A Four-dimensionalist Theory of Actions and Agents

Yasuo Nakayama
Emeritus professor, Graduate School of Human Sciences, Osaka University

Abstract

Donald Davidson proposed an event ontology and considered events as entities to which people can refer and over which people can quantify. Davidson also proposed to interpret actions as events that are intentional under some descriptions. Recent years, some philosophers further developed his theory of action and extended their analysis to collective actions. It is an aim of this paper to contribute to this research program.

In this paper, based on a four-dimensional event ontology, I propose an *axiomatic theory for actions and* agents and analyze collective actions and extended agents. This paper mainly investigates individual and collective actions that involve tools. This investigation also aims to give a metaphysical characterization of the ontological status of tools. In this investigation, it turns out that the four-dimensional ontology is a powerful framework to semantically describe temporally extended actions and collective actions. In two appendixes, I give a precise description of the proposed theories.

Keywords: collective action, extended agent, group agency, four-dimensional event ontology, ontology of artifacts

Introduction

Donald Davidson developed an event ontology and considered events as First-Order entities (FO-entities) as well as things. This means that we can quantify over events. Furthermore, he interpreted actions as events that are intentional under some descriptions (Davidson 1980). Since then, this event ontology of Davidson is widely accepted. For example, Kirk Ludwig further developed Davidson's event ontology and proposed a truth-conditional event ontology (Ludwig 2016; 2017).

In this paper, I extend the notion of *agent*. Some actions are performed by plural agents who use some tools. A play of symphony by an orchestra is a typical example for such actions. Recently, some philosophers started to investigate collective actions (Searle 2010; Tuomela 2013; Bratman 2014; Gilbert 2014; Ludwig 2016; 2017). However, actions performed with tools have been rarely studied, and one of topics in this paper is devoted for this problem. I propose to characterize a tool as a part of an extended agent. For this purpose, I use four-dimensional mereology as the formal framework. Nakayama (2013) proposed a notion of *extended agent*. However, this characterization of *extended agent* was not sufficient because notion of *joint action* (i.e., *collective action*) that is a key concept for this study was insufficiently characterized. To overcome this problem, this paper aims to clarify notions of *extended agent* and *collective action*.

There are several formal devices to represent collective expressions. One framework is Plural Logic[1] which is used in semantic investigations by Ludwig (2016; 2017). Another framework is mereology. To develop an event ontology, Nakayama (2017) proposed to give a four-dimensional interpretation of mereological *parthood* and developed a four-dimensional event ontology.

In this paper, I propose an *axiomatic theory for actions and agents* that formally characterizes actions and agents. In the main text, I explain the framework and its application examples. In two appendixes, I give a precise formalism of the theory.

1. Four-dimensional Event Ontology

Ontological studies tend to focus on things. By contrast, Nakayama (2017; 2019) proposed a four-dimensional mereological system and developed

1 For Plural Logic, see Linnebo (2022).

an ontological framework that considers events (or processes) as fundamental entities. This formal system is called *four-dimensional event ontology* (Th(4EO)) or *process ontology*. Th(4EO) is a theory for events and interprets things as a kind of events. In this framework, the universe is defined as the maximal event and all other concrete objects are considered as parts of the universe (see (Ap1.2.s)). In this section, I explain an essential part of Th(4EO) that is useful in this paper.

A mereology is an axiomatic system that has *part* as the single primitive relation. Core Mereology is the theory of partial ordering (see Ap1.1.a+b+c), and the standard mereological system is General Extensional Mereology (GEM)[2]. Core mereology is the weakest system and GEM has a strong expressive power. There are many mereological systems between them and above them. From GEM follows the following two important principles.

(S1.1.a) [Extensionality] If x and y have the same proper parts, then $x = y$.
(S1.1.b) [Unrestricted composition] Every plurality of objects possesses a sum (i.e., fusion).

Thus, to accept GEM means to accept all composed entities as existing. Some philosophers are against this inflation of entities and prefer a mereological system that is weaker than GEM. There are many criticisms on GEM, but some problems can be solved by taking four-dimensionalsm (see Sider 2001). In this paper, GEM is used as the fundamental ontological theory and the event parthood is four-dimensionally interpreted. In fact, the acceptance of GEM does not change our world but only requires a shift of our view on existence. If you accept GEM, then you say not only objects A and B but also object A+B exist.[3] Note that A+B does not add any new (atomic) entity into the world, but it only adds new referents. By accepting GEM, we can refer to more objects than before, and this gives GEM a strong expressive power.

A four-dimensionalist interprets the parthood relation as a relation between 4D (four-dimensional) entities.[4] In this paper, GEM is accepted as the basic system and the event parthood is four-dimensionally interpreted. In this section, I provide the four-dimensional mereology for events that gives a basis for analysis of actions and agents. In four-dimensionalism, the notion of *temporal part* plays an essential role, where there are different versions of definition of this notion (see Hawley 2020). Here, I modify the definition in Nakayama (2017) (see (S1.2.h+i)). Now, I explain an outline of *four-dimensional event ontology*.

(S1.2) Core of Th(4EO)[5]
(a)[(Ap1.2.a+b+c)] I use three parthood relations, namely *part*, $part_{st}$, and $part_T$ (see Table 1). I assume GEM for *part*, GEM for $part_{st}$, and GEM for $part_T$.

Parthood relation	Interpretation
part	four-dimensional parthood relation for entities in the universe
$part_{st}$	four-dimensional parthood relation for space-time objects
$part_T$	one-dimensional parthood relation for time objects

Table 1. Three parthood relations

(b) [(Ap1.2.d)] Every event $occupies_{st}$ exactly one space-time region.
(c) [(Ap1.2.e)] Every event $occupies_T$ exactly one time region.
(d) [(Ap1.2.f)] S is the space-time region of E iff E $occupies_{st}$ S. The space-time region of E is expressed by $st(E)$.
(e) [(Ap1.2.g)] T is the existence time of E iff E $occupies_T$ T. The existence time of E is expressed by *exist-time*(E).
(f) [(Ap1.2.h)] The event parthood determines the space-time parthood. In other words, if E_1 is a *part* of E_2, then $st(E_1)$ is a $part_{st}$ of $st(E_2)$.
(g) [(Ap1.2.i)] The event parthood determines the temporal parthood. In other words, if E_1 is a *part* of E_2, then *exist-time*(E_1) is a $part_T$ of *exist-time*(E_2).
(h) [(Ap1.2.j)] E_1 is a *temporal part* of E_2 iff for every event E_3 [if *exist-time*(E_3) = *exist-time*(E_1), then [E_3 is a *part* of E_1 iff E_3 is a *part* of E_2]]. In short, E_1 is a *temporal part* of E_2 iff E_1 and E_2 are indistinguishable within the existence time of E_1.
(i) [(Ap1.2.k)] If T is a $part_T$ of *exist-time*(E_1), then [the temporal part of E_1 at $T = E_2$ iff [E_2 is a temporal part of E_1 and T = *exist-time*(E_2)]]. The temporal part of E at T is expressed by *temporal-part*(E, T).
(j) [(Ap1.2.m)] x is a $part_{tp}$ of y in E iff *temporal-part*(x, *exist-time*(E)) is a part of *temporal-part*(y, *exist-time*(E)).
(k) [(Ap1.2.n] E_1 is a *spatial part* of E_2 iff [E_1 is a *part* of E_2 and *exist-time*(E_1) = *exist-time*(E_2)].
(l) [(Ap1.2.r)] $T_1 < T_2$ iff the latest time point of T_1 is

2 For mereology and GEM, see Varzi (2019). GEM is an axiomatic theory in First-Order Logic (FO-Logic) and is decidable (Tsai 2013; Varzi 2019, Sect 4.4). For formalization of GEM, see (Ap1.1).

3 Here, + is used as the symbol for *mereological sum* (see (Ap1.1.i)).

4 In general, a four-dimensionalist needs not accept GEM. In fact, there are many four-dimensionalists who reject GEM (see Sider 2001).

5 The related statement in appendixes is indicated through caption in form [(Ap*m*.*n*.*α*)].

earlier than the earliest time point of T_2.
(m) [(Ap1.2.s)] The universe is the maximal event.
(n) [(Ap1.2.t)] *now* is an indexical that denotes the current time point.

Th(4EO) delivers a basis for the following discussions in this paper.

