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Inversion of Large Loop Transient Electromagnetic Data over Layered Earth Models

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Abstract

The present paper describes an inversion scheme for the large loop transient electromagnetic (TEM) data that can be applied for all the measuring configurations due to a large loop source, such as central loop, in-loop and offset-loop configurations. The inversion is based on a non-linear least square method that generates a smooth layered earth model by minimizing the residual misfit function in an iterative process. It produces an inverted model from the data using the criteria of minimization of misfit function and/or convergence of residual in two successive iterations. The forward problem is formulated in frequency domain, and then it is transformed into the time domain using Fourier cosine or sine transform. The accuracy and robustness of algorithm is tested by inverting the large loop TEM data due to the central loop, in-loop and offset loop configurations over 3-layer synthetic models, with or without addition of random noise. Inverted results are in good accordance with theoretical models and demonstrate that different parameters are recovered with high accuracy. The program works satisfactorily with noisy data and produces inverted results with acceptable accuracy for synthetic data with 5% random noise. The program in its present form is meant for inversion of voltage response data, but it also possesses the option for inversion of apparent resistivity data with slight change of input parameters and modification in the program. It allows inversion of large loop TEM data with or without the displacement current factor.

1. Introduction

Transient electromagnetic (TEM) soundings are one of the standard geophysical methods used for mapping of subsurface geo-electrical structures, large-scale hydro-geological investigation, and engineering and environmental geophysics applications (Frischknecht and Rabb, 1984 ; Nabighian, 1984 ; Fittermann and Stewart, 1986 ; Buselli et al. 1990 ; Hoekstra and Blohm, 1990 ; Christensen and Sorensen, 1998). A large loop TEM system represents a class of TEM sounding methods, which consist of a large transmitter loop for

energizing the ground and a small receiver loop or magnetometer for recording the transient voltage or magnetic field either at center of the loop or at any arbitrary point inside or outside the loop source. In general, with a large loop source, one can acquire the data using one of the configurations, namely with receiver at center of the loop (central loop method), at an arbitrary in-loop point (in-loop method), and at an offset loop point (offset-loop method) respectively. Of the three configurations, the central loop system has been developed and used extensively for exploration of mineral and geothermal resources, for mapping contaminated ground water caused by hazardous waste and thickness of permafrost layer, because of mathematical simplicity associated with the expressions of EM fields, as compared to the in-loop and offset-loop systems. However, the greatest ability and advantage of a large loop system that one can measure inside as well as outside the loop with single layout of the source loop, can be fully recognized and utilized, if and only if there would be parallel development for the forward and inverse modeling solutions for the large in-loop and offset loop systems along with the well developed central loop system.

The work relating to large loop of finite size in frequency domain can be found in Patra and Mallick (1980) and Kaufman and Eaton (2001) for central loop soundings, in Morisson et al. (1969), Ryu et al. (1970), Poddar (1982, 1983), and Ward and Hohmann (1988) for induction depth sounding for points outside the loop, and in Singh and Mogi (2002) for central, in-loop and offset loop induction sounding, whereas those in time domain can be found in Newman et al. (1987), Buselli et al. (1990), Hoekstra and Blohm, (1990), and Nabighian and Macnae (1991). The TEM sounding data are often interpreted in terms of 1-D models because of computational intricacies involved in computing transient response of simple 2-D and 3-D structures (Newman, Hohmann and Anderson, 1986). Moreover, the layered earth interpretations can be used as priori or initial models for 2-D and 3-D inversion, and have relevance and use for having first hand information about the area under consideration.

Anderson (1982) presented an inversion algorithm, NLSTCI for inversion of layered earth central loop TEM data, based on a non-linear least square technique. The algorithm, NLSTCI produces inverted models, which are very much sensitive to the initial model and some time gives entirely different inverted model, if the initial model is away from the original model. The program NLSTCI is written for VAX computer system and in practice it suffers with local minima problem. Moreover, its forward solution makes use of program ZHANKS (Anderson, 1979), which is cumbersome and uses large number of filter weights and is only meant for the central loop system. Thus,

in view of these discussions it seems logical to develop an inversion technique, which is capable of performing inversion of large loop TEM data in all the possible source-receiver configurations, viz. central loop, in-loop and offset loop configurations, and overcomes limitations of earlier methods.

