



Title	A Method of Measuring Local Displacements with Desired Accuracy by Using a Simple Transmitter
Author(s)	MAEDA, Itaru
Citation	Journal of the Faculty of Science, Hokkaido University. Series 7, Geophysics, 9(3), 355-364
Issue Date	1993-03-15
Doc URL	<a href="http://hdl.handle.net/2115/8795">http://hdl.handle.net/2115/8795</a>
Type	bulletin (article)
File Information	9(3)_p355-364.pdf



[Instructions for use](#)

# **A Method of Measuring Local Displacements with Desired Accuracy by Using a Simple Transmitter**

**Itaru Maeda**

*Erimo Geophysical Observatory,  
Faculty of Science, Hokkaido University,  
Sapporo 060, Japan*

(Received November 19, 1992)

## **Abstract**

A new method to measure displacements of a point being dangerous to approach is proposed. The procedure of the method is to throw a simple transmitter with a single frequency into the point and to measure the frequency and phase variations at several fixed points. A theory to obtain displacements from the data is presented. A temporal estimate shows that by this method the resolution can be achieved to less than a millimeter.

## **1. Introduction**

Activity of a volcano in active is observed in many ways. Amongst other things observations of earthquakes and deformation of the volcano are the most important in relation to events of eruption and common. Earthquakes itself are discrete phenomena and are usually under 24-hours observation. Deformation is continuous phenomenon in space-time coordinate but observed at some (not many) discrete points on the volcano. Unfortunately the points are usually selected not on the summit or rim of a crater of the volcano because of danger but on places far from it for continuous measurement of strains and inclinations. This means that the observations are slanting eyed ones. Measurements of the movement of a crater rim is made by optical ways but at only limited times. They strongly depend on weather conditions and also the activities of the volcano. When eruption continues or is impending the measurements are impossible.

It is considered that the movement of the crater rim of a volcano in a very active state gives invaluable information on the volcanic activities as a whole.

To observe such a movement, installation of a permanent station is desirable but is nearly impossible. For want of better it can be considered to install an intricate instrument but usually such an instrument will be extremely expensive. Under a circumstance of an erupting volcano any instruments can be destroyed and once destroyed it is practically impossible to reinstall them if they are rather heavy, intricate, and also fragile (usually an intricate instrument will be fragile). For these reasons there is no observation of the movement with sufficient accuracy in relation to eruptions.

In this report we propose a system which makes it possible to measure the movement of the rim of a crater even when it erupting.

## 2. Method

To measure movement of a point is to determine the position as a function of time of the point relative to some fixed points. In a navigation system fixed points emit signals and an observer who is at the spot receives the signals to determine the position of the point. The present-day system called GPS uses satellites as emitters which are not fixed but the positions are known very accurately. The signals are coded or modulated visible light or radio waves.

On the other hand there are cases in which the spot we want to determine the position emits signals. The typical example is the determination of earthquake source locations. The signal is a sound wave without a known code. Primarily, arrival times are utilized to determine the location. The navigation case (type 1) and the case of hypocenter determination (type 2) give the typical ways to determine the position of a point.

We will at first set up several requirements from which we can judge which type and what point of the type is more advantageous for the new method. Under the circumstance given in the introduction, following requirements will be necessary ;

- (1) Continuous observation is possible.
  - (a) It should not depend on weather conditions.
  - (b) It should not depend on human labour except at the installation of the instruments.
- (2) The instrument which is placed at the spot on a crater should be simple, small, and expendable. It is also important that energy (electric power) consumption of the instrument is the minimum. Technically the above four conditions are synonyms.

A method of type 1 is not adequate by the requirement (2) because an

instrument placed at the spot must have very complicate functions which make the instrument complex, expensive, and high energy consumption. Therefore we adopt a method of type 2.

From the requirement (1) visible light is not adequate to transmit signals. Radio waves or sound waves will be possible candidates. To determine positions by receiving signals we need sufficient time resolution for the signals. In general the lower the frequency of signals is, the lower the time resolutions will be. For the sound waves, to make an instrument which can transmit sufficiently high frequency signals is not practical. The velocity of a sound wave can be severely affected by atmospheric conditions, which lowers the time resolution because we cannot know exactly the atmospheric conditions. From these reasons we discard sound waves.

We can now make a sketch of a system to measure the position of a perilous spot as a function of time, ie. the movement of the spot: a simple device transmitting a radio wave is placed at the spot. The signal is received at several fixed safe places by devices which have sufficient functions and stability to be able to analyze the signal with required accuracy though it is not given in advance.

