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Natural Frequencies of Isotropic Rectangular Plates in Improved Accuracy

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Abstract

The objectives of this paper are to re-visit one of the most important vibration problems and to present comprehensive lists of accurate natural frequencies for isotropic thin rectangular plates. For this purpose, a simple yet very accurate analytical approach is described to study the free vibration of the plates. In numerical computations, convergence and comparison studies are conducted to collaborate accuracy of the present solution and to demonstrate the improvement of numerical solutions in percentage from previous standard results. Twenty-one tables are then provided to list the lowest six frequency parameters for all possible combinations of three typical edge conditions (clamped, simply supported and free edges). Each table is given in five significant figures for non-Levy type problem and in six significant figures for Levy type problem (i.e., plate with two opposite edges simply supported). These results are presented for five different aspect ratios to follow the same format of the previous standard reference.

Keywords: Free vibration, rectangular plate, natural frequency, accuracy, boundary condition

1. Introduction

The free vibration of isotropic thin rectangular plates has been one of the most important problems for a long period of time in mechanical vibration. References up to 1970 on vibrations of general plate shapes are summarized in the famous monograph [1], and the wide coverage for natural frequencies of isotropic rectangular plates was made in 1973 for all possible twenty-one combinations of boundary conditions and five aspect ratios [2]. The numerical results in this reference have been widely accepted as accurate and comprehensive data, where Ritz method with beam functions is used for sixteen combinations not having opposite edges simply supported and the exact solution is employed for five remaining combinations having two opposite edges simply supported. Because of reasonable accuracy, the results are cited as the standard and reliable data in the vibration design book [3].

Besides these classical references, this topic has a long history to date back to an early work by Young in 1950 [4], and since then many papers have appeared including a series of Gorman's work [5-7] by using a method of superposition. Those related studies up to the year of 1980 are listed in the author's work [8]. The development in the 1980's is summarized also in review papers [9,10]. More recently, two papers [11,12] presented their approaches to solve this problem.

When one extends the problem to broader sets of the boundary conditions, namely clamped, simply supported and free edges (denoted by capital letters, C, S and F, respectively), there are 3^4=81 combinations for a rectangular plate fixed in the space. Physically, however, some plates have the identical natural frequencies, for example, in C-S-F-F and C-F-F-S square plates (note that the first letter indicates Edge(1) and the rest in counter-clock wise direction). Polya counting theory is applied to solve for the exact number of combinations in Ref.[13], and recently this approach is extended to calculate the number of physically meaningful combinations for generally shaped plates [14].

In this paper, natural frequencies of isotropic rectangular plates are listed in comprehensive way to serve as a new standard data in mechanical design, although the frequencies of the plates with typical edges, such as totally clamped or cantilevered plates, were already solved. For this purpose, Ritz method is employed by using polynomial functions with boundary index, not by the beam functions [2]. With this, better accuracy is realized as resulting in lower values in the upper-bound solutions.

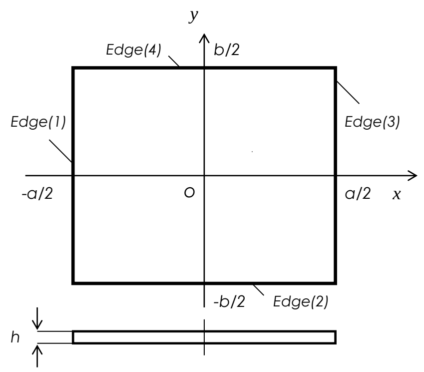

Figure 1. Rectangular plate in the coordinate system

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2. Outline of Analytical Method

A previous solution is used here as in [13] based on the method of Ritz within the classical thin plate theory. This analysis-based solution has a low computational cost and easiness in varying combination in boundary conditions, in contrast to numerical methods such as the finite element method. Figure 1 shows a geometry of rectangular plate and the coordinate system, and the dimension of the plate is given by $a \times b \times h$ (thickness).

The relation between stress and strain is written in

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (1)$$

where the elements are given for isotropic material by

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, Q_{12} = \nu Q_{11}, Q_{66} = G = \frac{E}{2(1+\nu)} \quad (2)$$

with E is Young's modulus, G is a shear modulus and ν is a Poisson's ratio. When Eq. (1) is integrated through the thickness after multiplying by a thickness coordinate z, one gets moment resultant

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3)$$

If one considers the small amplitude (linear) free vibration of a thin plate, the deflection w may be written by

$$w(x,y,t) = W(x,y)\sin\omega t \quad (4)$$

where W is the amplitude and ω is a radian frequency of the plate. Then, the maximum strain energy due to the bending is expressed by

$$U_{\max} = \frac{1}{2}\iint_A \{\kappa\}^T \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \{\kappa\}\, dA \quad (5)$$

where the D_{ij} are the bending stiffnesses and $\{\kappa\}$ is a curvature vector

$$\{\kappa\} = \left\{ -\frac{\partial^2 W}{\partial x^2} \quad -\frac{\partial^2 W}{\partial y^2} \quad -2\frac{\partial^2 W}{\partial x \partial y} \right\}^T \quad (6)$$

The maximum kinetic energy is given by

$$T_{\max} = \frac{1}{2}\rho h \omega^2 \iint_A W^2 dA \quad (7)$$

where ρ [kg/m^3] is the mass per unit volume.

For the sake of simplicity, non-dimensional quantities are introduced as

$$\xi = \frac{2x}{a}, \eta = \frac{2y}{b} \text{ (non-dimensional coordinates)},$$

$$\alpha = \frac{a}{b} \text{ (aspect ratio)},$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \text{ (reference stiffness)} \quad (8)$$

$$\Omega = \omega a^2 \sqrt{\frac{\rho h}{D}} \text{ (frequency parameter)}$$

The next step in the Ritz method is to assume that the amplitude as

$$W(\xi,\eta) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn} X_m(\xi) Y_n(\eta) \quad (9)$$

where A_{mn} are unknown coefficients, and $X_m(\xi)$, $Y_n(\eta)$ are the functions modified later so that any kinematical boundary conditions are satisfied at the edges.

