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An empirical approach to measure unobserved cultural relations using music trade data

Yuki Takara[1] · Shingo Takagi[2]

Abstract

Cultural relations between countries influence the exchange of cultural goods. This study provides novel knowledge of unobserved cultural relations by measuring the effect of cultural relations on the trade in recorded music compact discs, using the gravity model of international trade. We consider such relations as unobserved heterogeneity and introduce into the standard model a factor structure (multiple interactive fixed effect terms) to extract the features of unobserved relations, including cultural relations, between trading countries. We also consider the existence of multiple zero-trade country pairs and introduce a selectivity structure to account for zero flows. After the estimation procedure, we derive the implications of cultural relations from the estimated values of interactive terms using multivariate analysis. From the results of post-estimation analysis, the estimated values of our interactive terms could be interpreted as the effect of cultural relations. In addition to the positive effect of cultural proximity on trade, which existing studies have revealed, our inter- active terms could capture (i) the negative effect of cultural proximity on music trade, such as home consumption bias, (ii) the positive effect of modern music consumption trend on music trade, which is unexplained by cultural proximity based on traditional cultural studies.

Keywords Cultural relations, Music trade, Gravity equation, Interactive fixed effect, Expectation conditional maximization algorithm

JEL Classification F14 · C13 · Z10

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1 Introduction

Between two countries, cultural relations, whether observable—such as linguistics or religions (e.g., Marvasti (1994); Melitz (2008); Katz-Gerro et al. (2009))—or unobservable—such as embodied values (e.g., Throsby (1999); Kristjansdottir et al. (2017))—are important determinants of international trade in cultural goods. In general, the familiarity with, and approval of, one country's cultural goods by consumers in another country represent a positive relation. This relation could promote the import and domestic consumption of the other country's cultural goods. The goal of this study is to measure and analyze such effects on cultural goods trade, particularly, the trade in recorded music compact discs (CDs).

Theoretically, cultural proximity or differences can cause trade friction in various respects (e.g., Melitz (2008)), provide comparative advantage through effects on productivity (e.g., Belloc and Bowles (2017)), and create demand for cross-cultural varieties (see Helpman and Krugman (1985) for the "love of variety" model). In other words, cultural proximity could increase trading volume because of reduced trading costs due to international business administration or publicity campaigns (i.e., special treatment of prohibited goods and no need for translation) if countries share a common language or religious background. On the other hand, there could also be a strong demand for foreign goods from culturally different countries in terms of cross-cultural perceptual experience.

In the empirical study of trade, a strand of the literature defines cultural variables that embody cultural relations and confirms the effect of cultural factors on trade. For example, Tadesse and White (2010) calculate the cultural distance between two countries using the values provided by the World Values Survey and the European Values Survey. Giuliano et al. (2014) calculate cultural distance between two countries from gene data and introduce it into their empirical model. Takara (2018) discerns cultural proximity variables using country classifications based on ethnomusicology and political science and analyzes the effects of these cultural factors on music trade. These studies show that cultural proximity promotes trade between two countries and that the importance of cultural relations in bilateral trade should be emphasized [1].

Although these studies shed some light on the role of cultural relations in trade, we should pay more attention to the definition of culture. The definitions of cultural variables depend on authors' exogenous ideas on culture, and the results of measuring their effects on trade depend on the differences in their definitions. Therefore, we begin by clarifying the definition of "culture."

One of the most comprehensive definitions of culture is provided by UNESCO (2001), which defines culture as *the set of distinctive spiritual, material, intellectual, and emotional features of society or a social group, that encompasses, not only art and literature, but lifestyles, ways of living together, value systems, traditions, and beliefs*.

However, UNESCO (2009) notes that culture is not always measured directly, and it defines culture as follows: *Whereas it is not always possible to measure such beliefs and values directly, it is possible to measure associated behaviors and practices.*

[1] See also Jinji and Tanaka (2020) for an analysis of cultural goods trade in terms of cultural diversity.

