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# Behavior of a Disk-shaped Low Density Fluid under Gravity Placed in a Semi-infinite High Density Fluid

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**Abstract**

Mt. Usu erupted at the end of March, 2000. A preliminary analysis was made for the situation and proposed a possible physical mechanism on the initiation of the eruption. Existence of a magma body is assumed in advance. Observed data shows the mechanical response of the ground near eruption sites can be expressed as that of a viscous fluid.

Proposed process is as follow: By exchanging heat energy between the magma and the surrounding medium, vesiculation occurs in the magma and the medium. The consequence is to form a low density layer with lower viscosity. The low density layer is considered to be a liquid inclusion in a liquid medium.

Liquid-liquid boundary is unstable when a high density liquid is on a lower one in the gravitational field. The instability causes the initiation of the eruption. The instability results in a formation of a conduit of which configuration is a function of the density difference and viscosities of the medium and the vesiculated material. The pressure to open the conduit is the buoyancy and a pressure reduction called a conduit opening pressure. The conduit opening pressure is produced by a viscous flow from peripheral region to the conduit in the inclusion.

## 1. Introduction

At the end of March 2000, Mt. Usu erupted. There are already many reports about the eruption (e.g., Geological Survey of Hokkaido, 2000). Now the time is not to make "models" constituting mainly cartoons without mathematical and physics bases but to construct models expressed though mathematical equations such as differential equations which exhibit physical processes of the eruption. The almost all "models" proposed cannot be said a model in the sense of present day science, because we cannot test or examine quantitatively by comparing the values predicted by the "models", if they predict values, with observed data. They can be called stories at best if not fairy stories.

To construct a model based on physics, we must make many assumptions because the situation is so complex that simple applications of basic equations of physics is not realistic. For example, there is no pure elastic body with simple shape such as a sphere or no pure Newtonian fluid which occupies exclusively in a region comparable in size with that of the volcano in the present problem. Responses of multi-component medium, components of which are sufficiently different character, are so messy that we cannot handle the situation by applying equations simple-mindedly.

In order to make the construction of a model possible, we first evaluate qualitatively the behavior of the volcano and, then decide what equations should be applied to the eruption process. If we can construct a model, we must solve the system of the equations to obtain predictions which must be compared with the observed data. Finally, we can judge whether the assumptions are eligible or not.

In this report, we give the first stage arguments on the evaluation of the situation occurred around Mt. Usu and determine what kind of approximations are adequate to describe the system response for the eruption.

## 2. Mechanical response of the medium

Volcanic eruption is a phenomenon in that some material effuses through solid surface of the earth. The actual eruption process is initiated before the surface phenomena occurs. The initiation of the process is said to involve magma motions but no one knows what really happens deep in the ground at the time of the initiation.

Many physical processes can involve in the initiation. We consider cases in that a magma reservoir exists in advance. Although pressure increase in the reservoir is said to cause the initiation, no real physical process which increases the pressure have been proved to involve by observations in actual volcanic eruptions. At present, two physical processes are proposed with theoretical bases.

The first one involves cooling of a magma reservoir (Tait et al. 1989). Cooling causes partial crystallization of the magma and results in degassing, which increases the pressure in the reservoir. The pressure increase to be possible by this mechanism, the surrounding medium of the reservoir must behave elastically. The model is based on pure thermodynamic theory and has not been applied to any actual volcanic event.

The second one involves magma injection to a reservoir from below.

Although the idea is very old one, quantitative evaluations of this mechanism with observed data of an actual volcano is only recently made (Maeda, 2000). The injection does not mean pressurization of magma in a deeper place as people usually believe. The injection can be carried out by buoyancy of magma. The pressure increase by the injection requires again elastic responses of the surrounding medium of the reservoir.

If the response of the surrounding medium is not elastic, the mechanisms above are not applicable. Responses of materials are classified into two categories, elastic and viscous. Mathematically, whether the response is elastic or viscous depends on whether the stress is a function of deformation or a function of deformation rate. Although the response of a real material is never purely elastic or viscous, we classify the responses by the strength of the stresses caused by deformation or deformation rate: If the stress caused by deformation is larger than that of deformation rate, we call the material elastic. On the other hands, the stress as a function of deformation rate is greater, we call the material viscous fluid. There are many material or state of material which shows both characteristics with comparable amount.

The state (response) of material depends not only on the amount of deformation and also on the characteristic time of external actions. A material which behaves elastically under sufficiently small deformation may behaves as a viscous fluid when the amount of deformation exceeds its threshold. The typical example in our problem is the ground responses. The response is clearly elastic for small external actions such as the one caused by seismic waves. In this case, the amount of deformation is less than $10^{-5}$ in normal situations. It is well known, if a strong earthquake occurs, liquidizing of ground is observed at the deformation of $10^{-3}\sim10^{-2}$. The characteristic time of this example is rather short.

