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# Effect evaluation of drainage condition and water content on cyclic plastic deformation of aged ballast and its estimation models

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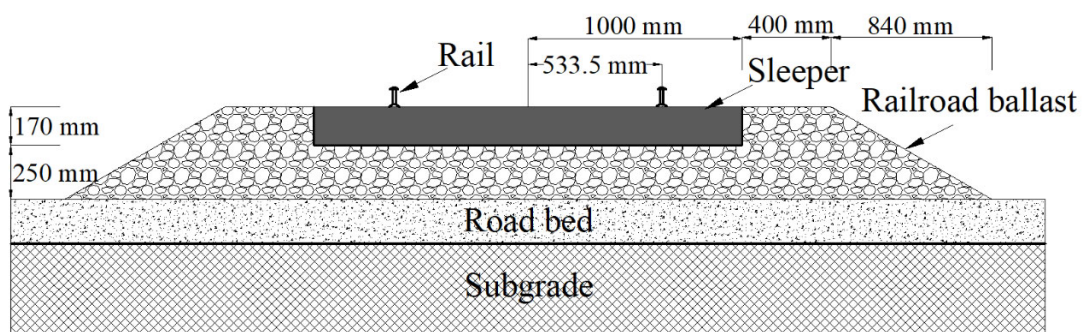
31 **Abstract**

32 With the ballast aging, the changes in the size and shape of the ballast particle reduce the drainage capacity, as  
33 well as cause a greater permanent deformation of the railroad ballast. Therefore, it is meaningful to investigate  
34 the effect of aging on the mechanical behavior of unsaturated ballast, and to estimate the cyclic plastic  
35 deformation by considering the aging effects. Here, "aging effect" means the increase in fine fraction content  
36 and the particle shape becomes rounded and smooth as compared with fresh ballast. In this study, the influence  
37 of aging on the cyclic plastic deformation of unsaturated ballast was evaluated through a series of cyclic loading  
38 triaxial compression tests. Test results indicate that the cyclic plastic deformation of ballast is seriously affected  
39 by water content, fine fraction content and drainage condition, and the increasing trend becomes more  
40 remarkable at the water-rich aged ballast under the fully undrained condition since the effective confining  
41 pressure decreases due to the generation of excess pore water pressure. Furthermore, the applicability of two  
42 types of estimation models (i.e., a semi-empirical model named University of Illinois at Urbana-Champaign  
43 (UIUC) model, and an elasto-plastic model named subloading surface extension (SSE) model) to the prediction  
44 for cyclic plastic deformation of unsaturated ballast is also verified in this study by comparing with results of  
45 cyclic loading triaxial compression tests. As the result, it is revealed that the UIUC model is suitable for  
46 predicting the cyclic plastic deformation of fresh ballast (or slightly aged ballast) with different water contents  
47 under the fully drained condition, and the SSE model shows good potential to estimate the cyclic plastic  
48 deformation of fresh and aged ballasts by considering the effects of water content and drainage condition. The  
49 findings of this study indicate that drainage conditions have a significant effect on predicting the cyclic plastic  
50 deformation of aged ballast, and appropriate test conditions for triaxial compression tests (i.e., CD and CU tests)  
51 should be selected according to hydraulic properties (i.e., permeability and water retentivity) of the aged ballast.

52  
53 Keywords: Aged ballast; Unsaturated soil; Cyclic plastic deformation; Laboratory element test; Estimation  
54 models.

55 **1 Introduction**

56 The ballasted track structure (Fig. 1) is one of the typical track structures for traditional railways in Japan.  
57 As passing tonnage of trains accumulates, clean or fresh ballast might be gradually contaminated with fouling  
58 materials, which are finer than fresh ballast aggregates, due to the ballast abrasion, breakdown, and external  
59 contamination (Indraratna et al., 2011; Selig and Waters, 1994). Selig and Waters (1994) pointed out that ballast  
60 fouling is mainly due to the degradation of ballast particle size and shape, which is called “aging” in this study.  
61 It is worth noting that the definition of “aging” used in this study is different from the definition of “fouling”  
62 commonly used in most previous studies. The aged ballast considers the changes in ballast particle size and  
63 shape, while the fouled ballast does not consider the change in particle shape. Some past studies indicated that  
64 the ballast fouling will cause a greater permanent deformation of ballasted tracks (Ebrahimi et al., 2015;  
65 Indraratna et al., 2013a; Ishikawa et al., 2019). In addition, the increase in fine fraction content caused by ballast  
66 fouling or aging affects the permeability and water retentivity of the ballast, which may cause an increase in  
67 water content of the ballasted layer and further increase the cyclic plastic deformation of the ballasted track  
68 (Indraratna et al. 2010; Tennakoon et al., 2012). The current Japanese design standard (RTRI, 2012) adopts a  
69 semi-empirical formula to estimate the cyclic plastic deformation of ballast for the construction of new lines.  
70 However, this formula cannot separately consider the adverse effects of water content and aging on the cyclic  
71 plastic deformation of the ballast.



72  
73 Fig. 1. Schematic diagram of a typical ballasted track structure in Japan.

74 For cyclic triaxial tests, previous studies indicated that the cyclic plastic deformation of the aggregate  
75 materials is affected by the stress level, water content, fine fraction content, grading (Lekarp et al., 2000; Le  
76 Pen et al., 2013). For example, the increase in fine fraction content seriously alters the deformation-strength  
77 characteristics of the ballast, depending on the amount of fouling materials mixed with clean ballast (Dareeju et

78 al., 2015; Lackenby et al., 2007). Besides, Indraratna et al. (2013a) pointed out that with the increase in the fine  
79 fraction content, the sleeper settlement of fouled ballasted track increases. Ebrahimi et al. (2015) indicated that  
80 the water content of fouling materials has a significant effect on accelerating the cyclic plastic deformation and  
81 changing the shape of deformational behavior of ballast from a small rate of plastic deformation to a high rate  
82 of plastic deformation. Ishikawa et al. (2019) revealed that both water content and fine fraction content may  
83 have serious influences on cyclic plastic deformation of the ballast. However, the effects of particle shape  
84 changes and drainage conditions on cyclic plastic deformation have not been evaluated in previous studies on  
85 fouled ballast so far.

86 For the semi-empirical model, many advanced phenomenological models (e.g., Abadi et al., 2016; Lekarp  
87 et al., 2000; Monismith et al., 1975) for estimating the accumulative strain of geomaterials under cyclic loading  
88 have been proposed by past researchers. In these models, the UIUC model is a simple model to predict the cyclic  
89 plastic deformation of pavements under repeated traffic loads, because its input parameters can be easily  
90 calculated by the shear strength parameters based on the monotonic triaxial test or the direct shear test instead  
91 of the cyclic triaxial test (Chow et al., 2014a, 2014b; Qamhia et al., 2016). After that, the applicability of the  
92 UIUC model was also verified for clean and fouled ballasts with different water contents and fine fraction  
93 contents (Ishikawa et al., 2019). However, the applicability of this model to the prediction for the cyclic plastic  
94 deformation of aged ballast has not been verified so far by considering the effect of drainage condition. On the  
95 other hand, various constitutive models for granular materials have been developed so far (Hashiguchi and Ueno,  
96 1977; Habiballah and Chazallon, 2005; Iwan, 1967; Mroz et al., 1981; Niemunis et al., 2005). In these models,  
97 the subloading surface model shows a good ability to predict the cyclic plastic deformation behavior for metals  
98 and clay (Hashiguchi and Ueno, 1977). After that, this model was modified by assuming an existence of the  
99 elastic domain surface inside the subloading surface to realistically describe the inelastic deformation behavior  
100 of coarse granular material under cyclic loading conditions. The validity of SSE model in predicting the  
101 permanent deformation of air-dried clean ballast under cyclic loading was verified by comparing the test results  
102 with the numerical results (Okayasu et al., 2014). However, the applicability of this model to predict the cyclic  
103 plastic deformation of the aged ballast with various water contents and drainage conditions has not been  
104 discussed so far.

105 To improve the track structure design and reduce maintenance costs, the main objective of this study is to  
 106 comprehensively investigate the effect of aging which includes changes in particle shape and particle size on  
 107 the cyclic plastic deformation of unsaturated ballast, and to estimate the cyclic plastic deformation of  
 108 unsaturated ballast by considering the aging effects. For these purposes, this study first evaluates the influence  
 109 of aging on cyclic plastic deformation of unsaturated ballast through a series of suction-controlled cyclic loading  
 110 triaxial compression tests for fresh and aged ballast. After that, the applicability of two types of estimation  
 111 models (i.e., UIUC model and SSE model) is verified to the prediction for the cyclic plastic deformation of fresh  
 112 and aged ballast with various water contents and drainage conditions.

## 113 2 Test materials

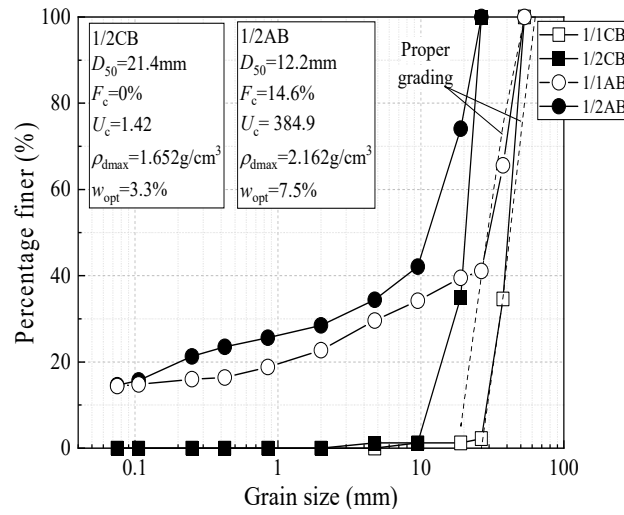
114 In general, most of the Japanese railroad ballasts are composed of angular, crushed, and hard andesite stone.  
 115 The grain size distribution of properly graded ballast in the Japanese railway standard ranges from 19 to 63 mm.  
 116 Fresh ballast or called clean ballast (CB) and aged ballast (AB) are used as test materials. In Fig. 2, the terms  
 117 “1/1CB” and “1/1AB” refer to the full-scale fresh ballast and full-scale aged ballast obtained from actual railway  
 118 tracks in Japan. Here, 1/1AB (Fig. 2) was extracted from heavily aged railway tracks. It should be noted that  
 119 field investigations on aged railway tracks in Japan show that the fine fraction content ( $F_c$ ) of in-situ aged ballast  
 120 is generally between 0 and 15%. Therefore, the maximum  $F_c$  value of 15% in the field investigation is used as  
 121 the basis for the laboratory test condition. However, in this study, the small-scale fresh ballast (1/2CB) and the  
 122 small-scale aged ballast (1/2AB) were employed as test samples due to the specimen size in the medium-size  
 123 triaxial apparatus. Both 1/2CB and 1/2AB have the half mean grain size of 1/1CB and 1/1AB with a parallel  
 124 gradation (Fig. 2). Some past researches (Indraratna et al., 1993; Le Pen et al., 2013; Wang et al., 2017) have  
 125 reported that a small-scaled ballast with the parallel gradation well reproduces the mechanical behavior of the  
 126 prototype ballast. It is noted that 1/2AB was prepared by the Los Angeles Abrasion (LAA) test using 1/2CB  
 127 with abrasion time ( $L$ ) of 50 min (number of turns ( $N_t$ ) is 1650). The validity of the LAA test for preparing aged  
 128 ballast can be referred to previous research (Yang et al., 2021). Physical properties of test samples were listed  
 129 in Table 1.

