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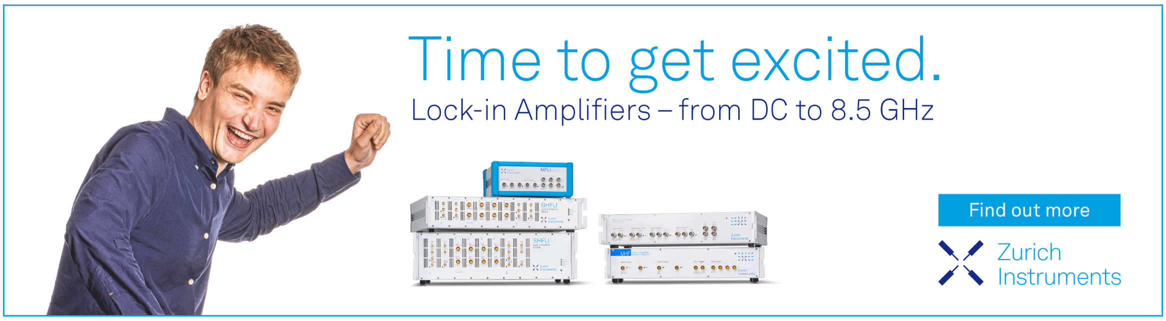

The Hoyle-analog states in light nuclei

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Abstract. We investigate resonance states in three-cluster continuum of some light nuclei ^{9}Be, ^{9}B, ^{10}B, ^{11}B and ^{11}C. These nuclei are considered to have a three-cluster configuration consisting of two alpha-particles and neutron, proton, deuteron, triton and nucleus ^{3}He. In this study, we make use two different microscopic three-cluster models. The first model employs the Hyperspherical Harmonics basis to numerate channels and describe three-cluster continuum. The second model is the well-known complex scaling method. Our main aim is to find the Hoyle-analog states in these nuclei or, in other words, whether it is possible to synthesize these nuclei in a triple collision of clusters. We formulate the criteria for selecting such states and apply them to resonance states, emerged from our calculations. We found that there are resonance states obeying the formulated criteria which make possible syntheses of these nuclei in a stellar environment.

INTRODUCTION

We are going to search and analyze properties of the Hoyle-like states in light nuclei. It is necessary to recall that the Hoyle state is a very narrow resonance state in ^{12}C, which was predicted by Fred Hoyle in 1954 [1]. Three years later this state was experimentally observed by studying beta decays of ^{12}B in Ref. [2]. It is interesting to point out that F. Hoyle predicted the energy of the 0^+ resonance state at E =0.33 MeV above the three alpha-particles threshold, and Cook *et al* in Ref. [2] determined the position of the resonance state at E= 0.372±0.002 MeV. One has to compare to the modern value of the energy which is E=0.3796±0.0002 MeV [3]. This resonance state created by a triple collision of three alpha-particles is the key element in syntheses of atomic nuclei starting from ^{12}C. The Hoyle state is a way for the nucleosynthesis of carbon in helium-burning red giant stars, which are rich of alpha-particles. Actually, F. Hoyle was the first who proclaimed that nuclear synthesis can take place in a triple collision of light nuclei, namely alpha-particles.

There are a very large number of publications devoted to the 0^+ and other resonance states in ^{12}C. Different methods have been used to determine parameters of the Hoyle state and to shed some light on the nature of this states and other resonances states, residing in the three-cluster continuum in ^{12}C. However, only few publications ([4, 5, 6, 7, 8, 9]) have been aimed at finding the Hoyle-analog states in light nuclei. They are mainly concentrated on closest neighbors of the ^{12}C nucleus, namely, ^{11}B and ^{11}C.

In the present paper we consider these nuclei and also ^{9}Be, ^{9}B and ^{10}B. We also consider a large number of states with different values of the total momentum J and both of negative and positive parities. Before starting in searching for the Hoyle-analog states, one needs to formulate clear criteria for selecting such states. By analyzing properties of the Hoyle state, one may suggest the following criteria for the Hoyle analog states in three-cluster systems:

1. Very narrow resonance state.
2. Resonance state which lies close to three-cluster threshold.
3. Resonance state which has the total orbital momentum L = 0.