If the characteristic time becomes long, in general, the threshold of deformation is lowered and the response tends to be viscous. This tendency is typically observed for wax. In the field, we frequently experience the landslide which consists of faults and, although a mass between two consecutive faults moves not as fluid but as a block, the motion can be described as a viscous deformation when averaged over some spacial extent and over some time span. Inversely speaking, if deformations around a volcano in action accompany faults, we would infer that the deformation of the volcano is not elastic but viscous.

Now, we return to the consideration for the initiation of volcanic eruption processes. There are cases in that the mechanical response of the volcanic body is considered to be viscous. We confine ourselves to the case of Mt. Usu erupted at the end of March, 2000. Many faults appeared and the difference of vertical displacements of the ground around the volcanic vents is reported to be more than 10 m over the spacial extent of 1,000 m [Geographical survey, 2000 et al]. These facts mean that the strain exceeds 1/100 and that the response of the ground can not be elastic but be viscous in the sense defined above.

Pressure buildup is not possible if the surrounding medium of a magma reservoir behaves as a viscous fluid. For this case, the initiation is caused not by the pressure increase but by the instability between two types of fluids, a heavier fluid placed over a lighter one. The lighter material may be produced by vesiculation of magma or of the surrounding medium containing water. We consider the situation as a bit more general settings.

## 3. Behavior of a fluid inclusion in a fluid medium

### *3.1 General consideration*

Behavior of a fluid inclusion in a semi-infinite different fluid under gravity relates many geophysical phenomena such as rain drops in the air or air bubbles in a pond. It goes without saying that cases consisting with a heavier fluid is placed on a lighter fluid are interesting.

Usually, cases with two fluids having largely different characteristics are analyzed. Densities and viscosities of water and air are largely different. Another related phenomenon is a formation of a rock salt dome. In this case, density and viscosity differences are not so large as those of air and liquid water. In this report, we consider a case of a small density difference and intermediate viscosity difference.

If the spacial scale is sufficiently small such as a rain drop, surface tension is important to control the shape of the inclusion. A sufficiently small inclusion would keep the shape nearly spherical by its surface tension and will be analyzed under fixed boundary assumption. An example of the case is given in the Landau-Lifshiz text book (1987). On the other hand, cases with sufficiently large scale, the boundary between two different fluid may be regarded as a plane boundary and analyses will be made under an assumption of two infinite or semi-infinite media with a plane boundary.

Consider the formation of rock-salt domes. The salt layer deposited in a geological formation has clearly finite spacial extent but may be analyzed as an infinitely extended medium with horizontal boundary. Instability of such cases is well known as Rayleigh-Taylor instability and analytical investigations are already made (Chandrasekhar 1961). For the rock salt dome, the medium surrounding the salt layer is considered to behave as a liquid. This situation has investigated experimentally (Whitehead 1986, Scott et al. 1986). These experiments illustrates how the instability grows and what will happen next. After the instability causes some disturbance at the boundary, the disturbance grows and upward streams of the lighter fluid establish.

What kinds of physical mechanisms determine the flow rate which may be expressed by factors such as the velocity and the radius of the stream? To clarify this problem, we make, in this report, so-called thought experiments based on the results given by real experiments referred above.

### *3.2. Factors determining the flow state*

Growth of a disturbance results in a dome and then becomes a plume. When the head of the plume goes upward, a conduit is formed behind the head as easily observed through simple experiments or as described by Whitehead (1986). It is evident that the size (radius in the horizontal cross section) of the plume head must be determined by the condition that the buoyancy exceeds viscous impedance exerted by the surrounding medium. The velocity of the upwelling may be approximated by the relation calculated for the motion of a liquid Stokes sphere, which given in the textbook by Landau and Lifshiz (1987).

On the other hand, the flow in the conduit may be approximated by the Poiseuille's flow driven by buoyancy. The conduit radius determines the flow rate which must be the same as the supply rate of the fluid to the conduit. There is only a little chance in that the flow rate in the conduit equals to the amount of mass conveyed by the head. Usually, it is probable that the latter is greater than the former. If it is the case, the radius of the conduit is smaller than the head size. To attain the configuration, the mass of the surrounding medium which once have been put aside by the head from the axis of the conduit squeeze the conduit inward. This means that the surrounding medium exerts to the conduit some extra pressure other than the equilibrium pressure caused by the density difference.

To make arguments a bit more clear, we give sketches of a vertical cross section of a probable situation (upper figure) and its idealized one (lower part) in figure 1. We are considering an axisymmetric situation. A piston-cylinder system is depicted in the lower figure, in that the block A is a piston with a vertical hole, that is a conduit. The piston is the representative of the medium located above the low density and low viscosity inclusion shown in the upper figure. The liquid filling inside the cylinder and under the piston is the representative of the inclusion.