Therefore, in present paper, an attempt is made to develop an inversion technique for inversion of large loop TEM data, which is suitable for all the possible source-receiver configurations, i.e. with receiver at center of the loop, at an arbitrary in-loop point and/or at an offset loop point. The method is based on minimization of misfit function and is equally capable of performing constrained as well as unconstrained inversion for large loop TEM data. It is presented for performing inversion with voltage response data, and has further option for inversion of apparent resistivity measurements over layered earth environment with slight modification of convergence limits and input parameters. Moreover, it is capable of inverting the large loop TEM data with or without the displacement current factor.

2. Theoretical Considerations

2.1 General

The plan view of large loop TEM method with different source-receiver configurations over a 3-layer earth model under consideration is shown in Figure 1. The large loop presents a source loop and smaller loops at center of the source loop (O), at an in-loop point (P_1) and at an offset loop point (P_3) depict receiver positions corresponding to the central loop, in-loop and offset loop configurations.

In general, the data collected from a large loop TEM system under consideration usually consist of vertical voltage measurements made at various time intervals after the current in transmitter is turned off. The voltage measurements are related to the time derivatives of vertical magnetic field ($\partial h/\partial t$) in accordance with relation given as following.

$$V(t) = -\mu_0(\partial h/\partial t)M \tag{1}$$

where M is the area-turns product of the receiving coil. The voltage data can be inverted directly for the layered earth models or it can be further transformed to the apparent resistivity and then inverted. Sometime it is preferable to use apparent resistivity transformation to have a direct relation with the geoelectrical section and for having an initial estimate of layer resistivities, which are often required for the non-linear inversion.

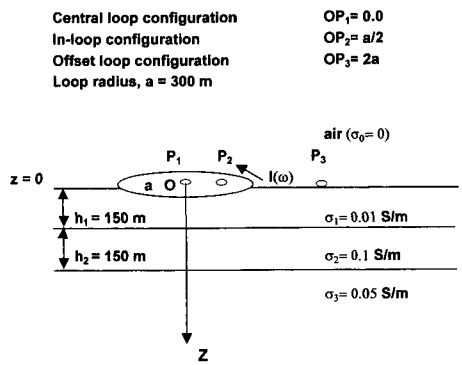

Fig. 1. Plan view of large loop system with different source-receiver configurations over a 3-layer earth model under consideration. The large loop in the figure, represent source loop, whereas the smaller loops at center (O), at an in-loop point (P_2) and at an offset-loop point (P_3) represents receiver loops for central loop, in-loop and offset loop configurations.

2.2 *Forward Problem*

The forward solution for the time derivative of vertical magnetic field ($\partial h_z/\partial t$) are obtained by transforming the frequency domain solutions for the vertical magnetic field into the time domain solutions using Fourier cosine or sine transform given as (Newman et. al. (1986)),

$$\frac{\partial h_z}{\partial t}(t) = -\frac{2}{\pi}\int_0^\infty \mathrm{Re}[H_z(\omega, \rho, h)]\cos(\omega t)d\omega \tag{2}$$

$$\frac{\partial h_z}{\partial t}(t) = \frac{2}{\pi}\int_0^\infty \mathrm{Im}[H_z(\omega, \rho, h)]\sin(\omega t)d\omega \tag{3}$$

where $\mathrm{Re}[H_z(\omega, \rho, h)]$ and $\mathrm{Im}[H_z(\omega, \rho, h)]$ are the real and imaginary parts of the vertical magnetic field over a layered earth in frequency domain. The components of vectors ρ and h are the resistivities and thickness of different layers of the layered earth model, and ω is the angular frequency.