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We consider the first criterion as the most important because in the case of very narrow (long-lived) resonance states, and a compound system has more chances to be reconstructed and transformed in to a bound state. However, we will analyze all resonance states from the point of view of the three criteria.

Our main aim is to find the Hoyle-analogue states in light nuclei ^{9}Be and ^{9}B, ^{10}B, ^{11}B and ^{11}C. In other words, we are going to study whether light nuclei can be created in triple collision of clusters. The necessary condition for such a process is the existence of a very narrow resonance state in three-cluster continuum. Actually we consider a chain of reactions

$$A_1 + A_2 + A_3 = A^* \Rightarrow A + \gamma$$

which consists of two steps. In the first step, an excited state (very narrow resonance state) of a compound nucleus is created in a triple collision of clusters consisting of A_1, A_2 and A_3 nucleons. In the second step, the compound nucleus by emitting a photon transits from the resonance state to the bound state. The narrower is a resonance state in the first step, the more is the probability to transit from the resonance to the bound state. For each nucleus we determine energy and width of resonance states. We select a resonance state with a very small width. We also analyze the wave function of selected resonance states. These investigations will be performed within a microscopic three-cluster model which involves the hyperspherical harmonics to distinguish channels of the three-cluster system. For this model, which was formulated in Ref. [10], we use the abbreviation AMHHB which means the algebraic model of scattering making use of the hyperspherical harmonics basis. In Ref. [11] this model has been applied to study bound and resonance states in ^{12}C. It fairly good reproduced the energy and width of the Hoyle state in ^{12}C.

The preliminary analysis of three-cluster resonance states in ^{9}B and ^{9}B has been carried out in Ref. [12], and in Ref. [13] resonance states have been investigated in the mirror nuclei ^{11}B and ^{11}C. In Ref. [14] the AMHHB model was applied to study the spectrum of bound states in ^{10}B. To make a systematic analysis of resonance states and to discover the Hoyle analog states in ^{9}Be, ^{9}B, ^{10}B, ^{11}B and ^{11}C we have to make additional calculations and thorough investigations of peculiarities of resonance wave functions.

Method

To study three-cluster systems we exploit a microscopic model which incorporates the resonating group method and the hyperspherical harmonics method. The standard ansatz of the RGM for representing the wave function of a three–s-cluster system is used

$$\Psi_{E,J} = \sum_{S,L} \widehat{A}\{[\Phi_1(A_1)\,\Phi_2(A_2)\,\Phi_3(A_3)]_S\,\psi_{E,LJ}(\mathbf{x},\mathbf{y})\}_J\,, \tag{1}$$

where the wave function $\psi_{E,LJ}(\mathbf{x},\mathbf{y})$ describes relative motion of clusters and the antisymmetric functions $\Phi_\nu(A_\nu)$ (ν=1, 2, 3) describes internal motion of nucleons inside the cluster with index ν. Two vectors $\mathbf{x}$ and $\mathbf{y}$ denote one of the possible sets of the Jacobi vectors. Within this paper, the vector $\mathbf{x}$ determines distance between two selected clusters, while the vector $\mathbf{y}$ represents displacement of the third cluster with respect to the center of mass of two selected clusters. The antisymmetrization operator $\widehat{A}$ provides full antisymmetrization of the wave function of a compound system.