2. Actions and Atomic Agents

Philosophy of action started with studies on simple actions that are performed by atomic agents. In this context, Davidson proposed to characterize agency in terms of intention: "a man is the agent of an act if what he does can be described under an aspect that makes it intentional" (Davidson 1980, 46). This thesis has been quite influential in philosophy of action, and many philosophers applied this principle not only to simple actions but also to collective actions (Ludwig 2017, Chapter 2). However, it is difficult to describe what collective intention is. At least, the existence of collective intention is not so obvious as individual intention. In this paper, I characterize actions as events that are brought about by agents, and I do not presuppose that there is unique explanation for agency. I only try to give some semantic characterizations of agency.

The second fundamental observation of this paper is that events and actions are, in general, *temporally extended*. For example, a walk has a duration and exists for certain time. In the same way, an agent who performs an action has a temporal duration. Only for the time span of a walk, *the agent of this walk* exists. I interpret this temporary extendedness of actions based on Th(4EO) and propose the following ontological theses of the four-dimensional action theory.

(S2.1) Characterizations of four-dimensional action theory
(a) [(Ap1.2.s)] The *universe* is the maximal 4D-entity. This means that any 4D-entity is a part of the universe.
(b) An *event* is a 4D-entity. Thus, an action is also a 4D-entity.
(c) An *agent* is a 4D-entity.
(d) [(Ap1.3.a)] x is an $agent_{tp}$ of E iff *temporal-part*(x, *exist-time*(E)) is an agent of E.
(e) [(Ap1.3.d)] E is an action iff there is x such that x is an $agent_{tp}$ of E.

Here, term *agent* is used as a primitive notion and *action* is defined based on this notion. Definition (S2.1.e) means that an action is an event that has an $agent_{tp}$. In other words, an $agent_{tp}$ is an entity that produces an event, and such an event can be interpreted as an action.

Some actions have their objects. For example, a *throw of a ball* has a ball as an object of this action. I propose the following characterization of $object_{tp}$.

(S2.2.a) [(Ap1.3.b)] x is an $object_{tp}$ of E iff *temporal-part*(x, *exist-time*(E)) is an object of E.
(S2.2.b) [(Ap1.3.j)] Any $object_{tp}$ of E has an $agent_{tp}$ of E and any musical composition has an $agent_{tp}$ of E, when it is an object of E.

According to (S2.2.b), there is no $object_{tp}$ of E without an $agent_{tp}$ of E. Thus, any $object_{tp}$ of an action requires its $agent_{tp}$. This requirement is reasonable because an action always presupposes its $agent_{tp}$ (see (S2.1.e)).

An *atomic* $agent_{tp}$ is a simple agent and this notion is a key concept of the agency.

(S2.3) Characterizations of *atomic* $agent_{tp}$
(a) [(Ap1.3.f)] An *atomic* $agent_{tp}$ of E is an $agent_{tp}$ of E.
(b) [(Ap1.3.g)] If x is an *atomic* $agent_{tp}$ of E_1, E_2 is a *spatial part* of E_1, y is an $agent_{tp}$ of E_2, and y is a *part* of x, then $y = x$ and $E_2 = E_1$. This means that an *atomic* $agent_{tp}$ of E_1 has no member who performs a sub-action of E_1. In other words, an *atomic* $agent_{tp}$ of E_1 is the single indivisible $agent_{tp}$ who performs E_1.
(c) [(Ap1.3.h)] Any $agent_{tp}$ x of E_1 has an *atomic* $agent_{tp}$ y of E_2 as its part so that E_2 is a part of E_1 and y is a $part_{tp}$ of x in E_2. In short, any $agent_{tp}$ of an action has an *atomic* $agent_{tp}$ of its sub-action.

According to (S2.3), an *atomic* $agent_{tp}$ is the smallest $agent_{tp}$ who can be a constituent of other complex $agent_{tp}$. Furthermore, any $agent_{tp}$ is herself an *atomic* $agent_{tp}$ or has an *atomic* $agent_{tp}$ as a part.

By using the introduced notions, we can analyze some simple English sentences (We use *tr* as a translation function from English sentences into sentences in FO-Logic).

(S2.4) Some English sentences
(a) Let core[S2.4.a](E) be an abbreviation of ($atomic\text{-}agent_{tp}$(Mary, E) $\wedge$ singing(E) $\wedge$ *exist-time*(E) $<$ *now*).
(b) *tr*(Mary was singing): $\exists E$ core[S2.4.a](E).
(c) core[S2.4.c](E, a): (core[S2.4.a](E) $\wedge$ *object*(a, E) $\wedge$ song(a)).
(d) *tr*(Mary sang a song): $\exists E$ $\exists a$ core[S2.4.c](E, a).[6]

6 A musical composition is an abstract object. For quantifications over musical compositions, see (Ap1.2.a+b). A set of abstract objects is often accepted as a domain in FO-Logic. For example, numbers and many mathematical objects are abstract objects that can be described in FO-Logic. However, I do not discuss this problem of abstract objects in this paper.

(e) core[S2.4.e](E, x, a): (core[S2.4.c](E, a) $\wedge$ in(st(E), st(x)) $\wedge$ school(x)).

(f) tr(Mary sang a song in a school): $\exists E$ $\exists x$ $\exists a$ core[S2.4.e](E, x, a).

(g) tr(Mary sang a song in a school yesterday): $\exists E$ $\exists x$ $\exists a$ (core[S2.4.e](E, x, a) $\wedge$ $part_T$($exist\text{-}time$(E), yesterday(now))).

(h) tr(Someone performs an action with a song): $\exists E$ $\exists x$ $\exists a$ ($agent$(x, E) $\wedge$ $action$(E) $\wedge$ $object_{tp}$(a, E) $\wedge$ song(a)).

(i) In FO-Logic, the following sentences are valid.

(i1) tr(Mary sang a song) $\rightarrow$ tr(Mary was singing)

(i2) tr(Mary sang a song in a school) $\rightarrow$ tr(Mary sang a song)

(i3) tr(Mary sang a song in a school yesterday) $\rightarrow$ tr(Mary sang a song in a school)

(i4) tr(Mary sang a song) $\rightarrow$ $\exists E$ $\exists a$ ($object$(a, E) $\wedge$ song(a))

(j) Based on (3.2.1.e) and (3.2.2.b), from Th(AT) follows: $\exists E$ $\exists a$ (($object$(a, E) $\wedge$ song(a)) $\rightarrow$ tr(Someone performs an action with a song)).

Davidson suggested that inferences in (S2.4.i1+i2+i3) are provable in his framework (Davidson 1967). According to (S2.4.i4), it appears that an action without an agent is possible. However, Th(AT) blocks this consequence because any object of an action implies the existence of an agent according to (S2.2.b).

English speakers share a belief base, and I claim that Th(AT) is included in this shared belief base.

3. Actions and Extended Agents

In this section, I would like to clarify some features of tools. Tools are artifacts that are used to support actions. To properly use a tool, we often need some exercises so that the tool is properly integrated into actions when we perform with it. Based on this observation, I propose (S3.1).

(S3.1) Characterizations of tools

(a) [(Ap1.3.c)] x is a $tool_{tp}$ for E iff $temporal\text{-}part$(x, $exist\text{-}time$(E)) is a tool for E.

(b) [(Ap1.3.k)] Any $tool_{tp}$ x for E has an $agent_{tp}$ of E which includes x as a part in E. In short, any $tool_{tp}$ x for E has an $agent_{tp}$ of E in which x is a constituent of this $agent_{tp}$ of E.

(c) [(Ap1.3.l)] If x+y is an $agent_{tp}$ of E_1 and y is a $tool_{tp}$ for E_1, then there is x's action E_2 such that E_2 is a *spatial part* of E_1 and x uses y as an $object_{tp}$ of E_2.

(d) [(Ap1.3.i)] An *extended* $agent_{tp}$ of E is an $agent_{tp}$ of E who is no *atomic* $agent_{tp}$ of E.

(S3.1.b) expresses a fundamental feature of tools. An entity becomes a tool only for the time in which an $agent_{tp}$ uses it to perform an action. For example, a knife is a simple entity when nobody uses it. Only when someone uses it, it functions as a tool.

According to (S3.1.d) and (S2.3.b), $agent_{tp}$ x+y of an action is an extended $agent_{tp}$ of this action when x+y is constructed from $agent_{tp}$ x and a $tool_{tp}$ y. A tool expands the power of an agent. A nearsighted person can safely drive a car when she wears glasses. People can easily communicate with each other in long distance when they have smart phones. As often mentioned, it is an essential capacity of humans to invent tools and to use them.