130 Table 1 Physical properties of test samples.

Test samples	$G_s$	$\rho_{dmax}$ (g/cm <sup>3</sup> )	$\rho_{dm}$ (g/cm <sup>3</sup> )	$D_{cm}$ (%)	$w_{opt}$ (%)	$D_{50}$ (mm)	$U_c$ (mm/mm)	$LL$ (%)	$PL$ (%)	$PI$ (%)	$F_c$ (%)	$k_s$ (m/s)
1/2CB	2.73	1.652	-	-	3.3	21.4	1.42	NP	-	-	0	$6.51 \times 10^{-2}$

1/2AB	2.73	2.162	-	-	7.5	12.2	384.9	21.4	17.6	3.8	14.6	$3.39 \times 10^{-4}$
1/1CB	2.70	1.682	1.581	94.0	3.0	40.7	1.33	NP	-	-	0	-
1/1AB	2.70	2.185	2.198	101.0	7.8	20.3	-	22.0	16.3	5.7	15.0	-

131  $G_s$ : specific gravity;  $\rho_{dmax}$  and  $w_{opt}$  are maximum dry density and optimum water content, obtained from compaction E-b  
132 method (JGS 0711-2009);  $\rho_{dm}$ : in-situ measured dry density;  $D_{cm}$ : in-situ measured degree of compaction;  $D_{50}$ : mean grain  
133 size;  $U_c$ : uniformity coefficient;  $LL$ ,  $PL$ , and  $PI$ , are liquid limit, plastic limit, and plasticity index, respectively, obtained  
134 from test method for liquid and plastic limit of soils (JGS 0141-2009);  $NP$ : non-plastic;  $F_c$ : fine fraction content, means the  
135 percentage by weight of ballast material passing the 0.075 mm sieve;  $k_s$ : saturated coefficient of permeability at degree of  
136 compaction ( $D_c$ ) of 94% for 1/2CB, and 101% for 1/2AB, fitted by the results of saturated permeability tests with different  
137  $D_c$  (Yang et al., 2021).



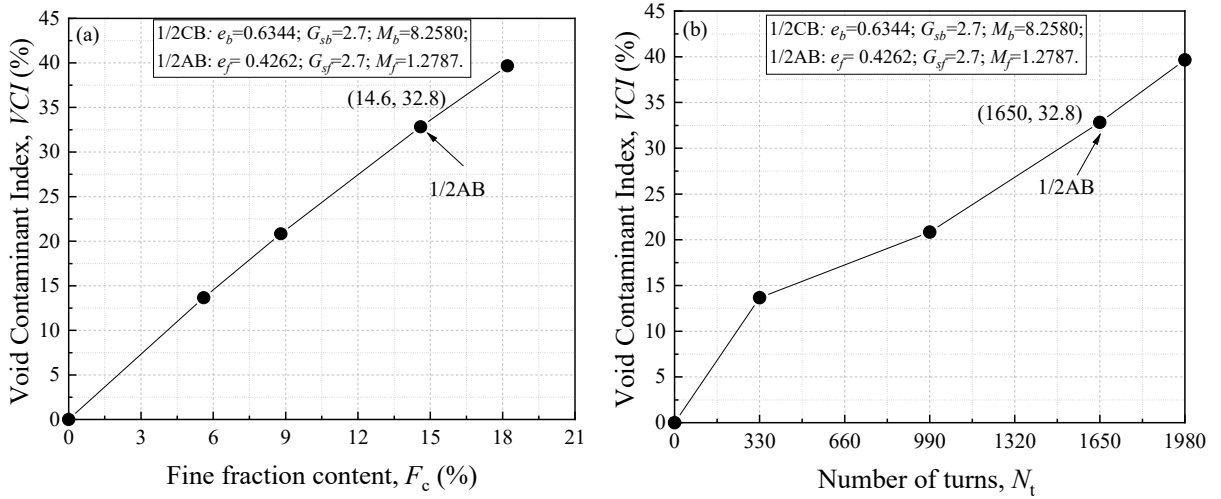
138  
139 Fig. 2. Grain size distributions of test samples.

140 In general, fresh ballast is a free draining material with large voids. However, the infiltration of fouling  
141 material reduces the voids and restricts water drainage. Indraratna et al. (2010) proposed the Void Contaminant  
142 Index ( $VCI$ ) to quantitatively evaluate the degree of fouling for fouled ballast, which can consider the differences  
143 in void ratio, specific gravity, and gradation between the fresh ballast and the fouling material.

$$144 \quad VCI = \frac{(1+e_f)}{e_b} \times \frac{G_{sb}}{G_{sf}} \times \frac{M_f}{M_b} \times 100 \quad (1)$$

145 where,  $e_b$ ,  $G_{sb}$ , and  $M_b$  are void ratio, specific gravity, and dry mass of fresh ballast, respectively;  $e_f$ ,  $G_{sf}$ , and  $M_f$   
146 are void ratio, specific gravity, and dry mass of fouling material, respectively. Based on this definition, Fig. 3  
147 shows the relationships between the  $VCI$  and  $F_c$  or  $N_t$  of 1/2AB. It should be noted that in addition to the 1/2AB  
148 used in this study, 1/2AB with three other different  $F_c$  was also prepared by the LAA tests with different  $N_t$  to  
149 examine the effectiveness of the LAA test for reproducing the lightly or heavily aged ballast with different  $F_c$

150 or  $VCI$ . Here, a lightly or heavily aged ballast refers to the difference in the aging level. For example, 1/2AB  
 151 ( $F_c = 6\%$ ) is considered a lightly aged ballast, and 1/2AB ( $F_c = 15\%$ ) is considered a heavily aged ballast.  
 152 Regarding the calculation parameters of  $VCI$ , the specific gravities ( $G_{sb}$ ,  $G_{sf}$ ) of fresh ballast and fouling material  
 153 are the same. Besides, the  $e_f$  and  $M_f$  of fouling material with different LAA test times can be calculated by  
 154 assuming that the initial dry mass of 1/2CB placed into the LAA test instrument is the same as the dry mass of  
 155 1/2AB regardless of abrasion time, and that the dry mass of fouling material for 1/2AB is the same as the dry  
 156 mass of the  $F_c$  for 1/2AB with the same abrasion time. As can be seen in Fig. 3(a),  $VCI$  of 1/2AB is about 32.8%,  
 157 and it increases approximately linearly with the increase of  $F_c$ . Besides,  $VCI$  also shows an overall increasing  
 158 trend as the  $N_t$  increases (Fig. 3(b)).



159  
160 Fig. 3. Relationship between  $VCI$  and  $F_c$ ,  $N_t$ : (a)  $VCI-F_c$ ; (b)  $VCI-N_t$ .

### 161 3 Test method for cyclic loading triaxial compression test

162 The cyclic loading triaxial compression (CL) test was performed by a medium-size triaxial apparatus (Fig.  
 163 4). The pressure membrane method was adopted in CL tests for unsaturated specimens to apply the matric  
 164 suction ( $s$ ). Here,  $s$  is defined as  $s = u_a - u_w$ ,  $u_a$  is pore air pressure, and  $u_w$  is pore water pressure. As for more  
 165 information about this triaxial apparatus, Ishikawa et al. (2014) and Yang et al. (2021) can be referred. The  
 166 preparation method of a cylindrical specimen ( $H = 300$  mm,  $D = 150$  mm) in this test was as follows: an air-  
 167 dried sample (1/2CB,  $w = 1.69\%$ ; 1/2AB,  $w = 2.01\%$ ) was put into the mold in five layers step by step. Each  
 168 layer was compacted by the vibrator to achieve the targeted  $D_c$  values similar to the actual track conditions  
 169 shown in Table 1. It is worth noting that according to field investigations, the measured  $D_c$  of aged ballast in  
 170 the field condition is close to or even greater than the maximum  $D_c$  obtained from the compaction E-b method.



171 In E-B method, the number of compacted layers for a test sample was 3 layers, and each layer was dropped 92  
172 times through a rammer (rammer mass is 4.5 kg) with the drop height of 450 mm. In this case, a larger  $D_c$  (i.e.,  
173  $D_c = 101\%$  for 1/2AB) in the laboratory element test than the maximum  $D_c$  can be obtained by controlling the  
174 vibration time of the sample. The uniformity of a specimen was ensured because the variations in the initial dry  
175 density of the same sample were less than 1%. The test conditions of the CL tests in this study were shown in  
176 Table 2. As shown in Table 2, CL tests were performed under three different water contents (air-dried,  
177 unsaturated ( $s = 5$  kPa), and saturated) according to the standards of Japanese Geotechnical Society (JGS 0523-  
178 2009; JGS 0524-2009; JGS 0527-2009). It is noted that the  $s$  of 5 kPa was selected under the unsaturated  
179 condition because results of water retention tests (Fig. 5) showed that the water content of the two test samples  
180 hardly decreases when the  $s$  was greater than 5 kPa. Besides, the effective confining pressure ( $\sigma'_c$ ) or net normal  
181 stress ( $\sigma_{net}$ ) of 20 kPa was selected in CL test, which was referred to that in monotonic loading triaxial  
182 compression (ML) test as shown in Table 3 (Yang et al., 2021). Here,  $\sigma_{net}$  is defined as  $\sigma_{net} = \sigma_c - u_a$  ( $\sigma_c$ : confining  
183 pressure), and the  $\sigma_{net}$  under the unsaturated condition was similar to the  $\sigma'_c$  ( $\sigma'_c = \sigma_c - u_a$ ) under the air-dried  
184 condition or  $\sigma'_c$  ( $\sigma'_c = \sigma_c - u_w$ ) under the saturated condition. More importantly, the  $\sigma'_c$  or  $\sigma_{net}$  in CL tests was close  
185 to the effective overburden pressure of the typical ballasted track in field conditions (Indraratna et al., 2013b;  
186 Suiker et al., 2005).

187 Table 2 Test conditions and results of CL tests.

Sample	Drainage condition	Water content	$\sigma'_c, \sigma_{net}$ (kPa)	$\rho_{dc}$ (g/cm <sup>3</sup> )	$D_c$ (%)	$S_r$ (%)	$\varepsilon_{p, max}$ (%)
1/2CB	CD test	Air-dried	20.0	1.571	95.1	4.22	1.08
1/2CB	CD test	Unsaturated ( $s = 5$ kPa)	20.0	1.539	93.2	10.62	2.97
1/2CB	CD test	Saturated	20.0	1.541	93.3	100	3.53
1/2AB	CD test	Air-dried	20.0	2.220	101.6	21.7	0.65
1/2AB	CD test	Unsaturated ( $s = 5$ kPa)	20.0	2.198	100.6	67.2	6.40
1/2AB	CD test	Saturated	20.0	2.205	100.9	100	12.09
1/2AB	CU test	Air-dried	20.0	2.196	100.5	21.3	0.97
1/2AB	CU test	Unsaturated ( $s = 5$ kPa)	20.0	2.195	100.5	68.5	8.24
1/2AB	CU test	Saturated	20.0	2.199	100.6	100	Failed

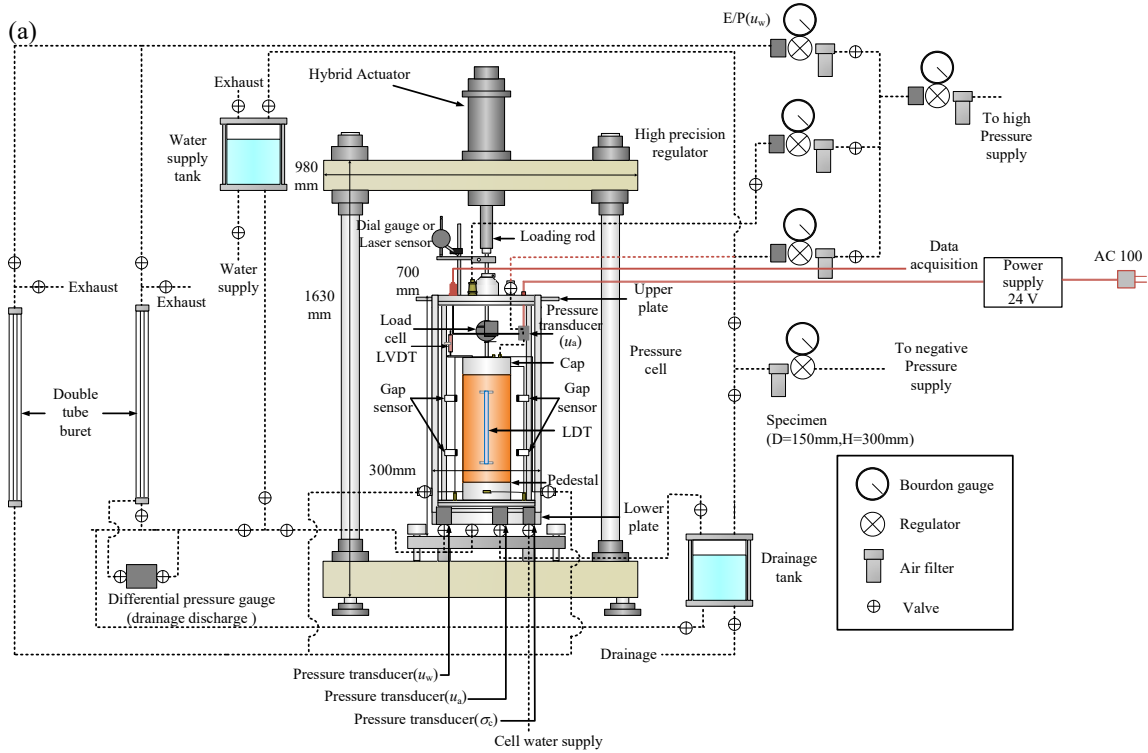
188  
189 Table 3 Test conditions and results of ML tests (Yang et al., 2021).