To simplify of obtaining wave functions of discrete and continuous spectrum states, we transit from the Jacobi vectors $\mathbf{x}$ and $\mathbf{y}$ to the hyperspherical coordinates which consist of hyperradius ρ and five hyperspherical angles which we denote as Ω_5. The hyperradius ρ is defined as

$$\rho = \sqrt{\mathbf{x}^2 + \mathbf{y}^2}. \tag{2}$$

We make use the most popular set of hyperspherical angles which was suggested by Zernike and Brinkman [18]. This set consists of the hyperspherical angle θ which determine relative lengths of the Jacobi vectors

$$x = \rho\cos\theta, \quad y = \rho\sin\theta, \tag{3}$$

two angles θ_x, ϕ_x, determining orientation of vector $\mathbf{x}$, and two other angles θ_y, ϕ_y, determining orientation of vector $\mathbf{y}$ in the space. Five hyperspherical angles are able to describe any shape and any orientation (i.e. rotation) of a triangle connecting centers of mass of three clusters, and hyperradius determines any size of that triangle.

Having introduced the hyperspherical coordinate, we can represent the three-cluster wave function (1) in the following form

$$\Psi_{E,J} = \sum_{c} \widehat{A} \{ [\Phi_1 (A_1) \Phi_2 (A_2) \Phi_3 (A_3)]_S \ \psi_{E,c} (\rho) \mathcal{Y}_c (\Omega_5) \}_J , \tag{4}$$

where c is a multiple index $c = \{K; \lambda, l; L, S\}$ classifying channels of the three-cluster system and involving the hypermomentum K, partial orbital momenta λ and l associated with the Jacobi vectors $\mathbf{x}$ and $\mathbf{y}$, respectively, and the total orbital momentum L. The hyperspherical harmonics $\mathcal{Y}_c (\Omega_5)$ form a complete set of functions on five-dimension sphere and thus account for all kinds of motion of a three-cluster system. Components of the many-channel hyperradial wave function $\{\psi_{E,c} (\rho)\}$ have to be determined by solving the Schrödinger equation with the selected nucleon-nucleon potential.

For three-cluster systems, when the internal structure of clusters and the Pauli principle are taking into account, the wave functions $\{\psi_{E,c} (\rho)\}$ obey the set of integro-differential equations.

Within the present model a wave function (1) of a three-cluster system is expanded over an infinite set of cluster oscillator functions $\left| n_\rho, c \right\rangle$

$$\Psi_{E,J} = \sum_{n_\rho, c} C^{E,J}_{n_\rho, c} \left| n_\rho, c \right\rangle ,$$

where

$$\left| n_\rho, c \right\rangle = \left| n_\rho, K; \lambda, l; L \right\rangle = \widehat{A} \left\{ \Phi_1 (A_1) \Phi_2 (A_2) \Phi_3 (A_3) R_{n_\rho K} (\rho, b) \mathcal{Y}_c (\Omega_5) \right\}, \tag{5}$$

$R_{n_\rho, K} (\rho, b)$ is an oscillator function

$$R_{n_\rho, K} (\rho, b) = (-1)^{n_\rho} \mathcal{N}_{n_\rho, K} r^K \exp\left\{ -\frac{1}{2} r^2 \right\} L^{K+3}_{n_\rho} \left(r^2 \right), \quad r = \rho / b, \quad \mathcal{N}_{n_\rho, K} = b^{-3} \sqrt{\frac{2 \Gamma \left(n_\rho + 1 \right)}{\Gamma \left(n_\rho + K + 3 \right)}}, \tag{6}$$

and b is an oscillator length. In this case, a set of the integro-differential equations is reduced to a set of the algebraic (matrix) equations

$$\sum_{\widetilde{n}_\rho, \widetilde{c}} \left[\left\langle n_\rho, c \left| \widehat{H} \right| \widetilde{n}_\rho, \widetilde{c} \right\rangle - E \left\langle n_\rho, c | \widetilde{n}_\rho, \widetilde{c} \right\rangle \right] C^{E,J}_{\widetilde{n}_\rho, \widetilde{c}} = 0, \tag{7}$$

which can be more easily solved by the numerical methods than the set of integro-differential equations. For continuous spectrum states one has to impose proper boundary conditions for expansion coefficients $\left\{ C^{E,J}_{n_\rho, c} \right\}$. These conditions have been discussed in Ref. [10] where relations between the discrete $\left\{ C^{E,J}_{n_\rho, c} \right\}$ and continuous $\{\psi_{E,c} (\rho)\}$ wave functions were established. By including the asymptotic form of expansion coefficients $\left\{ C^{E,J}_{n_\rho, c} \right\}$, which is valid for large values of hyperradial excitations $n_\rho \gg 1$, we obtain in a closed form the system of equations determining both wave functions of a continuous spectrum and the corresponding S matrix.