In natural languages, the existence of tools is often not explicitly expressed. In such cases, I propose to supplement a subject of a sentence with expression *with a tool*. For example, let us think about sentence *John buttered a piece of toast*. Obviously, John cannot butter a piece of toast without tool, so that I interpret the sentence as *John (with a tool) buttered a piece of toast*.

(S3.2.a) John-buttered: John (with a tool) buttered a piece of toast.

(S3.2.b) core[S3.2.b](E, x, y): ($agent_{tp}$(John+x, E) $\wedge$ buttering(E) $\wedge$ $tool_{tp}$(x, E) $\wedge$ $object_{tp}$(y, E) $\wedge$ toast(y) $\wedge$ $exist\text{-}time$(E) $<$ now).

(S3.2.c) tr(John-buttered): $\exists E$ $\exists x$ $\exists y$ core[S3.2.b](E, x, y).

(S3.2.d): tr(John-buttered, where John used the tool): $\exists E_1$ $\exists x$ $\exists y$ (core[S3.2.b](E_1, x, y) $\wedge$ $\exists E_2$ ($agent_{tp}$(John, E_2) $\wedge$ use(E_2) $\wedge$ $sp\text{-}part$(E_2, E_1) $\wedge$ $object_{tp}$(x, E_2))).

Here, *John (with a tool)* is an extended agent. Because of (S3.1.c), there is a sub-action of John that he performed using the tool.

(S3.2.e) Based on (S3.1.c), from Th(AT) follows: tr(John-buttered) $\rightarrow$ tr(John-buttered, where John used the tool).

Level 1	John (with a tool) buttered a piece of toast.	
Level 2	John used the tool.	

Table 2. Use of a tool

Here, a tool appears in two different modes, namely in the mode of a part of an extended agent and in the mode of an object of the atomic agent who is a part of the extended agent. John has a leading desire to butter a piece of toast with an object and John intends to move the object so that this leading desire will be satisfied. This John's action can be described as two layers of descriptions (Table 2). When John butters a piece of toast, he moves a tool so that this movement of the tool realizes the buttering a piece of toast. In this case, there are two actions, but the action on Level 2 constitutes an important part of the action of the extended agent on Level 1.

In this paper, I interpret sentence *John buttered a piece of toast with a knife* as *John (with a tool) buttered a piece of toast and this tool is a knife.*

(S3.3.a) *tr*(John buttered a piece of toast with a knife): $\exists E\ \exists x\ \exists y$ (core[S3.2.b](E, x, y) $\wedge$ knife(x)).
(S3.3.b) Based on (S3.2.c) and (S3.3.a), it is valid in FO-Logic: *tr*(John buttered a piece of toast with a knife) $\rightarrow$ *tr*(John (with a tool) buttered a piece of toast).

There are cases in which extended agents appear quite natural. Suppose that Paul lost his left leg by an accident, and since then he usually uses an artificial leg. When Paul is walking equipped with the artificial left leg, it is appropriate to say, "Paul with his artificial left leg is walking". Or, when we say, "Paul is walking", we mean that Paul with his artificial left leg is walking. Here, the agent of walking should be interpreted as *Paul with his artificial left leg*.

Now, let us consider a sentence in which two tools appear: "Booth shot Lincoln with a gun and pulled the trigger with his finger". It is known that the event semantics raises a problem when the shooting action and the pulling action are the same action under different descriptions (Ludwig 2018, 477). In our interpretation, the pulling action is a four-dimensional proper part of the shooting action so that the reported problem does not occur.

(S3.4) *tr*(Booth shot Lincoln with a gun and pulled the trigger with his finger): $\exists E_1\ \exists E_2\ \exists x\ \exists y\ \exists z$ ($agent_{tp}$(Booth+x, E_1) $\wedge$ shooting(E_1) $\wedge$ $tool_{tp}$(x, E_1) $\wedge$ gun(x) $\wedge$ $object_{tp}$(Lincoln, E_1) $\wedge$ *exist-time*(E_1) < *now* $\wedge$ $agent_{tp}$(Booth, E_2) $\wedge$ pulling(E_2) $\wedge$ *proper-part*(E_2, E_1) $\wedge$ $tool_{tp}$(y, E_2) $\wedge$ finger(y) $\wedge$ *part*(y, Booth) $\wedge$ $object_{tp}$(z, E_2) $\wedge$ trigger(z) $\wedge$ *part*(z, x)).

According to the translation (S3.4), the gun is a tool for shooting, Booth's finger is a tool for pulling the trigger of the pistol, and the action of *shooting the gun* includes the action of *pulling the trigger* as a proper part. To shoot Lincoln, Booth must exactly target him with the gun. Thus, shooting a gun is not just pulling a trigger. Here, we see that it is sometimes important to take four-dimensional extendedness of actions into consideration.

4. Collective Actions and Collective Agents

Ludwig (2017) pointed out that collective actions deal with the collective reading of plural and singular group action sentences (Chapter 3 and 4). To express the collective reading of plural and singular group action sentences in Th(AT), I introduce two notions, namely $group_{tp}$ and $member_{tp}$.

(S4.1) Characterizations of $group_{tp}$ and $member_{tp}$
(a) [(Ap1.3.m)] x is a $group_{tp}$ for E iff *temporal-part*(x, *exist-time*(E)) is a group for E.
(b) [(Ap1.3.n)] If x is a $group_{tp}$ for E and y is a $group_{tp}$ for E, then $x = y$.
(c) [(Ap1.3.o)] If x is a $group_{tp}$ for E_1, then [y is a $member_{tp}$ of x for E_1 iff [*temporal-part*(y, *exist-time*(E_1)) is a member of *temporal-part*(x, *exist-time*(E_1)), and there is E_2 such that E_2 is a proper part of E_1 and y is an $agents_{tp}$ of E_2]].
(d) [(Ap1.3.p)] At least two $members_{tp}$ for E belongs to a $group_{tp}$ for E.

I use *group for E* in the meaning of *group that is formed for the execution of E*. This idea is reflected in (S4.1.a). Corresponding to this idea, $members_{tp}$ of a $group_{tp}$ for E are stipulated as $agents_{tp}$ who contribute to a successful execution of E by performing sub-actions of E (see (S4.1.c)).

To explain collective actions, let us consider plural action sentence *They walked*. According to the distributing reading, the sentence means that each of them walked. By contrast, according to the collective reading, the sentence means that they walked together. In a language that includes Th(AT), these meanings can be expressed as follows.

(S4.2) Sentences with a walk
(a) core[S4.2.a](x, E): ($agent_{tp}$(x, E) $\wedge$ walking(E) $\wedge$ *exist-time*(E) < *now*).
(b) *tr*(Mary walked): $\exists E$ (core[S4.2.a](Mary, E) $\wedge$ *atomic-agent*$_{tp}$(Mary, E)).
(c) *tr**(They walked, distributive): $\forall x$ (*member*(x, They) $\rightarrow \exists E$ core[S4.2.a](x, E)).
(d) *tr**(They walked, collective): $\exists E$ (core[S4.2.a](They, E) $\wedge$ $group_{tp}$(They, E)).
(e) *tr**(Mary and Tom walked, distributive): *tr*(Mary walked) $\wedge$ *tr*(Tom walked).
(f) core[S4.2.f](E): (G = Mary+Tom $\wedge$ core[S4.2.a](G, E) $\wedge$ $group_{tp}$(G, E)).
(g) *tr**(Mary and Tom walked, collective): $\exists E$ core[S4.2.f](E).
(h) *tr**(Mary and Tom walked together as its members, collective): $\exists E$ (core[S4.2.f](E) $\wedge$ $member_{tp}$(Mary, G, E) $\wedge$ $member_{tp}$(Tom, G, E)).

According to the distributive readings (S4.2.c+e), every member of them performed a walking action in the past. By contrast, in the collective readings (S4.2.d+g), the action of each member is not explicitly mentioned. In a collective walking E, the $members_{tp}$ for E performed a sub-action of E so that the $group_{tp}$ for E could accomplish E. The sentence does not express what kind of actions the $members_{tp}$ for E performed.

Here, I propose to interpret a collective action as an action whose $agent_{tp}$ is a group of agents.