As such, the UNESCO Framework for Cultural Statistics defines culture through the identification and measurement of the behaviors and practices resulting from the beliefs and values of a society or a social group (UNESCO (2009)).

While some cultural factors used in existing studies can be observed, culture is hard to measure directly. Therefore, we consider cultural factors to be unobserved.

The current study measures the effect of unobserved cultural relations on cultural goods trade, specifically, recorded music. To introduce cultural relations as unobserved heterogeneous relations in our econometric trade model, we include multiple interactive terms of fixed effects to extract the features of unobserved relations (including cultural relations) in two countries. Through panel data analyses, the interactive effects capture the heterogeneous impacts of time on cross-sectional units (e.g., Gobillon and Magnac (2016); Shi and Lee (2017)). We include these interactive effects in the dyadic data between an exporter and importer (e.g., Cameron and Miller (2014); Cameron and Miller (2015)) where the interactive fixed term is a multiplicative term of fixed effects specific to an importer and exporter after controlling for traditional explanatory variables and the factors used in the empirical trade literature. These explanatory variables and factors successfully explain the economic status of two countries and the trading costs between them (for recent surveys using the gravity model, see Anderson (2011) and Head and Mayer (2014)).

We interpret the estimated interactive fixed effects as quantitative cultural relations, referring to the results of the post-estimation analysis using multivariate analysis. Based on the estimation results, we note that the major unexplained component of cultural goods trade in traditional econometric trade models (the gravity models) is the cultural relations between countries. This study fills this gap as the interactive terms in our model reflect the effect of such relations on trade.

2 Asymmetric Trading Flows

In their research, Helpman et al. (2008) derive a slightly modified gravity model with importer- and exporter-specific factors as additive two-way fixed effects [2]. The model is composed of two equations. The first determines whether the j-th country exports to the i-th (the selection equation); the second determines the trading volume (the outcome equation). The model is a generalized version of a sample selection model (Heckman (1979)). Positive trade flow is selective because of the fixed costs of serving a specific market. Moreover, low productivity is insufficient to serve a specific destination. The sources of selectivity induce linear and nonlinear selection correction terms into the outcome equation. Since the derivation of the correction terms heavily depends on the underlying assumptions, the validity of the inferences cannot be extended to cases with non-i.i.d. (independent and identically distributed) unobservable factors.

It is certain that heteroskedasticity in error terms can be partially alleviated by adding two-way fixed effects. However, the misspecifications of the unobserved terms' properties stem from the misspecified relations between the importer and the ex-

[2] Santos Silva and Tenreyro (2015) provide several remarks on the inference procedure.

porter; thus, the additive fixed effects have only limited mitigation effects on the misspecifications.

With regard to cultural goods trade, such misspecifications of unobserved terms are more serious because we find striking asymmetries in trade flows in our data set. These asymmetries cannot be explained only by the economic (or related) scales of trading countries or additive fixed effects. In the following, we first present the summary of our data set and then reveal the features of cultural goods trade flows and discuss the possibility of misspecifications.

We employ the amount of exported CDs (HS8524.32: discs for laser reading systems, for reproducing sound only) as representative of cultural goods traded [3]. Our data cover 187 countries from 2000 to 2006 and use averages by year. Table 1 lists the countries in our sample. The sample size of our data is 34,782, and we derive 6,328 country pairs that trade cultural goods.