After substituting Eq.(9) into the energies (5) and (7), the stationary value is obtained by

$$\frac{\partial}{\partial A_{\bar{m}\bar{n}}}(T_{\max} - U_{\max}) = 0 \quad (\bar{m} = 0,1,2,..;\bar{n} = 0,1,2,...) \quad (10)$$

Then the eigenvalue equation that contains the frequency parameter Ω is derived as

$$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left[d_{11} I^{(2200)} + \alpha^2 d_{12} (I^{(2002)} + I^{(0220)}) + \alpha^4 I^{(0022)} \right.$$

$$\left. +4\alpha^2 d_{66} I^{(1111)} - \Omega^2 I^{(0000)} \right]_{m\bar{m}n\bar{n}} \cdot A_{mn} = 0$$

$$(\bar{m} = 0,1,2,..;\bar{n} = 0,1,2,...) \quad (11)$$

where an integral I is the products

$$I_{m\bar{m}n\bar{n}}^{(pqrs)} = \phi_{m\bar{m}}^{(pq)} \cdot \phi_{n\bar{n}}^{(rs)} \quad (12)$$

of the two integrals defined by

$$\phi_{m\bar{m}}^{(pq)} = \int_{-1}^{1} \frac{\partial^{(p)} X_m}{\partial \xi^{(p)}} \frac{\partial^{(q)} X_{\bar{m}}}{\partial \xi^{(q)}} d\xi \quad (13)$$

Equation (11) is a set of linear simultaneous equations in terms of the coefficients A_{mn}, and the eigenvalues Ω may be extracted by using existing computer subroutines.

The analytical procedure developed thus far is a standard routine of the Ritz method, and the modification is explained next so as to incorporate arbitrary edge conditions into the amplitude $W(\xi,\eta)$. In the traditional approach, for example, using the beam functions for $X_m(\xi)$ and $Y_n(\eta)$, many different products of regular and hyper trigonometric functions exist for arbitrary conditions and it is difficult to make a unified subroutine to calculate all of the various kinds of integrals.

The present approach introduces a kind of polynomial

$$X_m(\xi) = \xi^m (\xi+1)^{B1} (\xi-1)^{B3}$$
$$Y_n(\eta) = \eta^n (\eta+1)^{B2} (\eta-1)^{B4} \quad (14)$$

where B_1, B_2, B_3 and B_4 are "boundary indices" [13,15,16] which are added to satisfy the kinematical boundary conditions and are used in such a way as B_i=0 for F (free edge), 1 for S (simply supported edge) and 2 for C (clamped edge). To the C-S-F-F plate, for instance, B_1=2, B_2=1 and B_3=B_4=0 are applied. With the boundary indices B_i's and Eqs.(14), the method of Ritz can accommodate arbitrary sets of the edge conditions, and the integrals (12) can be exactly evaluated.

3. Numerical Examples and Accuracy of Solution

3.1. Convergence and Comparison of the Solution

In numerical studies, the material constants are assumed for isotropic materials, and Young's modulus E and Poisson's ratio ν are included in the frequency parameters Ω in Eqs.(8). Poisson's ratio still affects values of the frequency parameters, and a constant of ν=0.3 is used throughout in the paper, except for the comparison in Table 2 with results obtained by another author who used ν=0.333 [5][7]. For thin plates, a value of plate thickness does not affect the frequency parameters, unlike in the analysis of shallow shells (panels) [15,16].

Table 1 presents convergence study of frequency parameters with increase of series terms in Eq.(9). Since the amplitude function satisfies kinematical conditions exactly, this Ritz method yields upper-bound solutions,

Table 1. Convergence characteristics of frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for square plates

Boundary Condition	mode				
$M\times N$	1	2	3	4	5
C-C-C-C					
6×6	35.986	73.395	73.395	108.22	131.78
8×8	35.985	73.394	73.394	108.22	131.58
10×10	35.985	73.394	73.394	108.22	131.58
12×12	35.985	73.394	73.394	108.22	131.58
14×14	35.985	73.394	73.394	108.22	131.58
S-S-S-S					
6×6	19.739	49.349	49.349	78.958	100.12
8×8	19.739	49.348	49.348	78.957	98.716
10×10	19.739	49.348	49.348	78.957	98.696
12×12	19.739	49.348	49.348	78.957	98.696
14×14	19.739	49.348	49.348	78.957	98.696
C-F-F-F					
6×6	3.4739	8.5128	21.313	27.461	30.980
8×8	3.4718	8.5091	21.292	27.200	30.965
10×10	3.4713	8.5077	21.288	27.199	30.960
12×12	3.4711	8.5071	21.286	27.199	30.958
14×14	3.4711	8.5029	21.285	27.198	30.945
C-C-F-F					
6×6	6.9247	23.924	26.592	47.671	62.746
8×8	6.9218	23.913	26.587	47.661	62.715
10×10	6.9201	23.908	26.586	47.657	62.710
12×12	6.9199	23.906	26.585	47.654	62.708
14×14	*	*	*	*	*
F-F-F-F					
6×6	13.469	19.726	24.541	35.288	35.288
8×8	13.468	19.596	24.271	34.801	34.801
10×10	13.468	19.596	24.270	34.801	34.801
12×12	13.468	19.596	24.270	34.801	34.801
14×14	13.467	19.596	24.271	34.801	34.801

*numerical instability occurs.

and therefore all the frequency parameters converge from above. For uniform edge conditions, such as C-C-C-C, S-S-S-S and F-F-F-F plates, the frequency parameters converge within five significant figures even for small number of terms as in 8×8 terms. When boundary conditions are mixed in one plate, however, the converge speed deteriorates as seen C-F-F-F plate. The worst result with respect to the convergence is given for C-C-F-F plate, where numerical instability occurs for the 14×14 solution, but these are still exceptions among twenty-one sets of edge conditions. For these two cases, the results in the following tables are obtained by using 12×12 solutions.

Table 2 is a comparison study with values of Gorman for C-C-C-C [6], C-F-F-F (ν=0.333) [5] and F-F-F-F plates (ν=0.333) [7]. The exact values can be obtained for S-S-S-S plate. In the all results presented in the table, good agreement is obtained with the present results. A method of Gorman, known as superposition method, is well known to yield accurate numerical results, although the formulation is a little cumbersome. Based on the convergence and comparison studies in these two tables, the accuracy of the present solutions is well established.