The accuracy will depends on frequencies being used and on the function of the transmitter. We assume to employ the simplest transmitter which will emit simple sinusoidal signal. With such a device we can utilize only phases of the wave. If we can employ more sophisticated devices, the accuracy will be improved. How to determine the transmitter's movement from the knowledge of phases of the signal will be given later.

At first we will estimate from the requirement (1) the upper bound of frequencies being able to use. In order not to be hampered by drops and particles in the air, the wavelength must be longer than the order of centimeter, which means that the frequencies should be less than  $10^{10}$  Hz. Selection of the frequency also depends on what amount of the movement we want to detect and on the theory which gives the movements from a knowledge of phase variations.

### 3. Theory

For the time being, we assume that each receiver has a sufficiently accurate clock and the clocks synchronize with each other. Observable quantities are the phase variations from a reference one which is arbitrarily fixed and the frequency of the signal. Let the true distance between the transmitter and  $i$ -th receiver be  $R_i$  and the wave-length be  $\lambda$  at time  $t=0$  which is arbitrarily fixed.

$\lambda$  is not a direct observable in this system but can be known. The relation among,  $R_i$ ,  $\lambda$  and phase  $\Phi_i$  is as follow :

$$R_i = \lambda \left( N_i + \frac{\Phi_i}{2\pi} \right) = \frac{c}{\nu} \left( N_i + \frac{\Phi_i}{2\pi} \right) \text{ at } t=0, \quad (1)$$

where  $N_i$  is an integer giving wave numbers contained in  $R_i$ ,  $\nu$  the frequency observed,  $c$  the velocity which can vary time to time. The phase  $\Phi_i$  divided by  $2\pi$  is defined as the residual of the quotient  $R_i/\lambda$ . We assumed that the velocities to each receiver are common and given independently from this system.

Let consider that at time  $t=t$  the distance changed to  $R_i' = R_i + \Delta R_i$ , the wavelength to  $\lambda'$ , and the phase to  $\Phi' = \Phi + \Delta\Phi$ . Then equation (1) reads

$$R_i + \Delta R_i = \lambda' \left( N_i' + \frac{\Phi_i'}{2\pi} \right) = \frac{c'}{\nu'} \left( N_i + \Delta N_i + \frac{\Phi_i + \Delta\Phi_i}{2\pi} \right), \quad (2)$$

where  $\Delta N_i$  is the change in wave numbers caused by the change in  $R_i$  and  $\lambda$ . In equation (2) observables are  $\nu'$  and  $\Delta\Phi_i$ .

In order to eliminate quantities being not observable, let consider the case that only the wavelength changed. In this case equation (2) reads

$$R_i = \lambda' \left( N_i + \Delta N_i^0 + \frac{\Phi_i + \Delta\Phi_i^0}{2\pi} \right), \quad (3)$$

where the superscript 0 indicates the values when the transmitter didn't move. Subtracting equation (3) from (2), we obtain

$$\Delta R_i = \lambda' \left( \Delta N_i - \Delta N_i^0 + \frac{\Delta\Phi_i - \Delta\Phi_i^0}{2\pi} \right). \quad (4)$$

The difference  $\Delta N_i - \Delta N_i^0$  is the change in wave numbers caused only by the displacement of the transmitter. If we select the frequency so that  $\lambda \gg \Delta R_i$  we can eliminate this difference. Then equation (4) becomes

$$\Delta R_i = \lambda' \frac{\Delta\Phi_i - \Delta\Phi_i^0}{2\pi}. \quad (5)$$

If we know the true value of the initial distance  $R_i$  we can calculate  $\Delta\Phi_i^0$  using the observed value of frequency  $\nu'$  and a given value of the velocity  $c'$ . Then we obtain the displacement from the set of equations of the form (5) for  $n$  receivers ( $n > 3$ ). Unfortunately we never know the true distances between the transmitter and the receivers. Let the assumed distance be  $r_i$  and  $r_i/R_i \simeq 1$ , ie.  $R_i = r_i + \delta r_i$ ,  $\delta r_i \ll [R_i \text{ or } r_i]$ . For this  $r_i$  equation (1) reads

$$r_i = \lambda \left( n_i + \frac{\Phi_i}{2\pi} \right). \tag{6}$$

From equation (1), divided by  $R_i$  and multiplied  $r_i$ , we obtain

$$r_i = \frac{r_i}{R_i} \lambda \left( N_i + \frac{\Phi_i}{2\pi} \right). \tag{7}$$