One important feature of our cultural goods trade data is the asymmetric trading flows between a pair of countries. In this regard, substantial differences are often found between the exports of country i to country j (y_{ji}) and those of j to i (y_{ij}). The gravity model of trade claims that the cause of such asymmetry is the difference between the economic conditions of two countries. We confirm the degree of trade asymmetry and the corresponding difference in gross domestic product per capita (GDPPC) between trading pairs. There are two types of asymmetric trade: (a) one country exports to another but the latter does not export to the former, and (b) two countries export to each other but the volume is asymmetric. Approximately 36% of in-trade pairs correspond to (a). The average value of trade asymmetry, $y_{ij} - y_{ji}$, is approximately 1,800. Moreover, pairs of economically large countries are not seen in the case of (a). With regard to (b), we prepare lists of highly asymmetric and symmetric pairs in Table 2 and Table 3, respectively. The column "Asymmetry" represents the value of $y_{ji} - y_{ij}$ and "Asymratio" represents y_{ij}/y_{ji}. We fix larger values to y_{ji}. The columns "GDPPC1" and "GDPPC2" represent the average values of GDPPC from 2000 to 2006 [4] in countries 1 and 2, respectively. The "GDPPC_Ratio" is the value of GDPPC1/GDPPC2.

We arrange the absolute values of "Asymmetry" in decreasing order; the top 30 pairs are shown in Table 2. With the exception of China, the Czech Republic, and Poland, Table 2 lists countries that have large GDPPC values. We expect that the values of "Asymmetry" become greater when both countries have larger GDPPC values. We also check the values of "Asymratio" to avoid the effect of trade volume size on "Asymmetry." With regard to "Asymratio," more than half the pairs show five to ten times larger trade asymmetry. However, the values of "GDPPC_Ratio" are not too asymmetric compared with the values of "Asymratio." In particular, UK–Netherlands and France–Germany have remarkable trade asymmetry, although they have similar GDPPC values. Symmetric pairs (where the value of "Asymratio" is close to 1) are listed in Table 3. With regard to "GDPPC_Ratio" in Table 3, no trend is evident; moreover, the pairs are highly asymmetric in "GDPPC_Ratio," although their trade volumes are symmetric. In the usual gravity model setting, trade asymmetry is ex-

[3] The data is from the United Nations Commodity Trade Statistics Database (*http://comtrade.un.org/*).

[4] Source: International Monetary Fund (*http://www.imf.org/external/index.htm*)

Table 1 Country List

Afghanistan	Cyprus	Kyrgyzstan	Rwanda
Albania	Czech Rep.	Lao People's Dem. Rep.	Saint Lucia
Algeria	Dem. People's Rep. of Korea	Latvia	Saint Vincent and the Grenadines
Andorra	Democratic Republic of the Congo	Lebanon	Samoa
Angola	Denmark	Lesotho	San Marino
Antigua and Barbuda	Djibouti	Liberia	Sao Tome and Principe
Argentina	Dominica	Libya	Saudi Arabia
Armenia	Dominican Rep.	Lithuania	Senegal
Aruba	Ecuador	Luxembourg	Seychelles
Australia	Egypt	Madagascar	Sierra Leone
Austria	El Salvador	Malawi	Singapore
Azerbaijan	Estonia	Malaysia	Slovakia
Bahamas	Ethiopia	Maldives	Slovenia
Bahrain	Faeroe Isds	Mali	Solomon Isds
Bangladesh	Fiji	Malta	Somalia
Barbados	Finland	Mauritania	South Africa
Belarus	France	Mauritius	Spain
Belgium	French Polynesia	Mexico	Sri Lanka
Belize	Gabon	Mongolia	Suriname
Benin	Gambia	Morocco	Swaziland
Bermuda	Georgia	Mozambique	Sweden
Bhutan	Germany	Myanmar	Switzerland
Bolivia	Ghana	N. Mariana Isds	Syria
Bosnia Herzegovina	Greece	Namibia	TFYR of Macedonia
Botswana	Greenland	Nauru	Thailand
Br. Virgin Isds	Grenada	Nepal	Timor-Leste
Brazil	Guatemala	Netherlands	Togo
Brunei Darussalam	Guinea	New Caledonia	Tonga
Bulgaria	Guinea-Bissau	New Zealand	Trinidad and Tobago
Burkina Faso	Guyana	Nicaragua	Tunisia
Burundi	Haiti	Niger	Turkey
Côte d'Ivoire	Honduras	Nigeria	Tuvalu
Cambodia	Hungary	Norway	Uganda
Cameroon	Iceland	Oman	Ukraine
Canada	India	Pakistan	United Arab Emirates
Cape Verde	Indonesia	Panama	United Kingdom
Central African Rep.	Iran	Papua New Guinea	United Rep. of Tanzania
Chad	Iraq	Paraguay	Uruguay
Chile	Ireland	Peru	USA
China	Israel	Philippines	Uzbekistan
China, Hong Kong SAR	Italy	Poland	Vanuatu
China, Macao SAR	Jamaica	Portugal	Venezuela
Colombia	Japan	Qatar	Viet Nam
Congo	Jordan	Rep. of Korea	Yemen
Costa Rica	Kazakhstan	Rep. of Moldova	Zambia
Croatia	Kenya	Romania	Zimbabwe
Cuba	Kuwait	Russian Federation	