3.2. Comprehensive Results for Non-Levy Type Problem

Tables 3 presents lists of frequency parameters in five significant figures of totally clamped rectangular plates (C-C-C-C plate) for the lowest six modes. Aspect ratio is varied from a/b=0.4, 2/3, 1(square), 1.5 and 2.5. This table uses the same format as in Ref.[2] and these results are presented by using the 12×12 solutions, together with the frequency parameters obtained by Leissa [2]. The difference is also calculated for each pair by using

$$\text{dif.}(\%)=\frac{\Omega_{\text{present}}-\Omega_{\text{Ref[2]}}}{\Omega_{\text{present}}}\times 100 \quad (\%) \qquad (15)$$

Table 2. Comparison study of frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for square plates

Boundary Condition	mode				
$M\times N$	1	2	3	4	5
C-C-C-C					
Present	35.985	73.394	73.394	108.22	131.58
Ref.[6]	35.98	73.40	73.40	108.2	131.6
S-S-S-S					
Present	19.7392	49.3480	49.3480	78.9568	98.6960
exact [2]	19.7392	49.3480	49.3480	78.9568	98.6960
C-F-F-F (ν =0.333)					
Present	3.4598	8.3578	21.093	27.065	30.559
Ref.[5]	3.459	8.356	21.09	27.06	30.55
F-F-F-F (ν =0.333)					
Present	13.169	19.224	24.423	34.233	34.233
Ref.[7]	13.17	19.22	24.42	34.23	34.23

Table 3 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for C-C-C-C plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	23.644	27.005	35.985	60.761	147.77
	Ref.[2]	23.648	27.010	35.992	60.772	147.80
	dif.(%)	-0.02	-0.02	-0.02	-0.02	-0.02
2	Present	27.807	41.704	73.394	93.833	173.79
	Ref.[2]	27.817	41.716	73.413	93.860	173.85
	dif.(%)	-0.04	-0.03	-0.03	-0.03	-0.03
3	Present	35.417	66.124	73.394	148.78	221.36
	Ref.[2]	35.446	66.143	73.413	148.82	221.54
	dif.(%)	-0.08	-0.03	-0.03	-0.03	-0.08
4	Present	46.671	66.522	108.22	149.67	291.70
	Ref.[2]	46.702	66.552	108.27	149.74	291.89
	dif.(%)	-0.07	-0.05	-0.05	-0.04	-0.07
5	Present	61.495	79.805	131.58	179.56	384.35
	Ref.[2]	61.554	79.850	131.64	179.66	384.71
	dif.(%)	-0.10	-0.06	-0.05	-0.06	-0.09
6	Present	63.083	100.81	132.20	226.82	394.27
	Ref.[2]	63.100	100.85	132.24	226.92	394.37
	dif.(%)	-0.03	-0.04	-0.03	-0.04	-0.03

Table 4 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for C-C-C-S plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	23.439	25.859	31.826	48.159	107.04
	Ref.[2]	23.440	25.861	31.829	48.167	107.07
	dif.(%)	0.00	-0.01	-0.01	-0.02	-0.02
2	Present	27.016	38.094	63.331	85.492	139.61
	Ref.[2]	27.022	38.102	63.347	85.507	139.66
	dif.(%)	-0.02	-0.02	-0.03	-0.02	-0.04
3	Present	33.785	60.304	71.076	123.95	194.26
	Ref.[2]	33.799	60.325	71.084	123.99	194.41
	dif.(%)	-0.04	-0.03	-0.01	-0.03	-0.08
4	Present	44.109	65.509	100.79	143.95	270.34
	Ref.[2]	44.131	65.516	100.83	143.99	270.48
	dif.(%)	-0.05	-0.01	-0.04	-0.03	-0.05
5	Present	58.001	77.533	116.36	158.27	322.45
	Ref.[2]	58.034	77.563	116.40	158.36	322.55
	dif.(%)	-0.06	-0.04	-0.04	-0.06	-0.03
6	Present	62.964	92.116	130.35	214.52	353.16
	Ref.[2]	62.971	92.154	130.37	214.78	353.43
	dif.(%)	-0.01	-0.04	-0.01	-0.12	-0.08

Table 5 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for C-C-C-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	22.512	22.930	23.921	26.628	37.565
	Ref.[2]	22.577	23.015	24.020	26.731	37.656
	dif.(%)	-0.29	-0.37	-0.41	-0.39	-0.24
2	Present	24.582	29.384	39.998	65.864	76.098
	Ref.[2]	24.623	29.427	40.039	65.916	76.407
	dif.(%)	-0.17	-0.15	-0.10	-0.08	-0.41
3	Present	29.211	44.327	63.221	65.917	134.51
	Ref.[2]	29.244	44.363	63.493	66.219	135.15
	dif.(%)	-0.11	-0.08	-0.43	-0.46	-0.48
4	Present	36.985	62.186	76.710	106.66	152.35
	Ref.[2]	37.059	62.417	76.761	106.80	152.47
	dif.(%)	-0.20	-0.37	-0.07	-0.14	-0.08
5	Present	48.204	68.814	80.572	124.82	192.75
	Ref.[2]	48.283	68.887	80.713	125.40	193.01
	dif.(%)	-0.16	-0.11	-0.18	-0.46	-0.14
6	Present	61.744	69.555	116.66	152.38	212.69
	Ref.[2]	61.922	69.696	116.80	152.48	213.74
	dif.(%)	-0.29	-0.20	-0.12	-0.07	-0.49

Table 6 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for C-C-S-S plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	16.848	19.951	27.054	44.890	105.30
	Ref.[2]	16.849	19.952	27.056	44.893	105.31
	dif.(%)	-0.01	0.00	-0.01	-0.01	-0.01
2	Present	21.357	34.020	60.538	76.545	133.48
	Ref.[2]	21.363	34.024	60.544	76.554	133.52
	dif.(%)	-0.03	-0.01	-0.01	-0.01	-0.03
3	Present	29.226	54.364	60.786	122.32	182.66
	Ref.[2]	29.236	54.370	60.791	122.33	182.73
	dif.(%)	-0.04	-0.01	-0.01	-0.01	-0.04
4	Present	40.493	57.508	92.836	129.39	253.08
	Ref.[2]	40.509	57.517	92.865	129.41	253.18
	dif.(%)	-0.04	-0.02	-0.03	-0.01	-0.04
5	Present	51.450	67.790	114.56	152.53	321.57
	Ref.[2]	51.457	67.815	114.57	152.58	321.60
	dif.(%)	-0.01	-0.04	-0.01	-0.03	-0.01
6	Present	55.096	90.051	114.70	202.61	344.35
	Ref.[2]	55.117	90.069	114.72	202.66	344.48
	dif.(%)	-0.04	-0.02	-0.01	-0.02	-0.04

Table 7 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for C-C-S-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	15.633	16.213	17.537	20.964	33.529
	Ref.[2]	15.696	16.287	17.615	21.035	33.578
	dif.(%)	-0.40	-0.46	-0.44	-0.34	-0.15
2	Present	18.338	24.172	36.023	54.916	66.363
	Ref.[2]	18.373	24.201	36.046	55.184	66.612
	dif.(%)	-0.19	-0.12	-0.06	-0.49	-0.38
3	Present	23.961	40.680	51.812	63.154	119.32
	Ref.[2]	23.987	40.701	52.065	63.178	119.90
	dif.(%)	-0.11	-0.05	-0.49	-0.04	-0.48
4	Present	32.744	50.600	71.077	98.897	150.78
	Ref.[2]	32.810	50.822	71.194	99.007	150.83
	dif.(%)	-0.20	-0.44	-0.17	-0.11	-0.03
5	Present	44.799	58.945	74.326	108.671	187.44
	Ref.[2]	44.862	59.071	74.349	109.22	187.61
	dif.(%)	-0.14	-0.21	-0.03	-0.51	-0.09
6	Present	50.073	66.213	105.79	150.86	192.23
	Ref.[2]	50.251	66.262	106.28	150.90	193.23
	dif.(%)	-0.35	-0.07	-0.47	-0.03	-0.52