The frequency domain integral expressions of EM field components at a point on or above the surface of an n-layered earth due to a finite horizontal circular loop of radius a, carrying a current $Ie^{i\omega t}$ and placed at height $z=-h$ above the surface of layered earth are derived in Ward and Hohmann (1988). The expressions of H_z field component at a measurement point on the surface of n-layered earth (i.e. at $z=0$) can be written as,

$$H_z(\omega, \rho, h)=\frac{Ia}{2}\int_0^\infty [e^{-u_0 h}(1+r_{TE})]\frac{\lambda^2}{u_0}J_1(\lambda a)J_0(\lambda r)d\lambda \tag{4}$$

where $r_{TE}=\dfrac{Y_0-\hat{Y}_1}{Y_0+\hat{Y}_1}$

with $Y_0=\dfrac{u_0}{i\omega\mu_0}$ (intrinsic admittance of free space)

and $\hat{Y}_1=\dfrac{H_y^{TE}}{E_x^{TE}}=-\dfrac{H_x^{TE}}{E_y^{TE}}$ (surface admittance at $z=0$)

For an n-layer case, the surface admittance are given by the recurrence relation,

$$\hat{Y}_1=Y_1=\frac{\hat{Y}_2+Y_1\tanh(u_1h_1)}{Y_1+\hat{Y}_2\tanh(u_1h_1)} \quad \hat{Y}_n=Y_n\frac{\hat{Y}_{n+1}+Y_n\tanh(u_nh_n)}{Y_n+\hat{Y}_{n+1}\tanh(u_nh_n)} \quad \text{and} \quad \hat{Y}_n=Y_n$$

with $Y_n=\dfrac{u_n}{i\omega\mu_n}$, $u_n=(k_x^2+k_y^2-k_n^2)^{1/2}=(\lambda^2-k_n^2)^{1/2}$, and $k_n^2=\omega^2\mu_n\epsilon_n-i\omega\mu_n\sigma_n$

Here, r is source-receiver offset (measured from center of the loop). For calculation purposes, $\tanh(u_nh_n)$ is used in its exponential form for stability reasons (Knight and Raiche, 1982).

Therefore, starting with computation of $H_z(\omega, \rho, h)$ field (as in equation (4)), using the method described in Singh and Mogi (2002), we have computed the time derivative of vertical magnetic field using the Fourier cosine and sine transforms (as in equations (2) and (3)). Thereafter, we have computed the voltage response (using equation (1)), which is needed as forward computation in this inversion scheme.

2.3 Inversion Approach

In general, for solving the TEM non-linear inverse problems, there exist two approaches. The first is to make no assumption about the conductivity distribution in the earth and find the classes of conductivity models that fit the data, like Occam's inversion (Constable et al, 1987). The second approach, which is more practical in many exploration problems, is to assume an initial model, which is supposed to represent that part of the earth under consideration. Thereafter, the parameters of model are estimated using an optimization technique.

The important aspect of the second approach is the assumption of correct class of model, like layered earth, dyke, cylinder or plate model. This approach allows for geological and geophysical information to be incorporated into the inverse problems. The major disadvantage of this approach is that by assum-

ing an initial model, there is always a chance of unknown bias introduced into the inverse problem (Draper and Smith, 1981).

In the present research, we have followed the second, model fitting approach, where the initial model is the layered earth with parameters consisting of resistivities and thickness of different layers. For the inversion, we have followed Anderson (1982) approach with some modification in scheme and changes in input parameters in accordance with need of present problem and to overcome the practical limitations associated with the NLSTCI program for the central loop case. The NLSTCI program is modification of a general non-linear least square algorithm of Dennis et al. (1981) to that of a constrained and unconstrained algorithm with weighted observations, and is more reliable than a Gauss-Newton or Levenberg-Marquardt algorithm when a large residual exists between data and forward solution. To overcome the problem of local minima associated with NLSTCI, in our program, we have made some adjustment in our program that in next step it recalls the program by replacing initial model parameters with inverted parameters obtained in the previous step and repeat the process for the desired number of steps or till we get a reasonable model parameters. This is achieved through the use of the fact that at each step program start with new value of λ (Marquardt parameter) depending upon the resulting residual at that step and some special procedures. Our forward problem is based on our earlier method (Singh and Mogi (2002)), which is entirely different to that in Anderson (1982) program, which uses Anderson (1979) algorithm for computation of central loop frequency domain response. The inversion is based on minimization of a residual misfit function in an iterative least square process. The misfit function, which is minimized in an iterative process, can be defined as following.

$$V(F)=\frac{1}{2}[V(RNORM)]^2 \tag{5}$$

and

$$V(RNORM)=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}R(I) \tag{6}$$

and

$$R(I)=SQWT(I)^*(Y(I)-F) \tag{7}$$

where

$$SQWT(I)=sqrt(WT(I))$$

where $Y(I)$ and $WT(I)$ are the Ith data point and corresponding weight factor, and F is corresponding calculated value.