plained by the difference in the economic conditions between two countries. This difference can be captured by additive fixed effect terms. However, from Tables 2 and 3, we confirm that economically equipollent countries do not necessarily export the same amount of cultural goods. This finding implies that additive fixed effects may not fully capture unobserved heterogeneity.

One incidental advantage of our framework is the asymmetry of the effect of cultural relations on trade. For example, the effect of cultural factors on the exports of country A to country B could differ from that of B to A after controlling for each country's economic condition. Asymmetry is naturally introduced because of the multiplicative nature of the interactive term. For example, the product of the fixed effect of A as an exporter and that of B as an importer could differ from the prod-

Table 2 Asymmetric Country Pairs

Rank	Country1	Country2	Asymmetry	Asymratio	GDPPC1(US$)	GDPPC2(US$)	GDPPC_Ratio
1	United Kingdom	Netherlands	46,419,096	0.05	31,118	32,196	0.97
2	USA	Canada	40,900,000	0.24	38,637	29,011	1.33
3	France	Netherlands	27,744,740	0.10	28,449	32,196	0.88
4	France	Austria	24,425,000	0.07	28,449	31,065	0.92
5	France	Germany	21,800,000	0.60	28,449	28,915	0.98
6	Germany	Czech Rep.	21,493,880	0.19	28,915	9,187	3.15
7	Belgium	Netherlands	15,926,488	0.08	29,523	32,196	0.92
8	Italy	Germany	15,146,773	0.09	25,450	28,915	0.88
9	United Kingdom	Czech Rep.	14,688,532	0.01	31,118	9,187	3.39
10	France	Belgium	14,048,012	0.16	28,449	29,523	0.96
11	United Kingdom	Poland	13,312,494	0.03	31,118	6,209	5.01
12	Ireland	United Kingdom	10,405,041	0.11	38,545	31,118	1.24
13	United Kingdom	Germany	9,800,000	0.63	31,118	28,915	1.08
14	Switzerland	Germany	9,757,083	0.30	43,480	28,915	1.50
15	Sweden	Germany	9,264,179	0.02	33,455	28,915	1.16
16	USA	Hong Kong SAR	9,119,023	0.03	38,637	24,764	1.56
17	USA	China	8,964,447	0.01	38,637	1,361	28.40
18	Norway	Sweden	8,900,131	0.21	51,013	33,455	1.52
19	Italy	Austria	8,830,225	0.02	25,450	31,065	0.82
20	Spain	Netherlands	8,820,569	0.01	21,420	32,196	0.67
21	Spain	United Kingdom	8,456,604	0.41	21,420	31,118	0.69
22	United Kingdom	Austria	8,391,598	0.02	31,118	31,065	1.00
23	Netherlands	Austria	7,883,073	0.17	32,196	31,065	1.04
24	Germany	Austria	7,500,000	0.62	28,915	31,065	0.93
25	Portugal	Spain	7,493,972	0.09	14,581	21,420	0.68
26	Spain	Germany	6,788,284	0.23	21,420	28,915	0.74
27	Sweden	Netherlands	6,208,189	0.05	33,455	32,196	1.04
28	Germany	Netherlands	5,900,000	0.73	28,915	32,196	0.90
29	New Zealand	Australia	5,698,775	0.06	19,552	27,030	0.72
30	United Kingdom	Israel	5,364,118	0.02	31,118	18,814	1.65