Sample	Drainage condition	Water content	$\sigma'_c, \sigma_{net}$ (kPa)	$\phi'$ (deg.)	$c', c$ (kPa)
1/2CB	CD test	Air-dried	20.0, 40.0	58.9	0
1/2CB	CD test	Unsaturated ( $s = 5$ kPa)	20.0, 40.0	55.3	1.6
1/2CB	CD test	Saturated	20.0, 40.0	55.3	0
1/2AB	CU test	Air-dried	20.0, 40.0	59.4	5.1
1/2AB	CU test	Unsaturated ( $s = 5$ kPa)	20.0	49.5	9.4
1/2AB	CU test	Saturated	20.0, 40.0	49.5	4.8

190  $\sigma'_c$  is the effective confining pressure under air-dried and saturated conditions;  $\sigma_{\text{net}}$  is the net normal stress in  
 191 unsaturated condition;  $\phi'$  is the effective friction angle;  $c'$  stands for effective cohesions under saturated and air-  
 192 dried conditions;  $c$  stands for total cohesion under unsaturated condition, which is composed of two parts: one  
 193 is the effective cohesion ( $c'$ ) of the saturated soil, and the other is related to the matric suction, as shown in Eq.  
 194 (2) (Fredlund et al., 1978).

$$195 \quad c = c' + (u_a - u_w) \tan \phi^b \quad (2)$$

196 where,  $\phi^b$  is the internal friction angle with respect to the matric suction.



197

198

199 Fig. 4. Medium-size triaxial test apparatus for unsaturated soils: (a) schematic diagram; (b) structures of cap and pedestal  
 200 (after Yang et al., 2021).  
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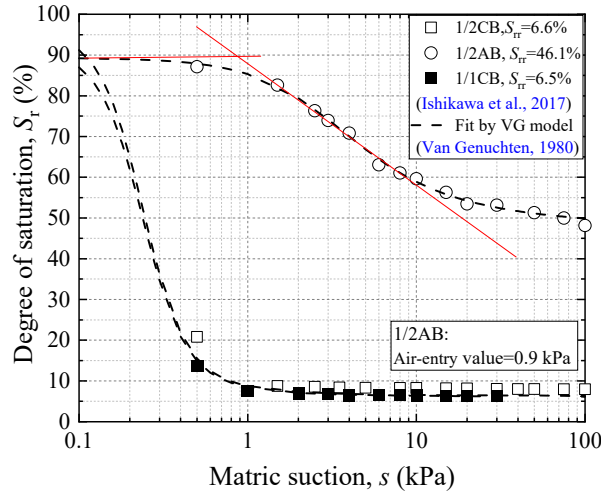


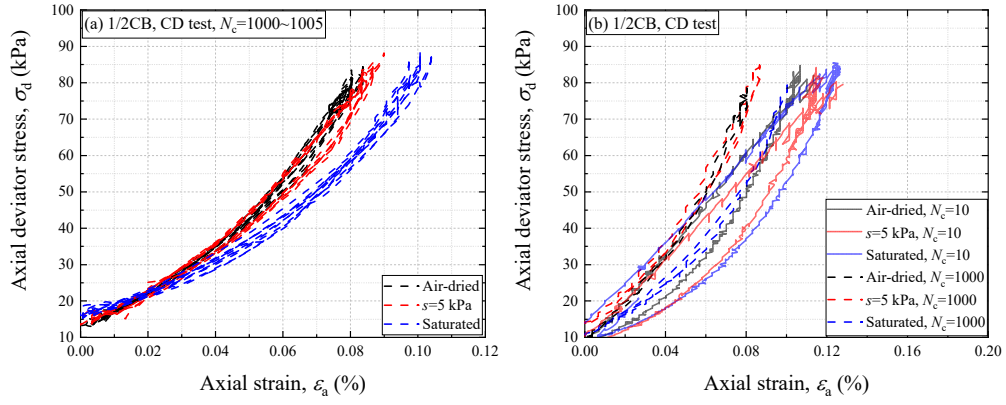
Fig. 5. Soil water characteristic curves of test samples (revised after Yang et al., 2021).

In a CL test, the consolidation process was conducted as follows. Under the air-dried condition, the specimen after the preparation was isotropically consolidated under a designated  $\sigma'_c$  of 20 kPa. Under the saturated condition, the de-aired water was supplied from the bottom end of the specimen, and the pore water pressure ( $u_w$ ) of 200 kPa was applied to ensure the pore water pressure coefficient B-value was above 0.96. Following the saturation process, the specimen was isotropically consolidated under a specified  $\sigma'_c$  of 20 kPa for 24 hours with  $\sigma_c$  of 220 kPa and the  $u_w$  of 200 kPa. Under the unsaturated condition, the de-aired water was immersed from the bottom of the specimen until the initial degree of saturation reached about 90%. Afterwards, the specimen was isotropically consolidated under a prescribed  $\sigma_{net}$  of 20 kPa for 24 hours with  $\sigma_c$  of 220 kPa, pore air pressure ( $u_a$ ) of 200 kPa, and  $u_w$  of 200 kPa. After the consolidation process, an unsaturated specimen with the  $s$  of 5 kPa was prepared by reducing  $u_w$  while keeping both  $\sigma_c$  and  $u_a$  unchanged. Next, the shearing process started after the equilibrium condition was reached in the suction process. The axial deviator stress ( $\sigma_d$ ) in sinusoidal waveform ranged from 10 kPa to 80 kPa was cyclically applied to the test specimen of 1/2CB and 1/2AB under the different drainage conditions. In this study, the CL tests for the test specimen of 1/2CB were conducted in CD tests due to its high permeability, while those of 1/2AB were conducted in both CD and CU tests due to its low permeability as shown in Table 1. It is noted that under the unsaturated condition, both pore water and pore air were allowed to drain in the CD test, while both were undrained in the CU test. The number of loading cycles ( $N_c$ ) in the CL test was 10, 000 or the specimen was failed (permanent axial strain,  $\epsilon_p > 15\%$ ), and the loading frequency of 1 Hz was selected by referring to previous research on simulated train loads (Cai et al., 2017; Chazallon et al., 2016).

## 223 4 Test results and discussions

### 224 4.1 Stress-strain relationship of aged ballast

225 Fig. 6 and Fig. 7 present the relationships between axial strain ( $\epsilon_a$ ) and axial deviator stress ( $\sigma_d$ ) of 1/2CB  
226 and 1/2AB with various water contents at different number of loading cycles ( $N_c$ ) in CD and CU tests. When  
227 other test conditions except the water content are constant, the elastic and plastic strains of both samples increase  
228 with the increment of water content. Besides, when comparing stress-strain relationships for both samples in  
229 CD tests under unsaturated and saturated conditions, the stiffness of 1/2AB is softer than that of 1/2CB, and the  
230 plastic strain rate of 1/2AB is greater than that of 1/2CB, though the stress-strain relationships of both samples  
231 are almost similar under air-dried condition. On the other hand, when comparing test results for 1/2AB in CD  
232 and CU tests, the difference in the stress-strain relation becomes clear with increasing the water content, and  
233 the saturated 1/2AB in the CU test has the largest plastic stain rate. It is worth noting that the stress-strain  
234 relationships for unsaturated samples are different in CD and CU tests, since both pore water pressure and pore  
235 air pressure change in the CU test due to the fully undrained conditions. In this case, the net normal stress (or  
236 effective confining pressure) is not constant in the CU test, while it remains constant in the CD test. Furthermore,  
237 as shown in Fig. 6(b), Figs. 7(c) and (d), when comparing the stress-strain relations at different  $N_c$  ( $N_c=10$  and  
238 1000), the area of the stress-strain hysteresis curve decreases as the  $N_c$  increases, regardless of test samples,  
239 water contents, and drainage conditions. More importantly, the changing trend of the stress-strain relationship  
240 is more significant for 1/2CB, which indicates that aging changes the development trend of plastic deformation  
241 of ballast. These results indicate that the water content, drainage condition, and ballast aging have significant  
242 influences on the plastic strain of ballast. As the water content increases, the aged ballast with much fine fraction  
243 content and rounded particle shape might reduce the interlock between particles, thereby the lubricating effect  
244 of water content and fine fraction content changes the stress-strain relationship of the ballast. Besides, the  
245 reduction of  $\sigma'_c$  in CU tests further increases the development of plastic strain, which will be discussed in a later  
246 section.

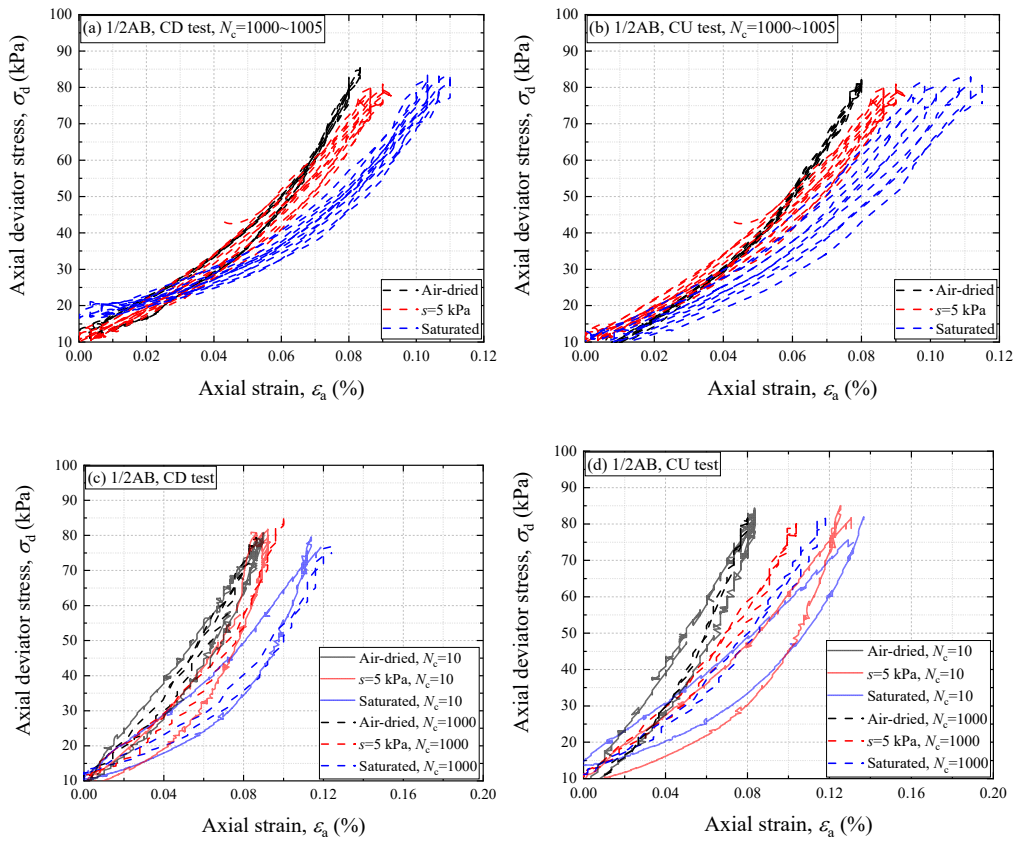


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Fig. 6. Stress-strain relationship for 1/2CB with different water contents /  $N_c$  in CD tests: (a)  $N_c = 1000\sim 1005$ ; (b)  $N_c = 10$  and 1000.