(S4.3) Characterizations for collective actions
(a) [(Ap1.3.q)] A $collective\text{-}action_{tp}$ is an action whose $agent_{tp}$ is a $group_{tp}$.
(b) [(Ap1.3.r)] If E is a $collective\text{-}action_{tp}$ and x is a $group_{tp}$ for E, then E consists of actions which are performed by the $agents_{tp}$ who are $members_{tp}$ of x for E.

Here, let us think again about the collective walk of Mary and Tom. In this case, because of (S4.3.b), from Th(AT) follows: tr^*(Mary and Tom walked together as its members, collective) $\rightarrow \exists E_1 \exists E_2 \exists E_3$ ($agent_{tp}$(Mary+Tom, E_1) $\wedge$ walking(E_1) $\wedge$ $E_1 = E_2+E_3$ $\wedge$ $agent_{tp}$(Mary, E_2) $\wedge$ $action(E_2)$ $\wedge$ $agent_{tp}$(Tom, E_3) $\wedge$ $action(E_3)$). Thus, when Mary and Tom are walking together, they try to perform individual actions that support the accomplishment of their collective walk. This individual action in a collective walk requires more than a simple individual walk and this further requirement distinguishes a collective walk from a simple fusion of plural individual walks (see Table 3 and Table 4).

Level 1	Mary and Tom walked together.	
Level 2	Mary performed a sub-action of the collective action in Level 1.	Tom performed a sub-action of the collective action in Level 1.

Table 3. Collective reading of *Mary and Tom walked*

Level 1	Mary walked.	Tom walked.

Table 4. Distributive reading of *Mary and Tom walked*

Next, let us consider a more complex example: A violinist and a pianist played Beethoven's *Spring Sonata* (BSS). In this example, a collective action of playing a sonata was performed by two extended agents who used music instruments.

(S4.4) Play of BSS
(a) core[S4.4.a](E, G, x, y): ($agent_{tp}(G, E) \wedge group_{tp}(G, E) \wedge part_{tp}(x+y, G, E)$ $\wedge$ violinist(x) $\wedge$ pianist(y) $\wedge$ playing(E) $\wedge$ $object$(BSS, E) $\wedge$ $exist\text{-}time(E) < now$).
(b) tr^*(A violinist and a pianist played BSS, collective): $\exists E \exists G \exists x \exists y$ core[S4.4.a](E, G, x, y).
(c) core[S4.4.c]($E_1, E_2, E_3, G, x, y, u, v$): (core[S4.4.a]($E_1, G, x, y$) $\wedge$ $G = x+u+y+v$ $\wedge$ $member_{tp}(x+u, G, E_1)$ $\wedge$ $member_{tp}(y+v, G, E_1)$ $\wedge$ $E_1 = E_2+E_3$ $\wedge$ $agent_{tp}(x+u, E_2)$ $\wedge$ $tool_{tp}(u, E_2)$ $\wedge$ violin(u) $\wedge$ $agent_{tp}(y+v, E_3)$ $\wedge$ $tool_{tp}(v, E_3)$ $\wedge$ piano(v)).
(d) tr^*(A violinist (with a violin) and a pianist (with a piano) played BSS, collective): $\exists E_1 \exists E_2 \exists E_3 \exists G \exists x \exists y \exists u \exists v$ core[S4.4.c]($E_1, E_2, E_3, G, x, y, u, v$).
(e) core[S4.4.e]($E_1, E_2, E_3, E_4, E_5, G, x, y, u, v$): (core[S4.4.c]($E_1, E_2, E_3, G, x, y, u, v$) $\wedge$ $sp\text{-}part(E_4, E_2)$ $\wedge$ $agent_{tp}(x, E_4)$ $\wedge$ use(E_4) $\wedge$ $object_{tp}(u, E_4)$ $\wedge$ $sp\text{-}part(E_5, E_2)$ $\wedge$ $agent_{tp}(y, E_5)$ $\wedge$ use(E_5) $\wedge$ $object_{tp}(v, E_5)$).
(f) Because of (S3.1.c), from Th(AT) follows: tr^*(A violinist (with a violin) and a pianist (with a piano) played BSS, collective) $\rightarrow \exists E_1 \exists E_2 \exists E_3 \exists E_4 \exists E_5 \exists G \exists x \exists y \exists u \exists v$ core[S4.4.e]($E_1, E_2, E_3, E_4, E_5, G, x, y, u, v$).

Level 1	A violinist and a pianist played BSS.			
Level 2	The violinist with a violin played a part of BSS as a sub-action of the collective action in Level 1.		The pianist with a piano played a part of BSS as a sub-action of the collective action in Level 1.	
Level 3	The violinist used the violin.			The pianist used the piano.

Table 5. Play of BSS

As this example shows, a collective action can have a quite complex structure. In the play of BSS, the violinist moves a string of the violin to play her part of BSS, and the pianist touches keys of the piano to play her part of BSS. Their plays of instruments must be coordinated so that the sum of both activities produces BSS. Otherwise, they cannot successfully play the sonata (see Table 5).

In some collective actions, different types of division of labor are possible. Let us consider a collective painting. Suppose that Peter and Tom decide to paint a house. They might divide the task of painting so that Peter (with a tool) paint a part of the house and Tom (with a tool) paint another part (see (S4.5.d+e)). In other situation, it might be the case that only Peter is a skillful painter and Tom devotes himself to support of Peter's painting (see (S4.5.f+g)).

(S4.5) Collective actions with different divisions of labor
(a) Sentence-painting: Peter and Tom painted a house.
(b) core[S4.5.b](E, x, y): (G(x) = Peter+Tom+x $\wedge$ $agent_{tp}$(G(x), E) $\wedge$ $group_{tp}$(G(x), E) $\wedge$ $tool_{tp}(x, E)$ $\wedge$ painting(E) $\wedge$ $object_{tp}(y, E)$ $\wedge$ house(y) $\wedge$ $exist\text{-}time(E) < now$).
(c) tr^*(Sentence-painting, collective): $\exists E \exists x \exists y$ core[S4.5.b](E, x, y).
(d) core[S4.5.d]($E_1, E_2, E_3, x, y, u, v, y1, y2$):

(core[S4.5.b](E_1, x, y) $\wedge$ $E_1 = E_2+E_3$ $\wedge$ $agent_{tp}$(Peter+u, E_2) $\wedge$ $member_{tp}$(Peter+u, G(x), E_1) $\wedge$ $tool_{tp}$(u, E_2) $\wedge$ $x = u+v$ $\wedge$ painting(E_2) $\wedge$ $object_{tp}$($y1$, E_2) $\wedge$ $y = y1+y2$ $\wedge$ $agent_{tp}$(Tom+v, E_2) $\wedge$ $member_{tp}$(Tom+v, G(x), E_1) $\wedge$ $tool_{tp}$(v, E_3) $\wedge$ painting(E_3) $\wedge$ $object_{tp}$($y2$, E_3)).

(e) *tr**(Sentence-painting and they divided the task, collective): $\exists E_1\ \exists E_2\ \exists E_3\ \exists x\ \exists y\ \exists u\ \exists v\ \exists y1\ \exists y2$ core[S4.5.d](E_1, E_2, E_3, x, y, u, v, $y1$, $y2$).

(f) core[S4.5.f](E_1, E_2, E_3, x, y): (core[S4.5.b](E_1, x, y) $\wedge$ $E_1 = E_2+E_3$ $\wedge$ $member_{tp}$(Peter+x, G(x), E_1) $\wedge$ $agent_{tp}$(Peter+x, E_2) $\wedge$ painting(E_2) $\wedge$ $object_{tp}$(y, E_2) $\wedge$ $member_{tp}$(Tom, G(x), E_1) $\wedge$ $agent_{tp}$(Tom, E_3) $\wedge$ supporting(E_3) $\wedge$ $object_{tp}$(Peter+x, E_3)).

(g) *tr**(Sentence-painting and Tom supported Peter during the work, collective): $\exists E_1\ \exists E_2\ \exists E_3\ \exists x\ \exists y$ core[S4.5.f](E_1, E_2, E_3, x, y).

This example shows that there are different forms of cooperation in collective actions. In some cases, tasks in a collective action are equally distributed among members of the collective agent. On the other hand, there are cases in which some agents perform main actions and other members support them.

There are collective actions that have a layered structure. Let us consider the following example of production in a factory.