3. Results and Discussions

For checking the accuracy and efficiency of the program for inverting the large loop TEM data, we have applied it for the inversion of large loop TEM data acquired using central loop, in-loop and offset loop configurations over the 3-layer earth model as shown in Figure 1. The inversion is tested for the noise free as well as for the noisy data (with addition of random noise), and some relevant results are shown in Figures 2-5.

Figures 2(a) and 2(b) present inversion results for the voltage response data over the 3-layer earth model (as shown in Figure 1) with 1% random (Gaussian) noise for a central loop TEM configuration with loop radius 300 m. The data points and inverted voltage response curve are shown by open circles and dotted curves in Figure 2(a), whereas the original model with which data was generated, the initial model with which inversion was started and the final inverted model are shown in Figure 2(b). The inversion was stared with an initial homogeneous model of conductivity 0.1 S/m. From Figure 2(a), it is observed that there is good matching between data points and inverted voltage response. Figure 2(b) depicts that final inverted model is in good agreement with the original model with which data was generated. The conductivity and thickness of top layer and conductivity of bottom layer are reproduced with difference as little as 0.1%, whereas the conductivities and thickness of middle layer differ by 0.3% and 1.8% respectively. The overall average variation of all the parameters is less than 1% and it may be due to addition of 1% random noise.

Figure 3 shows inversion results for the voltage response data over the 3-layer earth model (as shown in Figure 1) with 5% random noise for the central loop system with loop radius 300 m. Figure 3(a) shows data points and inverted voltage response curve, while Figure 3(b) shows original, initial and final inverted 3-layer models. The initial model with which inversion was started is a homogeneous model with conductivity 0.1 S/m. Figure 3(b) shows that there is good agreement between the observed and inverted voltage response data leading to the a final model as shown in Figure 3(b). From the Figure 3(b), it is observed that the inverted model is still in agreement with the original synthetic model with which data was generated, even after the addition of 5% random noise. The various inverted parameters, i.e. conductivities and thickness of different layers show an average variation of less than 5%, with largest difference in middle layer conductivities (33.5%). The inverted results are still in accordance with the theoretical results and depicts efficacy of the program to perform inversion with noisy data and possibility of its further application to the