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Fig. 7. Stress-strain relationships for 1/2AB with different water contents /  $N_c$  in CD and CU tests: (a) CD test,  $N_c = 1000\sim 1005$ ; (b) CU test,  $N_c = 1000\sim 1005$ ; (c) CD test,  $N_c = 10$  and 1000; (d) CU test,  $N_c = 10$  and 1000.

#### 4.2 Permanent axial deformation behavior of aged ballast

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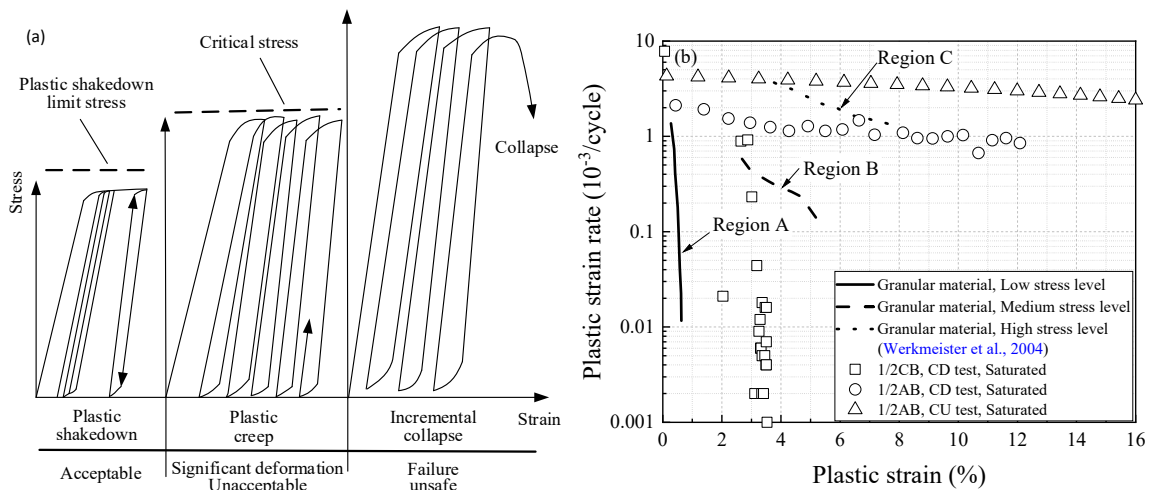
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For the plastic deformation of the granular material, the “shakedown theory” (Arnold et al., 2002; Werkmeister et al., 2004; Xiao et al., 2017) was commonly used to describe the plastic deformation and breakage rules of the granular material under cyclic loading. In general, the shakedown behavior of the granular material under cyclic loading includes three stages (Werkmeister et al., 2004): plastic shakedown (steady

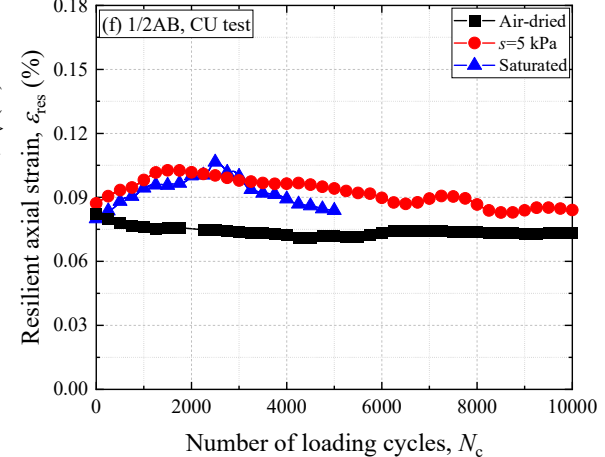
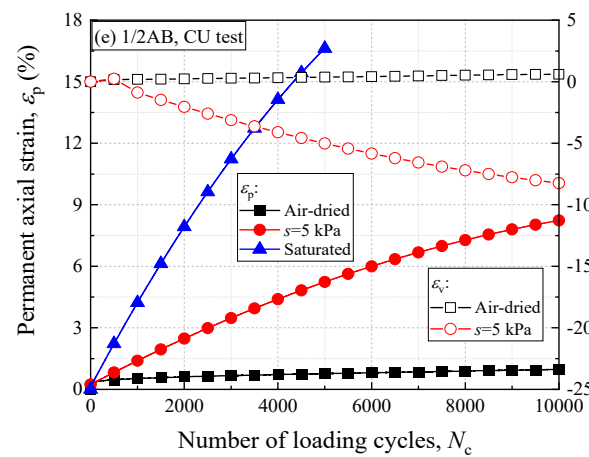
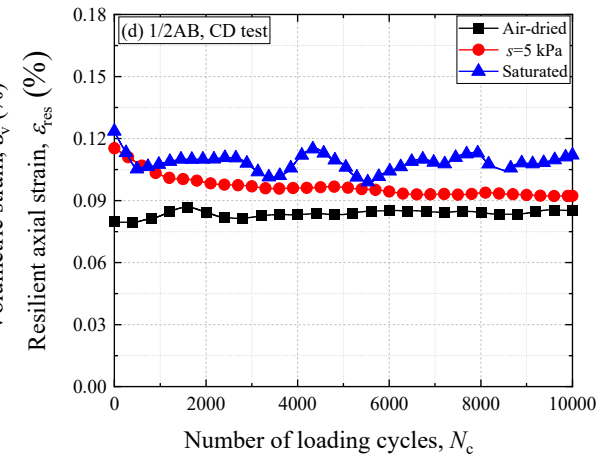
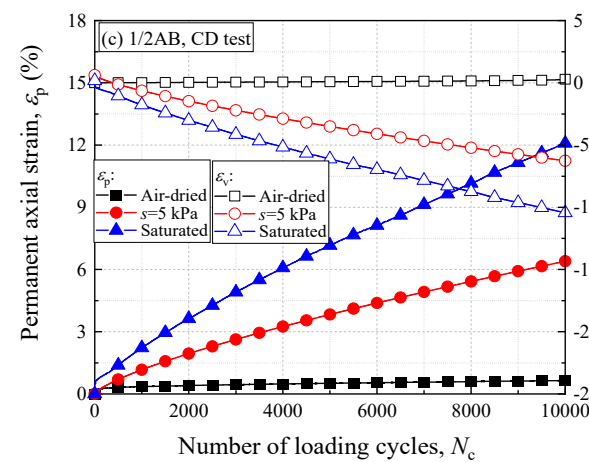
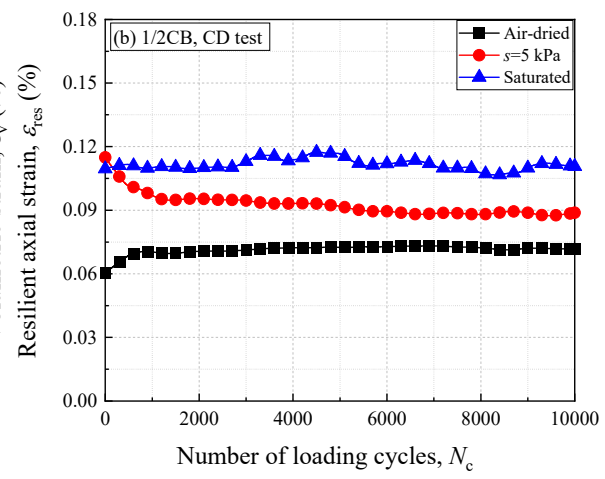
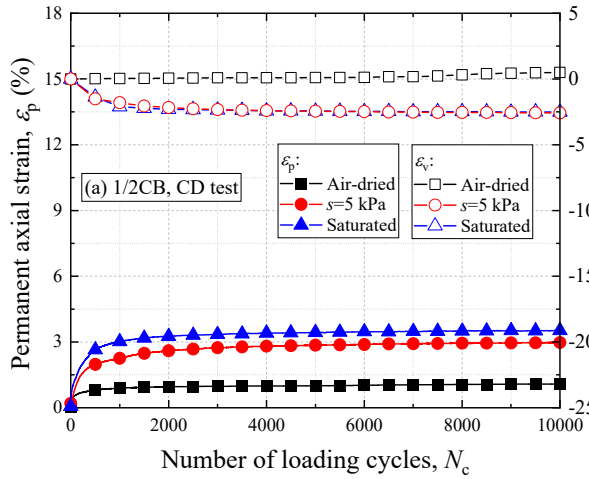
259 deformation behavior), plastic creep (failure at a large number of loading cycles), and incremental collapse  
 260 (failure at a small number of loading cycles), as shown in Fig. 8(a). In addition, Werkmeister et al. (2004)  
 261 pointed out that the relationship between the plastic strain rate and the plastic strain can be used to distinguish  
 262 these three types of shakedown behavior, as shown in Fig. 8(b). Region A indicates the plastic shakedown stage,  
 263 where the plastic strain rate decreases sharply as the plastic strain increases. Region B reveals the plastic creep  
 264 stage, where the plastic strain rate decreases sharply at first, and then tends to be stable with an increase in  
 265 plastic strain. Region C indicates the incremental collapse stage, where the plastic strain rate decreases slightly  
 266 as the plastic strain increases. For the plastic strain rate of railway ballast, previous studies (Mamou et al., 2017;  
 267 Sun et al., 2019) illustrated that there is a threshold stress level above which significant accumulation of plastic  
 268 strain and generation of excess pore pressure occurs, especially in undrained conditions. It is noted that Xiao et  
 269 al. (2017) pointed out that the fresh ballast is generally in a plastic shakedown or plastic creep state after  
 270 hundreds of loading cycles.



271  
 272 Fig. 8. Typical shakedown behaviors of granular materials under cyclic loading: (a) schematic diagram of the shakedown  
 273 theory (revised after Werkmeister et al., 2004); (b) relationship between plastic strain rate and plastic strain.

274 Fig. 9 shows the relationships between the permanent axial strain ( $\epsilon_p$ ), volumetric strain ( $\epsilon_v$ ), resilient axial  
 275 strain ( $\epsilon_{res}$ ) and number of loading cycles ( $N_c$ ) up to 10, 000 cycles for test specimens of 1/2CB and 1/2AB with  
 276 various water contents and drainage conditions. Here,  $\epsilon_p$  represents permanent axial strain (cyclic plastic strain)  
 277 at unloading, and  $\epsilon_{res}$  represents the resilient axial strain between the loading and unloading. The  $\epsilon_p$  of all  
 278 specimens shows a rapid growth trend in the initial cyclic loading process, and then the increasing trend of  $\epsilon_p$   
 279 becomes stable as the  $N_c$  increases, irrespective of test conditions. Besides, when comparing the  $\epsilon_p$  under the

280 same test conditions except the water content, the  $\varepsilon_p$  of all specimens increases with the increment of water  
281 content, irrespective of test samples. In CD tests for 1/2AB, the  $\varepsilon_p$  under the air-dried condition shows a stable  
282 trend with the increment of  $N_c$ , while the  $\varepsilon_p$  under unsaturated and saturated conditions reveal an approximately  
283 linear growth trend with the increment of  $N_c$ , as shown in Fig. 9(c). For the dilatancy behavior in CD tests, the  
284 volume of unsaturated and saturated specimens is dilated, while the volume of the air-dried specimen is  
285 compressed as the  $\varepsilon_p$  increases with the increment of  $N_c$ , irrespective of test samples. The reason for this  
286 phenomenon is that the volumes of the fresh and aged ballasts are firstly compressed at axial strain ( $\varepsilon_a$ ) about  
287 1%, followed by dilation with the increase in  $\varepsilon_a$  during the shear process (Yang et al., 2021). When other  
288 conditions are unchanged, the specimen under the higher degree of saturation shows an upward trend in the  
289 dilation due to the larger  $\varepsilon_p$ . Besides, the  $\varepsilon_{res}$  of both samples in CD tests increases by the sequences in air-dried,  
290 unsaturated, and saturated specimens, though the differences in  $\varepsilon_{res}$  under different water contents are not  
291 remarkable as those in the  $\varepsilon_p$ . Moreover, with the increment of  $N_c$ , the  $\varepsilon_{res}$  shows an increasing trend under the  
292 air-dried condition, while it decreases under the unsaturated and saturated conditions. This is because the  
293 unsaturated and saturated specimens have larger plastic deformation, which leads to an increase in density and  
294 thus greater stiffness. The above results indicate that the permanent/residual axial and volumetric strains of fresh  
295 and aged ballast in CD tests are affected by the water content. On the other hand, as shown in Fig. 9(e), the  $\varepsilon_p$   
296 of 1/2AB under the air-dried condition presents a stable trend with the increment of  $N_c$  in CU tests, while  $\varepsilon_p$   
297 under unsaturated and saturated conditions shows an approximately linear increasing trend with the increment  
298 of  $N_c$ . It is noted that under the saturated condition, 1/2AB is failed at the  $N_c$  of 5, 000 in the CU test. Besides,  
299 as shown in Fig. 9(f),  $\varepsilon_{res}$  in CU tests increases by in the order of air-dried, unsaturated, and saturated specimens.  
300 These results indicate that the cyclic plastic deformation of aged ballast is significantly affected by the water  
301 content in both CD and CU tests, and the effects of water content on the cyclic plastic deformation are more  
302 significant in CU tests.