We make thought experiments using the piston-cylinder system to determine the situation. First, consider that the outlet of the conduit B is closed. The system is in an equilibrium state. The pressure at just under the inclusion must be the overburden pressure of the surrounding medium at the depth of h+D in order to prevent horizontal motion of the surrounding medium. On the other hand, the pressure at the bottom of the inclusion must equal to overburden pressure P3 plus $\rho'$gh, where $\rho'$ is the density of the inclusion. The difference between the two pressures, $(\rho-\rho')$gh, where $\rho$ is the density of the surrounding medium, works as buoyancy of the inclusion. The buoyancy force acting on the bottom surface of the piston will open a conduit in an actual situation or make the head to go up as described above.

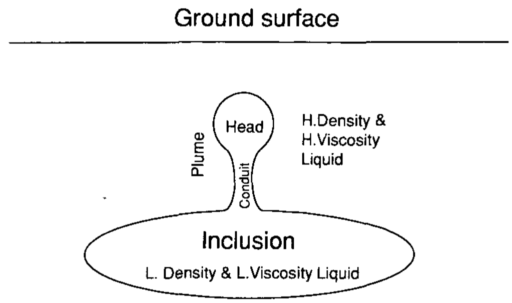
Now, we open the outlet B. Motion of the piston is very slow because of high viscosity nature of the surrounding medium. The pressure in the conduit will be released gradually from the top to bottom. At this moment, the static pressure P1 at the bottom of the conduit is $\rho'$gD, and the overburden pressure at the bottom of the piston is P3. Then, the liquid in the inclusion would effuse out from the outlet B. For simplicity, we consider that the effusion rate is steady, meaning that the piston goes down at a constant velocity.

The pressure distribution over the top of the inclusion must be uniform at the initial time, i.e.., P2=P5 in Fig. 1, but becomes non-uniform. It is evident that P2<P5. To keep supplying liquid to the conduit, there must be a flow from the peripheral region of the inclusion to the inlet of the conduit. The existence of the flow means that there exists a pressure gradient, high at the peripheral and low at the inlet of the conduit. This means that P2 is reduced but P5 is fixed constant approximately, therefore, the surrounding medium around the inlet squeezes the conduit. We call the pressure difference w between the original P2 and the reduced P2 a conduit opening pressure because the buoyancy force $(\rho-\rho')$gh must exceeds w in order to keep the conduit being open.

It is evident that this pressure difference depends on h. If h is small, the flow in the inclusion is mainly horizontal and can be approximated by a flow between walls. The flow resistance becomes large as the distance between walls decreases. This means that the conduit opening pressure w increases as h decreases. At least w is a function of h.

## 4. Conclusions

In order to construct a physical model for the eruption, 2000, of Mt. Usu, we



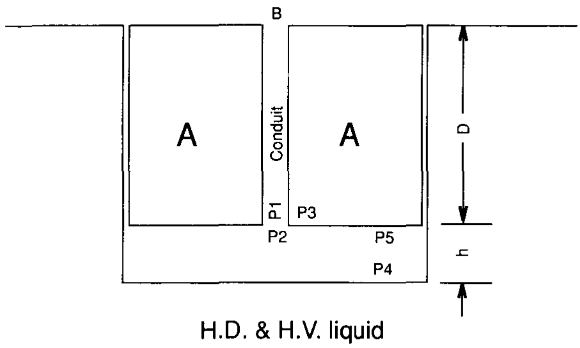




Fig. 1. Upper figure is an image of a liquid inclusion which represents a low density zone formed by vesiculation. The upper boundary is unstable and, at some point, an upward motion occurs to form a plume. The neck of the plume may be called a conduit.
Lower figure is a diagram corresponding to the upper one for thought experiments.

made a preliminary analysis for the situation and proposed a possible physical mechanism on the initiation of the eruption. A magma body appears on the scene from somewhere. We assume the existence of a magma body in advance. Observed data shows the mechanical response of the ground near eruption sites can be expressed as that of a viscous fluid.

By exchanging heat energy between the magma and the surrounding medium, vesiculation occurs in the magma and the medium. The consequence is to form a low density layer with lower viscosity. It is evident that the spacial extent of the layer is confined near the boundary between the magma top and the medium. The low density layer is considered to be a liquid inclusion in a liquid medium.

Liquid-liquid boundary is unstable when a high density liquid is on a lower one in the gravitational field. The instability causes the initiation of the eruption. The instability results in a formation of a conduit of which configuration is a function of the density difference and viscosities of the medium and the vesiculated material. The pressure to open the conduit is the buoyancy and a pressure reduction called as a conduit opening pressure. The conduit opening pressure is produced by a viscous flow from peripheral region to the conduit in the inclusion.

The adequacy of the proposed mechanism must be examined by comparisons with observed data. Preliminary calculations show very positive results. We will report later the equations constructed according to the mechanism given in this report.

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