Having obtained the expansion coefficients for any state of the three-cluster continuum, we can easily construct its wave function in the coordinate space:

$$\psi_{E,c} (\rho) = \sum_{n_\rho} C^{E,J}_{n_\rho, c} R_{n_\rho, K} (\rho, b), \quad \psi_{E,LJ} (\mathbf{x}, \mathbf{y}) = \sum_{n_\rho, c} C^{E,J}_{n_\rho, c} R_{n_\rho, K} (\rho, b) \mathcal{Y}_c (\Omega_5). \tag{8}$$

To get more information about the state under consideration we will study different quantities which can be obtained with the wave function in discrete or coordinate spaces. With wave functions in the discrete oscillator quantum number representation we can determine a weight W_{sh} of the oscillator function belonging to the oscillator shell N_{sh} in this wave function:

$$W_{sh} (N_{sh}) = \sum_{n_\rho, c \in N_{sh}} \left| C^{E,J}_{n_\rho, c} \right|^2 . \tag{9}$$

where the summation is performed over all hyperspherical harmonics and hyperradial excitations obeying the following condition $N_{os} = 2n_\rho + K$. Here N_{os} is fixed. Basis wave functions (6) belongs to the oscillator shell with the number of oscillator quanta $N_{os} = 2n_\rho + K$. It is convenient to numerate the oscillator shells by N_{sh} ($= 0, 1, 2, \ldots$), which we determine as $N_{os} = 2n_\rho + K = 2N_{sh} + K_{\min}$, where $K_{\min} = L$ for normal parity states $\pi = (-1)^L$ and $K_{\min} = L + 1$ for abnormal parity states $\pi = (-1)^{L+1}$. Thus we account oscillator shells starting from a "vacuum" shell ($N_{sh} = 0$) with minimal value of the hypermomentum $K_{\min}$ compatible with a given total orbital momentum L.

The weights W_{sh} we will calculate both for bound and resonance states. For a bound state, the wave function of which is normalized by the condition

$$\langle \Psi_{E,J} | \Psi_{E,J} \rangle = \sum_{n_\rho, c} \left| C^{E,J}_{n_\rho, c} \right|^2 = 1, \tag{10}$$

and this quantity W_{sh} determines the probability. For the continuous spectrum state, when the wave function is normalized by the condition

$$\left\langle \Psi_{E,J} | \Psi_{\widetilde{E},J} \right\rangle = \sum_{n_\rho, c} C^{E,J}_{n_\rho, c} C^{\widetilde{E},J}_{n_\rho, c} = \delta\left(k - \widetilde{k}\right), \tag{11}$$

this quantity has a different meaning. It determines the relative contribution of the different oscillator shells and also the shape of the resonance wave function in the oscillator representation.

By employing the wave function in the coordinate space we determine the average distances between clusters R_1 and R_2

$$R_1 = \sqrt{\frac{A}{(A_1 + A_2) A_3}} \sqrt{\int y^2 \left| \psi_{E,LJ} (\mathbf{x}, \mathbf{y}) \right|^2 d\mathbf{x} d\mathbf{y}}, \quad R_2 = \sqrt{\frac{(A_1 + A_2)}{A_1 A_2}} \sqrt{\int x^2 \left| \psi_{E,LJ} (\mathbf{x}, \mathbf{y}) \right|^2 d\mathbf{x} d\mathbf{y}}. \tag{12}$$

In our notations, R_2 determines average distance between alpha-particles, while R_1 determines distance of the third cluster to the center of mass of two alpha particles. It is obvious, that the average distances R_1 and R_2 can be calculated for the bound state only, since for resonance states integrals in Eq. (12) diverge. In Ref. [12] we suggested to extent to resonance states the definition of average distances R_1 and R_2. For this aim we restricted the integration within the internal part of the resonance wave functions which was normalized to unity. Recall that the internal part of a wave function is represented in the region ($0 \leq \rho \leq \rho_{\max}$ in the coordinate space or $0 \leq n_\rho \leq N^{(i)}$ in the oscillator space) where distances between clusters are relatively small and effects intercluster interactions are very strong.