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Fig. 9.  $\epsilon_p$ ,  $\epsilon_v$ , and  $\epsilon_{res}$  of test samples with different water contents: (a)  $\epsilon_p$  and  $\epsilon_v$  of 1/2CB in CD tests; (b)  $\epsilon_{res}$  of 1/2CB in CD tests; (c)  $\epsilon_p$  and  $\epsilon_v$  of 1/2AB in CD tests; (d)  $\epsilon_{res}$  of 1/2AB in CD tests; (e)  $\epsilon_p$  and  $\epsilon_v$  of 1/2AB in CU tests; (f)  $\epsilon_{res}$  of 1/2AB in CU tests.



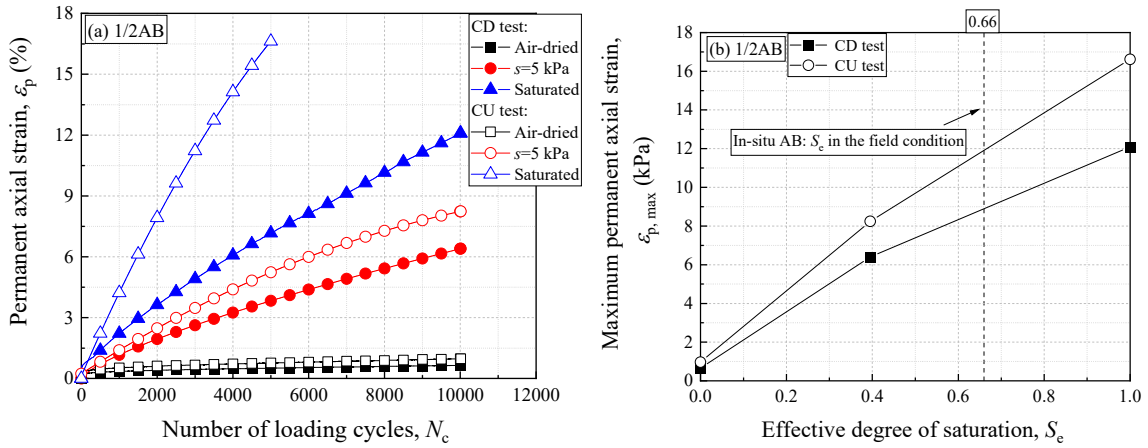
309 4.3 Effects of drainage condition, water content, and aging on permanent axial deformation

310 4.3.1 Effect of drainage condition

311 Considering that the drainage condition may affect the permanent axial deformation of aged ballast due to  
 312 its low permeability, thus both CD and CU tests were conducted for the test specimen of 1/2AB. In this case,  
 313 Fig. 10(a) shows the comparisons of  $\varepsilon_p$  for 1/2AB between CD tests and CU tests. The  $\varepsilon_p$  of 1/2AB in CU tests  
 314 is larger than those in CD tests, irrespective of water contents. Besides, under the air-dried condition,  $\varepsilon_p$  of all  
 315 specimens is small, irrespective of drainage conditions. However, the differences in  $\varepsilon_p$  under unsaturated and  
 316 saturated conditions are remarkable between the two types of test conditions, especially under the saturated  
 317 condition. These findings indicate that in the high degree of saturation condition, the drainage condition has a  
 318 significant influence on the permanent axial deformation of aged ballast. For a clearer explanation, Fig. 10(b)  
 319 presents the relationship between the  $\varepsilon_{p, \max}$  and  $S_e$  of 1/2AB. Here,  $\varepsilon_{p, \max}$  means the maximum permanent axial  
 320 strain at  $N_c$  of 10, 000, and the effective degree of saturation ( $S_e$ ) is the normalized water content between the  
 321 saturated state and the residual degree of saturation (Brooks and Corey, 1964).

322 
$$S_e = \frac{S_r - S_{rr}}{S_{rs} - S_{rr}} \quad (3)$$

323 where,  $S_{rs}$  is the degree of saturation under saturated conditions, and  $S_{rr}$  is the residual degree of saturation. It is  
 324 noted that the  $S_e$  of the soil in the air-dried state in this study is considered to be zero. As can be seen in this  
 325 figure, the  $\varepsilon_{p, \max}$  in CU tests is larger than those in CD tests, regardless of  $S_e$ . More importantly, the difference  
 326 in  $\varepsilon_{p, \max}$  is greater as the water content increases. This indicates that the effect of drainage condition on the  
 327 permanent axial strain becomes more significant at the higher water content.

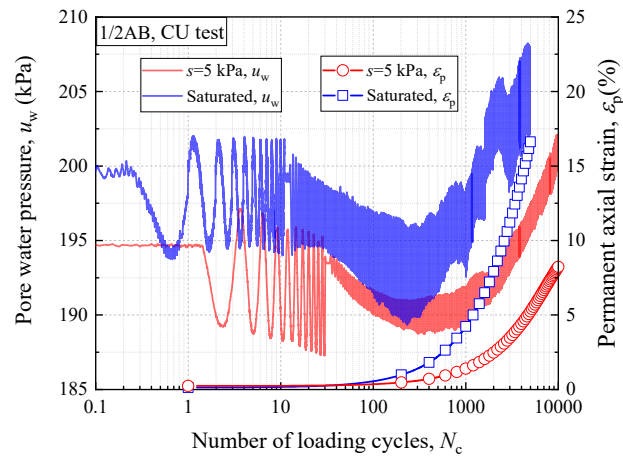


328  
329 Fig. 10. Comparison of  $\varepsilon_p - N_c$  and  $\varepsilon_{p, \max} - S_e$  of 1/2AB between CD test and CU test: (a)  $\varepsilon_p - N_c$ ; (b)  $\varepsilon_{p, \max} - S_e$ .

330 Fig. 11 shows the changes in  $u_w$  and  $\varepsilon_p$  for 1/2AB in CU tests. It is worth mentioning that the increasing  
331 trend of excess pore water pressure in the saturated condition is larger than that in the unsaturated condition,  
332 which is a probable reason why the saturated specimen is failed at the  $N_c$  of 5, 000. It can be inferred that the  
333 excess pore water pressure generated in the CU test is one of the factors that affect the permanent axial  
334 deformation of the aged ballast. In addition, the water content itself can reduce the shear strength of aged ballast,  
335 and induce a greater permanent axial deformation. Besides, the excess pore water pressure shows a whole  
336 increasing trend with the  $N_c$ , though it shows a downward trend in the initial loading process, irrespective of  
337 water content. It is noted that the excess pore water pressure is generated even for the unsaturated sample, since  
338 the degree of saturation of 1/2AB under the unsaturated condition is about 70% (Fig. 5). This indicates that due  
339 to the low permeability of the heavily aged ballast, the excess pore pressure is easily generated even in an  
340 unsaturated state under the repeated train loads, which is consistent with the phenomenon of aged railway tracks  
341 under the field condition.

342 Similar to the increasing trend of excess pore water pressure with the increment of  $N_c$  in CU tests, the  $\sigma'_c$   
343 of 1/2AB shows a whole decreasing trend with the increment of  $N_c$ , though an increasing trend is observed in  
344 the initial loading process, as shown in Fig. 12. To be specific, when the  $N_c$  is greater than 500, the  $\sigma'_c$  decreases  
345 as the increment of the  $N_c$  and the final values of  $\sigma'_c$  in CU tests are smaller than those in CD tests, regardless of  
346 water contents. The decrease in  $\sigma'_c$  in CU tests is a reason why the  $\varepsilon_p$  in CU tests under the same initial test  
347 condition is higher than that in CD tests. Besides, the change in  $\sigma'_c$  in CU tests is related to the dilatancy trend  
348 of the test specimen. When comparing the final values of  $\sigma'_c$  under different water content conditions, the final  
349 value of  $\sigma'_c$  of the saturated specimen is lower than those of unsaturated and air-dried specimens. This is because  
350 the positive dilatancy trends become stronger by the sequences in air-dried, unsaturated, and saturated  
351 specimens. These results demonstrate that the permanent axial strain of aged ballast is affected by the drainage  
352 condition. Considering that the high  $S_r$  and low permeability of the aged ballast, the in-situ aged ballast is almost  
353 under a partially or fully undrained condition with a high degree of saturation. In general, the cyclic loading  
354 tests for fresh and fouled ballasts in an air-dried condition have been carried out in the past researches so far.  
355 However, these results indicate that the cyclic loading tests for unsaturated and saturated specimens in partially  
356 and fully undrained conditions should be adopted for the accurate estimation of cyclic plastic deformation of

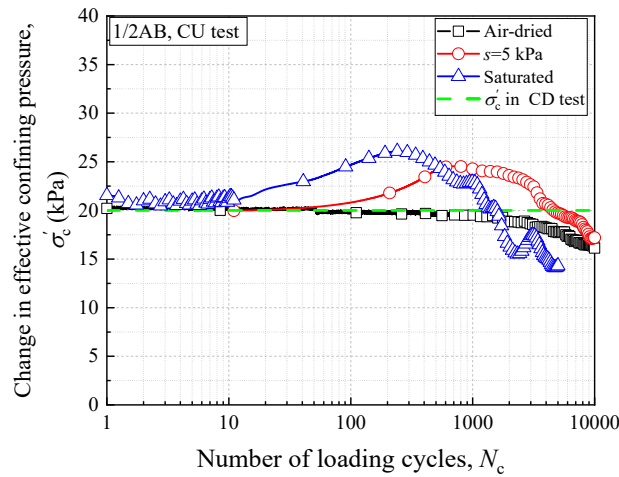
357 heavily aged ballast, though the cyclic loading tests for an air-dried specimen in fully drained condition is  
 358 suitable for evaluating the permanent deformation of fresh or slightly aged ballast.



359

360

Fig. 11. Changes in pore water pressure/permanent axial strain of 1/2AB in CU tests.



361

362

Fig. 12. Changes in effective confining pressure of 1/2AB in CU tests.

#### 363 4.3.2 Effect of water content

364 To evaluate the effect of water content on the  $\epsilon_p$ , Fig. 13(a) shows the effect of water content on  $\epsilon_p$  for both  
 365 samples in CD tests. When the other initial test conditions are constant except for the water content, the  $\epsilon_p$   
 366 increases with the increase in water content, irrespective of test samples. Furthermore, the wetting causes aged  
 367 ballast to increase  $\epsilon_p$  dramatically as compared with fresh ballast. As shown in Fig. 13(b), the  $\epsilon_{p, \max}$  appears a  
 368 remarkable increasing trend under the higher  $S_e$ , regardless of test samples and drainage conditions. These  
 369 results reveal that the water content has serious influence on permanent axial deformation of both samples,  
 370 especially for the aged ballast.