Comparing equations (6) and (7), we obtain

$$\phi_i = \frac{r_i}{R_i} \Phi_i. \tag{8}$$

At time  $t = t'$ , equation (3) reads

$$r_i = \lambda' \left( n_i' + \frac{\phi_i'}{2\pi} \right) = \frac{r_i}{R_i} \lambda' \left( N_i' + \frac{\Phi_i'}{2\pi} \right).$$

From this we obtain

$$\phi_i' = \frac{r_i}{R_i} \Phi_i'. \tag{9}$$

As  $\phi_i = \phi_i + \Delta\phi_i^0$ ,  $\Phi_i' = \Phi_i + \Delta\Phi_i^0$  when  $\Delta R_i = 0$ , using equations (8) and (9) we obtain

$$\phi_i' = \phi_i + \Delta\phi_i^0 = \frac{r_i}{R_i} (\Phi_i + \Delta\Phi_i^0) = \phi_i + \frac{r_i}{R_i} \Delta\Phi_i^0,$$

which gives

$$\Delta\Phi_i^0 = \frac{R_i}{r_i} \Delta\phi_i^0 = \left( 1 + \frac{\delta r_i}{r_i} \right) \Delta\phi_i^0. \tag{10}$$

This  $\Delta\phi_i^0$  can be calculated. Inserting equation (10) into equation (5), we obtain

$$\Delta R_i = \frac{\lambda'}{2\pi} \left( \Delta\Phi_i - \left( 1 + \frac{\delta r_i}{r_i} \right) \Delta\phi_i^0 \right) \approx \frac{\lambda'}{2\pi} (\Delta\Phi_i - \Delta\phi_i^0) = \frac{c'}{2\pi\nu'} (\Delta\Phi_i - \Delta\phi_i^0) \tag{11}$$

From the set of  $n$  equations of (11), we can calculate the displacement of the transmitter, that is, of the spot we want to measure, with sufficient accuracy under the assumption that  $\delta r_i < r_i$ .

Next we will present a procedure to determine the true position of the transmitter, which will give  $\delta r_i$ . We assume that the coordinates of each receiver are given. Figure 1 shows the relation among one of the receivers which is at the origin, the assumed position of the transmitter  $T'$ , and the true one  $T$ . One of the coordinate axes is taken in the horizontal direction of  $T'$ .

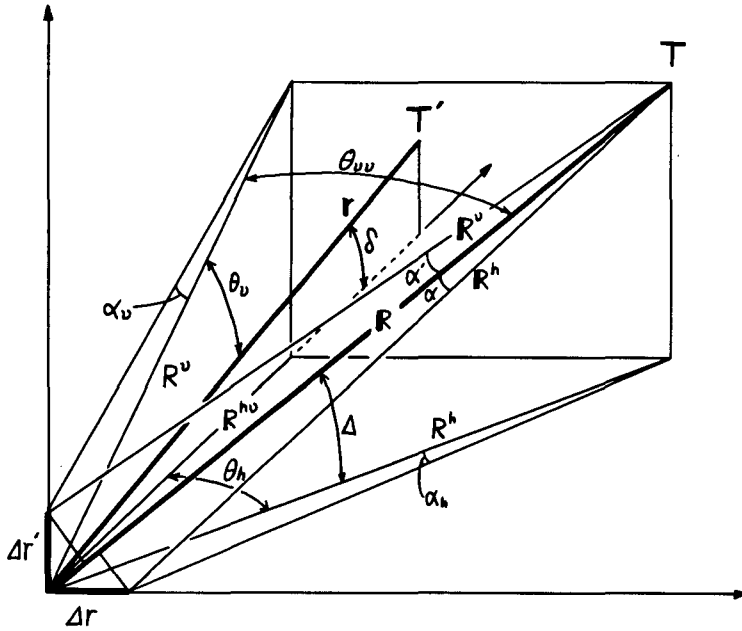


Fig. 1. Geometrical relations among the position of one of the receivers, the true position of the transmitter  $T$ , and assumed one  $T'$ . The origin of the coordinate is taken at the receiver.

The position vector of  $T'$  is  $\mathbf{r} = (r, \delta, 0)$  and that of  $T$  is  $\mathbf{R} = (R, \Delta, \theta_h)$ . The angle between  $\mathbf{R}$  and its projection onto the vertical plane, of which length is  $R^v$ , is  $\theta_{vv}$ . The angle between the projection and  $\mathbf{r}$  is  $\theta_v$ . The horizontal component of the projection is  $R^{hv}$ . The horizontal component of  $\mathbf{R}$  is  $R^h$ .