Table 8 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for C-C-F-F plates. (10x10 solution)

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	3.9731	4.9688	6.9201	11.180	24.832
	Ref.[2]	3.9857	4.9848	6.9421	11.216	24.911
	dif.(%)	-0.32	-0.32	-0.32	-0.32	-0.32
2	Present	7.1312	13.237	23.908	29.783	44.571
	Ref.[2]	7.1551	13.289	24.034	29.901	44.719
	dif.(%)	-0.33	-0.39	-0.53	-0.40	-0.33
3	Present	13.042	23.278	26.586	52.376	81.510
	Ref.[2]	13.101	23.384	26.681	52.615	81.879
	dif.(%)	-0.45	-0.45	-0.36	-0.46	-0.45
4	Present	21.733	30.118	47.657	67.766	135.83
	Ref.[2]	21.844	30.262	47.785	68.090	136.52
	dif.(%)	-0.51	-0.48	-0.27	-0.48	-0.51
5	Present	22.798	34.139	62.710	76.812	142.49
	Ref.[2]	22.896	34.240	63.039	77.041	143.10
	dif.(%)	-0.43	-0.30	-0.52	-0.30	-0.43
6	Present	26.404	52.245	65.539	117.55	165.03
	Ref.[2]	26.501	52.398	65.833	117.90	165.63
	dif.(%)	-0.37	-0.29	-0.45	-0.30	-0.37

Table 9 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for C-S-C-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	22.483	22.778	23.371	24.678	28.466
	Ref.[2]	22.544	22.855	23.460	24.775	28.564
	dif.(%)	-0.27	-0.34	-0.38	-0.39	-0.34
2	Present	24.261	27.930	35.571	53.682	70.270
	Ref.[2]	24.296	27.971	35.612	53.731	70.561
	dif.(%)	-0.15	-0.15	-0.11	-0.09	-0.41
3	Present	28.314	40.651	62.875	64.682	113.89
	Ref.[2]	28.341	40.683	63.126	64.959	114.00
	dif.(%)	-0.10	-0.08	-0.40	-0.43	-0.09
4	Present	35.301	62.093	66.762	97.121	130.26
	Ref.[2]	35.345	62.310	66.808	97.257	130.84
	dif.(%)	-0.12	-0.35	-0.07	-0.14	-0.45
5	Present	45.634	62.642	77.374	123.95	159.30
	Ref.[2]	45.710	62.695	77.502	124.48	159.54
	dif.(%)	-0.17	-0.08	-0.16	-0.43	-0.15
6	Present	59.455	68.562	108.87	127.83	209.34
	Ref.[2]	59.562	68.683	108.99	127.92	210.32
	dif.(%)	-0.18	-0.18	-0.11	-0.07	-0.47

Table 10 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for C-S-S-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	15.591	15.999	16.792	18.469	23.004
	Ref.[2]	15.649	16.067	16.865	18.540	23.067
	dif.(%)	-0.37	-0.42	-0.43	-0.39	-0.27
2	Present	17.915	22.421	31.114	50.419	59.725
	Ref.[2]	17.946	22.449	31.138	50.442	59.969
	dif.(%)	-0.17	-0.13	-0.08	-0.05	-0.41
3	Present	22.880	36.683	51.397	53.467	111.90
	Ref.[2]	22.902	36.703	51.631	53.715	111.95
	dif.(%)	-0.09	-0.05	-0.46	-0.46	-0.04
4	Present	30.855	50.486	64.021	88.699	114.57
	Ref.[2]	30.892	50.696	64.043	88.802	115.11
	dif.(%)	-0.12	-0.42	-0.03	-0.12	-0.47
5	Present	42.045	57.798	67.540	107.67	153.08
	Ref.[2]	42.108	57.908	67.646	108.19	153.24
	dif.(%)	-0.15	-0.19	-0.16	-0.48	-0.10
6	Present	50.053	59.808	101.12	126.06	188.57
	Ref.[2]	50.222	59.840	101.21	126.09	189.49
	dif.(%)	-0.34	-0.05	-0.09	-0.03	-0.49

Table 11 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for CSFF plates. (10x10 solution)

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	3.8437	4.4125	5.3512	6.9187	10.087
	Ref.[2]	3.8542	4.4247	5.3639	6.9309	10.100
	dif.(%)	-0.27	-0.28	-0.24	-0.18	-0.12
2	Present	6.4020	10.871	19.076	27.182	35.048
	Ref.[2]	6.4198	10.912	19.171	27.289	35.157
	dif.(%)	-0.28	-0.37	-0.50	-0.39	-0.31
3	Present	11.523	22.853	24.672	38.386	74.664
	Ref.[2]	11.576	22.958	24.768	38.586	74.990
	dif.(%)	-0.46	-0.46	-0.39	-0.52	-0.44
4	Present	19.660	25.573	43.090	64.048	99.380
	Ref.[2]	19.767	25.698	43.191	64.254	99.928
	dif.(%)	-0.55	-0.49	-0.23	-0.32	-0.55
5	Present	22.430	32.349	52.708	67.189	127.23
	Ref.[2]	22.521	32.425	53.000	67.467	127.69
	dif.(%)	-0.40	-0.24	-0.55	-0.41	-0.36
6	Present	25.954	48.281	63.763	107.75	134.76
	Ref.[2]	26.024	48.467	64.050	108.02	135.45
	dif.(%)	-0.27	-0.39	-0.45	-0.26	-0.51

Table 12 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for C-F-C-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	22.283	22.228	22.168	22.093	21.983
	Ref.[2]	22.346	22.314	22.272	22.215	22.130
	dif.(%)	-0.28	-0.39	-0.47	-0.55	-0.67
2	Present	22.991	24.193	26.407	30.784	41.568
	Ref.[2]	23.086	24.309	26.529	30.901	41.689
	dif.(%)	-0.41	-0.48	-0.46	-0.38	-0.29
3	Present	25.608	31.636	43.597	60.964	60.608
	Ref.[2]	25.666	31.700	43.664	61.303	61.002
	dif.(%)	-0.23	-0.20	-0.15	-0.56	-0.65
4	Present	30.574	46.754	61.176	70.869	92.032
	Ref.[2]	30.633	46.820	61.466	70.960	92.384
	dif.(%)	-0.19	-0.14	-0.47	-0.13	-0.38
5	Present	38.481	61.328	67.176	73.883	119.13
	Ref.[2]	38.687	61.566	67.549	74.259	119.88
	dif.(%)	-0.54	-0.39	-0.56	-0.51	-0.63
6	Present	49.680	64.004	79.817	118.08	156.95
	Ref.[2]	49.858	64.343	79.904	118.33	157.76
	dif.(%)	-0.36	-0.53	-0.11	-0.21	-0.51

