An Estimate of the Velocity and Stress in the Deep Ocean Bottom Boundary Layer

Sei-ichi Kanari, Chikashi Kobayashi

Department of Geophysics, Faculty of Science, Hokkaido University, Sapporo 060, Japan

and

Takayuki Ishikawa

Institute of Low Temperature Science, Hokkaido University, Sapporo 060, Japan

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Abstract

In order to estimate the near-bottom velocity and the boundary shear stress, a one-dimensional unsteady boundary layer model was applied for the observed time series of the 6-level current observation conducted from Oct. 26 through to Nov. 3, 1988 at the site of the north western end of the Kyushu-Palau Ridge. Current meters were deployed 6, 10.5, 18.5, 39.0, 75.0 and 151.0 meters respectively up from the sea floor of 3519 m depth for seven days. The applied model, developed by McLean and Yean, predicts fairly well the near-bottom velocity field and stress, with the adjusted roughness of $z_0=0.025$ cm. The estimated time-averaged drag coefficient is $4.5 \times 10^{-3}$. Thickness of the logarithmic layer is inferred as the order of 1 meter, which overlies the viscous sublayer of a thickness of about 1 cm.

1. Introduction

In recent observations of salinity and potential temperature taken at the bottom of the deep ocean, profiles often exhibit a well-mixed region just above the bottom, which is often referred to as the benthic boundary layer (Weatherly and Kelley, 1982; Armi and Millard, 1976). The thickness of the benthic boundary layer (hereafter referred to as BBL) varies in time and space, and the thickness extends from ten meters to a hundred meters. Armi and Millard (1976) have correlated the thickness of the BBL with the velocity observed in a water column above the layer, and showed that the BBL thickness in the Hatteras Abyssal Plain extending above the bottom to about 6 times the turbu-
lent Ekman layer height.

Weatherly and Kelley (1982) have examined near-bottom CTD data and current meter records obtained on the continental rise south of Nova Scotia, and found that the bottom layer of thickness of about 60 m is part of a continuous ribbon or filament with a width of about 100 km (they referred to this feature as the Cold Filament) found on the continental margin near its base, and the Cold Filament is a distinct layer of water flowing along the bottom with a thickness comparable to that associated with bottom mixed layers.

Our main interest is in understanding of the physical process associated with the variations of the thickness of BBL. We are also expecting that the Cold Filament theory is one of the major processes associated with the variations of bottom mixed layers. In order to clarify the physical processes and its mechanism near the deep ocean bottom as mentioned above, two sets of 6-levels current meter system have been moored at different sites. One is at the site of 3519 m depths of north western end of Kyushu- Palau Ridge (referred as KPR in Fig. 1.), and the other is at the site of 4,540 m depths of north western end of

Fig. 1. Topographic map of the north western part of Shikoku Basin. Locations of mooring site are indicated by the solid circles. Depths are in meters.
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Abyssal Plain in Shikoku Basin (referred as EAP in Fig. 1.) as shown in Fig. 1.
The system at KPR was moored between 26 October 1988 and 3 November 1988
as a preliminary observation, and the system at EAP was moored on 19
September 1989 and is under operation (this system is expected to be retrieved
in September 1990).

In the present study, preliminary current data at KPR was examined using
a one-dimensional, time-dependent model as described by McLean and Ye"an

2. Description of the mooring system

For the purpose of near-bottom observation, compact-sized electro-magneti"c
current meters (ACM-5000S), which has been newly developed by Alec
Electronics Co., were adopted. The Current meter, length 50 cm, 9 cm diameter,
15 kg weight in water has a maximum operating depth of 5,000 m. The
instrument has a two axes electro-magnetic type velocity sensor and a tempera-
ture sensor with a range from -2 to +12°C with an accuracy of ±0.01°C
(resolution of 0.005°C), which could possibly measure weak temperature changes
in the deep ocean.

In the current observation at the site of KPR, every data sampling scheme
used was 100 samples in a burst every 10 minutes. As shown in Fig. 2, six
current meters were deployed vertically with an equal interval in logarithmic
vertical scale. The current meter sensors were positioned at 6 m, 10.5 m, 18.5
m, 39 m, 75 m and 151 m respectively above the bottom.

3. Current meter data

Observed time series of current vectors are shown in Fig. 3. The time
series of current vector at respective layer is dominated with $M_2$ component of
tides and inertial period oscillation. The maximum velocity of composed
current vector of those two major components reaches about 15 cm/s, while the
mean velocity is less than 2 cm/s to the NE.

Fig. 4 shows the mean velocity profile. The magnitude of the flow is almost
constant above the height of 39 m, but gradually decreases below it to the
bottom. The dashed line in Fig. 4 represents a presumed profile, and it suggests
that the thickness of BBL is less than 39 m.

