



Title	AC response of spin-pseudospin current in a double quantum well
Author(s)	Ishikawa, Toshiya; Akera, Hiroshi
Citation	Japanese Journal of Applied Physics (JJAP), 61(6), 63002 https://doi.org/10.35848/1347-4065/ac6644
Issue Date	2022-05-24
Doc URL	http://hdl.handle.net/2115/89320
Rights	© [2022] The Japan Society of Applied Physics; This is the Accepted Manuscript version of an article accepted for publication in Applied Physics Express. IOP Publishing Ltd is not responsible for any errors or omissions in this version of the manuscript or any version derived from it. The Version of Record is available online at https://doi.org/10.35848/1347-4065/ac6644 .
Type	article (author version)
File Information	AC_DQW_JJAP.pdf



[Instructions for use](#)

AC response of spin-pseudospin current in a double quantum well

Toshiya Ishikawa¹ and Hiroshi Akera²

¹*Division of Applied Physics, Graduate School of Engineering, Hokkaido University, Sapporo, Hokkaido, 060-8628, Japan*

²*Division of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo, Hokkaido, 060-8628, Japan*

The spin Hall effect due to the skew scattering is studied using the Boltzmann equation in a double quantum well when the inplane electric field with angular frequency ω is applied. The two wells have opposite signs of impurity potential so that the skew-scattering spin Hall current is antiparallel and carries a pseudospin, which is formed by $|L\rangle$ and $|R\rangle$, the ground states of the two wells. The pseudospin precession is induced by the interwell tunneling in the strength of $\hbar\omega_{\text{SAS}}$, the energy difference between the symmetric and antisymmetric states. It is found that the dynamics of the spin-pseudospin current, described by the pseudospin analogue of the Bloch equation, is equivalent in form to the classical cyclotron resonance. Consequently the antiparallel spin Hall current exhibits the resonance peak at $\omega \sim \omega_{\text{SAS}}$. Such spin-pseudospin coupling is expected to be useful in controlling the spin polarization in many electronic systems.

1. Introduction

Spintronics¹⁾ has been continuously providing interesting quantum phenomena and potentially useful engineering applications. Research has been extended to pseudospintronics,²⁾ which utilizes the pseudospin formed by two orbital states. A leading example is valleytronics in graphene and transition metal dichalcogenides, in which the valley Hall effect has been studied theoretically^{3,4)} and experimentally.^{5,6)} In this circumstance the interplay of spin and pseudospin has been attracting much attention.²⁾

One of the simplest systems with pseudospin is a double quantum well^{7–14)} in which the pseudospin is formed by the ground states in left and right wells, denoted by $|L\rangle$ and $|R\rangle$. The interwell tunneling gives rise to the formation of the symmetric and antisymmetric states. The energy splitting $\hbar\omega_{\text{SAS}}$ between these states can be thought of as the pseudospin splitting due to the effective magnetic field. Thus the pseudospin precesses in the angular frequency ω_{SAS} .

In our previous theory,¹⁵⁾ we have studied the spin Hall effect^{16–24)} in a double-quantum-well system in which the spin currents due to the skew scattering^{25–27)} in the two wells are in opposite directions. Such antiparallel spin current is generated by introducing impurity po-

tentials with different signs between the two layers. In a GaAs well layer, for example, Si and Be impurities give attractive and repulsive potentials, respectively, as has been experimentally demonstrated.²⁸⁾ Since the spin-orbit interaction induced by such impurity potentials creates the coupling of spin and pseudospin, the spin current in this system carries pseudospin. Although such spin-pseudospin current is also generated in the intrinsic spin Hall effect^{16,17)} in a system consisting of two layers with opposite directions of the effective magnetic field,^{29–38)} we focus on the spin-pseudospin current due to the skew scattering because the dynamics is much simpler in the skew scattering in that spin-up and spin-down states are not mixed.

The spin accumulation generated by the spin Hall effect in a single quantum well has been experimentally observed³⁹⁾ and explained by the extrinsic mechanism,²³⁾ and recently the theory and the experiment have been extended to a wide quantum well in which the ground state and the first excited state are occupied by electrons.^{31,35)} However, the pseudospin dynamics in the spin Hall effect in the AC electric field has not been explored theoretically and experimentally.

In this paper we theoretically study the AC spin Hall effect due to the skew scattering in a double quantum well in which the two wells have opposite signs of impurity potential to generate the antiparallel spin Hall current. Although we focus on the double quantum well as the simplest system with pseudospin, the present theory can be extended to a bilayer consisting of two atomic layers by appropriate modifications. In Sect. II, we describe the Hamiltonian of the double-quantum-well system with use of the pseudospin operator. In Sect. III, we describe the Boltzmann equation derived in our previous paper¹⁵⁾ and introduce the equation for the time evolution of pseudospin components of the distribution function. In Sect. IV, we derive the equation of motion of the spin-pseudospin current, which is the pseudospin analogue of the Bloch equation, and find that the dynamics of the spin-pseudospin current is the same in form as that in the classical cyclotron resonance. We analytically obtain the dependence of the antiparallel spin Hall conductivity on the AC frequency and the pseudospin precession frequency. In Sect. V, we give the conclusion.

2. Hamiltonian

Our system is a double quantum well structure with translational symmetry in the xy -direction. We describe the electron state in the conduction band in the effective mass approximation.^{32,40,41)}

Our Hamiltonian is the sum of the double-quantum-well Hamiltonian H_w and the impurity

term V ,

$$H = H_w + V, \quad (1)$$

where H_w consists of the term describing the in-plane motion, H_{\parallel} , and that describing the interwell transfer, H_{\perp} ,

$$H_w = H_{\parallel} + H_{\perp}. \quad (2)$$

H_{\parallel} is given by

$$H_{\parallel} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m}, \quad (3)$$

where $\hat{\mathbf{p}} = [\hat{p}_x, \hat{p}_y, \hat{p}_z]$ is the momentum operator and m is the effective mass of conduction band. The eigenvector $|\mathbf{k}\rangle$ and the eigenvalue ε_k of H_{\parallel} satisfy

$$H_{\parallel} |\mathbf{k}\rangle = \varepsilon_k |\mathbf{k}\rangle, \quad (4)$$

with

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}, \quad (5)$$

where $\mathbf{k} = (k_x, k_y)$ and $k = \sqrt{k_x^2 + k_y^2}$. $|\mathbf{k}\rangle$ is the eigenvector of \hat{p}_x and \hat{p}_y ,

$$\hat{p}_x |\mathbf{k}\rangle = \hbar k_x |\mathbf{k}\rangle, \quad \hat{p}_y |\mathbf{k}\rangle = \hbar k_y |\mathbf{k}\rangle. \quad (6)$$