Table 13 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for C-F-S-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	15.316	15.258	15.192	15.115	15.012
	Ref.[2]	15.382	15.340	15.285	15.217	15.128
	dif.(%)	-0.43	-0.54	-0.61	-0.68	-0.77
2	Present	16.282	17.854	20.584	25.635	37.230
	Ref.[2]	16.371	17.949	20.673	25.711	37.294
	dif.(%)	-0.54	-0.53	-0.43	-0.30	-0.17
3	Present	19.607	26.689	39.736	49.231	48.870
	Ref.[2]	19.656	26.734	39.775	49.550	49.226
	dif.(%)	-0.25	-0.17	-0.10	-0.65	-0.73
4	Present	25.498	43.147	49.449	63.691	83.049
	Ref.[2]	25.549	43.190	49.730	64.012	83.325
	dif.(%)	-0.20	-0.10	-0.57	-0.50	-0.33
5	Present	34.314	49.605	56.280	68.083	102.43
	Ref.[2]	34.507	49.840	56.617	68.126	103.14
	dif.(%)	-0.56	-0.47	-0.60	-0.06	-0.69
6	Present	46.282	52.689	77.324	103.08	143.00
	Ref.[2]	46.435	53.013	77.368	103.70	143.68
	dif.(%)	-0.33	-0.61	-0.06	-0.60	-0.47

Table 14 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for C-F-F-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	3.4984	3.4846	3.4719	3.4551	3.4378
	Ref.[2]	3.5107	3.5024	3.4917	3.4772	3.4562
	dif.(%)	-0.35	-0.51	-0.57	-0.64	-0.54
2	Present	4.7672	6.3882	8.5065	11.658	17.967
	Ref.[2]	4.7861	6.4062	8.5246	11.676	17.988
	dif.(%)	-0.40	-0.28	-0.21	-0.16	-0.12
3	Present	8.0683	14.467	21.286	21.468	21.398
	Ref.[2]	8.1146	14.538	21.429	21.618	21.563
	dif.(%)	-0.57	-0.49	-0.67	-0.70	-0.77
4	Present	13.805	21.916	27.199	39.330	57.225
	Ref.[2]	13.882	22.038	27.331	39.492	57.458
	dif.(%)	-0.56	-0.56	-0.48	-0.41	-0.41
5	Present	21.523	25.914	30.958	53.542	60.130
	Ref.[2]	21.638	26.073	31.111	53.876	60.581
	dif.(%)	-0.53	-0.62	-0.49	-0.62	-0.75
6	Present	23.047	31.448	54.189	61.619	105.94
	Ref.[2]	23.731	31.618	54.443	61.994	106.54
	dif.(%)	-2.97	-0.54	-0.47	-0.61	-0.56

Table 15 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for S-S-F-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	1.3198	2.2334	3.3676	5.0256	8.2505
	Ref.[2]	1.3201	2.2339	3.3687	5.0263	8.2506
	dif.(%)	-0.02	-0.02	-0.03	-0.01	0.00
2	Present	4.7308	9.5393	17.317	21.464	29.566
	Ref.[2]	4.7433	9.5749	17.407	21.544	29.646
	dif.(%)	-0.26	-0.37	-0.52	-0.37	-0.27
3	Present	10.316	16.679	19.293	37.527	64.473
	Ref.[2]	10.362	16.764	19.367	37.718	64.760
	dif.(%)	-0.45	-0.51	-0.39	-0.51	-0.44
4	Present	15.789	24.544	38.211	55.223	98.681
	Ref.[2]	15.873	24.662	38.291	55.490	99.206
	dif.(%)	-0.53	-0.48	-0.21	-0.48	-0.53
5	Present	18.845	26.994	51.035	60.736	117.78
	Ref.[2]	18.930	27.058	51.324	60.882	118.31
	dif.(%)	-0.45	-0.24	-0.57	-0.24	-0.45
6	Present	20.100	44.059	53.487	99.132	125.62
	Ref.[2]	20.171	44.172	53.738	99.388	126.07
	dif.(%)	-0.35	-0.26	-0.47	-0.26	-0.35

Table 16 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for S-F-F-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	2.6886	4.4774	6.6433	9.8458	14.813
	Ref.[2]	2.6922	4.4810	6.6480	9.8498	14.939
	dif.(%)	-0.14	-0.08	-0.07	-0.04	-0.85
2	Present	6.4730	12.943	14.902	14.888	16.238
	Ref.[2]	6.5029	13.009	15.023	15.013	16.242
	dif.(%)	-0.46	-0.51	-0.81	-0.84	-0.03
3	Present	12.574	15.571	25.376	33.913	48.443
	Ref.[2]	12.637	15.674	25.492	34.027	48.844
	dif.(%)	-0.50	-0.66	-0.46	-0.34	-0.83
4	Present	15.234	20.246	26.001	47.953	51.950
	Ref.[2]	15.337	20.373	26.126	48.332	52.089
	dif.(%)	-0.67	-0.63	-0.48	-0.79	-0.27
5	Present	17.371	30.391	48.449	54.781	96.684
	Ref.[2]	17.510	30.548	48.711	55.066	97.225
	dif.(%)	-0.80	-0.52	-0.54	-0.52	-0.56
6	Present	21.597	33.234	50.579	70.270	101.52
	Ref.[2]	21.699	33.411	50.849	70.695	102.34
	dif.(%)	-0.47	-0.53	-0.53	-0.60	-0.80