Results and discussions.

For all nuclei under consideration we employ the Minnesota potential ([20, 21]) (MP) or the modified Hasegawa-Nagata potential [22, 23] (MHNP). Both the central and spin-orbital components of these potentials are taken into account.

In such a type of calculations we have only one free parameter to be selected. This is the oscillator length b which is common for all clusters of a compound nucleus. In our calculations the oscillator length b is fixed by minimizing the energy of the three-cluster threshold.

The Majorana parameter m of the MHNP and the exchange parameter u of the MP are very often used as an adjustable parameter. We adjust parameters m and u to reproduce the energy of the ground state of a compound system measured from the three-cluster threshold.

In all our calculations we use a standard set of the hyperspherical harmonics and hyperradial excitations. Positive parity states are calculated with the hyperspherical harmonics $K_{\min} \leq K \leq K_{\max}$. where $K_{\max} = 14$ for the positive parity states and $K_{\max} = 13$ for the negative parity states. The minimal value of the hypermomentum $K_{\min}$ equals the total orbital momentum L for normal parity states $\pi = (-1)^L$ and $K_{\min} = L + 1$ for the non-normal parity states. The total number of channels N_{ch} depends on the total angular momentum J and the total orbital moment L. To achieve the asymptotic region and to provide sufficient precision of our calculations we take into account the hyperradial excitation up to 70. This value of hyperradial excitations and the number of the hyperspherical channels cover a large range of intercluster distances and different shapes of the three-cluster triangle.

In this section we are going to reexamine some results obtained in previous papers concentrating our much interest to properties of the Hoyle state in ^{12}C.

In Table 1 we compare parameters of resonance states obtained within AMHHB [11] and CSM [24]. There are some consistencies in these two different methods of obtaining resonance states in the three-cluster continuum. Energy and total width of the first 0^+ resonance state (the Hoyle state) are very close in both methods. The same is observed for other narrow 1^- resonance states in ^{12}C.

TABLE 1. Low-lying resonance states in ^{12}C calculated within the AMHHB and CSM.

	CSM [24]		**AMHHB** [11]	
J^π	E, **MeV**	Γ, **keV**	E, **MeV**	Γ, **keV**
0^+	0.76	2.4	0.68	2.9
	1.66	1480	5.16	534
2^+	2.28	1100	2.78	10
1^-	3.65	0.30	3.52	0.21
3^-	1.51	2.0×10^{-3}	0.67	8.34

In Fig. 1 we display the structure of the wave function of the Hoyle state. As we see the weights of oscillator shells have very large amplitudes and main contribution to the wave function in the internal region comes from the oscillator shells $0 \le N_{sh} \le 30$. In the asymptotic region, this function has an oscillatory behavior with much smaller amplitude. We consider such a behavior of a resonance wave function as a 'standard' or pattern for the Hoyle analog states.