H_{\perp} is given by

$$H_{\perp} = \frac{\hat{p}_z^2}{2m} + V_{\text{well}}(z), \quad (7)$$

where $V_{\text{well}}(z)$ is the potential of the double quantum well. In order to focus on the extrinsic spin Hall effect, we neglect the spin-orbit interaction (SOI) originating from the well potential⁴²⁻⁴⁴⁾ and the Dresselhaus SOI.^{45,46)} We express the motion in the z direction by the linear combination of the ground states of the left and right wells, $|L\rangle$ and $|R\rangle$, and introduce the pseudospin operator $\hat{\tau}_{\gamma}$ ($\gamma = 0, 1, 2, 3$),

$$\begin{aligned} \hat{\tau}_0 &= |L\rangle \langle L| + |R\rangle \langle R|, \\ \hat{\tau}_1 &= |L\rangle \langle R| + |R\rangle \langle L|, \\ \hat{\tau}_2 &= -i |L\rangle \langle R| + i |R\rangle \langle L|, \\ \hat{\tau}_3 &= |L\rangle \langle L| - |R\rangle \langle R|. \end{aligned} \quad (8)$$

Then H_{\perp} is expressed by

$$H_{\perp} = -\frac{\hbar\omega_{\text{SAS}}}{2} \hat{\tau}_1, \quad (9)$$

where $\hbar\omega_{\text{SAS}} = \Delta_{\text{SAS}} (> 0)$ represents the interwell coupling. In double quantum wells, the interwell coupling strength can be controlled by varying the width of the barrier between wells. On the other hand, when we extend the theory to a bilayer consisting of atomic layers, the interlayer coupling strength is determined by the fixed interlayer distance. The eigenvector $|n\rangle$ and the eigenvalue ε_n of H_{\perp} satisfy

$$H_{\perp} |n\rangle = \varepsilon_n |n\rangle \quad (n = G, E), \quad (10)$$

with

$$\varepsilon_G = -\frac{\hbar\omega_{\text{SAS}}}{2}, \quad \varepsilon_E = \frac{\hbar\omega_{\text{SAS}}}{2}. \quad (11)$$

The symmetric state $|G\rangle$ and the antisymmetric one $|E\rangle$ are

$$|G\rangle = \frac{1}{\sqrt{2}} (|L\rangle + |R\rangle), \quad |E\rangle = \frac{1}{\sqrt{2}} (|L\rangle - |R\rangle). \quad (12)$$

We take eigenvectors of $\hat{\sigma}_z$ as basis vectors of spin

$$\hat{\sigma}_z |\sigma\rangle = \sigma |\sigma\rangle \quad (\sigma = \pm 1). \quad (13)$$

Then, the eigenvector $|nk\sigma\rangle$ and the eigenvalue ε_{nk} of H_w satisfy

$$H_w |nk\sigma\rangle = \varepsilon_{nk} |nk\sigma\rangle, \quad (14)$$

with

$$\varepsilon_{nk} = \varepsilon_n + \varepsilon_k. \quad (15)$$

The perturbation V is the sum of the impurity potential $V_{\text{imp}}(\mathbf{r})$ with $\mathbf{r} = [x, y, z]$ and the impurity-induced SOI, $H_{\text{imp}}^{\text{SO}}$,

$$V = V_{\text{imp}}(\mathbf{r}) + H_{\text{imp}}^{\text{SO}}, \quad (16)$$

$$H_{\text{imp}}^{\text{SO}} = -\frac{\eta}{\hbar} \hat{\boldsymbol{\sigma}} \cdot (\nabla V_{\text{imp}} \times \hat{\mathbf{p}}), \quad (17)$$

where η is the effective coupling constant of the SOI for an electron in the conduction band. We assume that $V_{\text{imp}}(\mathbf{r})$ is the sum of the individual impurity potentials.

$$V_{\text{imp}}(\mathbf{r}) = \sum_{i=1}^N u_i(\mathbf{r} - \mathbf{r}_i), \quad (18)$$

where N is the total number of impurities and \mathbf{r}_i is the coordinate of the i th impurity. $u_i(\mathbf{r} - \mathbf{r}_i)$ is a repulsive or attractive potential depending on z_i ,

$$\begin{aligned} u_i(\mathbf{r} - \mathbf{r}_i) &= u_L(\mathbf{r} - \mathbf{r}_i) < 0 \quad \text{for } z_i \text{ in } L, \\ u_i(\mathbf{r} - \mathbf{r}_i) &= u_R(\mathbf{r} - \mathbf{r}_i) > 0 \quad \text{for } z_i \text{ in } R. \end{aligned} \quad (19)$$

When charged donor and acceptor impurities are in L and R layers, respectively, the corresponding potentials, $u_L(\mathbf{r})$ and $u_R(\mathbf{r})$, are well described by the screened Coulomb potential²²⁾ and therefore $u_L(\mathbf{r}) = -u_R(\mathbf{r})$ is approximately satisfied. We ignore the interlayer matrix elements of the impurity potential,

$$u_i(\mathbf{r} - \mathbf{r}_i) = |L\rangle \langle L| u_i(\mathbf{r} - \mathbf{r}_i) |L\rangle \langle L| + |R\rangle \langle R| u_i(\mathbf{r} - \mathbf{r}_i) |R\rangle \langle R|. \quad (20)$$

This can be achieved by setting $u_i(\mathbf{r} - \mathbf{r}_i) = 0$ in the region where $\langle R|z\rangle \times \langle z|L\rangle$ is not negligible. In this case we also have

$$\langle L| u_i(\mathbf{r} - \mathbf{r}_i) |L\rangle \langle R| u_i(\mathbf{r} - \mathbf{r}_i) |R\rangle = 0. \quad (21)$$

3. Transport equation

3.1 Assumptions

There are two mechanisms of the extrinsic spin Hall effect: skew scattering^{25–27)} and side jump.^{47–50)} In this study, we consider systems with long momentum relaxation time, where the skew scattering is dominant, and neglect the side jump. In addition, we neglect Δ_{SAS} in the collision term of the Boltzmann equation, that is we do not consider the combined action of Δ_{SAS} and V_{imp} . This assumption on the pseudospin splitting Δ_{SAS} corresponds to that on the spin splitting employed in the Dyakonov-Perel spin-relaxation theory.⁵¹⁾ This paper⁵¹⁾ states that the spin splitting can be neglected in the collision integral when the spin splitting is small in comparison with the electron energy. By neglecting Δ_{SAS} , the unperturbed Hamiltonian and the perturbation in calculating the collision term are H_{\parallel} and V , respectively.