Table 17 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for F-F-F-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	3.4328	8.9314	13.468	20.096	21.454
	Ref.[2]	3.4629	8.9459	13.489	20.128	21.643
	dif.(%)	-0.88	-0.16	-0.16	-0.16	-0.88
2	Present	5.2783	9.5171	19.596	21.413	32.987
	Ref.[2]	5.2881	9.6015	19.789	21.603	33.050
	dif.(%)	-0.19	-0.89	-0.99	-0.89	-0.19
3	Present	9.5407	20.599	24.270	46.347	59.628
	Ref.[2]	9.6220	20.735	24.432	46.654	60.137
	dif.(%)	-0.85	-0.66	-0.67	-0.66	-0.85
4	Present	11.329	22.182	34.801	49.910	70.803
	Ref.[2]	11.437	22.353	35.024	50.293	71.484
	dif.(%)	-0.96	-0.77	-0.64	-0.77	-0.96
5	Present	18.628	25.651	34.801	57.713	116.42
	Ref.[2]	18.793	25.867	35.024	58.201	117.45
	dif.(%)	-0.89	-0.84	-0.64	-0.84	-0.88
6	Present	18.923	29.791	61.093	67.030	118.27
	Ref.[2]	19.100	29.973	61.526	67.494	119.38
	dif.(%)	-0.94	-0.61	-0.71	-0.69	-0.94

Table 18 Frequency parameters $\Omega = \omega a(\rho h/D)^{1/2}$ for S-S-S-S plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	11.4488	14.2561	19.7392	32.076	71.5543
	Ref.[2]	11.4487	14.2561	19.7392	32.0762	71.5564
	dif.(%)	0.00	0.00	0.00	0.00	0.00
2	Present	16.1862	27.4155	49.3480	61.6849	101.164
	Ref.[2]	16.1862	27.4156	49.3480	61.6850	101.163
	dif.(%)	0.00	0.00	0.00	0.00	0.00
3	Present	24.0818	43.8649	49.3480	98.6960	150.511
	Ref.[2]	24.0818	43.8649	49.3480	98.6960	150.512
	dif.(%)	0.00	0.00	0.00	0.00	0.00
4	Present	35.1358	49.3480	78.9568	111.033	219.599
	Ref.[2]	35.1358	49.3480	78.9568	111.033	219.599
	dif.(%)	0.00	0.00	0.00	0.00	0.00
5	Present	41.0575	57.0244	98.6960	128.305	256.610
	Ref.[2]	41.0576	57.0244	98.6960	128.305	256.610
	dif.(%)	0.00	0.00	0.00	0.00	0.00
6	Present	45.7950	78.9568	98.6960	177.653	286.219
	Ref.[2]	45.7950	78.9568	98.6960	177.653	286.218
	dif.(%)	0.00	0.00	0.00	0.00	0.00

Table 19 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for S-C-S-C plates.

mode	*a/b*	0.4	2/3	1	1.5	2.5
1	Present	12.1347	17.3730	28.9509	56.3481	145.484
	Ref.[2]	12.1347	17.3730	28.9509	56.3481	145.484
	dif.(%)	0.00	0.00	0.00	0.00	0.00
2	Present	18.3647	35.3445	54.7431	78.9836	164.739
	Ref.[2]	18.3647	35.3445	54.7431	78.9836	164.739
	dif.(%)	0.00	0.00	0.00	0.00	0.00
3	Present	27.9657	45.4294	69.3270	123.172	202.227
	Ref.[2]	27.9657	45.4294	69.3270	123.172	202.227
	dif.(%)	0.00	0.00	0.00	0.00	0.00
4	Present	40.7500	62.0544	94.5853	146.268	261.105
	Ref.[2]	40.7500	62.0544	94.5853	146.268	261.105
	dif.(%)	0.00	0.00	0.00	0.00	0.00
5	Present	41.3782	62.3131	102.216	170.111	342.165
	Ref.[2]	41.3782	62.3131	102.216	170.111	342.144
	dif.(%)	0.00	0.00	0.00	0.00	0.01
6	Present	47.0009	88.8047	129.096	189.122	392.875
	Ref.[2]	47.0009	88.8047	129.096	189.122	392.875
	dif.(%)	0.00	0.00	0.00	0.00	0.00

Table 20 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for S-C-S-S plates.

mode	*a/b*	0.4	2/3	1	1.5	2.5
1	Present	11.7502	15.5783	23.6464	42.528	103.922
	Ref.[2]	11.7502	15.5783	23.6463	42.5278	103.923
	dif.(%)	0.00	0.00	0.00	0.00	0.00
2	Present	17.1872	31.0723	51.6742	69.0031	128.338
	Ref.[2]	17.1872	31.0724	51.6743	69.0031	128.338
	dif.(%)	0.00	0.00	0.00	0.00	0.00
3	Present	25.9171	44.5644	58.6463	116.267	172.380
	Ref.[2]	25.9171	44.5644	58.6464	116.267	172.380
	dif.(%)	0.00	0.00	0.00	0.00	0.00
4	Present	37.8317	55.3926	86.1345	120.996	237.250
	Ref.[2]	37.8317	55.3926	86.1345	120.996	237.250
	dif.(%)	0.00	0.00	0.00	0.00	0.00
5	Present	41.2071	59.4627	100.270	147.635	320.792
	Ref.[2]	41.2070	59.4627	100.270	147.635	320.792
	dif.(%)	0.00	0.00	0.00	0.00	0.00
6	Present	46.3620	83.6060	113.228	184.101	322.986
	Ref.[2]	46.3620	83.6060	113.228	184.101	322.964
	dif.(%)	0.00	0.00	0.00	0.00	0.01

Table 21 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for S-C-S-F plates.

mode	*a/b*	0.4	2/3	1	1.5	2.5
1	Present	10.1888	10.9752	12.6872	16.8219	30.6297
	Ref.[2]	10.1888	10.9752	12.6874	16.8225	30.6277
	dif.(%)	0.00	0.00	0.00	0.00	0.01
2	Present	13.6037	20.3357	33.0650	45.3022	58.0819
	Ref.[2]	13.6036	20.3355	33.0651	45.3024	58.0804
	dif.(%)	0.00	0.00	0.00	0.00	0.00
3	Present	20.0972	37.9553	41.7019	61.0178	105.548
	Ref.[2]	20.0971	37.9552	41.7019	61.0178	105.547
	dif.(%)	0.00	0.00	0.00	0.00	0.00
4	Present	29.6219	40.2717	63.0148	92.3072	149.457
	Ref.[2]	29.6219	40.2717	63.0148	92.3073	149.457
	dif.(%)	0.00	0.00	0.00	0.00	0.00
5	Present	39.6382	49.7317	72.3976	93.8294	173.106
	Ref.[2]	39.6382	49.7317	72.3976	93.8293	173.106
	dif.(%)	0.00	0.00	0.00	0.00	0.00
6	Present	42.2426	64.1890	90.6113	141.783	182.811
	Ref.[2]	42.2425	64.1889	90.6114	141.783	182.811
	dif.(%)	0.00	0.00	0.00	0.00	0.00

Table 22 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for S-S-S-F plates.