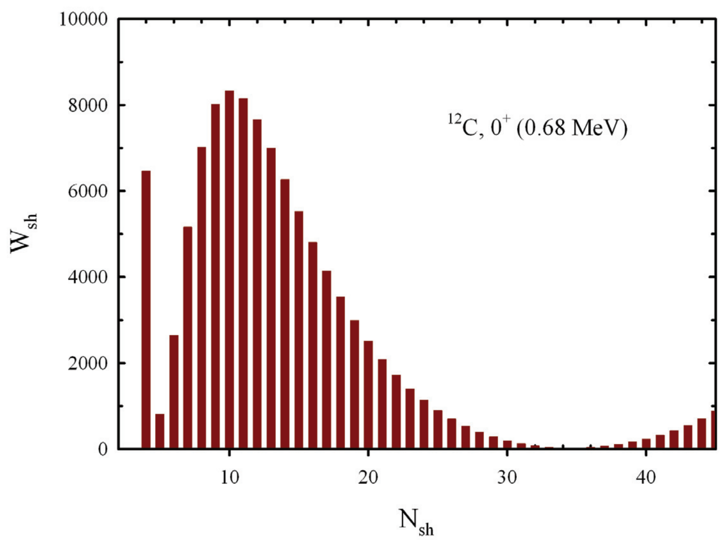

FIGURE 1. Weights of different oscillator shells in the wave function of the first 0^+ resonance state in ^{12}C.

We determined the shape of the triangle comprised of three alpha-particles in bound and resonance states. The average distances between clusters are displayed in Table 2. It is interesting to note that the shape of resonance states, shown in Table 2, is almost independent on the energy and total width of the resonance state, and the structure of resonance wave functions. The main conclusion one may deduce from Table 2 is that the average distances between alpha-particles are rather large. The ground state of ^{12}C shows a compact three-cluster configuration, as it is expected.

Having reanalyzed properties of the Hoyle state and other resonance states in ^{12}C, we suggest the following criteria for the Hoyle-analog states:

- The Hoyle-analog state is a very narrow resonance state in the three-cluster continuum.
- The wave function of the Hoyle-analog state has large values of amplitudes W_{sh} in the internal region.

As we pointed out above, we consider the first criterion is the most important one. We believe that the more long-lived resonance state has more chances that the system transits from a resonance state into a bound states, and vise versa. It is well-known that a resonance state could substantially increase a cross section of a processes if the total width of this resonance state is very small. To quantify the "narrowness" of a resonance state we will calculate the ratio Γ/E. For the original Hoyle state this ratio is 2.24×10^{-7}.

TABLE 2. The energy, width and average distances R_1, R_2 between clusters for the ground state and for the 0^+ and 1^- resonance states in ^{12}C.

J^π	E, MeV	Γ, keV	R_1, fm	R_2, fm
0^+	-11.37	-	3.12	3.60
	0.68	2.9	6.95	8.02
	5.16	534	6.43	7.43
1^-	3.52	0.21	6.07	7.00

Now we consider the spectra of resonance states in ^{11}B and ^{11}C. The energy and width of resonance states in the three-cluster continua of ^{11}B and ^{11}C, respectively, were calculated in Ref. [13]. By using the criteria for selecting the candidate to the Hoyle-analog states, formulated above, we selected four resonance states in ^{11}B and four resonance states in ^{11}C. In Table 3 we display the properties of the selected resonance states in ^{11}B and ^{11}C, and compare them with some bound states.

TABLE 3. Parameters of resonance states in ^{11}B and ^{11}C selected as candidates to the Hoyle-analog states.

Nucleus	J^π	E, MeV	Γ, keV	Γ/E	R_1, fm	R_2, fm
	$3/2^-$	-11.055			2.60	2.88
^{11}B	$1/2^+$	0.437	15.26	3.49×10^{-2}	10.48	6.77
	$5/2^-$	0.583	5.14×10^{-4}	8.81×10^{-7}	4.71	7.20
	$3/2^-$	0.755	0.58	7.7×10^{-4}	5.36	7.75
	$5/2^+$	1.047	1.54	1.47×10^{-3}	4.98	7.47
	$3/2^-$	-9.073			2.64	2.90
	$1/2^+$	0.906	162.94		10.75	7.08
^{11}C	$5/2^-$	0.783	9.64×10^{-5}	1.23×10^{-7}	3.20	3.87
	$3/2^-$	0.805	9.93×10^{-3}	1.23×10^{-5}	5.02	6.86
	$5/2^+$	1.460	0.90	6.16×10^{-4}	5.00	6.69

Figure 2 demonstrating wave functions of the $5/2^-$ resonance states in ^{11}B and ^{11}C explicitly indicate that these resonance states can be considered as the Hoyle analog state. Both resonance states have very large amplitudes of weights W_{sh}. Structure of the wave functions of the $5/2^-$ resonance states in ^{11}C looks like as a wave function of a bound state. These results also show that the average distances between clusters R_1 and R_2 in these resonance states are very close to average distances for bound states.