Temperature data of respective current meter showed a constant tempera-
ture of 1.55°C through the whole layer, and we concluded that there was no
significant thermal stratification between the toplayer and bottom.

The thickness of bottom Ekman layer is given by a friction depth when eddy viscosity is constant. Wimbush and Munk (1971), and Weatherly (1972, 1975) have mentioned that the thickness of turbulent Ekman layer can be given by the
friction depth derived by Ekman theory. For the depth and time-dependent eddy viscosity, the friction depth can not be expected to give a reasonable thickness of the bottom boundary layer. Based on the laboratory experiments, Caldwell et al. (1972), and also Howroyd and Slawson (1975) have presented the thickness of turbulent bottom Ekman layer $h_e$ as follows,
where, \( u_* \) is the friction velocity, and \( f = 2\omega \sin \varphi \) is Coriolis parameter. Biscaye and Eittreim (1974) have empirically estimated the friction velocity as \( 1/30 \) of a representative velocity.

In the present case, replacing the root mean square of the velocity in the

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**Fig. 4.** The vertical profile of time averaged mean flow. The horizontal lines show the probable error of current measurement. The dashed line shows the presumed mean flow profile.

**Fig. 5.** Progressive vector diagrams based on the lower three current meters.
present time series as a representative velocity, the thickness of the bottom
Ekman boundary layer is estimated as 29 m, which is not quite inconsistent with
the boundary height inferred from the mean profile as given in Fig. 4.

As seen in Fig. 5, the progressive vector of the flow of lower three layers
show a tendency of anticlockwise bearing of flow direction with the decreasing
height. The bearing angle of the progressive vectors between 6 m-layer and
10.5 m-layer is about 6° on an average.

4. Bottom boundary layer model

Assuming a horizontally uniform flow, linear balance of the horizontal flow
in the \( x, iy, z \) coordinate can be expressed by the following form,

\[
\partial_t q(t, z) + i f q(t, z) = \rho^{-1} (\partial_n P(t, z) + \partial_z \tau(t, z))
\]

where, \( q = u + iv \), \( \tau = \tau_x + i \tau_y \), \( \partial_m = \partial / \partial_m \) is an operator of partial derivative, \( \partial_n P = \partial_x P + i \partial_y P \), and \( i = \sqrt{-1} \).

Further, assuming a quasi-geostrophic balance outside the BBL, the current
field above the boundary can be given by

\[
\partial_t q_G + i f q_G = -\rho^{-1} \partial_n P
\]

where, \( q_G = v_G + iv_G \) is a quasi-geostrophic current.

Eliminating the pressure term using (2) and (3), we have

\[
\partial_z \tau(t, z) = (\partial_t + f) (q_G(t, h) - q(t, z))
\]

The shear stress in a flow field with eddy viscosity \( \nu \) is given by

\[
\tau(t, z) = \nu \partial_z q(t, z)
\]

Using (4) and (5), one can estimate the internal shear stress and flow field
alternately with the finite difference method, if the eddy coefficient is \textit{a priori}
given.

After McLean and Yean (1987), we adopted the form of eddy viscosity \( \nu_e \) for
channel flow, given by Long (1981), that is, \( \nu = \nu_e + \nu_m \) and

\[
\nu_e = k u_*(t, z) z \exp (-zf/0.6 u_*(t, z))
\]

where, \( \nu_m \) is the molecular viscosity, \( k \) is karman's constant given by 0.4, \( z \) is the
height above the bottom.

In the present model, we assumed the layer of 39 m is just above the BBL,
and adopted the time series of flow velocity at 39 m level as \( q_G(t, h) \).

Time integration from (4) to (5) with (6) was carried out by a finite
difference scheme in the frequency domain on the Fourier transformed quantities, $r^*(\omega, z)$, $q^*(\omega, z)$ and $q_c^*(\omega, z)$. The vertical grid size $\Delta z$ for the difference scheme was selected such that the vertical grid has equal intervals on the logarithmic scale. For this, the vertical axis $z$ was transformed to $\xi$ axis using the following form.

$$\xi = \ln(z/z_0)/\ln(h/z_0)$$

(7)

where, $z_0$ is a roughness parameter, $h$ is the thickness of BBL, assuming $h=39$ m. The transformation leads to

$$\Delta z = \lambda \xi \Delta \xi$$

(8)

where, $\lambda = 1/1.443 \ln(h/z_0)$.

Thus, the fourier transformed stress for the $j+\frac{1}{2}$ layer can be calculated with the stress at the level of $j$ and with the flow velocities $q_c^*(\omega, h)$ and $q^*(\omega, j+1)$ using the following finite difference form.

$$r^*(\omega, j+\frac{1}{2}) = r^*(\omega, j-\frac{1}{2}) + i(\omega+f)\Delta z \left\{ q^*(\omega, j) - q_c^*(\omega, h) \right\}$$

(9)

The level $j=1$ corresponds to the level just below the boundary, where the stress $r^*(\omega, 1)$ is assumed as $r^*(\omega, \frac{1}{2})=0$. The flow velocity $q^*(\omega, 2)$ can be given by the Fourier transform of the measured time series at the height of 18.5 m above the bottom.