(S4.6) Collective action with a layered structure of actions

(a) Example: There was a factory F_1 and there were two machines M_1 and M_2 in F_1. Workers of F_1 produced packed flue masks with the machines. Group G_1 of workers produced flue masks with machine M_1 and group G_2 of workers packed the products with machine M_2.

(b) core[S4.6.b]: (factory(F_1) $\wedge$ machine(M_1) $\wedge$ machine(M_2) $\wedge$ M = M_1+ M_2 $\wedge$ in(st(M), st(F_1)) $\wedge$ $exist\text{-}time$(F_1) < now).

(c) [The first sentence] *tr*(1): core[S4.6.b].

(d) core[S4.6.d](E, x): (core[S4.6.b] $\wedge$ G = σu worker-of(u, F_1) $\wedge$ $agent_{tp}$(G+M, E) $\wedge$ $group_{tp}$(G+M, E) $\wedge$ producing(E) $\wedge$ $tool_{tp}$(M, E) $\wedge$ $object_{tp}$(x, E) $\wedge$ packed-flue-masks(x) $\wedge$ $exist\text{-}time$(E) < now).[7]

(e) [The first two sentences] *tr*(1+2): $\exists E\ \exists x$ core[S4.6.d](E, x).

(f) core[S4.6.f](E_1, E_2, E_3, x, y): (core[S4.6.d](E_1, x) $\wedge$ $E_1 = E_2+E_3$ $\wedge$ G = G_1+G_2 $\wedge$ $agent_{tp}$(G_1+M_1, E_2) $\wedge$ $group_{tp}$(G_1+M_1, E_2) $\wedge$ producing(E_2) $\wedge$ $object_{tp}$(y, E_2) $\wedge$ flue-masks(y) $\wedge$ $part$(y, x) $\wedge$ $agent_{tp}$(G_2+M_2, E_3) $\wedge$ $group_{tp}$(G_2+M_2, E_3) $\wedge$ packing(E_3) $\wedge$ $object_{tp}$(x, E_3)).

(g) [Three sentences] *tr*(1+2+3): $\exists E_1\ \exists E_2\ \exists E_3\ \exists x\ \exists y$ core[S4.6.f](E_1, E_2, E_3, x, y).

(h) Based on (S4.3.b), from Th(AT) follows: *tr*(1+2+3) $\rightarrow$ $\exists E_1\ \exists E_2\ \exists E_3\ \exists x\ \exists y$ (core[S4.6.f](E_1, E_2, E_3, x, y) $\wedge$ $\forall z$ ($member_{tp}$(z, G_1+M_1, E_2) $\rightarrow$ $\exists E_4$ ($part$(E_4, E_2) $\wedge$ $agent_{tp}$(z, E_4))) $\wedge$ $\forall z$ ($member_{tp}$(z, G_2+M_2, E_3) $\rightarrow$ $\exists E_4$ ($part$(E_4, E_3) $\wedge$ $agent_{tp}$(z, E_4)))).

Level 1	G produced pfms (packed flue masks) with M.			
Level 2	G_1 produced fms with M_1 as a sub-action of the collective action in Level1.		G_2 packed pfms with M_2 as a sub-action of the collective action in Level1.	
Level 3	Workers in G_1 with M_1 performed sub-actions of the collective action in Level 2.		Workers in G_2 with M_2 performed sub-actions of the collective action in Level 2.	
Level 4	Each worker in G_1 performed an action with a part of M_1 as a part of action described in Level 3.			Each worker in G_2 performed an action with a part of M_2 as a part of action described in Level 3.

Table 6. Production with two machines

In this example, the production of packed flue masks is divided into two parts, namely production of flue masks and packing of them. This is *division of labor* whose notion was introduced by sociologist Émil Durkheim (Durkheim 1893). In this case, the form of the division was determined by the features of two machines. The structure of machines restricts the range of possible distributions of workers. In the example, we can identify four levels of working stages (see Table 6). As Table 6 shows, a collective action can have a complex layered structure. Note that we can construct more complex layered structure of collective actions.

Based on Th(AT), we can properly express the content of a sentence which is a conjunction of a plural subject action sentence with the collective reading and one with the distributive reading.

(S4.7) Collective reading and distributive reading

(a)Example[S4.7] (Ludwig (2016), 143): They carried the piano upstairs and got a cookie as a reward.

(b) core[S4.7.b](E, x): ($agent_{tp}$(They, E) $\wedge$ $group_{tp}$(They, E) $\wedge$ carrying-upstairs(E) $\wedge$ $object_{tp}$(x, E) $\wedge$ piano(x) $\wedge$ $exist\text{-}time$(E) < now).

(c) *tr**(They carried the piano upstairs, collective): $\exists E$ $\exists^{=1} x$ core[S4.7.b](E, x).

(d) core[S4.7.d](E_1, E_2, x): (core[S4.7.b](E_1, x) $\wedge$ $\forall y$ ($member$(y, $temporal\text{-}part$(They, E_2)) $\rightarrow$ $\exists E_3$ $\exists z$ (get-as-a-reward($temporal\text{-}part$(y, E_3), E_3) $\wedge$ $part$(E_3, E_2) $\wedge$ $object_{tp}$(z, E_3) $\wedge$ cookie(z) $\wedge$ $exist\text{-}time$(E_1) < $exist\text{-}time$(E_3) $\wedge$ $exist\text{-}time$(E_3) < now))).

7 σu worker-of(u, F_1) denotes the sum of workers of factory F_1. For this notation, see (Ap1.1.g).

(e) tr*(Example[S4.7], collective): $\exists E_1 \exists E_2 \exists x$ core[S4.7.d](E_1, E_2, x).

In this example, the plural subject "they" in the collective reading denotes a temporal part of a plural entity which is a $group_{tp}$ for a carrying action and the second plural subject in the distributive reading denotes a different temporal part of the same plural entity.

The examples in this section suggest that Th(AT) can be used to analyze complex human activities.

5. Characterizations of Actions

As a summary of this paper, I describe translation schemata for sentences with transitive verbs. Translation schemata for sentences with intransitive verbs can be constructed in a similar manner (see (S4.2)).

(S5.1) Translation schemata for sentences with transitive verbs

(a) core[S5.1.a](E, x, y): ($agent_{tp}(x, E) \wedge \mathrm{act}_{\mathrm{transitive}}(E) \wedge object_{tp}(y, E) \wedge \mathrm{obj}(y) \wedge exist\text{-}time(E) < now$).

(b) [Singular sentence] tr(S [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj): $\exists E \exists x$ (core[S5.1.a] (E, S, x) $\wedge$ $atomic\text{-}agent_{tp}$(S, E)).

(c) [Singular sentence + tool] tr(S [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj with a device): $\exists E \exists x \exists y$ (core[S5.1.a](E, S+y, x) $\wedge$ $atomic\text{-}agent_{tp}$(S, E) $\wedge$ $tool_{tp}(y, E)$ $\wedge$ device(y)).

(d) [Plural sentence] tr*(G [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj, distributive): $\forall x$ ($member(x$, G) $\rightarrow \exists E \exists y$ core[S5.1.a](E, x, y)).

(e) [Plural sentence] tr*(G [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj, collective): $\exists E \exists x$ (core[S5.1.a](E, G, x) $\wedge$ $group_{tp}$(G, E)).

(f) [Plural sentence + tool] tr*(G [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj with a device, distributive): $\forall x$ ($member(x$, G) $\rightarrow$ $\exists E \exists y \exists z$ (core[S5.1.a](E, x+z, y) $\wedge$ $tool_{tp}(z, E)$ $\wedge$ device(z))).

(g) [Plural sentence + tool] tr*(G [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj with a device, collective): $\exists E \exists x \exists y$ (core[S5.1.a] (E, G+y, x) $\wedge$ $group_{tp}$(G+y, E) $\wedge$ $tool_{tp}(y, E)$ $\wedge$ device(y)).