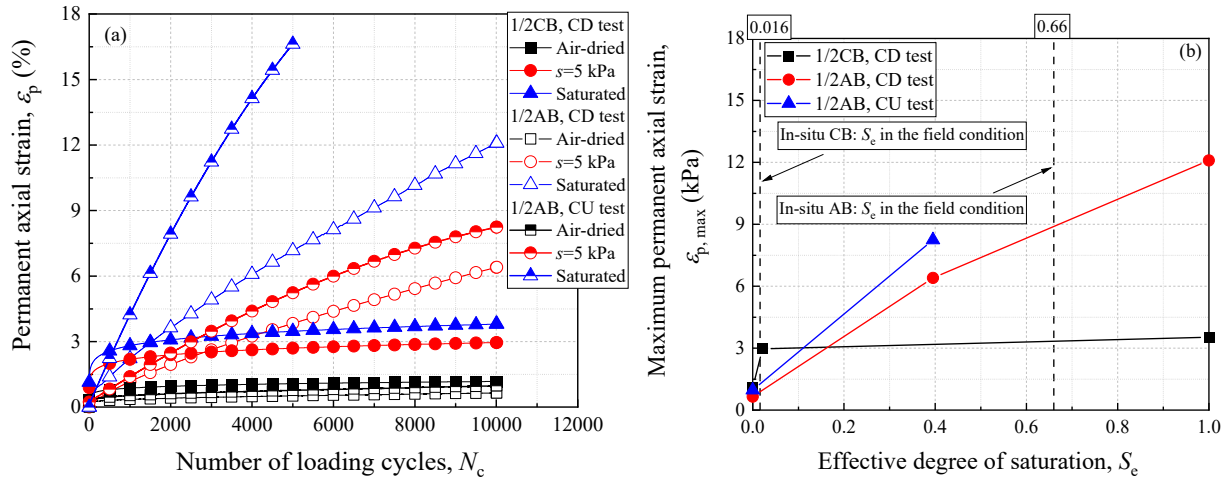


Fig. 13. Effect of water content on  $\epsilon_p$  and  $\epsilon_{p,max}$ : (a)  $\epsilon_p - N_c$ ; (B)  $\epsilon_{p,max} - S_e$ .

### 4.3.3 Effect of aging

As for the effect of aging on the  $\epsilon_p$  of ballast, the different tendency of  $\epsilon_p$  between fresh and aged ballasts is due to their plastic strain rate being in different “Regions”, as shown in Fig. 8(b). It is noted that the plastic strain rate for the fresh and aged ballast was calculated by the increment of plastic strain at each loading cycle during the whole cyclic loading. As illustrated in this figure, the plastic strain rate of the fresh ballast is generally in the Region A, while the plastic strain rate of the aged ballast is generally in the Region B or the Region C. These indicate that the aging significantly changes the tendency of the plastic strain of the ballast from plastic shakedown to incremental collapse stage, though the cyclic stress level of different samples is the same in this study. Furthermore, Fig. 14(a) presents the relationship between the  $\epsilon_{p,max}$  and  $F_c$ . As can be seen in this figure,  $\epsilon_{p,max}$  of 1/2AB is higher than those of 1/2CB in unsaturated and saturated conditions, though the opposite trend appears in the air-dried condition. The changing trend of the  $\epsilon_{p,max}$  with various water contents is similar to that of maximum principal stress ratio ( $(\sigma'_1/\sigma'_3)_{max}$ ) in the same condition shown in Fig. 14(b) (Yang et al., 2021). These results indicate that the both water content and aging synergistically affect the permanent axial strain of the ballast, and the increase in water content caused by ballast aging negatively enhances the adverse effects on the permanent axial strain. In addition, it should be emphasized that the effects of particle shape changes and increase in fine fraction content on the permanent axial strain of aged ballast are considered simultaneously in this study, though the effect of particle shape changes was not evaluated in previous studies on fouled ballast (Indraratna et al., 2011; Ishikawa et al., 2019). Compared with previous studies, test results in this study indicate that aged ballast has an unexpected smaller permanent axial strain than those of fresh and fouled ballasts in the

392 air-dried condition, while it has a larger permanent axial strain than those of fouled ballast in unsaturated and  
 393 saturated conditions. The reasonable explanation for this phenomenon is that the smaller and rounder particles  
 394 produced in the process of ballast breakdown and abrasion might provide a weakening effect in the aggregate  
 395 matrix by preventing contact between large particles thus greatly reducing the aggregate matrix in the  
 396 unsaturated and saturated conditions, though the fine particles can also provide a stabilizing effect by filling the  
 397 voids among larger particles in the air-dried condition. Conclusively, the difference in permanent axial strain  
 398 between fouled ballast and aged ballast demonstrates that when evaluating the permanent axial strain of aged  
 399 ballast, it is essential to consider the influence of particle shape, fine fraction content, and water content  
 400 comprehensively.

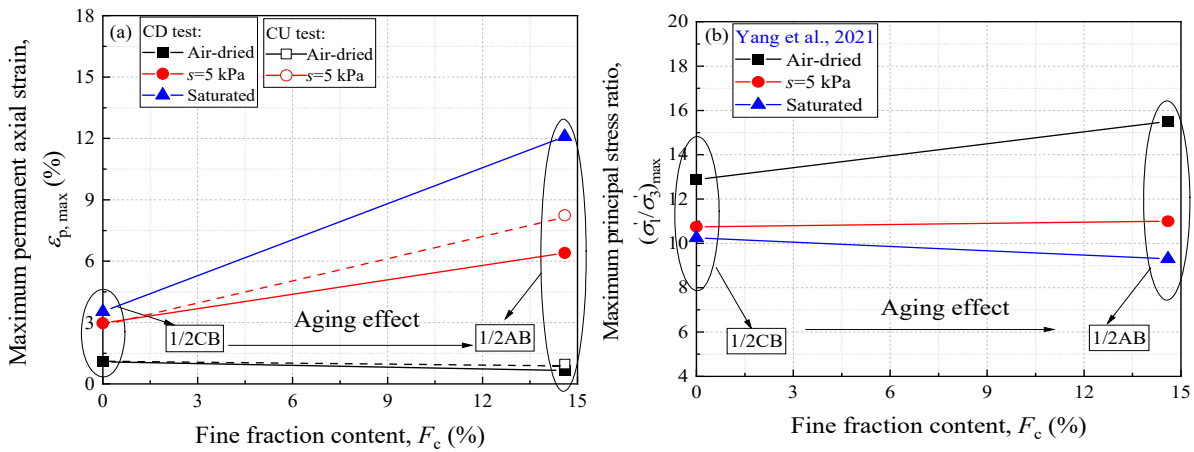


Fig. 14. Effect of aging on  $\epsilon_{p, \max}$ ; and  $(\sigma'_1/\sigma'_3)_{\max}$ : (a)  $\epsilon_{p, \max}$ ; (b)  $(\sigma'_1/\sigma'_3)_{\max}$  (after Yang et al., 2021).

403 From the above analysis, the results of ML tests and CL tests illustrate that the  $\tau_f$  and  $\epsilon_{p, \max}$  are affected by  
 404 the water content and ballast aging. Thus, the relationships between the  $\epsilon_{p, \max}$  and  $\tau_f$  with various water contents  
 405 and drainage conditions are shown in Fig. 15. As expected, the samples with a lower  $\tau_f$  appear a higher  $\epsilon_{p, \max}$ ,  
 406 irrespective of test samples and drainage conditions. This indicates that the water content and aging have similar  
 407 effects on the shear strength and permanent axial strain of ballast, that is the increase in water content or aging  
 408 might cause a decrease in shear strength and an increase in permanent axial deformation of ballast, and the  
 409 downward/upward trend of shear strength/permanent axial deformation is more remarkable for aged ballast  
 410 under the high-water content condition.

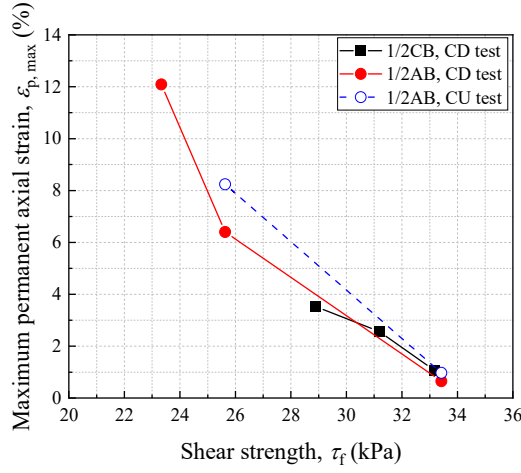


Fig. 15. Relationship between  $\varepsilon_{p, \max}$  and  $\tau_f$ .

411  
412  
413 **5 Estimation of permanent axial deformation of unsaturated ballast**

414 5.1 Estimation of permanent axial deformation by UIUC model

415 The UIUC model belongs to a semi-empirical model, and its input parameters can be obtained from ML  
416 tests (Qamhia et al., 2016). Therefore, in this section, the applicability of the UIUC model is evaluated by  
417 comparing the predicted results by this model with measured results from CL tests. For this model, the  
418 permanent axial strain can be calculated as follows:

419 
$$\varepsilon_p = AN_c^B \sigma_d^C \left( \frac{\tau_f}{\tau_{\max}} \right)^D \quad (4)$$

420 where,  $\varepsilon_p$  is the permanent axial strain corresponding to  $N_c$  applications (%);  $\sigma_d$  is the axial deviator stress (kPa);  
421  $A, B, C, D$  are regression parameters. Besides,  $\tau_f$  and  $\sigma_f$  are the applied effective shear stress and applied effective  
422 normal stress acting on failure plane calculated by Eq. (5) and Eq. (6), respectively;  $\tau_{\max}$  is the maximum  
423 effective shear stress at  $\sigma_f$  calculated by Eq. (7) determined by ML tests.

424 
$$\tau_f = \sqrt{\left( \frac{\sigma_d}{2} \right)^2 - \left( \sigma_f - \left( \sigma'_3 + \frac{\sigma_d}{2} \right) \right)^2} \quad (5)$$

425 
$$\sigma_f = \frac{2\sigma'_3 + 2\tan^2\phi'\sigma'_3 + \sigma_d + \tan^2\phi'\sigma_d - \sqrt{\tan^2\phi'\sigma_d^2 + \tan^4\phi'\sigma_d^2}}{2(1 + \tan^2\phi')} \quad (6)$$

426 
$$\tau_{\max} = c + \sigma_f \tan\phi' \quad (7)$$

427 where  $\sigma'_3$  is the minimum effective principal stress, which is equal to  $\sigma'_c$  in CL tests;  $c$  stands for total cohesions  
 428 of both samples with different water contents, as shown in Table 3. It is noted that the  $c$  of test samples in air-  
 429 dried and saturated conditions is equal to the effective cohesion ( $c'$ ).

430 The input parameters and the corresponding regression parameters of UIUC model for 1/2CB and 1/2AB  
 431 are shown in Table 4 and Table 5, respectively. It is noted that the input parameters of  $c$  and  $\phi'$  for 1/2CB were  
 432 determined by CD tests, while  $c$  and  $\phi'$  for 1/2AB are determined by CU tests. The regression parameters in  
 433 Table 5 were obtained by fitting the results of CL tests at different water contents for each test sample. As shown  
 434 in Table 5, 1/2AB has the higher  $A$  and  $D$  values than those of 1/2CB. It means that the aged ballast shows a  
 435 higher initial permanent axial deformation, and that the effect of  $(\tau_f/\tau_{max})$  on permanent axial strain becomes  
 436 more pronounced for the aged ballast as compared with fresh ballast. These indicate that it is feasible to predict  
 437 the permanent axial strain of fresh and aged ballasts with various water contents by the UIUC model.

