a^2)(s+b^2)(s+c^2)}$$

for an ellipsoid having principal axis lengths a, b, and c in x, y, and z directions, respectively. Then, the average electric flux density is expressed using the inverse tensor of S_{mn}, S_{mn}^{-1} ;

$$\bar{D}_m=\sum_n[(1-c)\varepsilon^e\delta_{mn}+c\varepsilon^i S_{mn}^{-1}]\bar{E}_n$$
$$\equiv\sum_n\bar{\varepsilon}_{mn}\bar{E}_n$$

The coefficient $\bar{\varepsilon}_{mn}$ is the dielectric tensor of the medium containing ellipsoidal cracks.

3. Discussion and conclusion

Here we estimate the possible degree of anisotropy due to the aligned cracks. The porosity defined above must be regarded as the ratio of the volume of the cracks contributing to the anisotropy to the volume of the medium. This means that the volume of randomly oriented cracks is neglected. In the coordinate system in which the the principal axes of the ellipsoid (crack) coincide with the coordinate axes, the average dielectric tensor has the form of

$$\bar{\varepsilon}_{lm}=\left[(1-c)\varepsilon^e+\frac{c\ \varepsilon^i}{1+n^l}\left(\frac{\varepsilon^i}{\varepsilon^e-1}\right)\right]\delta_{lm}$$

where suffix l and m take x, y, and z.

For simplicity, we consider a penny shape crack, that is $a=b\gg c$. Then, the depolarization factors become

$$n^x=n^y\approx 0,\ n^z=1$$

Table 1. Dielectric constants of rocks and possible anisotropies. The dielectric constants are selected from the table given by Beblo (1982) according to two criteria ; the measurement is made on dry rocks and the measuring frequencies are above 10^5 Hz. The anisotropies presented in percentages are calcuated for the case of water filled cracks, using the smallest value of each rock type and assuming a porosity of 2 percent.

Rock		frequency Hz	Dielectric constant	possible anisotropy (%)
Anorthosite		10^5-10^7	10.9−9.03	16
Basalt	1	5.10^5	15.6	8
	2	5.10^5	10.3	14
Diabase	1	10^2-10^7	23.5−8.5	17
	2	10^2-10^7	13.4−7.76	19
	3	5.10^5	11.6	12
Diorite	1	10^2-10^7	17.0−8.57	17
	2	10^5-10^7	6.3−5.9	25
	3	10^4-10^7	11.5−8.5	17
Dunite		10^2-10^7	10.0−7.18	21
Gabbro	1	10^2-10^7	15.0−8.78	16
	2	10^2-10^7	15.6−7.30	20
Granite	1	$>10^6$	4.80−18.9	32
	2	10^2-10^7	9.63−5.23	29
	3	10^2-10^7	8.47−6.68	22
	4	5.10^5	4.74−5.42	32
Peridotite	1	10^5-10^7	18.8−15.7	8
	2	5.10^5	12.1	11
Augite porphyry		10^5-10^7	12.6−9.5	15
Syenite		5.10^5	6.83	22
Dolomite	1	10^2-10^7	11.9−7.72	19
	2	10^2-10^7	8.6−8.0	18
Kaolinite		10^2-10^7	7.65−4.49	34
Limestone	1	10^2-10^7	10.4−8.56	17
	2	10^2-10^7	15.4−9.22	16
Novaculite		10^2-10^7	5.93−4.86	31
Sandstone		$>10^6$	4.69−4.99	30
Arkose sandstone		10^2-10^7	5.94−5.31	29
Jurassic sandstone		5.10^5	13.96−4.66	33
Shale	1	10^2-10^6	7.0−4.0	39
	2	10^2-10^6	10.0−9.5	15
Amphibolite		10^5-10^7	8.9−7.9	19
Gneiss		10^2-10^7	9.73−8.07	18
Marble		10^3-10^7	9.0−8.9	16
Quartzite	1	5.10^5	4.36	35
	2	5.10^5	4.85	31
Hornblend schist		10^2-10^7	10.3−8.88	16
Talc schist		10^2-10^7	31.5−7.57	19
Slate	1	$50-5.10^7$	34.0−7.5	20
	2	$50-5.10^7$	10.0−9.0	16

Substituting these n's into the average dielectric tensor, we obtain

$$\bar{\varepsilon}=\begin{bmatrix} (1-c)\varepsilon^e+c\varepsilon^i & 0 & 0 \\ 0 & (1-c)\varepsilon^e+c\varepsilon^i & 0 \\ 0 & 0 & \varepsilon^e \end{bmatrix}$$

The degree of the anisotropy A is the ratio of the xx-component or the yy-component to the zz-component, which is the same as that of the matrix ;

$$A=1+c\left(\frac{\varepsilon^i}{\varepsilon^e}-1\right)$$

It is seen that the anisotropy A is linealy dependent on the porosity c.

In Table 1, dielectric constants of various rocks are listed. These we slected from the table edited by Beblo (1982) according to two criteria : (1) the measurement was made on a dry sample and (2) the measured frequency range was above 10^5 Hz. The first criterion means that the values can be used as the dielectric constant of matrix. The reason for the second criterion will be mentioned below.

Roughly speaking, the dielectric constant of rocks, ε^e, is nearly 10 in Gaussian units. When cracks are empty, that is, $\varepsilon^i=1$ and the porosity is on the order of several percent or less, the anisotropy A is almost 1, which means that no anisotropy will be observed. On the other hand, when cracks are filled with water, that is, $\varepsilon^i=81$, a fair amount of anisotropy appears. In the table, values of A, presented in percentages, which are calculated for rocks containing cracks filled with water assuming a porosity of 2 percent, are listed in the last column. In the calculation for each rock, the smallest walue of the dielectric constant obtained at highest frequency is used.

In general, the dielectric constant decreases with increasing frequency. The use of the smaller dielectric constant results in a larger value for anisotropy when the cracks are filled with water. Insofar as we are interested in the anisotropy, we should use higher frequencies. This is the reason for the second criterion mentioned above.

It can be seen from the table and from the linear dependence of the anisotropy on porosity that most rocks show considerable aninotropy even when the porosity is one percent. Although, taking into account the definition of the porosity, one percent might still be considered large for many rocks not under differential stress, it is not necessarily unnatural for rocks under differential stress.

In conclusion, it can be said that, in a field where rocks are subjected to tectonic stress and their cracks are filled with water, dielectric anisotropy will be detected.

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