These schemata show that the following FO-sentence is included in every translation: $\exists E \exists x \exists y$ ($agent_{tp}(x, E) \wedge \mathrm{act}_{\mathrm{transitive}}(E) \wedge object_{tp}(y, E) \wedge \mathrm{obj}(y) \wedge exist\text{-}time(E) < now$). This FO-sentence corresponds to tr(someone [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj) and means: For some E, x, and y, [x is an $agent_{tp}$ of E, E is an $\mathrm{act}_{\mathrm{transitive}}$, y is an $object_{tp}$ of E, y is an obj, and E is past]. In a simple singular sentence, the following condition is added to this basic FO-sentence: S is an atomic $agent_{tp}$ of E (see (S5.1.b)). Similarly, in a plural sentence in the collective reading, the following condition is added: G is a $group_{tp}$ for E (see (S5.1.e)). The distributive reading of a plural sentence "G [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj" interprets its content as follows: each member x of G performs an action expressed in simple sentence "x [$\mathrm{act}_{\mathrm{transitive}}$]$_{\mathrm{past}}$ an obj" (see (S5.1.d)). In this distributive case, all individual actions are independently performed so that no collective agent exists in this context. An action using a device as a tool can be easily expressed by interpreting $agent_{tp}$ as an *extended* $agent_{tp}$ with the tool and adding conditions that u is a $tool_{tp}$ for E and that u is a device for a proper variable u (see (S5.1.c+f+g)). As you can see, all FO-translation schemata in (S5.1) are straightforward. The complexity of meaning of action sentences with extended and collective agents emerges from implications that some fundamental relations involve. Based on this consideration, the meaning of fundamental relations such as $agent_{tp}$, $tool_{tp}$, and $group_{tp}$ is axiomatically characterized in Th(AT).[8]

This paper provides a semantic analysis of action sentences based on Th(AT). However, we did not answer the following fundamental question: How do atomic agents realize a collective action? A task of this paper was to deliver a semantic basis for investigations on collective intentionality. The next step will be to develop a theory of collective intentionality based on Th(AT) and the presupposition that Th(AT) is shared by members of a linguistic community.[9]

Concluding Remarks

In this paper, I proposed a *four-dimensionalist axiomatic theory of actions and agents* (called Th(AT)) and analyzed collective actions and extended agents. In this paper, I have shown that Th(AT) is quite useful to investigate temporally extended complex (collective) actions. I have also suggested that tools extend the range of actions. In fact, inventions of tools have a potential to change societies and their environments.[10] [11]

Appendix 1

Here, I precisely describe some definitions and characterizations discussed in the main text.

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8 The characterization of agency in Th(AT) gives only a necessary condition for agency. A description of intentionality is needed for a full characterization of agency.

9 A formal model of agents proposed in Nakayama (2022) might be useful to describe the relationship between agents and the society.

10 For discussions on the development of technology, see Nakayama (2016: Chapter 8).

11 I would like to thank two reviewers for many valuable suggestions.

(Ap1.1) General Extensional Mereology (GEM) (modified from Varzi (2019))

(a) [Reflexivity] $\forall x\, P(x, x)$.

(b) [Anti-symmetry] $\forall x\, \forall y\, ((P(x, y) \wedge P(y, x)) \rightarrow x = y)$.

(c) [Transitivity] $\forall x\, \forall y\, \forall z\, ((P(x, y) \wedge P(y, z)) \rightarrow P(x, z))$.

(d) [Overlap] $\forall x\, \forall y\, (O(x, y) \leftrightarrow \exists z\, (P(z, x) \wedge P(z, y)))$.

(e) [Strong Supplementation] $\forall x\, \forall y\, (\neg P(y, x) \rightarrow \exists z\, (P(z, y) \wedge \neg O(z, x)))$.

(f) [Unrestricted Fusion] For any formula $\varphi(x)$, $\exists x\, \varphi(x) \rightarrow \exists z\, \forall y\, (O(y, z) \leftrightarrow \exists x\, (\varphi(x) \wedge O(y, x)))$, when variables y and z do not occur free in $\varphi(x)$.

(g) [Notation for fusion] $\exists x\, \varphi(x) \rightarrow \forall z\, (z = \sigma x\, \varphi(x) \leftrightarrow \forall y\, (O(y, z) \leftrightarrow \exists x\, (\varphi(x) \wedge O(y, x))))$, when variables y and z do not occur free in $\varphi(x)$.

(h) [Proper Part] $\forall x\, \forall y\, (PP(x, y) \leftrightarrow (P(x, y) \wedge \neg P(y, x)))$.

(i) [Sum] $\forall x\, \forall y\, (x+y = \sigma z\, (P(z, x) \vee P(z, y)))$.

(Ap1.2) Theory for four-dimensional event ontology (Th(4EO))

(a)In this paper, I express quantifications over different sorts of objects by relativizations. Let α and β be two sorts of variables and *Domain* be a *sub-domain predicate*. Then, $\forall\beta\, \varphi(\beta)$ and $\exists\beta\, \varphi(\beta)$ are used as abbreviations of $\forall\alpha\, (Domain(\alpha) \rightarrow \varphi(\alpha))$ and $\exists\alpha\, (Domain(\alpha) \wedge \varphi(\alpha))$, respectively. Here, I use *Space-time*, *Time*, *Event*, and *Music* as sub-domain predicates (see Table 7).

Sub-domain	Variables	Sub-domain predicates
Space-time objects	$S, S_1, S_2, \ldots$	*Space-time*
Time objects	$T, T_1, T_2, \ldots$	*Time*
Events	$E, E_1, E_2, \ldots$	*Event*
Musical compositions	$a, a_1, a_2, \ldots$	*Music*

Table 7. Variables and predicates for relativizations in Th(4EO)

(b) $\neg\exists\alpha\, (Sub\text{-}domain_1(\alpha) \wedge Sub\text{-}domain_2(\alpha))$ for any two different sub-domain predicates from {*Space-time*, *Time*, *Event*, *Music*}. Additionally, we require: $\forall\alpha\, (Space\text{-}time(\alpha) \vee Time(\alpha) \vee Event(\alpha) \vee Music(\alpha))$.

(c) GEM for *part*, GEM for $part_T$, and GEM for $part_{st}$.

(d) $\forall E\, \exists^{=1} S\, occupy_{st}(E, S)$.

(e) $\forall E\, \exists^{=1} T\, occupy_T(E, T)$.

(f) $\forall E\, \forall S\, (S = st(E) \leftrightarrow occupy_{st}(E, S))$.

(g) $\forall E\, \forall T\, (T = exist\text{-}time(E) \leftrightarrow occupy_T(E, T))$.

(h) $\forall E_1\, \forall E_2\, (part(E_1, E_2) \rightarrow part_{st}(st(E_1), st(E_2)))$.

(i) $\forall E_1\, \forall E_2\, (part(E_1, E_2) \rightarrow part_T(exist\text{-}time(E_1), exist\text{-}time(E_2)))$.

(j) [Relation *temp-part*] $\forall E_1\, \forall E_2\, (temp\text{-}part(E_1, E_2) \leftrightarrow \forall E_3\, (exist\text{-}time(E_3) = exist\text{-}time(E_1) \rightarrow (part(E_3, E_1) \leftrightarrow part(E_3, E_2))))$.

(k) [Function *temporal-part*] $\forall T\, \forall E_1\, (part_T(T, exist\text{-}time(E_1)) \rightarrow \forall E_2\, (temporal\text{-}part(E_1, T) = E_2 \leftrightarrow (temp\text{-}part(E_2, E_1) \wedge T = exist\text{-}time(E_2))))$.

(l) $\forall x\, \varphi(x)$ and $\exists x\, \varphi(x)$ are abbreviations of $\forall E\, (Thing(E) \rightarrow \varphi(E))$ and $\exists E\, (Thing(E) \wedge \varphi(E))$, respectively. Thus, *Thing* denotes a sub-domain of events.[12] I use $x, y, z, \ldots$ as variables for things.

(m) $\forall x\, \forall y\, \forall E\, (part_{tp}(x, y, E) \leftrightarrow part(temporal\text{-}part(x, exist\text{-}time(E)), temporal\text{-}part(y, exist\text{-}time(E))))$.

(n) [Spatial Part] $\forall E_1\, \forall E_2\, (sp\text{-}part(E_1, E_2) \leftrightarrow (part(E_1, E_2) \wedge exist\text{-}time(E_1) = exist\text{-}time(E_2)))$.

(o) $\forall T_1\, (atomic(T_1) \leftrightarrow \forall T_2\, (part_T(T_2, T_1) \rightarrow T_2 = T_1))$.

(p) $\forall t\, \varphi(t)$ and $\exists t\, \varphi(t)$ are used as abbreviations of $\forall T\, (atomic(T) \rightarrow \varphi(T))$ and $\exists T\, (atomic(T) \wedge \varphi(T))$, respectively.