mode	*a/b*	0.4	2/3	1	1.5	2.5
1	Present	10.1258	10.6712	11.6846	13.7114	18.8017
	Ref.[2]	10.1259	10.6712	11.6845	13.7111	18.8009
	dif.(%)	0.00	0.00	0.00	0.00	0.00
2	Present	13.0570	18.2995	27.7563	43.5722	50.5404
	Ref.[2]	13.0570	18.2995	27.7563	43.5723	50.5405
	dif.(%)	0.00	0.00	0.00	0.00	0.00
3	Present	18.8390	33.6974	41.1966	47.8571	100.232
	Ref.[2]	18.8390	33.6974	41.1967	47.8571	100.232
	dif.(%)	0.00	0.00	0.00	0.00	0.00
4	Present	27.5580	40.1307	59.0655	81.4788	110.226
	Ref.[2]	27.5580	40.1307	59.0655	81.4789	110.226
	dif.(%)	0.00	0.00	0.00	0.00	0.00
5	Present	39.3389	48.4082	61.8606	92.6924	147.632
	Ref.[2]	39.3377	48.4082	61.8606	92.6925	147.632
	dif.(%)	0.00	0.00	0.00	0.00	0.00
6	Present	39.6119	57.5930	90.2941	124.563	169.103
	Ref.[2]	39.6118	57.5929	90.2941	124.564	169.103
	dif.(%)	0.00	0.00	0.00	0.00	0.00

Table 23 Frequency parameters $\Omega=\omega a(\rho h/D)^{1/2}$ for S-F-S-F plates.

mode	a/b	0.4	2/3	1	1.5	2.5
1	Present	9.75995	9.6984	9.63127	9.55886	9.48072
	Ref.[2]	9.7600	9.6983	9.6314	9.5582	9.4841
	dif.(%)	0.00	0.00	0.00	0.01	-0.04
2	Present	11.0369	12.9812	16.135	21.6194	33.6228
	Ref.[2]	11.0368	12.9813	16.1348	21.6192	33.6228
	dif.(%)	0.00	0.00	0.00	0.00	0.00
3	Present	15.0626	22.9535	36.7257	38.7216	38.3628
	Ref.[2]	15.0626	22.9535	36.7256	38.7214	38.3629
	dif.(%)	0.00	0.00	0.00	0.00	0.00
4	Present	21.7065	39.1052	38.9450	54.8443	75.2042
	Ref.[2]	21.7064	39.1052	38.9450	54.8443	75.2037
	dif.(%)	0.00	0.00	0.00	0.00	0.00
5	Present	31.1779	40.3560	46.7381	65.7921	86.9680
	Ref.[2]	31.1771	40.3560	46.7381	65.7922	86.9684
	dif.(%)	0.00	0.00	0.00	0.00	0.00
6	Present	39.2387	42.6847	70.7401	87.6262	130.358
	Ref.[2]	39.2387	42.6847	70.7401	87.6262	130.358
	dif.(%)	0.00	0.00	0.00	0.00	0.00

As seen in the table, all the differences are between -0.10% -0.02%, and all the present values are very slightly lower than those in Ref.[2], and due to the fact that all the present solution satisfies exactly the kinematical condition at the edges, it is observed that the present results are more accurate and closer to the exact values (assuming that they exist). The mode numbers (i.e., number of half-waves in the mode shape) are listed for each frequency in Ref.[2].

Tables 4 to 17 are in the same format as in Table 3, and the frequency parameters are tabulated for fourteen sets of edge conditions, where the exact solutions are not available due to the condition that two opposite edges are not simply supported (non-Levy type problem).

In all pairs of the present and reference values (i.e., 450 pairs= (5 aspect ratios)×(6 modes)×(15 tables)), all the differences are negative (the present values are lower) and the differences are all within one percent.

3.3. Comprehensive Results for Levy Type Problem

It is widely known that the exact solution is available for vibration of isotropic rectangular plates when a pair of opposite edges are simply supported. The solution is already presented in Ref. [2]. This means that it is not necessary to use approximate method to calculate natural frequencies. In this paper, however, six tables are prepared in the same format to demonstrate that this Ritz solution can provide frequency parameters in the accuracy with same degree. The present results are given with six significant figures, and compared to the exact values in Ref. [2].

Table 24 Correspondance of the present tables with those in Ref.[2].

Present tables	Tables in Ref.[2]	Represen-ting B.C.	B.C.s to give the identical frequency parameters
(1) Plates *not* having two opposite edges simply supported			
Table 3	**C1**	**C-C-C-C**	
Table 4	**C2**	**C-C-C-S**	**S-C-C-C, C-S-C-C, C-C-S-C**
Table 5	**C3**	**C-C-C-F**	**F-C-C-C, C-F-C-C, C-C-F-C**
Table 6	**C4**	**C-C-S-S**	**S-S-C-C, S-C-C-S, C-S-S-C**
Table 7	**C5**	**C-C-S-F**	**F-S-C-C, F-C-C-S ,S-F-C-C, S-C-C-F, C-F-S-C, C-S-F-C, C-C-F-S**
Table 8	**C6**	**C-C-F-F**	**F-F-C-C, F-C-C-F, C-F-F-C**
Table 9	**C7**	**C-S-C-F**	**F-C-S-C, S-C-F-C, C-F-C-S**
Table 10	**C8**	**C-S-S-F**	**F-S-S-C, F-C-S-S, S-F-C-S, S-S-F-C, S-S-C-F, S-C-F-S, C-F-S-S**
Table 11	**C9**	**C-S-F-F**	**F-F-S-C, F-F-C-S, F-S-C-F, F-C-S-F, S-F-F-C, S-C-F-F, C-F-F-S**
Table 12	**C10**	**C-F-C-F**	**F-C-F-C**
Table 13	**C11**	**C-F-S-F**	**F-S-F-C, F-C-F-S, S-F-C-F**
Table 14	**C12**	**C-F-F-F**	**F-F-F-C, F-F-C-F, F-C-F-F**
Table 15	**C13**	**S-S-F-F**	**F-F-S-S, F-S-S-F, S-F-F-S**
Table 16	**C14**	**S-F-F-F**	**F-F-F-S, F-F-S-F, F-S-F-F**
Table 17	**C15**	**F-F-F-F**	
(2) Plates having two opposite edges simply supported			
Table 18	**A1**	**S-S-S-S**	
Table 19	**A2**	**S-C-S-C**	**C-S-C-S**
Table 20	**A3**	**S-C-S-S**	**S-S-S-C, S-S-C-S, C-S-S-S**
Table 21	**A4**	**S-C-S-F**	**F-S-C-S, S-F-S-C, C-S-F-S**
Table 22	**A5**	**S-S-S-F**	**F-S-S-S, S-F-S-S, S-S-F-S**
Table 23	**A6**	**S-F-S-F**	**F-S-F-S**

Tables 18 to 23 list up the frequency parameters for rectangular plates with S-S-S-S, S-C-S-C, S-C-S-S, S-C-S-F, S-S-S-S-F and S-F-S-F, respectively. Among 180 pairs of the present and exact frequencies (i.e., 180 pairs= (5 aspect ratios)×(6 modes)×(6 tables)), almost all pairs show the exact match with six significant figures, only with four exceptions showing very slight difference over 0.01 percent.