Additionally, we focus on the transport phenomena, which does not depend on individual configurations of impurities randomly distributed in the xy plane. Therefore, we take the ensemble average⁵²⁾ of each term in the Boltzmann equation, which is defined, for the physical quantity $A(x_1, y_1, \dots, x_N, y_N)$, by

$$\bar{A} \equiv \frac{1}{S} \int_S \cdots \frac{1}{S} \int_S A(x_1, y_1, \dots, x_N, y_N) dx_1 dy_1 \cdots dx_N dy_N, \quad (22)$$

where S is the in-plane area of the double quantum well. Furthermore, we make the following assumption with respect to the matrix element of impurity potential,

$$\langle L\mathbf{k}| V_{\text{imp}} |L\mathbf{k}\rangle = \langle R\mathbf{k}| V_{\text{imp}} |R\mathbf{k}\rangle = 0. \quad (23)$$

3.2 Boltzmann equation

We employ the Boltzmann equation, which has been derived¹⁵⁾ for the present double-quantum-well system with use of the density operator $\hat{\rho}$,⁵²⁾ and calculate the spin Hall current in the first order of the SOI and in the first order of the electric field. In deriving the Boltz-

mann equation,¹⁵⁾ we have neglected spin-flip terms with $\hat{\sigma}_x$ and $\hat{\sigma}_y$ of $H_{\text{imp}}^{\text{so}}$ in Eq. (17), which give contributions of, at least, the second order in the SOI, to the spin Hall current (as can be seen from the spin Hall current expressed by the trace of the operator [Eq. (52)] since contributions of spin-flip terms to the trace involve, at least, two spin flips). Then we only need diagonal-in-spin elements of the density matrix. On the other hand, we take into account the pseudospin coherence described by off-diagonal-in-pseudospin elements of the density matrix. In this case it is convenient to introduce an operator in pseudospin space,

$$\hat{\rho}_{k\sigma} = \langle k\sigma | \hat{\rho} | k\sigma \rangle, \quad (24)$$

which can be regarded as distribution operator in the state with wavenumber \mathbf{k} and spin σ . With use of the quantum Liouville equation for $\hat{\rho}$, the following Boltzmann equation for the temporal evolution of $\hat{\rho}_{k\sigma}$ in the AC electric field with frequency ω , is obtained (see, for the derivation, Appendix)

$$\frac{\partial \hat{\rho}_{k\sigma}}{\partial t} + \frac{\mathbf{F}(t)}{\hbar} \cdot \frac{\partial \hat{\rho}_{k\sigma}}{\partial \mathbf{k}} = \frac{1}{i\hbar} [\langle k\sigma | H_w | k\sigma \rangle, \hat{\rho}_{k\sigma}] + \hat{C}^{(2)} + \hat{C}^{(3)}, \quad (25)$$

where $\mathbf{F}(t) = (-e)\mathbf{E}e^{-i\omega t}$. $\mathbf{E} = [E_x, E_y, 0]$ is the in-plane electric field and $e > 0$ is the absolute value of the electronic charge. The first term on the right-hand side reduces to $\frac{1}{i\hbar} [H_{\perp}, \hat{\rho}_{k\sigma}]$ and describes the precession of the pseudospin. $\hat{C}^{(2)}$ is the collision term of the second order in impurity potential:

$$\hat{C}^{(2)} = \sum_{k'} W_{k'k}^{(2)} \left(-\hat{\rho}_{k\sigma} + |L\rangle \rho_{k'\sigma}^{LL} \langle L| + |R\rangle \rho_{k'\sigma}^{RR} \langle R| \right). \quad (26)$$

Here we abbreviate matrix elements of the density matrix $\langle \ell | \hat{\rho}_{k\sigma} | \ell \rangle$ as $\rho_{k\sigma}^{\ell\ell}$ with $\ell = L, R$. $W_{k'k}^{(2)}$ in $\hat{C}^{(2)}$ is the transition rate in the second order of the impurity potential, and is given by

$$W_{k'k}^{(2)} \equiv W_{Lk'Lk}^{(2)} = W_{Rk'Rk}^{(2)}, \quad (27)$$

with

$$W_{Lk'Lk}^{(2)} = \frac{2\pi}{\hbar} \overline{|\langle Lk' | V_{\text{imp}} | Lk \rangle|^2} \delta(\varepsilon_{k'} - \varepsilon_k), \quad (28)$$

where we assumed the following symmetry of the ensemble average,

$$\overline{|\langle Lk' | V_{\text{imp}} | Lk \rangle|^2} = \overline{|\langle Rk' | V_{\text{imp}} | Rk \rangle|^2}, \quad (29)$$

which is true when $u_L(\mathbf{r}) = -u_R(\mathbf{r})$. Even when there is a significant deviation from $u_L(\mathbf{r}) = -u_R(\mathbf{r})$, Eq. (29) can be satisfied by adjusting the difference in impurity number between L and R layers. Owing to the assumptions, Eqs. (20) and (21), the impurity scattering occurs within the same layer, that is $L \rightarrow L$ and $R \rightarrow R$. Since the unperturbed Hamiltonian is H_{\parallel}

and the perturbation is V in calculating the collision term, the eigenvalue of $H_{||}$, ε_k , and the corresponding eigenvector $|Lk\rangle$ appear in Eq. (28). $\hat{C}^{(3)}$ in Eq. (25) is the collision term of the third order in impurity potential:

$$\hat{C}^{(3)} = \sum_{k'} \left(|L\rangle W_{LkLk'}^{\sigma(3)ss} \rho_{k'\sigma}^{LL} \langle L| + |R\rangle W_{RkRk'}^{\sigma(3)ss} \rho_{k'\sigma}^{RR} \langle R| \right). \quad (30)$$

$W_{LkLk'}^{\sigma(3)ss}$ and $W_{RkRk'}^{\sigma(3)ss}$ in $\hat{C}^{(3)}$ represent the skew scattering in the lowest order of the impurity potential and of the SOI, and are given by

$$W_{\ell k \ell k'}^{\sigma(3)ss} = \text{first-order-in-}\eta \text{ terms of } W_{\ell k \ell k'}^{\sigma(3)} \quad (31)$$

and

$$W_{\ell k' \ell k}^{\sigma(3)} = \frac{2\pi}{\hbar} \sum_{k''} \left[\frac{(\ell k k' k'' k \sigma)}{\varepsilon_k - \varepsilon_{k''} + i\delta} + \text{c.c.} \right] \delta(\varepsilon_{k'} - \varepsilon_k), \quad (32)$$

$$(\ell k k' k'' k \sigma) = \overline{\langle \ell k \sigma | V | \ell k' \sigma \rangle \langle \ell k' \sigma | V | \ell k'' \sigma \rangle \langle \ell k'' \sigma | V | \ell k \sigma \rangle},$$

with

$$\langle \ell k' \sigma | V | \ell k \sigma \rangle \equiv \left[1 - i\eta\sigma (k'_x k_y - k'_y k_x) \right] \langle \ell k' | V_{\text{imp}} | \ell k \rangle, \quad (33)$$