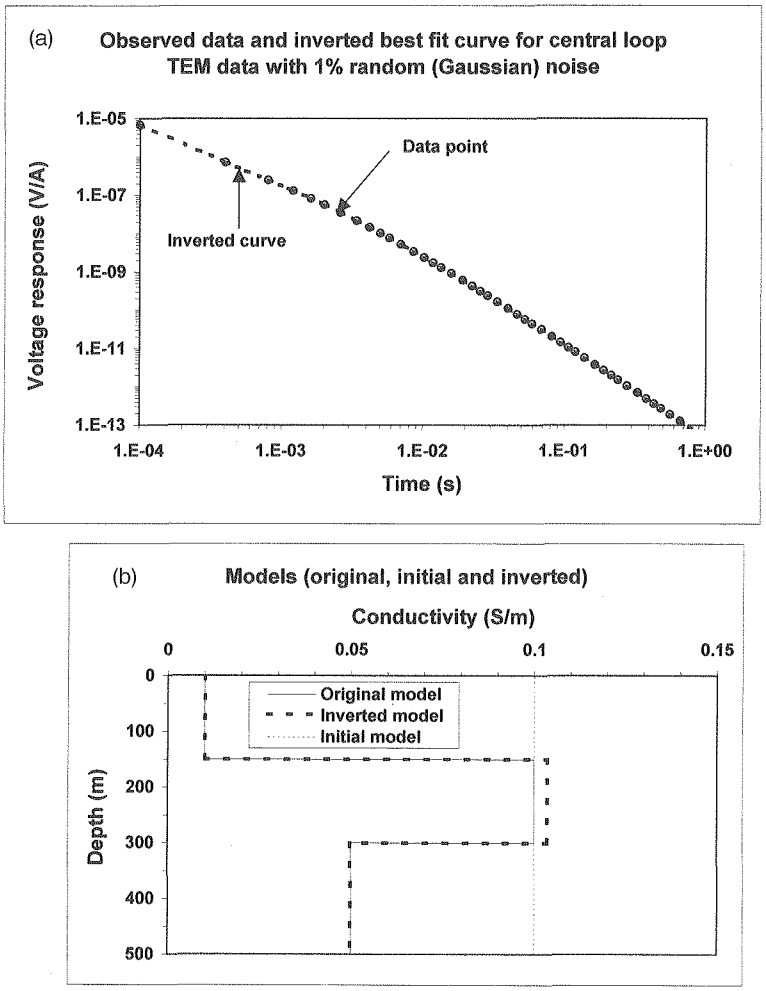

Fig. 2. Inversion results for the central loop TEM data (with loop radius 300 m) over a 3-layer earth model with 1% random noise. (a) The synthetic data points and inverted best fit curve, (b) The original model with which data was generated, the initial model with which inversion was started and the final inverted model obtained after the inversion.

real field data.

Figure 4 depicts inversion results for voltage response data over the 3-layer earth model (as in Figure 1) for an in-loop configuration with 1% random noise. The radius of source loop is 300 m and the receiver loop lies at an in-loop point at 150 m away from the centre of the loop. Figure 4(a) shows data points and

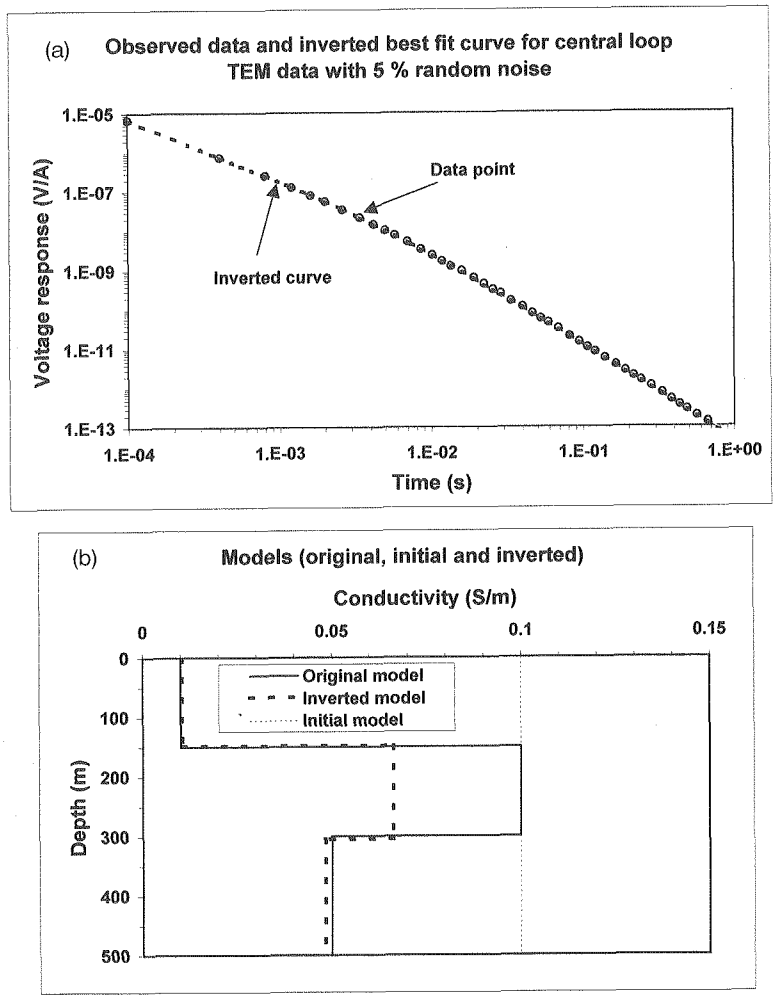

Fig. 3. Inversion results for the central loop TEM data (with loop radius 300 m) over 3-layer earth model with 5% random noise. (a) The data points and inverted best fit curve, (b) The original model with which data was generated, the initial model with which inversion was initiated, and the last inverted model.