438 Table 4 Input parameters of UIUC model.

Input Parameters	1/2CB	1/2CB	1/2CB	1/2AB	1/2AB	1/2AB
	Air-dried, CD test	$s=5$ kPa, CD test	Saturated, CD test	Air-dried, CD test	$s=5$ kPa, CD test	Saturated, CD test
$\sigma_d$ (kPa)	80	80	80	80	80	80
$\sigma_f$ (kPa)	25.77	27.11	27.11	25.57	29.58	29.58
$\tau_f$ (kPa)	20.69	22.77	22.77	20.36	25.98	25.98
$c$ (kPa) (Yang et al., 2021)	0	1.6	0	5.1	9.4	4.8
$\phi'$ (deg.) (Yang et al., 2021)	58.9	55.3	55.3	59.4	49.5	49.5
$\tau_{max}$ (kPa)	42.64	40.73	39.16	48.34	44.04	39.44
$\tau_f/\tau_{max}$	0.485	0.559	0.581	0.421	0.590	0.659

439 Table 5 Regression parameters of UIUC model.

Sample	Drainage condition	Regression parameters				
		$A$	$B$	$C$	$D$	$R^2$
1/2CB	CD test	0.1938	0.1306	1.2130	6.5239	0.9801
1/2AB	CD test	0.3162	0.6843	0.1066	7.5684	0.9960
1/2AB	CU test (S)	0.3162	0.6843	0.1066	7.5684	-
1/2AB	CU test (D)	0.6020	0.6266	0.1022	6.8378	0.9397

440 CU test (S) and CU test (D) mean that the same and different regression parameters from CD tests were adopted in CU  
 441 tests.

442 Fig. 16 compares the measured and estimated permanent axial strains of 1/2CB with various water contents.  
 443 The estimated values by the UIUC model using the same regression parameters (Table 5) are consistent with  
 444 measured values even though the water contents are different. This indicates that the effect of water content on  
 445 permanent axial deformation of fresh ballast can be effectively estimated by UIUC model. Then, in order to  
 446 evaluate the applicability of this model to the prediction for the permanent axial deformation of aged ballast,

447 Fig. 17 compares the measured and estimated permanent axial strains of 1/2AB with different water contents  
 448 and drainage conditions. It is noted that the regression parameters of 1/2AB are different from those of 1/2CB  
 449 as shown in Table 5. In addition, the input parameters of  $\sigma_d$ ,  $\tau_f$ , and  $\tau_{max}$  in Eq. (4) in CU tests are changed due  
 450 to the change in the  $\sigma'_3$ , which can be calculated by the Eqs. (5) ~ (7). Subsequently, the permanent axial strain  
 451 ( $\varepsilon_{p,i}$ ) at different  $N_{c,i}$  in CU tests can be calculated with different  $\sigma_{d,i}$ ,  $\tau_{f,i}$ , and  $\tau_{max,i}$  by Eq. (8). The reason for the  
 452 fluctuation of the predicted values seen in Fig. 17(b) is that the input parameters of  $\sigma_{d,i}$ ,  $\tau_{f,i}$ , and  $\tau_{max,i}$  are changed  
 453 as the  $\sigma'_3$  changes in CU tests, while these input parameters are constants adopted for the estimation of CD tests  
 454 (Table 4).

$$455 \quad \varepsilon_{p,i} = \varepsilon_{p,i-1} + A(N_{c,i})^B \sigma_{d,i}^C \left( \frac{\tau_{f,i}}{\tau_{max,i}} \right)^D - A(N_{c,i-1})^B \sigma_{d,i}^C \left( \frac{\tau_{f,i}}{\tau_{max,i}} \right)^D \quad (8)$$

456 where,  $N_{c,i}$  represents the  $N_c$  is  $i$ th;  $N_{c,i-1}$  represents the  $N_c$  is  $(i-1)$ th;  $\varepsilon_{p,i}$  is the permanent axial strain at the  $N_{c,i}$ ;  
 457  $\varepsilon_{p,i-1}$  is the permanent axial strain at the  $N_{c,i-1}$ ;  $\sigma_{d,i}$ ,  $\tau_{f,i}$ , and  $\tau_{max,i}$  are the axial deviator stress, effective shear stress,  
 458 and effective shear strength at the  $N_{c,i}$ . It can be seen that the estimated permanent axial strains of 1/2AB by  
 459 UIUC model are consistent with measured values in CD tests (Fig. 17(a)), irrespective of water contents, while  
 460 the estimated values are different from measured values in CU tests (Fig. 17(b)). The possible reason is that the  
 461 UIUC model cannot reproduce the effect of the change in excess pore water pressure (Fig. 11) caused by cyclic  
 462 loading in CU test on the mechanical behavior of ballast. On the other hand, when using different regression  
 463 parameters from those for CD tests, the estimated values by the UIUC model are consistent with measured  
 464 values in air-dried and unsaturated conditions, though the estimated value is smaller than the measured value in  
 465 the saturated condition, as shown in Fig. 17(c). These results prove that the permanent axial deformation of  
 466 fresh and aged ballasts with various water contents in CD tests can be well estimated by UIUC model by using  
 467 the same regression parameters, while the applicability of UIUC model to the prediction for permanent axial  
 468 deformation of aged ballast may be limited in CU tests, since it is difficult to predict the change in excess pore  
 469 pressure under cyclic loading.



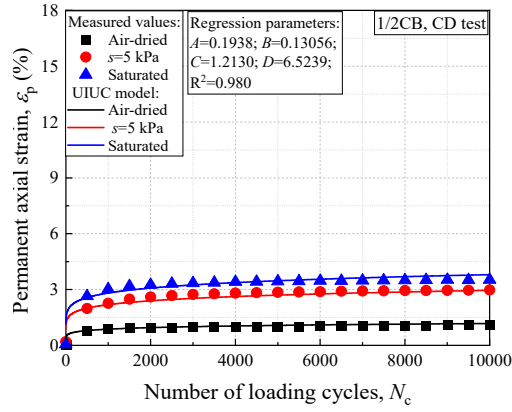


Fig. 16. Comparison of  $\epsilon_p$  for 1/2CB with various water contents in CD tests.

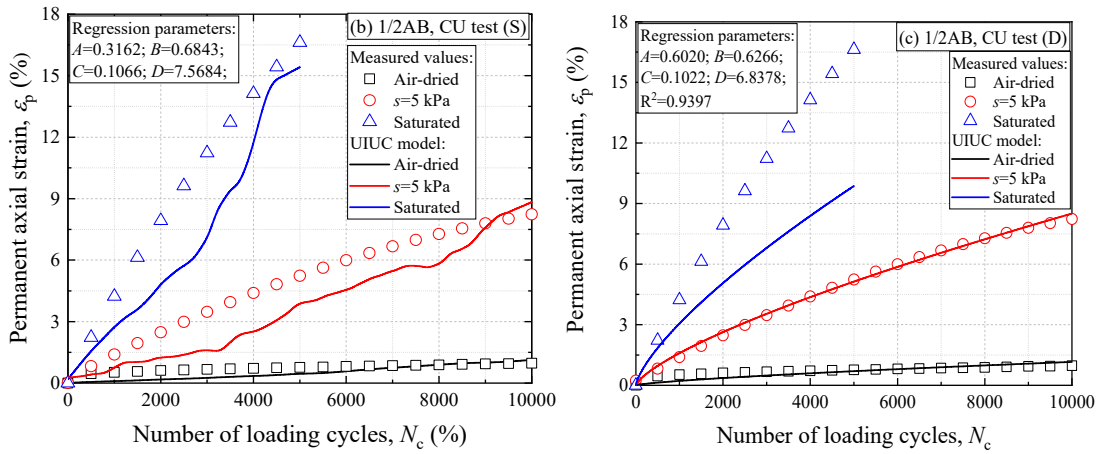
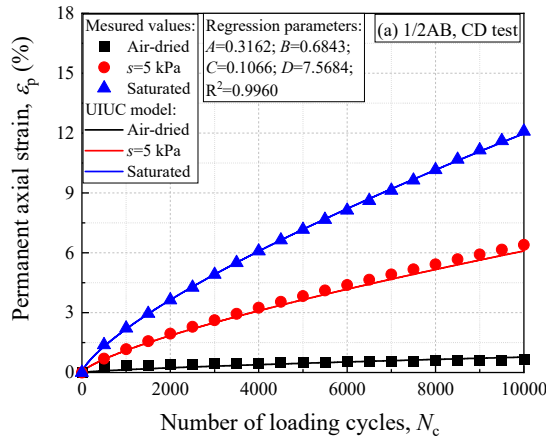


Fig. 17. Comparison of  $\epsilon_p$  for 1/2AB with various water contents: (a) CD test; (b) CU test (S); (c) CU test (D).

## 5.2 Estimation of permanent axial deformation by SSE model

The reliability of the SSE model is discussed by comparing the results of CL tests for fresh and aged ballasts under various water contents with the numerical results. The input parameters of SSE model for 1/2CB and 1/2AB under various water contents and drainage conditions were listed in Table 6. Here, the  $\sigma'_c$  can be determined from the CL test.  $F_0$  can be determined from the ML test (Wood, 1990).  $\mu$  can be determined from

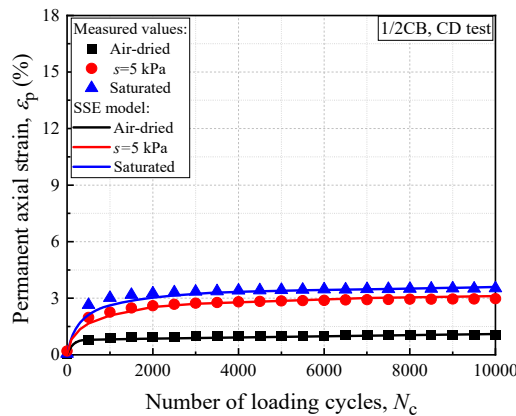
480 the ML test (Qi et al., 2020).  $\phi'$  can be obtained from the ML test.  $\lambda$  and  $\kappa$  can be determined from the isotropic  
 481 compression test, which refers to previous study on air-dried ballast (Okayasu et al., 2014). In accordance with  
 482 observations reported in Alonso et al. (1990),  $\lambda$  depends on the matric suction ( $s$ ), which can be determined  
 483 from CL tests. Besides,  $p_i$ ,  $U(R)$ ,  $U(R_e)$ , and  $R_e$  are material constants independent on water content, while  $R_{e\max}$   
 484 changes depending on the water content, all of which can be fitted by the results of CL tests.

485 Table 6 Input parameters of SSE model.

Sample	1/2CB	1/2CB	1/2CB	1/2AB	1/2AB	1/2AB
Drainage condition	CD test			CD test, CU test		
Water content	Air-dried	$s=5$ kPa	Saturated	Air-dried	$s=5$ kPa	Saturated
$\sigma'_c$	20	20	20	20	20	20
$F_0$	1078	887	842	1186	895	792
$\mu$	0.3	0.3	0.3	0.3	0.3	0.3
$\phi'$	58.9	55.3	55.3	59.4	49.5	49.5
$\lambda$	0.05	0.07	0.09	0.05	0.07	0.09
$\kappa$	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
$p_i$	45	45	45	45	45	45
$U(R)$	20	20	20	150	150	150
$U(R_e)$	4	4	4	25	25	25
$R_e$	0.001	0.001	0.001	0.001	0.001	0.001
$R_{e\max}$	0.150	0.067	0.060	0.047	0.037	0.022

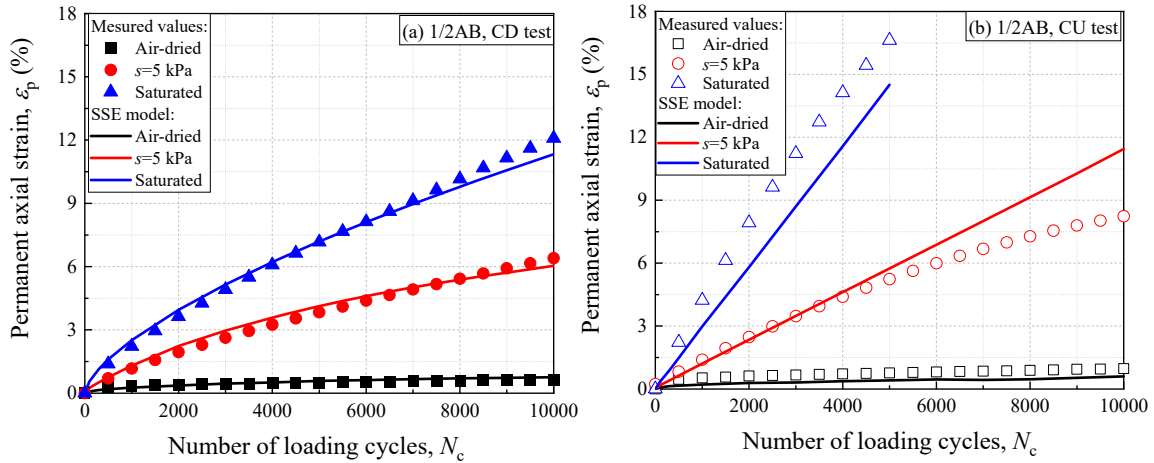
486  $\sigma'_c$  is the effective confining pressure;  $F_0$  is an initial value of the size of the normal yield surface;  $\mu$  is the Poisson ratio;  $\lambda$   
 487 and  $\kappa$  are the slopes of the normal-consolidation and swelling line;  $p_i$ ,  $U(R)$ , and  $U(R_e)$  are material constant;  $R_e$  is a ratio  
 488 of the size of the elastic boundary surface to the size of the normal-yield surface;  $R_{e\max}$  is the maximum value of the  $R_e$ .