(This expression of $\langle \ell k' \sigma | V | \ell k \sigma \rangle$ shows that the second-order transition rate proportional to $|\langle \ell k' \sigma | V | \ell k \sigma \rangle|^2$ has no dependence on σ and therefore does not produce spin-dependent transport). Here we introduce an operator in pseudospin space representing the skew scattering:

$$\begin{aligned} \hat{W}_{kk'}^{\sigma(3)ss} &= |L\rangle W_{LkLk'}^{\sigma(3)ss} \langle L| + |R\rangle W_{RkRk'}^{\sigma(3)ss} \langle R| \\ &= W_{0kk'}^{\sigma(3)ss} \hat{\tau}_0 + W_{3kk'}^{\sigma(3)ss} \hat{\tau}_3, \end{aligned} \quad (34)$$

which has $\hat{\tau}_0$ (symmetric) and $\hat{\tau}_3$ (antisymmetric) components with strengths given by

$$W_{0kk'}^{\sigma(3)ss} = \frac{1}{2} (W_{LkLk'}^{\sigma(3)ss} + W_{RkRk'}^{\sigma(3)ss}), \quad W_{3kk'}^{\sigma(3)ss} = \frac{1}{2} (W_{LkLk'}^{\sigma(3)ss} - W_{RkRk'}^{\sigma(3)ss}). \quad (35)$$

Because the sign of the impurity potential is opposite between L and R layers, the third-order transition rates $W_{LkLk'}^{\sigma(3)ss}$ and $W_{RkRk'}^{\sigma(3)ss}$ are opposite in sign. In the case of $u_L(\mathbf{r}) = -u_R(\mathbf{r})$, $W_{LkLk'}^{\sigma(3)ss} = -W_{RkRk'}^{\sigma(3)ss}$ and then the $\hat{\tau}_0$ component $W_{0kk'}^{\sigma(3)ss}$ is absent.

Here, we find it convenient to decompose the distribution operator $\hat{\rho}_{k\sigma}$ into the linear combination of pseudospin operator $\hat{\tau}_\gamma$,^{53–55)}

$$\hat{\rho}_{k\sigma} = \frac{1}{2} \sum_{\gamma=0}^3 \tau_{\gamma k\sigma} \hat{\tau}_\gamma, \quad (36)$$

where $\tau_{\gamma k\sigma}$, twice the γ component of $\hat{\rho}_{k\sigma}$, is given by

$$\tau_{\gamma k\sigma} = \text{tr}_\tau \left[\hat{\rho}_{k\sigma} \hat{\tau}_\gamma \right] \equiv \sum_\ell \langle \ell | \hat{\rho}_{k\sigma} \hat{\tau}_\gamma | \ell \rangle. \quad (37)$$

By virtue of this decomposition, all the terms of the Boltzmann equation Eq. (25) are expressed in a linear combination of $\hat{\tau}_\gamma$. In particular the skew scattering term is written as

$$\begin{aligned} \hat{C}^{(3)} &= \sum_{k'} \left(|L\rangle \langle L| \hat{W}_{kk'}^{\sigma(3)ss} \hat{\rho}_{k'\sigma} |L\rangle \langle L| + |R\rangle \langle R| \hat{W}_{kk'}^{\sigma(3)ss} \hat{\rho}_{k'\sigma} |R\rangle \langle R| \right) \\ &= \sum_{k'} \frac{1}{2} \left[\left(W_{0kk'}^{\sigma(3)ss} \tau_{0k'\sigma} + W_{3kk'}^{\sigma(3)ss} \tau_{3k'\sigma} \right) \hat{\tau}_0 + \left(W_{0kk'}^{\sigma(3)ss} \tau_{3k'\sigma} + W_{3kk'}^{\sigma(3)ss} \tau_{0k'\sigma} \right) \hat{\tau}_3 \right]. \end{aligned} \quad (38)$$

Then we obtain the following equations for $\tau_{\gamma k\sigma}$

$$\begin{aligned} \frac{\partial \tau_{0k\sigma}}{\partial t} + \frac{\mathbf{F}(t)}{\hbar} \cdot \frac{\partial \tau_{0k\sigma}}{\partial \mathbf{k}} \\ = \sum_{k'} \left[W_{k'k}^{(2)} (-\tau_{0k\sigma} + \tau_{0k'\sigma}) + W_{0kk'}^{\sigma(3)ss} \tau_{0k'\sigma} + W_{3kk'}^{\sigma(3)ss} \tau_{3k'\sigma} \right], \end{aligned} \quad (39)$$

$$\frac{\partial \tau_{1k\sigma}}{\partial t} + \frac{\mathbf{F}(t)}{\hbar} \cdot \frac{\partial \tau_{1k\sigma}}{\partial \mathbf{k}} = - \sum_{k'} W_{k'k}^{(2)} \tau_{1k\sigma}, \quad (40)$$

$$\frac{\partial \tau_{2k\sigma}}{\partial t} + \frac{\mathbf{F}(t)}{\hbar} \cdot \frac{\partial \tau_{2k\sigma}}{\partial \mathbf{k}} = \omega_{\text{SAS}} \tau_{3k\sigma} - \sum_{k'} W_{k'k}^{(2)} \tau_{2k\sigma}, \quad (41)$$

$$\begin{aligned} \frac{\partial \tau_{3k\sigma}}{\partial t} + \frac{\mathbf{F}(t)}{\hbar} \cdot \frac{\partial \tau_{3k\sigma}}{\partial \mathbf{k}} &= -\omega_{\text{SAS}} \tau_{2k\sigma} \\ &+ \sum_{k'} \left[W_{k'k}^{(2)} (-\tau_{3k\sigma} + \tau_{3k'\sigma}) + W_{0kk'}^{\sigma(3)ss} \tau_{3k'\sigma} + W_{3kk'}^{\sigma(3)ss} \tau_{0k'\sigma} \right]. \end{aligned} \quad (42)$$

4. AC response of spin-pseudospin current

4.1 Assumptions and calculation procedure

In equilibrium, $\tau_{\gamma k\sigma}$ reduces to $\tau_{\gamma k\sigma}^{(0)} = \text{tr}_\tau \left[\hat{\rho}_{k\sigma}^{(0)} \hat{\tau}_\gamma \right]$ where $\hat{\rho}^{(0)} = f_0(H_w)$ with $f_0(\varepsilon)$ the Fermi-Dirac distribution function. The equation for $\tau_{\gamma k\sigma}^{(1)}$, the component in the first order of \mathbf{E} , is obtained from Eqs. (39) - (42) by replacing $\tau_{\gamma k\sigma}$ in the force term with $\tau_{\gamma k\sigma}^{(0)}$ and $\tau_{\gamma k\sigma}$ in other terms with $\tau_{\gamma k\sigma}^{(1)}$. We evaluate \mathbf{k}' summations of the collision terms in Eqs. (39) and (42) using the known form of the first-order solution,

$$\tau_{\gamma k\sigma}^{(1)} = \mathbf{b}_{\gamma k\sigma} \cdot \mathbf{k}, \quad (43)$$

where $\mathbf{b}_{\gamma k\sigma}$ does not depend on the direction of \mathbf{k} .