inverted voltage response, and Figure 4(b) shows original, initial and final inverted 3-layer models. The inversion was started with a homogeneous model of conductivity 0.1 S/m. From Figure 4(b) it is clear that there is good agreement between the observed and inverted voltage response data, resulting in a final model as shown in Figure 4(b). From Figure 4(b), it is clear that the inverted

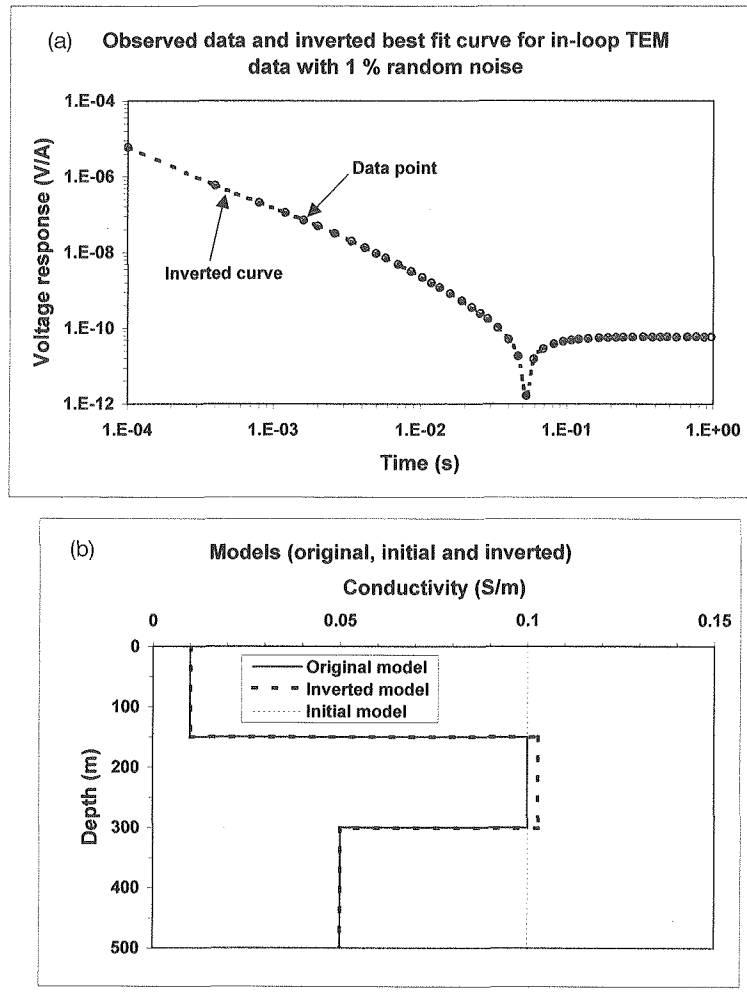

Fig. 4. Inversion results for the in-loop TEM data (with loop radius 300 m and source receiver offset 150 m) over 3-layer earth model with 1% random noise. (a) The data points and inverted best fit curve, (b) The original model with which data was generated, the initial model with which inversion was initiated, and the last inverted model.

model is in good agreement with the original synthetic model. The inverted parameters are recovered with significantly high accuracy, i.e. conductivities and thickness of different layers show that there is an average variation of less than 1%, with maximum deviation of 2.4% in conductivity of middle layer.