489 Fig. 18 compares the permanent axial strain of 1/2CB with various water contents between simulations and  
 490 tests. It can be found that the predicted values by SSE model are consistent with measured values, regardless of  
 491 water contents. These results prove that the applicability of the SSE model to the prediction for the permanent  
 492 axial deformation of fresh ballast by considering the effect of water content.



493  
 494 Fig. 18. Comparison of  $\varepsilon_p$  for 1/2CB in CD tests between simulations and tests.

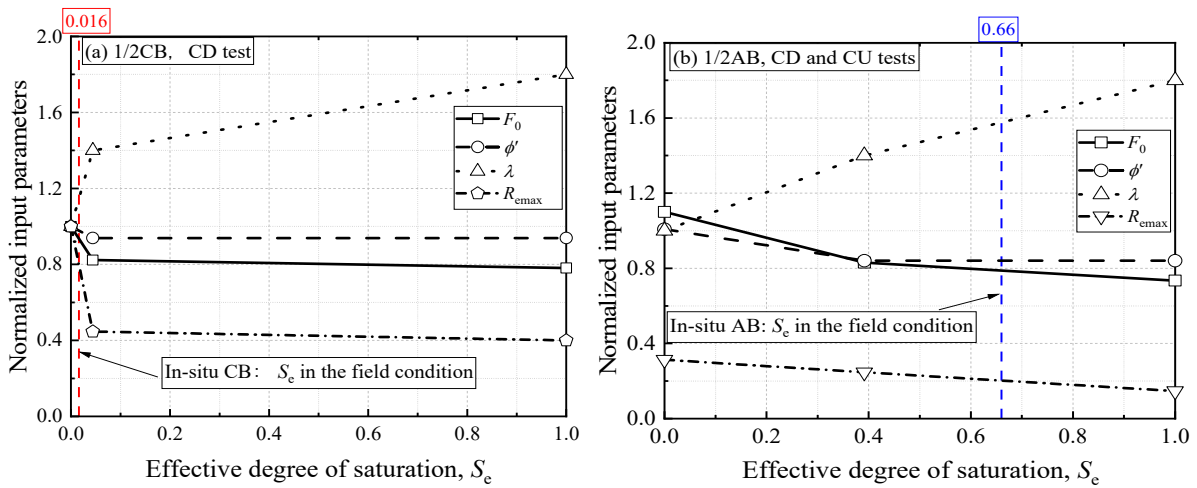
495 On the other hand, Fig. 19 compares the simulated and measured permanent axial strains of 1/2AB in  
 496 different drainage conditions. As can be seen in Fig. 19(a), the permanent axial strains of 1/2AB with various  
 497 water contents in CD tests can be well predicted by SSE model. However, as shown in Fig. 19(b), there is a  
 498 certain difference between the estimated and measured permanent axial strain of 1/2AB in CU tests, and with  
 499 increasing  $S_r$ , the difference becomes larger due to the generation of excess pore water pressure. This is because  
 500 only a mechanical model in numerical simulations is adopted in this study, which causes the SSE model cannot  
 501 accurately predict the change in excess pore water pressure under cyclic loading. These results prove that the  
 502 SSE model can well predict the permanent axial deformation of fresh and aged ballasts with various water  
 503 contents in CD tests, and there is a possibility for improving the prediction accuracy of the SSE model for aged  
 504 ballast in CU tests by employing a hydro-mechanical model.



505  
 506 Fig. 19. Comparisons of  $\epsilon_p$  for 1/2AB between simulations and tests: (a) CD test (b) CU test.

507 To analyze the influence of water content on input parameters of SSE model for both samples, the changes  
 508 in normalized input parameters with the effective degree of saturation ( $S_e$ ) for 1/2CB and 1/2AB are shown in  
 509 Fig. 20. It is noted that this figure only plots input parameters that change with  $S_e$ , instead of all input parameters.  
 510 Here, the value of each input parameter of 1/2CB in the air-dried condition is taken as the standard value, and  
 511 the values of normalized input parameters under various test conditions are the ratio of original values to  
 512 standard values. It shows that the  $F_0$  decreases with the increase in water content, regardless of test samples. As  
 513 for the  $U(R)$ ,  $U(R_e)$  and  $R_e$ , they are unchanged with various water contents, regardless of test samples. Besides,  
 514 the  $R_{e\max}$  decreases with the increment of  $S_e$ , irrespective of test samples and drainage conditions. As for the  $\phi'$   
 515 in the unsaturated condition, Fredlund et al. (1978) proposed that an increase in matric suction ( $s$ ) results in an

516 increase in the apparent cohesion while maintaining the same  $\phi'$  as the saturated soils. In this case, when other  
 517 test conditions are constant, the  $\phi'$  maintains constant, even though the  $S_e$  changes, except in the air-dried  
 518 condition. Moreover, as the  $S_e$  increases, the  $\lambda$  increases while the  $\kappa$  does not change, which is consistent with  
 519 the results of Alonso et al. (1990). On the other hand, previous study (Yang et al., 2021) revealed that the  $S_e$  of  
 520 the fresh ballast is generally below 0.016 and the  $S_e$  of the aged ballast is generally above 0.66 in the field  
 521 conditions. In this case, the input parameters of the fresh ballast under the air-dried condition can reproduce  
 522 actual conditions for a new railway line, while the input parameters of the aged ballast under the saturated  
 523 condition can simulate actual conditions for an old railway line since when the  $S_e$  is greater than 0.66, the  
 524 normalized input parameters of the aged ballast show a small change with the water content.



525  
 526 Fig. 20. Relationship between normalized input parameters and  $S_e$  of test samples: (a): 1/2CB in CD tests; (b) 1/2AB in  
 527 CD and CU tests.

## 528 6 Conclusions

529 In this study, the influence of aging on cyclic plastic deformation of unsaturated ballast was evaluated by  
 530 performing a series of the CL tests. After that, the applicability of the UIUC model and the SSE model was  
 531 verified to the prediction for the cyclic plastic deformation of unsaturated ballast by considering aging effects.  
 532 The following findings can be mainly obtained:

- 533 • The permanent axial deformation of ballast increases with the increase in water content, irrespective of test  
 534 samples and drainage conditions, and wetting has a more significant impact on the permanent axial  
 535 deformation of aged ballast as compared with fresh ballast. It can be inferred that the water content and aging  
 536 effect synergistically affect the permanent axial deformation of ballast, and this is the reason why the aged

537 ballast with a high degree of saturation would appear a greater permanent axial deformation than that of fresh  
538 ballast with a low degree of saturation at the field condition.

539 • For the aged ballast, the permanent axial deformation in CU tests is larger than that in CD tests, regardless  
540 of water contents. Besides, the effect of drainage conditions on the cyclic plastic deformation for aged ballast  
541 becomes more significant as the degree of saturation increases since the excess pore water pressure increases  
542 significantly at higher degree of saturation.

543 • The shear strength and permanent axial deformation are affected by the water content and aging. When other  
544 test conditions are constant, the unsaturated ballast with the lower shear strength in the monotonic loading  
545 condition has the higher permanent axial deformation in the cyclic loading condition, irrespective of test  
546 samples and drainage conditions. The decreasing/increasing trend of shear strength/permanent axial  
547 deformation is more remarkable, in case of both the water content and fine fraction content increase.

548 • Aged ballast has smaller permanent axial strain than that of fouled ballast under the air-dried condition, while  
549 it has a larger permanent axial strain than that of fouled ballast under unsaturated and saturated conditions.  
550 Thus, the difference in permanent axial strain between fouled ballast and aged ballast demonstrates that when  
551 evaluating the permanent axial strain of aged ballast, it is essential to consider the influence of particle shape,  
552 fine fraction content, and water content comprehensively.

553 • In CD tests, the influence of water content on the permanent axial deformation of fresh and aged ballasts can  
554 be well estimated by the UIUC model with the same regression parameters, even though water contents are  
555 different. However, in CU tests, the applicability of the UIUC model to the prediction for the permanent  
556 axial deformation of aged ballast is limited, since it is difficult to predict the change in excess pore pressure  
557 under cyclic loading.

558 • The SSE model can well predict the permanent axial deformation of fresh and aged ballasts with different  
559 water contents in CD tests. Thus, the input parameters of the fresh ballast under the air-dried condition can  
560 reproduce actual conditions for a new railway line. On the other hand, the effect of water content on the  
561 permanent axial deformation of aged ballast in CU tests could not accurately predict by the SSE model, since  
562 a mechanical model was adopted in this study. However, the input parameters of the aged ballast under the

563 saturated condition may simulate actual conditions for an old railway line when the hydro-mechanical  
564 analysis is adopted.

565 The findings of this research indicate that it is necessary to consider the effects of the drainage condition  
566 and water content on the hydro-mechanical behavior of the aged ballast, due to the low permeability and high  
567 residual degree of saturation. For the fresh ballast, the air-dried specimen in triaxial tests under the fully drained  
568 condition can reproduce the actual conditions. However, for the aged ballast, the unsaturated and saturated  
569 specimens in triaxial tests under partially and fully undrained conditions can reproduce the actual conditions.  
570 Therefore, for the actual design, when evaluating the cyclic plastic deformation of the aged ballast, it is  
571 important to first consider the hydraulic properties (i.e., permeability and water retentivity), and select  
572 appropriate test conditions (i.e., CD and CU tests) based on these properties. For example, when the fine fraction  
573 content of aged ballast exceeds a threshold (i.e.,  $F_c > 15$ ) or when the coefficient of permeability of aged ballast  
574 is less than a threshold (i.e.,  $k_s < 10^{-4}$ ), the undrained test conditions should be selected. Besides, the UIUC model  
575 is suitable for predicting the permanent axial deformation of fresh ballast or slightly aged ballast under the  
576 drained condition, and the SSE model shows good potential to estimate the permanent axial deformation of  
577 fresh and aged ballasts under different drainage conditions when the hydro-mechanical analysis is adopted,  
578 although the applicability of the SSE model in this study is verified only based on the mechanical analysis under  
579 the drained condition. However, the findings in this study were obtained under limited test conditions (only one  
580 unsaturated state, fine fraction content, and confining pressure). Further laboratory element tests (triaxial  
581 compression test, water retention test, and permeability test) considering more different degrees of saturation,  
582 fine fraction contents, and confining pressures need to be conducted in the future to investigate the hydro-  
583 mechanical behavior of aged ballast, and to further determine the threshold value between heavily and slightly  
584 aged ballast. More importantly, the applicability of the UIUC model and the SSE model to the prediction for  
585 the permanent axial deformation of aged ballast in the undrained condition still needs further study.

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