(q) [Linearity] $\forall t\, \neg(t < t) \wedge \forall t_1\, \forall t_2\, \forall t_3\, ((t_1 < t_2 \wedge t_2 < t_3) \rightarrow t_1 < t_3) \wedge \forall t_1\, \forall t_2\, (t_1 < t_2 \vee t_1 = t_2 \vee t_2 < t_1)$.

(r) $\forall T_1\, \forall T_2\, (T_1 < T_2 \leftrightarrow \forall t_3\, \forall t_4\, ((part_T(t_3, T_1) \wedge part_T(t_4, T_2)) \rightarrow t_3 < t_4))$.

(s) $Universe = \sigma E\, part(E, E)$.

(t) *now* is an indexical that denotes the current time point.

(u) Th(4EO) consists of the requirements from (Ap1.2.a) to (Ap1.2.t).[13]

Th(AT) presupposes Th(4EO). In Th(AT), *agent*, *object*, *tool*, *member* are introduced as primitive relations.

(Ap1.3) Theory for actions and agents (Th(AT))).

(a) $\forall x\, \forall E\, (agent_{tp}(x, E) \leftrightarrow agent(temporal\text{-}part(x, exist\text{-}time(E)), E))$.

(b) $\forall x\, \forall E\, (object_{tp}(x, E) \leftrightarrow object(temporal\text{-}part(x, exist\text{-}time(E)), E))$.

(c) $\forall x\, \forall E\, (tool_{tp}(x, E) \leftrightarrow tool(temporal\text{-}part(x, exist\text{-}time(E)), E))$.

(d) $\forall E\, (action(E) \leftrightarrow \exists x\, agent_{tp}(x, E))$.

(e) $\forall x\, \forall y\, \forall E\, ((agent_{tp}(x, E) \wedge agent_{tp}(y, E)) \rightarrow x = y)$.

(f) $\forall x\, \forall E\, (atomic\text{-}agent_{tp}(x, E) \rightarrow agent_{tp}(x, E))$.

(g) $\forall x\, \forall y\, \forall E_1\, \forall E_2\, ((atomic\text{-}agent_{tp}(x, E_1) \wedge sp\text{-}part(E_2, E_1) \wedge agent_{tp}(y, E_2) \wedge part(y, x)) \rightarrow (y = x \wedge E_2 = E_1))$.

(h) $\forall x\, \forall E_1\, (agent_{tp}(x, E_1) \rightarrow \exists y\, \exists E_2\, (part(E_2, E_1) \wedge part_{tp}(y, x, E_2) \wedge atomic\text{-}agent_{tp}(y, E_2)))$.

(i) $\forall x\, \forall E\, (extended\text{-}agent_{tp}(x, E) \leftrightarrow (agent_{tp}(x, E) \wedge$

12 For characterizations of things, see Section 4 in Nakayama (2017). For example, a person is stipulated as the whole life of the person.

13 This description of Th(4EO) is based on Nakayama (2017). Nakayama (2017) proposes an extension of the axiomatic system in this paper.

$\neg atomic\text{-}agent_{tp}(x, E)))$.

(j) $\forall x\ \forall E\ (object_{tp}(x, E) \rightarrow \exists y\ agent_{tp}(y, E)) \wedge \forall a\ \forall E\ (object(a, E) \rightarrow \exists y\ agent_{tp}(y, E))$.

(k) $\forall x\ \forall E\ (tool_{tp}(x, E) \rightarrow \exists y\ (agent_{tp}(y, E) \wedge part_{tp}(x, y, E)))$.

(l) $\forall x\ \forall y\ \forall E_1\ ((agent_{tp}(x{+}y, E_1) \wedge tool_{tp}(y, E_1)) \rightarrow \exists E_2\ (\mathrm{use}(E_2) \wedge sp\text{-}part(E_2, E_1) \wedge agent_{tp}(x, E_2) \wedge object_{tp}(y, E_2)))$.

(m) $\forall x\ \forall E\ (group_{tp}(x, E) \leftrightarrow group(temporal\text{-}part(x, exist\text{-}time(E)), E))$.

(n) $\forall x\ \forall y\ \forall E\ ((group_{tp}(x, E) \wedge group_{tp}(y, E)) \rightarrow x = y)$.

(o) $\forall x\ \forall E_1\ (group_{tp}(x, E_1) \rightarrow \forall y\ (member_{tp}(y, x, E_1) \leftrightarrow (member(temporal\text{-}part(y, exist\text{-}time(E_1)), temporal\text{-}part(x, exist\text{-}time(E_1))) \wedge \exists E_2\ (proper\text{-}part(E_2, E_1) \wedge agent_{tp}(y, E_2)))))$.

(p) $\forall x\ \forall E\ (group_{tp}(x, E) \rightarrow \exists^{\geq 2} y\ member_{tp}(y, x, E))$.

(q) $\forall E\ (collective\text{-}action_{tp}(E) \leftrightarrow \exists x\ (agent_{tp}(x, E) \wedge group_{tp}(x, E)))$.

(r) $\forall x\ \forall E_1\ ((collective\text{-}action_{tp}(E_1) \wedge group_{tp}(x, E_1)) \rightarrow E_1 = \sigma E_2\ \exists y\ (member_{tp}(y, x, E_1) \wedge part(E_2, E_1) \wedge agent_{tp}(y, E_2)))$.

(s) Th(AT) consists of the requirements in Th(4EO) and the requirements from (Ap1.3.a) to (Ap1.3.r).

Appendix 2

In this part, I sketch proofs of consistency of GEM, Th(4EO), and Th(AT).

(Ap2.1) [Proposition] In FO-Logic, the consistency of theory T can be proved by showing that there is a model for T.

PROOF. For FO-Logic, the strong completeness holds. This means: φ follows from T iff every model for T is a model for $\{\varphi\}$. Thus, $\varphi\wedge\neg\varphi$ does not follow from T iff there is a model for T that is not a model for $\{\varphi\wedge\neg\varphi\}$. However, according to the semantic definition of $\wedge$ and $\neg$, there is no model for $\{\varphi\wedge\neg\varphi\}$. Thus, T is consistent iff there is a model for T. Q.E.D.

(Ap2.2) [Proposition] Let S be a set of simple elements. We define $\mathrm{U}(S) = P(S) - \{\varnothing\}$, where $P(S) = \{\mathrm{X}: \mathrm{X}\subseteq S\}$ and $\varnothing$ is the empty set. Then, structure $\langle \mathrm{U}(S), \mathrm{I}\rangle$ with $\mathrm{I}(part) = \subseteq$ is a model for GEM, where I is an interpretation function.

PROOF. We can prove that $\langle \mathrm{U}(S), \mathrm{I}\rangle$ with $\mathrm{I}(part) = \subseteq$ satisfies all axioms of GEM. Thus, this structure is a model for GEM. Q.E.D.

GEM is a subsystem of Boolean algebras. The following proposition suggests this fact.[14]

(Ap2.3) [Proposition] A model of Boolean algebras can be constructed from structure $\langle P(S), \subseteq\rangle$.

PROOF. Suppose that $\langle P(S), \subseteq\rangle$ is given. We introduce some functions through the following explicit definitions: $\forall \mathrm{a}\ \forall \mathrm{b}\ \forall \mathrm{c}\ (\mathrm{c} = \mathrm{a}\cup\mathrm{b} \leftrightarrow \forall \mathrm{d}\ (\mathrm{d}\subseteq\mathrm{c} \leftrightarrow (\mathrm{d}\subseteq\mathrm{a} \vee \mathrm{d}\subseteq\mathrm{b})))$, $\forall \mathrm{a}\ \forall \mathrm{b}\ \forall \mathrm{c}\ (\mathrm{c} = \mathrm{a}\cap\mathrm{b} \leftrightarrow \forall \mathrm{d}\ (\mathrm{d}\subseteq\mathrm{c} \leftrightarrow (\mathrm{d}\subseteq\mathrm{a} \wedge \mathrm{d}\subseteq\mathrm{b})))$, and $\forall \mathrm{a}\ \forall \mathrm{b}\ (\mathrm{b} = complement(\mathrm{a}) \leftrightarrow \forall \mathrm{c}\ (\mathrm{c}\subseteq\mathrm{b} \leftrightarrow \neg\ \mathrm{c}\subseteq\mathrm{a}))$. Then, we can easily prove that structure $\langle P(S), \cup, \cap, complement, \varnothing, \mathrm{S}\rangle$ satisfies all axioms of Boolean algebras. Q.E.D.