Here we assume, for simplicity, the short-range impurity potential, in which $W_{k'k}^{(2)}$ also does not depend on directions of \mathbf{k}' and \mathbf{k} . Then $\sum_{k'} W_{k'k}^{(2)} \tau_{\gamma k'\sigma}^{(1)}$ vanishes in Eqs. (39) and (42). In such short-range impurity potential, the momentum relaxation time τ_p is equal to the life

time:

$$\tau_p^{-1} = \sum_{k'} W_{k'k}^{(2)} \quad (44)$$

On the other hand, skew scattering terms in Eqs. (39) and (42) become, in the first order of \mathbf{E} ,

$$\sum_{k'} W_{\gamma k k'}^{\sigma(3)ss} \tau_{\gamma' k' \sigma}^{(1)} = \sum_{k'} W_{\gamma k k'}^{\sigma(3)ss} \mathbf{b}_{\gamma' k \sigma} \cdot \mathbf{k}', \quad (45)$$

and $\mathbf{k}' = \mathbf{k} \cos \phi + (\mathbf{e}_z \times \mathbf{k}) \sin \phi$ where ϕ is the angle of \mathbf{k}' relative to that of \mathbf{k} and \mathbf{e}_z is the unit vector in the z direction. Since $W_{\gamma k k'}^{\sigma(3)ss}$ is an odd function of ϕ , skew scattering terms reduce to

$$\sum_{k'} W_{\gamma k k'}^{\sigma(3)ss} \tau_{\gamma' k' \sigma}^{(1)} = -\sigma \tau_{\gamma ss}^{-1} \mathbf{b}_{\gamma' k \sigma} \cdot (\mathbf{e}_z \times \mathbf{k}). \quad (46)$$

Here the skew scattering time corresponding to the γ component $W_{\gamma k k'}^{\sigma(3)ss}$ is defined by

$$\tau_{\gamma ss}^{-1} = \sum_{k'} W_{\gamma k' k}^{\uparrow(3)ss} \sin \phi, \quad (\gamma = 0, 3), \quad (47)$$

and $W_{\gamma k k'}^{\sigma(3)ss} = \sigma W_{\gamma k k'}^{\uparrow(3)ss}$ and $W_{\gamma k k'}^{\sigma(3)ss} = -W_{\gamma k' k}^{\sigma(3)ss}$ are used.

With use of τ_p and $\tau_{\gamma ss}$, coupled equations for $\tau_{\gamma k \sigma}^{(1)} = \mathbf{b}_{\gamma k \sigma} \cdot \mathbf{k}$ ($\gamma = 0, 2, 3$) become

$$\frac{\mathbf{F}(t)}{\hbar} \cdot \frac{\partial \tau_{0k\sigma}^{(0)}}{\partial \mathbf{k}} = (i\omega - \tau_p^{-1}) \tau_{0k\sigma}^{(1)} - \sigma (\tau_{0ss}^{-1} \mathbf{b}_{0k\sigma} + \tau_{3ss}^{-1} \mathbf{b}_{3k\sigma}) \cdot (\mathbf{e}_z \times \mathbf{k}), \quad (48)$$

$$0 = \omega_{SAS} \tau_{3k\sigma}^{(1)} + (i\omega - \tau_p^{-1}) \tau_{2k\sigma}^{(1)}, \quad (49)$$

$$0 = -\omega_{SAS} \tau_{2k\sigma}^{(1)} + (i\omega - \tau_p^{-1}) \tau_{3k\sigma}^{(1)} - \sigma (\tau_{0ss}^{-1} \mathbf{b}_{3k\sigma} + \tau_{3ss}^{-1} \mathbf{b}_{0k\sigma}) \cdot (\mathbf{e}_z \times \mathbf{k}). \quad (50)$$

Here we used $\frac{\partial \tau_{\gamma k \sigma}^{(1)}}{\partial t} = -i\omega \tau_{\gamma k \sigma}^{(1)}$. Since $\tau_{2k\sigma}^{(0)} = \tau_{3k\sigma}^{(0)} = 0$, the force term is absent in $\gamma = 2, 3$ components. In this paper we evaluate the spin current in the first order of the SOI and of \mathbf{E} . Therefore in Eqs. (48) and (50) we evaluate $\mathbf{b}_{\gamma' k \sigma}$ in the zeroth order of the SOI and in the first order of \mathbf{E} . The above equations in this order give

$$\mathbf{b}_{0k\sigma} = \frac{\hbar \mathbf{F}}{m(\tau_p^{-1} - i\omega)} \left(-\frac{\partial \tau_{0k\sigma}^{(0)}}{\partial \varepsilon_k} \right), \quad \mathbf{b}_{3k\sigma} = 0. \quad (51)$$

Obtained equations for $\tau_{\gamma k \sigma}^{(1)}$ are employed to derive equations for the spin-pseudospin current in the next subsection.

4.2 Equation of motion of the spin-pseudospin current

Here, we introduce the spin-pseudospin current carrying the z component of spin and the y component of pseudospin in the direction $\mu = x, y$, which is given by

$$j_{\mu}^{sy} \equiv \frac{1}{2S} \text{tr} \left[\hat{\rho} \hat{\sigma}_z \hat{\tau}_y \frac{\hat{p}_{\mu}}{m} \right] = \frac{1}{2S} \sum_{k\sigma} \tau_{\gamma k\sigma} \sigma \frac{\hbar k_{\mu}}{m}. \quad (52)$$

In this study, we calculate the j_y^{sy} in the first order of the electric field $\mathbf{E} = [E_x, 0, 0]$ and of the SOI in the region where $k_B T \ll \varepsilon_F$ and $\hbar\omega_{\text{SAS}} \ll \varepsilon_F$ with ε_F the Fermi energy. By performing the integration with respect to \mathbf{k} in Eq. (52) of each term in Eqs. (48) - (50), we obtain the equation of motion of the spin-pseudospin current

$$\frac{dj_y^{s0}(t)}{dt} = -\tau_p^{-1} j_y^{s0}(t) + \tau_p^{-1} \frac{\sigma_0^{\text{SH}}}{1 - i\omega\tau_p} E_x e^{-i\omega t}, \quad (53)$$

$$\frac{dj_y^{s2}(t)}{dt} = \omega_{\text{SAS}} j_y^{s3}(t) - \tau_p^{-1} j_y^{s2}(t), \quad (54)$$

$$\frac{dj_y^{s3}(t)}{dt} = -\omega_{\text{SAS}} j_y^{s2}(t) - \tau_p^{-1} j_y^{s3}(t) + \tau_p^{-1} \frac{\sigma_3^{\text{SH}}}{1 - i\omega\tau_p} E_x e^{-i\omega t}. \quad (55)$$

Here σ_y^{SH} is the spin Hall conductivity when the DC electric field is applied in a single layer with the skew scattering time $\tau_{\gamma\text{ss}}$,

$$\sigma_{\gamma}^{\text{SH}} = -\frac{2N_e e \tau_p^2}{m \tau_{\gamma\text{ss}}}, \quad (56)$$

where N_e is the electron density per spin in the single layer, and τ_p and $\tau_{\gamma\text{ss}}$ in these equations are to be evaluated at ε_F .