The finite difference form of equation (5) for the grids of $j+1$ and $j+2$ gives the prediction of flow at $j+2$, that is

$$q(j+2) = q(j+1) - \Delta z \cdot \tau(j+1)/\nu(j+1)$$

(10)

Fig. 6. Relative change of the root-mean square error of velocity between the observed and estimated flow against the roughness parameter $z_0$. The vertical dashed line shows the optimum value of $z_0$, adopted in the calculation described in the text.
where, \( r(t, j + \frac{1}{2}) \) is the stress time series given by the inverse transform of eq. (9).

The boundary condition at the bottom becomes

\[
q(t, z) = 0 \text{ at } z = z_0
\]  

(11)

To carry out the above successive calculation, the roughness parameter \( z_0 \) must be previously determined. As a result of trials for various values of \( z_0 \), we found that the fraction of r.m.s. error between the observed time series of currents and estimated one becomes minimum at a certain \( z_0 \)-value, and we selected the value of \( z_0 = 0.025 \) cm with the minimum fraction as shown in Fig. 6.

5. Estimated flow and stress

The result of flow estimation at the height of 6 m is shown in the lower in Fig. 7 comparable to the observed vector time series in the same figure (upper). Fig. 8 shows the result of further estimates at the heights from 3 m to 9.375 cm, though there are no observed data to compare. The flow velocity gradually decreases as close to the bottom floor, but change of the bearing angle of the flow does not seem to be so significant.

An example of the estimated time series of stress components, and corresponding parameters at the height of 14 cm are shown in Fig. 9. The time series of drag coefficient \( C_d \) is calculated from the time series of absolute stress \(|T|\) divided by the r.m.s. quasi-geostrophic current velocity \( \langle q_c \rangle \). The time average of \( C_d \) is about 0.0045. However, the large amount of variability ranges

- Fig. 7. Time series of velocity vector at 6 m above the bottom. Observed velocity vector (a), and estimated (b).
Fig. 8. Estimated time series of current vector at 3 m, 1.5 m, 0.375 m, and 0.094 m above the bottom respectively.

from $10^{-4}$ to 0.008. McLean and Yeay (1987) have suggested that such a wild fluctuation of $C_d$ is attributable to the low flow velocity outside frictional layer.

The time series, denoted by $P_h$ which is the bottom line in the same figure shows the bearing angle between the flow vector and stress vector $-\tau$ rotated by 180°. Accordingly, the line of $P_h=0$ means that the vector of bottom stress acting in the opposing direction to the outside flow. Though the total time length of the opposed stress vectors is only about 40% of the time span at the height of 14 cm, the fraction increases to more than 90% within the viscous sublayer of thickness of about 1 cm, as shown in Fig. 10.

There can be seen intermittent large fluctuations of the deflection angle in the respective time series of $P_h$. The angle $P_h$ near $\pm 180^\circ$ means that the stress vector is acting in the same direction of outside flow, and that in such case, the stress locally accelerates the fluid on the sea floor. The duration time of such an intermittent fluctuation becomes shorter with the decreasing height from the bottom, and the feature seems to suggest the “Turbulent burst” event.

The vertical profiles of the time averaged viscosity, stress, friction velocity, and drag coefficient are shown in Fig. 11. As seen from the assumed form, the viscosity reaches its maximum (80 cm²/s) at 10 m above the bottom and tends to be its minimum (0.01 cm²/s) below it. The stress, friction velocity, and therefore the drag coefficient gradually increases from the upper boundary, and
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Fig. 9. Estimated time series of near bottom \((z=0.14 \text{ m})\) shear stress components \(T_x, T_y, \text{magnitude of stress } |T|, \text{friction velocity } u_*, \text{drag coefficient } C_d, \text{and the bearing angle of stress vector } \Phi \text{ against the quasi-geostrophic current vector outside the Ekman boundary layer. The angle } \Phi \text{ is measured with rotating the stress vector by } 180^\circ.\)

become almost constant below the height of 1 m, which corresponds to the constant stress bottom boundary layer with logarithmic velocity profile.

6. Viscous sublayer

As seen in Fig. 8, there remains a considerable flow velocity of 3 cm/s even at 9 cm above the bottom. The vertical profile of stress in Fig. 11 suggests a logarithmic profile of flow at the level below 1 m.

Estimated vertical structure of the fourier amplitude velocity for the three components of the inertial, the semidiurnal, and the mean are plotted in Fig. 12. As presumed in Fig. 11, each profile shows a logarithmic velocity distribution between the height of 3 m and 0.01 m above the bottom.