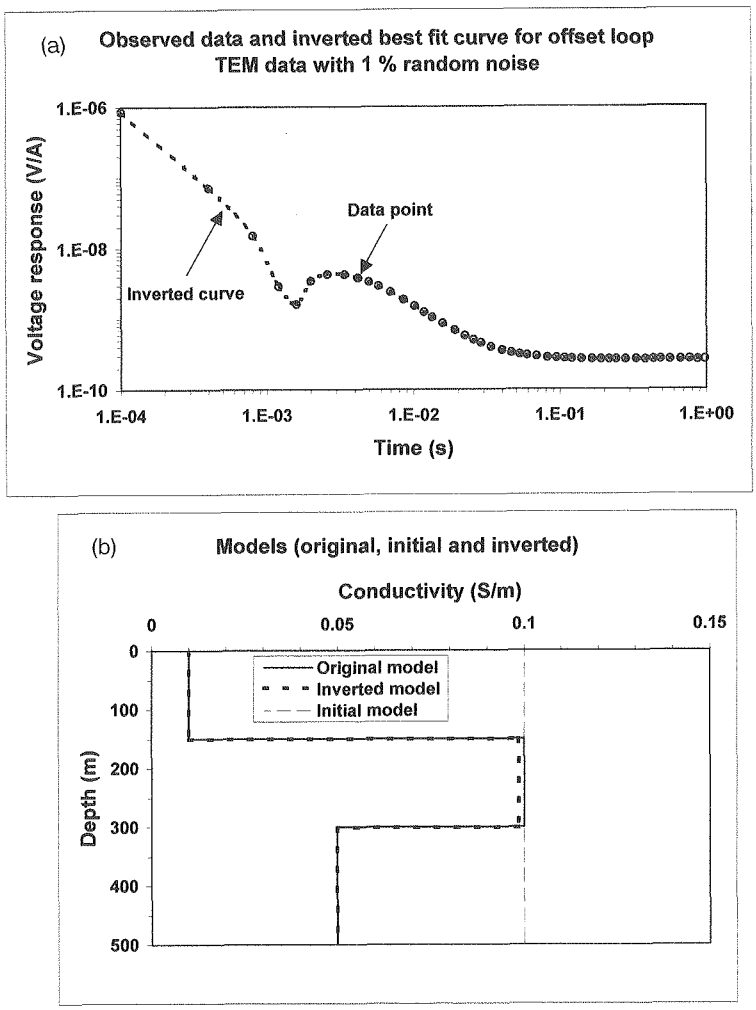

Fig. 5. Inversion results for the offset loop TEM data (with loop radius 300 m and source-receiver offset 600 m) over 3-layer earth model with 1% random noise. (a) The data points and inverted best fit curve, (b) The original model with which data was generated, the initial model with which inversion was initiated, and the final inverted model.

The inverted results are in accordance with the theoretical results and thus depict efficacy of the scheme to invert large loop TEM data acquired using in-loop configuration.

Figure 5 shows inversion results for the voltage response data over the 3-

layer earth model (as in Figure 1) for an offset loop configuration with 1% random noise. The loop radius and source-receiver offset are 300 m and 600 m respectively. Figure 5(a) shows data points and inverted voltage response curve, and depicts that there is good matching between the data points and inverted curve. Figure 5(b) shows original, initial and inverted 3-layer models. The inversion was started with a homogeneous model as shown in Figure 5(b). Figure 5(b), depicts that there is good agreement between inverted and original model with which data was generated. The inverted parameters, i.e. conductivities and thickness of different layers are close to the original parameters. The average variation of inverted parameters is less than 1%. These results depict accuracy and capability of the method for inversion of large loop TEM data acquired using offset loop configuration.

4. Conclusions

An inversion scheme based on minimization of residual misfit function is presented for the inversion of large loop transient electromagnetic data acquired using central loop, in-loop and/or offset-loop configurations. It generates a smooth layer earth model using the model fitting approach in an iterative least square process. In forward solution, it first performs frequency domain computation for the vertical magnetic field, and then transforms it into the time derivative of vertical magnetic field using Fourier cosine and sine transforms. It is equally suitable for the data with or without the displacement current factors. The theoretical examples illustrating the accuracy and efficacy of the inversion program for inverting the large loop TEM data with and/or without random noise demonstrate the potential of the program for its further application for interpretation of real field data. In addition, this program reduces the local minima problem faced by NLSTCI program (Anderson (1982)), and as shown in illustrations it gives satisfactory results with initial model parameters far away from the original models, even with the noisy data. The scheme works satisfactorily and produces reliable inversion results for data with 5% random noise. However, the program in its present form is designed for inversion of voltage response data, but it can be further modified for inversion of apparent resistivity data with slight change in tolerance limits, input parameters and some modifications in its forward subroutine.

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