Equations (54) and (55) are the pseudospin analogue of the Bloch equation, which is originally written for spin. In these equations ω_{SAS} is the precession frequency of pseudospin, τ_p^{-1} is the relaxation rate due to collisions, and the last term in the right-hand side of Eq. (55) generates the spin Hall current. When we take into account the dephasing of pseudospin with the dephasing time τ_{ps} as in our previous paper,¹⁵⁾ τ_p^{-1} in Eqs. (54) and (55) becomes $\tau_p^{-1} + \tau_{\text{ps}}^{-1}$. Here, we find that the dynamics represented by Eqs. (54) and (55) is equivalent to the cyclotron resonance with the correspondence of $j_y^{s2}(t)$ and $j_y^{s3}(t)$ to the velocity of the electron, ω_{SAS} to the cyclotron frequency, and the generation term to the force due to AC electric field. In Eqs. (54) and (55) the pseudospin precession around ‘‘axis 1’’ corresponding to the cyclotron motion is induced by the antisymmetric component of skew scattering, $\tau_{3\text{ss}}^{-1}$, which we introduce by placing impurity potentials of opposite signs in L and R layers.

On the other hand, Eq. (53) shows that the symmetric component of skew scattering, $\tau_{0\text{ss}}^{-1}$, gives the symmetric component of spin Hall current j_y^{s0} in which spin currents flow parallel

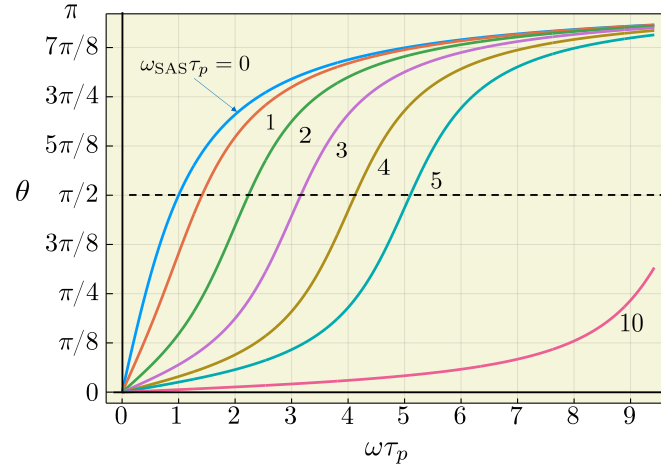


Fig. 1. The dependence on the $\omega\tau_p$ of θ , the argument of $\sigma_{yx}^{s3}(\omega)$.

in L and R layers and no pseudospin precession is induced.

4.3 The spin-pseudospin Hall conductivity

We define the spin-pseudospin Hall conductivity by $j_y^{sy}(t) = \sigma_{yx}^{sy}(\omega)E_x e^{-i\omega t}$. We focus on the $\gamma = 3$ component, since its observation in the experiment is easier than that of $\gamma = 2$. Eqs. (54) and (55) lead to

$$\sigma_{yx}^{s3}(\omega) = \frac{1}{(1 - i\omega\tau_p)^2 + \omega_{SAS}^2\tau_p^2} \sigma_3^{SH}. \quad (57)$$

First, we consider the decoupled-well case of $\omega_{SAS} = 0$, in which $\sigma_{yx}^{s3}(\omega) = (1 - i\omega\tau_p)^{-2} \sigma_3^{SH}$. Although the system of $\omega_{SAS} = 0$ represents an ideal case of the infinite separation between wells, $\sigma_{yx}^{s3}(\omega)$ in this case is useful in extracting effects of ω_{SAS} in $\sigma_{yx}^{s3}(\omega)$ of coupled wells. The argument of $\sigma_{yx}^{s3}(\omega)$, θ , represents the delay of the response to the AC electric field. As shown in Fig 1, θ approaches π in the $\omega \rightarrow \infty$ limit. This is in contrast with the electrical conductivity $\sigma_{xx}(\omega) \propto (1 - i\omega\tau_p)^{-1}$ the argument of which never exceeds $\frac{\pi}{2}$. As ω_{SAS} is increased, the larger value of ω is necessary to reach a fixed value of θ , say $\theta = \frac{\pi}{2}$. This is because the spin-pseudospin conductivity is determined by the relative strength between ω , ω_{SAS} , and τ_p^{-1} . We note that the real part of the spin-pseudospin Hall conductivity $\sigma_{yx}^{s3}(\omega)$ is negative when $\theta > \frac{\pi}{2}$, without violating the non-negative entropy production.

Figure 2 shows the dependence on the $\omega\tau_p$ of the absolute value of the spin-pseudospin Hall conductivity $\sigma_{yx}^{s3}(\omega)$ divided by σ_3^{SH} . In the case of $\omega_{SAS} = 0$, the absolute value of the spin-pseudospin Hall conductivity shows the Lorentzian decay with the increase of $\omega\tau_p$. On the other hand, in the case of $\omega_{SAS}\tau_p > 1$, a peak appears at $\omega\tau_p = \sqrt{\omega_{SAS}^2\tau_p^2 - 1}$ and the value at the peak is $(2\omega_{SAS}\tau_p)^{-1}$. This is the consequence of the resonance at $\omega \sim \omega_{SAS}$ which

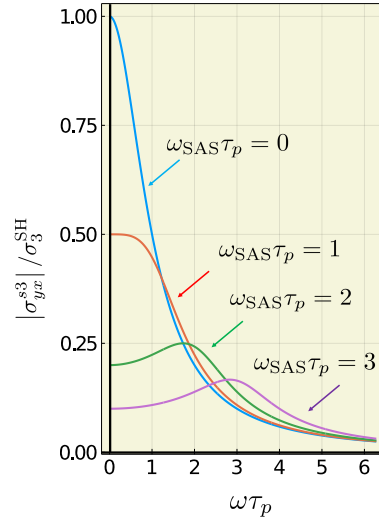


Fig. 2. The dependence on the $\omega\tau_p$ of the absolute value of $\sigma_{yx}^{s3}(\omega)$ normalized by σ_3^{SH} .

manifests itself in the spin-pseudospin current.