In Table 4 we show the three-cluster resonance states in ^{10}B calculated with the MP. Details of these calculations can be found in Ref. [14]. As we can see in Table 4, there are a few narrow resonance states which can be considered as candidates to the Hoyle-analog states. Three resonance states have the total width less than 12 keV.

TABLE 4. Parameters of resonance states in ^{10}B.

J^π	E, MeV	Γ, keV	Γ/E	R_1, fm	R_2, fm
1^+	0.604	232.30	0.384		
	0.987	7.08	7.17×10^{-3}	6.67	10.67
2^+	1.055	12.063	11.43×10^{-3}	6.64	10.83
	2.810	170.74	60.76×10^{-3}		
3^+	1.062	11.73	11.05×10^{-3}	6.43	10.35
	2.202	526.47	0.239		
1^-	1.100	76.75	69.77×10^{-3}	9.31	10.84
	1.820	562.71	0.309		

In Table 4 we also show the average distances between interacting clusters.

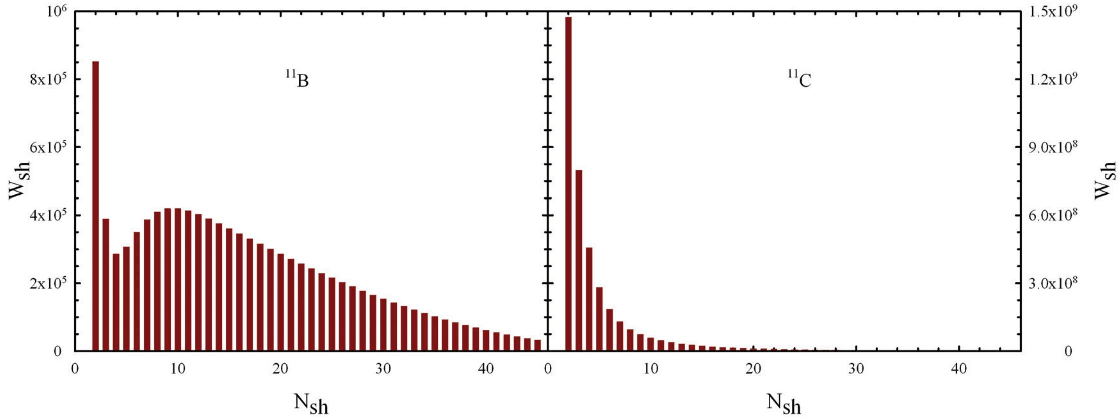

FIGURE 2. Weights of oscillator shells in the wave functions of the $5/2^-$ resonance states in ^{11}B and ^{11}C.

As we see in Table 4, all resonance states selected as the candidates to the Hoyle-analog states have a dispersed configuration with a large distance between alpha particles.