Now we displace the receiver in two definite directions, say in horizontal direction perpendicular to  $\mathbf{r}$  and in vertical direction, with amounts of  $\Delta r$  and  $\Delta r'$ , respectively. The angle between  $\mathbf{R}$  and the displaced vector when horizontally displaced is  $\alpha$  of which projection onto horizontal plane is  $\alpha_h$ . When vertically displaced, they are  $\alpha'$  and  $\alpha_v$ , respectively.

Let the changes of distance between the receiver and the transmitter be  $\Delta R$  and  $\Delta R'$  when the displacement be horizontal and vertical respectively. The displacements should not be too fast to be affected by Doppler shift and too slow to be affected by the change of light velocity and the frequency change caused by instability of the transmitter. We can know  $\Delta R$  and  $\Delta R'$  by measuring the phase variations;

$$\Delta R = \lambda \left( \Delta N^h + \frac{\Delta \Phi^h}{2\pi} \right), \Delta R' = \lambda \left( \Delta N^v + \frac{\Delta \Phi^v}{2\pi} \right). \tag{12}$$

The projection of  $\Delta R$  onto horizontal plane,  $\Delta R^h$ , and that of  $\Delta R'$  on to vertical plane,  $\Delta R^v$  are given as

$$\Delta R^h = \Delta R \cos \Delta, \Delta R^v = \Delta R' \cos \theta_{vv}. \tag{13}$$

On the other hand, geometrical considerations show following relations for  $\Delta R^h$  and  $\Delta R^v$  :

$$\begin{aligned} \Delta R^h &= -\Delta r (\sin \theta_h - \cot \eta_h \cos \theta_h), \\ \Delta R^v &= -\Delta r' \{ \sin(\theta_v + \delta) - \cot \eta_v \cos(\theta_v + \delta) \}. \end{aligned} \tag{14}$$

where

$$\cot \eta_h = \frac{1}{\Delta r \cos \theta_h} \sqrt{4(R - \Delta R)^2 \sin^2 \frac{\alpha}{2} - \Delta R^2 \sin^2 \Delta - \Delta r^2 \cos^2 \theta_h}, \tag{15}$$

and a similar expression for  $\cot \eta_v$ .

Equations from (12) to (15) are, in principle, two independent equations corresponding to two observations with three unknowns. Three displacements in orthogonal directions will be sufficient to determine the true position of the transmitter. Instead, we will make some approximation to reduce the number of unknowns. At first we estimate the size of  $\cot \eta_h$ . We change variables in equation (15) by using following geometrical relations ;

$$\begin{aligned} (R - \Delta R) \sin \Delta' &= R \sin \Delta, \\ \frac{\Delta r}{\sin \alpha_h} &= \frac{(R - \Delta R) \cos \Delta'}{\cos \theta_h}, \end{aligned}$$

where  $\Delta'$  is the dip angle of the position vector  $\mathbf{R}^h$  of the transmitter from the receiver which is horizontally displaced. Introducing these expressions to equation (15), we obtain

$$\cot \eta_h = \frac{1}{\cos \Delta' \sin \alpha_h} \sqrt{4 \sin^2 \frac{\alpha}{2} - \frac{\Delta R^2}{R^2} \sin^2 \Delta' - \cos^2 \Delta' \sin^2 \alpha_h} \tag{16}$$

We consider the case that the dip angle of  $\mathbf{R}$  is happen to be the same with that of  $\mathbf{r}$ , ie.,  $\delta = \Delta = \Delta'$ . In this case we can take the 'horizontal' plane as the plane which contains  $\mathbf{r}$  and  $\mathbf{R}$  and also the displacement vector of which length is  $\Delta r$ . Then we can put

$$\alpha = \alpha_h, \Delta' = 0.$$

Substituting these relations to equation (16) we obtain



$$\cot \eta_h = \tan \frac{\alpha}{2}.$$

Because the distance  $R$  must be sufficiently greater than the displacement  $\Delta r$ , the angle  $\alpha$  must be extremely small. For  $\cot \eta_v$  we can apply the same argument on the vertical plane. We can, therefore, neglect the second term on the right-hand side of equations (14).