5. Conclusion

We have theoretically studied the AC response of the spin Hall current generated by the skew scattering in a double-quantum-well structure in which the two wells have opposite sign of the impurity potential so as to generate the antiparallel spin Hall current. We have shown that the dynamics of the pseudospin components of the spin Hall current is described by the pseudospin analogue of the Bloch equation, which consists of the precession term proportional to the interwell tunneling strength $\hbar\omega_{SAS}$, the damping term proportional to the scattering strength τ_p^{-1} , and the term generating the antiparallel spin Hall current. We have found that the derived equation has the same form as that describing the classical cyclotron resonance. The analytical solution for the antiparallel spin Hall conductivity shows that the absolute value as a function of the AC frequency exhibits the resonance peak when $\omega_{SAS} > \tau_p^{-1}$, while the argument approaches π with increasing the frequency.

These findings demonstrate that the spin current can be strongly modified by changing the AC frequency in the presence of the pseudospin degree of freedom which is coupled to the AC electric field and to the spin. In a variety of electronic systems with pseudospin, the effective magnetic field acts on the pseudospin and induces the pseudospin precession. Varying the AC frequency in the vicinity of the frequency of this precession may significantly change the spin current, as demonstrated in Fig. 2 for double-quantum-well systems, and is therefore expected to be useful for the electrical control of spin. In particular, in a bilayer having a *fixed nonzero* value of ω_{SAS} such as that consisting of atomic layers, changing the AC frequency

will be an effective method in controlling the spin current.

Acknowledgment

This work was supported by Grant-in-Aid for Scientific Research (C) Grant Number JP21K03413 from the Japan Society for the Promotion of Science (JSPS).

Appendix: Derivation of the Boltzmann equation

The second term on the left-hand side of Eq. (25) expresses the temporal evolution of $\hat{\rho}_{k\sigma}$ by the electric field, which is given by the derivative of $\hat{\rho}_{k\sigma}$ with respect to k according to Ref. 56. To derive terms on the right-hand side of Eq. (25), we start with

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[H, \hat{\rho}], \quad (\text{A}\cdot 1)$$

with $H = H_w + V$. Here we treat H_w and V as the unperturbed Hamiltonian and the perturbation, respectively and employ the interaction representation:

$$\hat{\rho}_I(t) = U^\dagger(t)\hat{\rho}(t)U(t), \quad V_I(t) = U^\dagger(t)V(t)U(t), \quad (\text{A}\cdot 2)$$

with

$$U(t) = \exp\left(\frac{1}{i\hbar}H_w t\right). \quad (\text{A}\cdot 3)$$

Then we obtain, from Eq. (A.1),

$$\frac{d\hat{\rho}_I}{dt} = \frac{1}{i\hbar}[V_I, \hat{\rho}_I]. \quad (\text{A}\cdot 4)$$

We turn on the interaction V at time t_0 and perform the integration of both sides of Eq. (A.4) from t_0 to t , which leads to $\hat{\rho}_I(t) = \hat{\rho}_I(t_0) + (i\hbar)^{-1} \int_{t_0}^t dt' [V_I(t'), \hat{\rho}_I(t')]$. Substituting this expression into Eq. (A.4) repeatedly, we obtain, up to the third order in V ,

$$\frac{d\hat{\rho}_I}{dt} = J_1(t) + J_2(t) + J_3(t), \quad (\text{A}\cdot 5)$$

with

$$\begin{aligned} J_1(t) &= \frac{1}{i\hbar}[V_I(t), \hat{\rho}_I(t_0)], \\ J_2(t) &= \frac{1}{(i\hbar)^2} \int_{t_0}^t dt' [V_I(t), [V_I(t'), \hat{\rho}_I(t_0)]], \\ J_3(t) &= \frac{1}{(i\hbar)^3} \int_{t_0}^t dt' \int_{t_0}^{t'} dt'' [V_I(t), [V_I(t'), [V_I(t''), \hat{\rho}_I(t_0)]]]. \end{aligned} \quad (\text{A}\cdot 6)$$

Here we employ a widely-used approximation, $\hat{\rho}_I(t_0) \approx \hat{\rho}_I(t)$, which is valid when the variation of $\hat{\rho}_I(t)$ is small within the collision time. Substituting $\hat{\rho}_I(t) = U^\dagger(t)\hat{\rho}(t)U(t)$ [Eq. (A.2)] into Eq. (A.5) leads to the equation describing the temporal evolution of $\hat{\rho}(t)$ up to the third order in V ,

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[H_w, \hat{\rho}] + \tilde{J}_1(t) + \tilde{J}_2(t) + \tilde{J}_3(t), \quad (\text{A}\cdot 7)$$

with

$$\tilde{J}_n(t) = U(t)J_n(t)U^\dagger(t), \quad (n = 1, 2, 3). \quad (\text{A}\cdot 8)$$

Diagonal-in- $k\sigma$ components of $[H_w, \hat{\rho}]$ in Eq. (A·7) gives $[\langle k\sigma | H_w | k\sigma \rangle, \hat{\rho}_{k\sigma}] (= [H_\perp, \hat{\rho}_{k\sigma}])$ in Eq. (25), which describes the pseudospin precession, while those of $\tilde{J}_2(t)$ and $\tilde{J}_3(t)$ give $\hat{C}^{(2)}$ and $\hat{C}^{(3)}$, respectively, in Eq. (25). With use of Eq. (33), $\langle k\sigma | \tilde{J}_n(t) | k\sigma \rangle$ is expressed by products of $\langle \ell k' | V_{\text{imp}} | \ell k \rangle$. The average of such products over inplane impurity configurations is calculated, using Eqs. (22), (23), and (21), to be

$$\overline{\langle \ell k' | V_{\text{imp}} | \ell k \rangle} = 0, \quad (\text{A}\cdot 9)$$

$$\overline{\langle \ell' k'' | V_{\text{imp}} | \ell' k' \rangle \langle \ell k' | V_{\text{imp}} | \ell k \rangle} \propto \delta_{\ell' \ell} \delta_{k'' k}, \quad (\text{A}\cdot 10)$$

and

$$\overline{\langle \ell'' k''' | V_{\text{imp}} | \ell'' k'' \rangle \langle \ell' k'' | V_{\text{imp}} | \ell' k' \rangle \langle \ell k' | V_{\text{imp}} | \ell k \rangle} \propto \delta_{\ell'' \ell} \delta_{\ell' \ell} \delta_{k''' k}. \quad (\text{A}\cdot 11)$$

In calculating $\langle k\sigma | \tilde{J}_n(t) | k\sigma \rangle$, we neglect the pseudospin precession given by ω_{SAS} and replace H_w in $U(t)$ with H_\parallel as explained in Sect. 3.1. Then the integration with respect to time in Eq. (A·6) leads to the conservation of the inplane kinetic energy ε_k .