At the height from 0.01 m to \(z_o\), the flow velocity deviates from the line of
Fig. 10. Variations of the bearing angle $\Phi$ within the viscous sub-layer. At 0.37 mm above the bottom, the stress vector becomes to be in the opposed direction to the outside flow except that the large angle at the intermittent burstlike events of short time duration.

Fig. 11. Time averaged vertical profiles of turbulent parameters within the assumed Ekman boundary layer.

log-profile and approaches zero at $z_0$.

Chriss and Caldwell (1984) have discussed the observed profile of near bottom flow over the slope of continental shelf off Oregon Coast, and concluded that the thickness of the viscous sublayer of flow, deviating from log-profile, is about 1 cm.

Denoting the thickness of viscous sublayer as $\delta$, the velocity profile of logarithmic layer can be given by
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\[ u(z) = \frac{u_*}{k} \ln(z) + C \]  
(12)

where \( C \) is constant.

Assuming a constant viscosity (\( \nu = \nu_c \)) and friction velocity, the flow velocity in the viscous sublayer can be simply written with another constant \( C' \) as

\[ u(z) = \frac{u_*^2}{\nu_c} z + C' \]  
(13)

As the flow must be continuous at \( z = \delta \), then, from (12) and (13), we have

\[ C - C' = \frac{u_*^2}{\nu_c} \delta - \frac{u_*}{k} \ln(\delta) \]  
(14)

No flow condition at \( z = z_0 \) leads to

\[ C' = \frac{u_*^2}{\nu_c} z_0 \]  
(15)

From (12) to (15), the velocity profiles in respective layer can be given as follows.

Fig. 12. Vertical structure of the Fourier amplitude velocity for the inertial (1), the semidiurnal (2), and the mean (3) components respectively within a logarithmic layer and viscous sublayer.
\[ \bar{u}(z) = \frac{u_*}{k} \ln\left(\frac{z}{\delta}\right) + \frac{u_*^2}{\nu_c} (\delta - z_0) \; ; \; \delta \leq z \leq \delta_t \]  

(16)

and

\[ \bar{u}(z) = \frac{u_*^2}{\nu_c} (z - z_0) \; ; \; z_0 \leq z < \delta \]  

(17)

Where, \( \delta_t \) is the thickness of the log-layer.

Fitting eq. (17) to the mean flow in the sublayer in Fig. 12 leads to the constant viscosity of 1.2 cm\(^2\)/s.

7. Conclusion

Structure of the near bottom flow and boundary shear stress were estimated from the measured time series of currents, using a time dependent linear bottom boundary layer model described by McLean and Yean (1987). The thickness of bottom boundary layer inferred from the mean current profile nearly corresponds to that given by eq. (1).

Calculation by the present scheme is required to give the value of \( z_0 \) \textit{a priori}. The value of \( z_0 \) was determined by an iterative least square method, so that the r.m.s. of the difference between the estimated time series of the flow and measured one at the lowest height (6 m) becomes minimum at a certain \( z_0 \)-value. The determined \( z_0 \)-value was 0.025 cm.

Following McLean and Yean (1987), we adopted the Long's eddy viscosity formula with the length scale of 0.44 \( u_*/f \), but we used it in the form of Long's formula plus molecular viscosity.

The scheme described above is certainly available for estimation of the flow and stress in the bottom boundary layer (BBL) if the thickness of BBL has been properly determined. In the present data, the thickness, derived from mean velocity profile is consistent with the turbulent Ekman height, given by \( h = 0.4 u_*/f \).

Since the time averaged friction velocity is 0.4 cm/s the bottom surface the Rossby number \( u_*/fz_0 \) becomes \( 2.22 \times 10^5 \). The result of calculation shows that the log-layer thickness begins at 3 m above the bottom. The resultant thickness of log-layer almost coincides with the thickness given by \( \delta_t = 0.057 \; u_*/f \), accordingly the data point of the nondimensionalized log-layer thickness \( \delta_t/z_0 \) versus surface Rossby number \( u_*/fz_0 \) was very close to the line given by \( \delta_t = 0.057 \; u_*/f \) which was presented by Lavelle and Mofjeld (1983).

The eddy viscosity assumed in the calculation decreases almost exponential-
ly within the log-layer and becomes almost constant value, represented by additional molecular viscosity term, at the height of about 1 cm above the sea floor \((z = z_0)\). Below this height, the time averaged velocity profiles deviate significantly from the logarithmic profile. It is suggestive that the thin non-logarithmic layer with the thickness of about 1 cm corresponds to the viscous sublayer described by Chriss and Caldwell (1984).

In this paper, we could not deal with the variations of the boundary layer thickness. Such arguments will be given in future using the long-term mooring data at the site EAP which is now under operation.

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