Within this approximation, equations (12), (13), and (14) become

$$\Delta r \sin \theta_h = -\lambda \left( \Delta N^h + \frac{\Delta \Phi^h}{2\pi} \right) \cos \Delta \quad (17)$$

$$\Delta r' \sin (\theta_v + \delta) = -\lambda \left( \Delta N^v + \frac{\Delta \Phi^v}{2\pi} \right) \cos \theta_{vv} \quad (18)$$

The relations among the angles  $\theta_h$ ,  $\theta_v$ ,  $\theta_{vv}$ , and  $\Delta$  are

$$\tan (\theta_v + \delta) = \frac{\tan \Delta}{\cos \theta_h} \quad (19)$$

$$\cos \theta_{vv} = \frac{R^v}{R} = \frac{\sin \Delta}{\sin (\theta_v + \delta)}, \quad (20)$$

which are obtained from following geometrical relations ;

$$\begin{aligned} R^{hv} &= R^h \cos \theta_h = R \cos \theta_h \cos \Delta, \\ R_v \cos (\theta_v + \delta) &= R^{hv}, \\ R_v \sin (\theta_v + \delta) &= R \sin \Delta, \end{aligned}$$

and their definitions.

Substituting equation (20) into (18), we obtain

$$\Delta r' \sin (\theta_v + \delta) = -\lambda \left( \Delta N^v + \frac{\Delta \Phi^v}{2\pi} \right) \frac{\sin \Delta}{\sin (\theta_v + \delta)}. \quad (21)$$

We have obtained three equations (17), (19), and (21) for three unknowns,  $\theta_h$ ,  $\theta_v$ , and  $\Delta$ , with quantities being given or observed. Then we can, in principle, obtain true direction of the transmitter. Although equations (17) and (21) are approximate ones, it is possible to improve the values by repeated application of this procedure.

Once known the true direction, in principle, we can calculate  $R$  using equation (15) with a relation

$$\sin^2 \frac{\alpha}{2} = \frac{\Delta r^2 - \Delta R^2}{4R(R - \Delta R)}.$$

There is a reason to have discarded to perform three independent displacements to determine  $(R, \Delta, \theta_h)$  without approximation. The reason is based on the fact

that the distance  $R$  is quite a large quantity comparing the displacements  $\Delta r$  and generally  $\theta_h$  and  $\theta_v$  are small quantities. Small errors in  $\theta_h$  and  $\theta_v$  will result in an extremely large error in  $R$ . Because of this reason we would not recommend to determine  $R$  using data obtained at only one receiver's point. In order to determine  $R$ , it is desirable to apply the above procedure at several receiver's points.

There is no reason that the directions of the displacement should be taken in the directions prescribed above. Instead of vertical displacement, another horizontal displacement in the direction of  $\mathbf{r}$  may be more effective to reduce errors.

#### 4. Practical considerations

In this section we will consider matters which will arise in actual operation of this system. From the assumptions made in the preceding section we must determine at first what amount of the displacement of the transmitter, that is the movement of a crater, we want to measure. Then we take the wavelength of the signal to be longer than the amount of the displacement, from which the frequency of the transmitter is determined.

As an example we assume that the displacement is an order of centimeter, say 3 centimeters. Then we will take the frequency to be  $10^8$  Hz, corresponding wavelength being 3 meters. If we can measure the phase differences to the fourth figure, ie, the resolution being  $10^{-4}$  radian, and the other factors are perfectly known, we can measure the displacement with the accuracy of 0.3 mm.

Although how to make the other factors being perfectly known is purely a technical problem and outside the scope of present report, some points can be presented here. The assumption that the velocities to receivers from a transmitter have a common value may hold good generally around a volcano. The velocity change can be monitored if there is a mountain near the volcano, on which we install the same transmitter with slightly different frequency and work out the same procedure given in the preceding section with this transmitter, assuming that the mountain would not move. In this case, frequency variation of the transmitter has to be determined by comparing data obtained at different receiving points.

The transmitter on a volcano needs not to be highly stable but should not be too much unstable in order not the position determination procedure to be cumbersome. If the position determination procedure is repeatedly applied, it will take some time, during which it is desirable the frequency not to change

largely.

As for the installation of the transmitter, we propose to throw it into the point we want to measure the displacement. By this way if the transmitter is destroyed, we can reinstall new one even when eruption continues. Though the new transmitter will be thrown into a different point from the old one's, the difference can be known by the position determination procedure and it would not make the data useless in the sense of continuity.

In general the accuracy of measurements of vertical movement will be worse than that of horizontal one, because the receivers will be allocated nearly in a horizontal plane. To improve the situation, distances from the transmitter to receivers should be taken differently, that is, some receivers are placed close and some others distant from the transmitter.