References

- 1) I. Žutić, J. Fabian, and S. D. Sarma, *Rev. Mod. Phys.* **76**, 323 (2004).
- 2) D. Pesin and A. H. MacDonald, *Nat. Mater.* **11**, 409 (2012).
- 3) D. Xiao, W. Yao, and Q. Niu, *Phys. Rev. Lett.* **99**, 236809 (2007).
- 4) T. Ando, *J. Phys. Soc. Jpn.* **84**, 114705 (2015).
- 5) R. V. Gorbachev, J. C. W. Song, G. L. Yu, A. V. Kretinin, F. Withers, Y. Cao, A. Mishchenko, I. V. Grigorieva, K. S. Novoselov, L. S. Levitov, and A. K. Geim, *Science* **346**, 448 (2014).
- 6) K. F. Mak, K. L. McGill, J. Park, and P. L. McEuen, *Science* **344**, 1489 (2014).
- 7) T. Chakraborty and P. Pietiläinen, *Phys. Rev. Lett.* **59**, 2784 (1987).
- 8) D. Yoshioka, A. H. MacDonald, and S. M. Girvin, *Phys. Rev. B* **39**, 1932 (1989).
- 9) H. A. Fertig, *Phys. Rev. B* **40**, 1087 (1989).
- 10) S. He, X. C. Xie, S. Das Sarma, and F. C. Zhang, *Phys. Rev. B* **43**, 9339 (1991).
- 11) Y. W. Suen, L. W. Engel, M. B. Santos, M. Shayegan, and D. C. Tsui, *Phys. Rev. Lett.* **68**, 1379 (1992).
- 12) J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, K. W. West, and S. He, *Phys. Rev. Lett.* **68**, 1383 (1992).
- 13) X.-G. Wen and A. Zee, *Phys. Rev. Lett.* **69**, 1811 (1992).
- 14) Z. F. Ezawa and A. Iwazaki, *Phys. Rev. B* **47**, 7295 (1993).
- 15) T. Ishikawa and H. Akera, *Phys. Rev. B* **100**, 125307 (2019).
- 16) S. Murakami, N. Nagaosa, and S.-C. Zhang, *Science* **301**, 1348 (2003).
- 17) J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, *Phys. Rev. Lett.* **92**, 126603 (2004).
- 18) M. I. Dyakonov and V. I. Perel, *Sov. Phys. JETP* **13**, 467 (1971).
- 19) M. I. Dyakonov and V. I. Perel, *Phys. Lett. A* **35**, 459 (1971).
- 20) J. E. Hirsch, *Phys. Rev. Lett.* **83**, 1834 (1999).
- 21) S. Zhang, *Phys. Rev. Lett.* **85**, 393 (2000).
- 22) H.-A. Engel, B. I. Halperin, and E. I. Rashba, *Phys. Rev. Lett.* **95**, 166605 (2005).
- 23) E. M. Hankiewicz and G. Vignale, *Phys. Rev. B* **73**, 115339 (2006).
- 24) W. K. Tse and S. Das Sarma, *Phys. Rev. Lett.* **96**, 056601 (2006).
- 25) N. F. Mott, *Proc. R. Soc. A* **124**, 425 (1929).
- 26) J. Smit, *Physica* **21**, 877 (1955).
- 27) J. Smit, *Physica* **24**, 39 (1958).

- 28) R. J. Haug, R. R. Gerhardt, K. von Klitzing, and K. Ploog, *Phys. Rev. Lett.* **59**, 1349 (1987).
- 29) T. Koga, J. Nitta, H. Takayanagi, and S. Datta, *Phys. Rev. Lett.* **88**, 126601 (2002).
- 30) P.-Q. Jin and Y.-Q. Li, *Phys. Rev. B* **76**, 235311 (2007).
- 31) E. Bernardes, J. Schliemann, M. Lee, J. C. Egues, and D. Loss, *Phys. Rev. Lett.* **99**, 076603 (2007).
- 32) R. S. Calsaverini, E. Bernardes, J. C. Egues, and D. Loss, *Phys. Rev. B* **78**, 155313 (2008).
- 33) S. Nakosai, Y. Tanaka, and N. Nagaosa, *Phys. Rev. Lett.* **108**, 147003 (2012).
- 34) M. Akabori, S. Hidaka, H. Iwase, S. Yamada, and U. Ekenberg, *J. Appl. Phys.* **112**, 113711 (2012).
- 35) F. G. G. Hernandez, L. M. Nunes, G. M. Gusev, and A. K. Bakarov, *Phys. Rev. B* **88**, 161305 (2013).
- 36) S. Souma, A. Sawada, H. Chen, Y. Sekine, M. Eto, and T. Koga, *Phys. Rev. Applied* **4**, 034010 (2015).
- 37) A. Khaetskii and J. C. Egues, *EPL* **118**, 57006 (2017).
- 38) K. Hayashida and H. Aker, *Phys. Rev. B* **101**, 035306 (2020).
- 39) V. Sih, R. C. Myers, Y. K. Kato, W. H. Lau, A. C. Gossard, and D. D. Awschalom, *Nat. Phys.* **1**, 31 (2005).
- 40) P. Nozières and C. Lewiner, *J. Phys. (Paris)* **34**, 901 (1973).
- 41) R. Lassnig, *Phys. Rev. B* **31**, 8076 (1985).
- 42) F. J. Ohkawa and Y. Uemura, *J. Phys. Soc. Jpn.* **37**, 1325 (1974).
- 43) Y. A. Bychkov and E. I. Rashba, *J. Phys. C* **17**, 6039 (1984).
- 44) Y. A. Bychkov and E. I. Rashba, *JETP Lett.* **39**, 78 (1984).
- 45) G. Dresselhaus, *Phys. Rev.* **100**, 580 (1955).
- 46) M. I. Dyakonov and V. Y. Kachorovskii, *Sov. Phys. Semicond.* **20**, 110 (1986).
- 47) L. Berger, *Phys. Rev. B* **2**, 4559 (1970).
- 48) L. Berger, *Phys. Rev. B* **5**, 1862 (1972).
- 49) S. K. Lyo and T. Holstein, *Phys. Rev. Lett.* **29**, 423 (1972).
- 50) D. Culcer, E. M. Hankiewicz, G. Vignale, and R. Winkler, *Phys. Rev. B* **81**, 125332 (2010).
- 51) M. I. Dyakonov and V. I. Perel, *Sov. Phys. Solid State* **13**, 3023 (1972).
- 52) W. Kohn and J. M. Luttinger, *Phys. Rev.* **108**, 590 (1957).
- 53) E. L. Ivchenko, *Sov. Phys. Solid State* **15**, 1048 (1973).

- 54) M. M. Glazov and E. Ivchenko, *J. Supercond.* **16**, 735 (2003).
- 55) E. G. Mishchenko and B. I. Halperin, *Phys. Rev. B* **68**, 045317 (2003).
- 56) M. I. Dyakonov and A. V. Khaetskii, *Sov. Phys. JETP* **59**, 1072 (1984).