Table 24 provides classification of physically meaningful sets of edge conditions to give the identical natural frequencies. For example in this table, “Table 4” gives the frequency parameters of C-C-C-S plate and the results are identical with S-C-C-C, C-S-C-C and C-C-S-C.

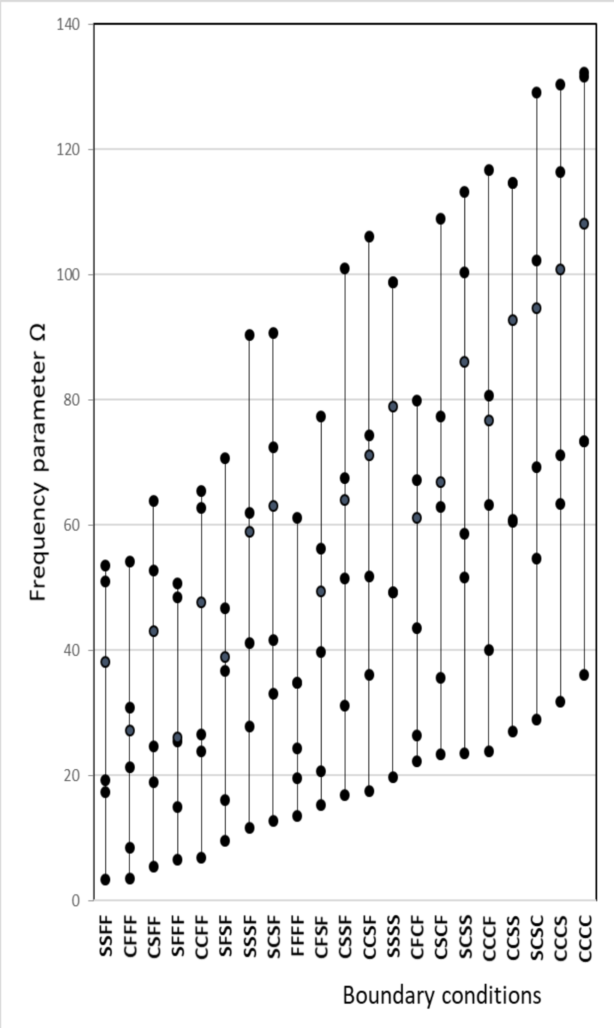

Figure 2. Distribution of lowest six frequency parameters of square plates with twenty-one sets of boundary conditions

Figure 2 presents the distribution of lowest six frequency parameters of square plates for twenty-one sets of boundary conditions. The boundary conditions are aligned in the increasing order of the fundamental frequencies, and the fundamental frequencies may appear almost linearly increasing. Among them, SFFF and FFFF plates have rigid body motions and only the self-equilibrium modes (i.e, modes with some nodal lines) exist. Also, SSSS, CCSS and CCCC plates may appear to have only four distinct modes because two modes include a pair of identical frequencies, and FFFF plate does five distinct modes due to the same reason.

4. Conclusions

The aim of this study was to establish a new standard for natural frequencies of isotropic rectangular plates and to summarize the lists of frequencies for all possible combinations of three classical boundary conditions (i.e., free, simply supported and clamped edges). After the accuracy of the Ritz solution was established for non-Levy type problems (i.e., plates not having two opposite sides simply supported), these frequency parameters were given in five significant figures in the lists and were compared to the other list [2]. The differences were all less than one-percent. For Levy type (i.e., plates with two opposite sides simply supported) problems where the exact solutions are available, two sets of frequency parameters were in good agreement showing the differences lower than 0.001 percent.

The present results for most commonly needed information in mechanical vibration will be valuable standard for comparison with numerical methods in future studies and design data.

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References

1. Leissa AW, Vibration of Plates, NASA-SP-160, 1969. https://ntrs.nasa.gov/citations/19700009156
2. Leissa AW, The free vibration of rectangular plates, J. Sound Vib., 31(1973), 257-293.
3. Blevins RD, Formulas for natural frequency and mode shape, Krieger Pub. Com. (2001).
4. Young D, Vibration of rectangular plates by the ritz method, Trans. ASME J. Applied Mechanics, 17 (1950), 448-453.
5. Gorman DJ, Free vibration analysis of cantilever plates by the method of superposition, J. Sound Vibr., 49 (1976), 453-467.
6. Gorman DJ, Free-vibration analysis of rectangular plates with clamped-simply supported edge conditions by the method of superposition, Trans.ASME, J. Applied Mech., 44 (1977), 743-749.
7. Gorman DJ, Free vibration analysis of the completely free rectangular plates by the method of superposition, J. Sound Vibr., 57 (1978), 437-447.
8. Narita Y, Free vibration of elastic plates with various shapes and boundary conditions, Doctoral Dissertation, Hokkaido University (1980), https://eprints.lib.hokudai.ac.jp/dspace/handle/2115/32630
9. Leissa AW, Plate vibration research 1976-1980: classical theory, Shock Vib. Dig., 13-9 (1981), 11-22.
10. Leissa AW, Recent studies in plate vibrations: 1981-85, Part.1 classical theory, 1981-85, Shock Vib. Dig., 19-2 (1987), 11-18.
11. Ramu I, Mohanty SC, Study on free vibration analysis of rectangular plate structures using finite element method, Procedia Eng. 38 (2012), 2758-2766.
12. Senjanovic I, Tomic M, Vladimir N, Hadzic N, an analytical solution to free rectangular plate natural vibrations by beam modes-ordinary and missing plate modes, Trans. FAMENA 40 (2016), 1-18.
13. Narita Y, Combinations for the free-vibration behaviors of anisotropic rectangular plates under general edge conditions, Trans. ASME, J. Appl. Mech., 67 (2000), 568-573.
14. Narita Y, Polya counting theory applied to combination of edge conditions for generally shaped isotropic plates, EPI Int. J. Eng., 2 (2019), 194-202.
15. Narita Y, Robinson P, Maximizing the fundamental frequency of laminated cylindrical panels using layerwise optimization, Intl. J. Mech. Sci., 48 (2006) 1516–1524.
16. Narita D, Narita, Y., Accurate results for free vibration of doubly curved shallow shells of rectangular planform (Part 1), EPI Int. J. Eng., 4 (2021), 29-36.