(Ap2.4) [Corollary] GEM is consistent.

PROOF. This proposition follows from (Ap2.1+2). Q.E.D.

(Ap2.5) [Proposition] Th(4EO) is consistent.

PROOF. At first, we construct a structure. Let S_T be the set of real numbers and $\mathrm{U}(\mathrm{S}_T) = P(\mathrm{S}_T) - \{\varnothing\}$. We stipulate: For all T_1, T_2 with $\mathrm{T}_1\subseteq\mathrm{S}_T$ and $\mathrm{T}_2\subseteq\mathrm{S}_T$, [$\mathrm{T}_1 <_{\mathrm{T}} \mathrm{T}_2$ iff [for all real numbers t_3 and t_4, if $\{t_3\}\subseteq\mathrm{T}_1$ and $\{t_4\}\subseteq\mathrm{T}_2$, then $t_3 < t_4$]]. A structure for time objects is defined as $\langle \mathrm{U}(\mathrm{S}_T), \mathrm{I}_{\mathrm{T}}\rangle$ with $\mathrm{I}_{\mathrm{T}}(part_T) = \subseteq$ and $\mathrm{I}_{\mathrm{T}}(<) = <_{\mathrm{T}}$. For space-time objects, we define: $\mathrm{S}_{st} = \{ \langle s_1, s_2, s_3, t\rangle : s_1, s_2,$ and s_3 are real numbers and $t\in\mathrm{S}_T\}$, $\mathrm{U}(\mathrm{S}_{st}) = P(\mathrm{S}_{st}) - \{\varnothing\}$, and $\mathrm{I}_{st}(part_{st}) = \subseteq$. Then, according to (Ap.2.2), $\langle \mathrm{U}(\mathrm{S}_T), \mathrm{I}_{\mathrm{T}}\rangle$ and $\langle \mathrm{U}(\mathrm{S}_{st}), \mathrm{I}_{st}\rangle$ are models for GEM. For the sake of simplicity, we accept only one simple four-dimensionally extended trajectory $E_{\mathrm{trajectory}}$. We stipulate: $\mathrm{U}(E_{\mathrm{trajectory}}) = \{E$: E is a constituent of $E_{\mathrm{trajectory}}\}$ and $\mathrm{I}_{\mathrm{E}}(part) = constituent\text{-}of$. Then, we can prove that $\langle \mathrm{U}(E_{\mathrm{trajectory}}), \mathrm{I}_{\mathrm{E}}\rangle$ is a model for GEM by proving that this structure satisfies all axioms of GEM. Now, we introduce structure $\langle \mathrm{U}(\mathrm{S}_T)\cup\mathrm{U}(\mathrm{S}_{st})\cup\mathrm{U}(E_{\mathrm{trajectory}})\cup\mathrm{U}(\mathrm{M}), \mathrm{I}\rangle$ with $\mathrm{I}(Time) = \mathrm{U}(\mathrm{S}_T)$, $\mathrm{I}(Space\text{-}time) = \mathrm{U}(\mathrm{S}_{st})$, $\mathrm{I}(Event) = \mathrm{U}(E_{\mathrm{trajectory}})$, $\mathrm{I}(Music) = \mathrm{U}(\mathrm{M}) = \{$Beethoven's spring sonata (BSS)$\}$, $\mathrm{I}(part_{\mathrm{T}}) = \mathrm{I}_{\mathrm{T}}(part_{\mathrm{T}})$, $\mathrm{I}(part_{st}) = \mathrm{I}_{st}(part_{st})$, $\mathrm{I}(<) = \mathrm{I}_{\mathrm{T}}(<)$, $\mathrm{I}(part) = \mathrm{I}_{\mathrm{E}}(part)$, $\mathrm{I}(occupy_{st}) = \{ \langle E, s\rangle : s$ is the four-dimensional region occupied by E & $s\in\mathrm{U}(\mathrm{S}_{st})$ & $E\in\mathrm{U}(E_{\mathrm{trajectory}})\}$, and $\mathrm{I}(occupy_T) = \{ \langle E, T\rangle :$ T is the temporal region occupied by E & $T\in\mathrm{U}(\mathrm{S}_T)$ & $E\in\mathrm{U}(E_{\mathrm{trajectory}})\}$. You can prove that this structure satisfies all axioms of Th(4EO). Thus, according to (Ap2.1), Th(4EO) is consistent. Q.E.D.

A trajectory in a four-dimensional space can be interpreted as a thing. Such a thing represents a worm discussed in the worm theory, which is a version of the four-dimensionalism.

(Ap2.6) [Proposition] Th(AT) is consistent.

PROOF. In this proof, notions of Th(4EO) are used, where (Ap2.5) justifies this treatment. Now, let us consider a small world consisting of Mary, Tom, Mary's piano, and Tom's violin. Now, suppose that the only complete event in this world is their play of BSS and that all other events are parts of this complete event. Let $\mathrm{TP}(E_1, E_2)$ be an abbreviation of "E_1 is a temporal part of E_2". Now, let us define structure

14 Tsai (2009) gives a short overview of models of mereological theories and Boolean algebras.

$\langle U(S_T) \cup U(S_{st}) \cup U(E_{BSS}) \cup U(M), I_{BSS} \rangle$, where $U(M) = \{BSS\}$ and $U(E_{BSS})$ is the set of all events (including things) that deal with this play of BSS. I characterize the interpretation function I_{BSS} as follows.

(a) G_{BSS} = Mary+piano$_{Mary}$+Tom+violin$_{Tom}$, $E_{play\text{-}BSS} = E_{play\text{-}piano} + E_{play\text{-}violin}$, $E_{use\text{-}piano}$ is a part of $E_{play\text{-}piano}$, and $E_{use\text{-}violin}$ is a part of $E_{play\text{-}violin}$.

(b) $I_{BSS}(Thing)$ = {d: d is a mereological entity constructed from elements of {Mary, Tom, piano$_{Mary}$, violin$_{Tom}$}}, where the existence time of these entities is identical with the existence time of $E_{play\text{-}BSS}$.

(c) $I_{BSS}(atomic\text{-}agent_{tp}) = \{\langle Mary, E\rangle: TP(E, E_{use\text{-}piano})\} \cup \{\langle Tom, E\rangle : TP(E, E_{use\text{-}violin})\}$.

(d) $I_{BSS}(extended\text{-}agent_{tp}) = \{\langle Mary+piano_{Mary}, E\rangle : TP(E, E_{play\text{-}piano})\} \cup \{\langle Tom+violin_{Tom}, E\rangle : TP(E, E_{play\text{-}violin})\} \cup \{\langle G_{BSS}, E\rangle : TP(E, E_{play\text{-}BSS})\}$.

(e) $I_{BSS}(agent_{tp}) = I_{BSS}(atomic\text{-}agent_{tp}) \cup I_{BSS}(extended\text{-}agent_{tp})$.

(f) $I_{BSS}(object_{tp}) = \{\langle piano_{Mary}, E\rangle : TP(E, E_{use\text{-}piano})\} \cup \{\langle violin_{Tom}, E\rangle : TP(E, E_{use\text{-}violin})\}$ & $\{\langle BSS, E\rangle : TP(E, E_{play\text{-}BSS})\} \subseteq I_{BSS}(object)$.

(g) $I_{BSS}(tool_{tp}) = \{\langle piano_{Mary}, E\rangle : TP(E, E_{play\text{-}piano})\} \cup \{\langle violin_{Tom}, E\rangle : TP(E, E_{play\text{-}violin})\}$.

(h) $I_{BSS}(group_{tp}) = \{\langle G_{BSS}, E\rangle : TP(E, E_{play\text{-}BSS})\}$.

(i) $I_{BSS}(member_{tp}) = \{\langle Mary+piano_{Mary}, G_{BSS}, E\rangle : TP(E, E_{play\text{-}BSS})\} \cup \{\langle Tom+violin_{Tom}, G_{BSS}, E\rangle : TP(E, E_{play\text{-}BSS})\}$.

(j) $I_{BSS}(collective\text{-}action_{tp}) = \{E: TP(E, E_{play\text{-}BSS})\}$.

Then, you can prove that this structure satisfies all axioms of Th(AT). Thus, $\langle U(S_T) \cup U(S_{st}) \cup U(E_{BSS}) \cup U(M), I_{BSS}\rangle$ is a model for Th(AT). Then, according to (Ap2.1), Th(AT) is consistent. Q.E.D.

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