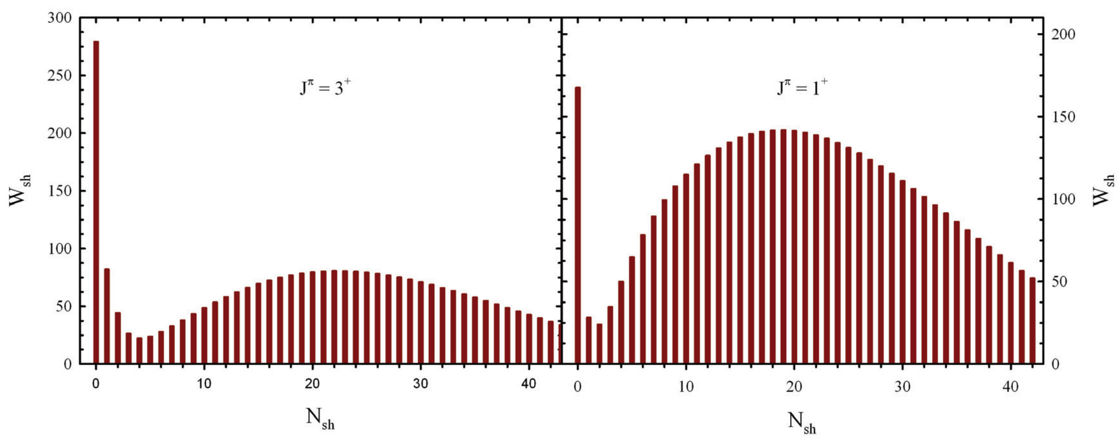

FIGURE 3. Weights of different oscillator shells in wave functions of the 3^+ and 1^+ resonance states in ^{10}B.

Let us turn our attention to the wave functions of the selected resonance states. In Fig. 3 we display shell weights in wave functions of the narrow 3^+ and 1^+ resonance states in ^{10}B. One notices, that the compact three-cluster configuration (N_{sh} =0) has a relatively large contribution to these wave functions. The shapes of the curves are similar to the shape of the Hoyle state (Fig. 1), however the amplitudes are much more smaller. We assume that the interplay of the attractive potential, created by the central and spin-orbital parts of the nucleon-nucleon interaction, and repulsive potential, formed by the Coulomb interaction, does not create a favorable situation for very narrow resonance states in ^{10}Be.

Conclusion

We have performed a systematic investigation of the three-cluster resonance states in light nuclei ^{9}Be, ^{9}B, ^{10}B, ^{11}B, ^{11}C and ^{12}C. These nuclei have been considered to have a three-cluster structure composed of two alpha particles and an *s*-shell nucleus. A microscopic three-cluster model was applied to search and to study resonance states embedded in the three-cluster continuum. This model imposes proper boundary conditions by employing hyperspherical coordinates and hyperspherical harmonics. Having reanalyzed properties of the Hoyle state, we formulated criteria for the Hoyle-analog states. Among these resonances, we have found the Hoyle-analog states in these nuclei. The Hoyle-analog states are created by a collision of two alpha particles and a neutron, proton, triton and nucleus ^{3}He. These resonance states have very small width.

TABLE 5. Parameters of the Hoyle-analog states in light nuclei ^{9}Be, ^{9}B, ^{11}B and ^{11}C.

Nucleus	Configuration	J^π	E, MeV	Γ, keV	Γ/E
^9Be	$\alpha+\alpha+n$	$5/2^-$	0.897	$2.36{\cdot}10^{-2}$	$2.63{\cdot}10^{-5}$
^9B	$\alpha+\alpha+n$	$3/2^-$	0.379	$1.08{\cdot}10^{-3}$	$2.84{\cdot}10^{-6}$
		$5/2^-$	2.805	$18.0{\cdot}10^{-3}$	$6.42{\cdot}10^{-6}$
^{11}B	$\alpha+\alpha+^3H$	$5/2^-$	0.583	$5.14{\cdot}10^{-4}$	$8.87{\cdot}10^{-7}$
		$3/2^-$	0.755	0.58	7.70×10^{-4}
		$5/2^+$	1.047	1.54	1.47×10^{-3}
^{11}C	$\alpha+\alpha+^3He$	$5/2^-$	0.783	$9.64{\cdot}10^{-5}$	$1.23{\cdot}10^{-7}$
		$3/2^-$	0.805	$9.93{\cdot}10^{-3}$	$1.23{\cdot}10^{-5}$
		$5/2^+$	1.460	0.90	6.16×10^{-4}

In Table 5 we collect the parameters of the Hoyle-analog states in light nuclei under consideration. From this Table we deduced new criterion for the Hoyle-analog states. A three-cluster resonance state can be treated as the Hoyle-analog state if the ratio $E/\Gamma < 1.47\times10^{-3}$ for this resonance state.

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