uct of the fixed effect of B as an exporter and that of A as an importer. Moreover, the fixed effects of an exporting country are not always the same as those of an importing country. Asymmetry helps to explain the difference in trading volumes between two equipollent countries in terms of asymmetric unobserved heterogeneous relations such as cultural relations. Most pairs of countries in our data set show asymmetric trade flows. For example, the exports from country A to country B are much larger (smaller) than those from B to A. Institutional differences such as tariff and copyright systems (e.g., Towse (2017)) account for some of the asymmetry. However, such differences are not always observable, at least in a unified way. We expect that the product forms of importer- and exporter-specific fixed effects can capture unobservable heterogeneities in the relations between two countries.

Although such asymmetry in trade is frequently observed, most studies on the cultural effects of trade do not consider it because of a common limitation: Except for a few studies (e.g., Disdier et al. (2010); Felbermayr and Toubal (2010); Shin and McKenzie (2019)), the "cultural distance" from A to B is usually treated in the same way as that from B to A and is always symmetrically measured. Thus, using regression analysis, the marginal effect of the cultural variable on the exports from country A to country B is the same as that on exports from B to A.

Given this gap, we generalize the estimation method by using a factor structure in the form of interactive fixed effects. This structure can flexibly capture the asym-

Table 3 Symmetric Country Pairs

Rank	Country1	Country2	Asymmetry	Asymratio	GDPPC1(US$)	GDPPC2(US$)	GDPPC_Ratio
1	Switzerland	Oman	0	1.00	43,480	10,636	0.24
1	Ireland	Cameroon	0	1.00	38,545	800	48.19
1	Finland	Azerbaijan	0	1.00	31,227	1,147	27.22
4	Ireland	Spain	595	1.00	38,545	21,420	1.80
5	Barbados	Canada	0	0.99	10,290	29,011	0.35
6	USA	Bulgaria	23	0.99	38,637	2,585	14.95
7	USA	Grenada	2	0.99	38,637	4,290	9.01
8	India	France	446	0.99	575	28,449	0.02
9	Israel	Greece	193	0.99	18,814	15,815	1.19
10	Guatemala	Italy	0	0.99	1,868	25,450	0.07
11	Kazakhstan	Belgium	0	0.98	2,533	29,523	0.09
12	France	Hong Kong SAR	3,897	0.98	28,449	24,764	1.15
13	Greece	Hungary	13	0.98	15,815	7,920	2.00
14	Spain	Croatia	28	0.98	21,420	6,675	3.21
15	United Arab Emirates	New Zealand	11	0.98	24,266	19,552	1.24
16	Argentina	South Africa	5	0.98	5,150	3,831	1.34
17	USA	France	34,212	0.97	38,637	28,449	1.36
18	Ethiopia	Ghana	0	0.97	134	390	0.34
19	USA	Sweden	8,808	0.97	38,637	33,455	1.15
20	Ecuador	Panama	59	0.96	2,222	4,351	0.51
21	Poland	Australia	23	0.96	6,209	27,030	0.23
22	Lithuania	Greece	2	0.96	5,507	15,815	0.35
23	Bosnia Herzegovina	Australia	2	0.95	1,879	27,030	0.07
24	Finland	Norway	1,404	0.95	31,227	51,013	0.61
25	Germany	Maldives	2	0.95	28,915	2,309	12.52
26	United Kingdom	Turkey	4,269	0.95	31,118	3,687	8.44
27	Spain	Canada	1,313	0.94	21,420	29,011	0.74
28	Georgia	Finland	6	0.94	1,069	31,227	0.03
29	Denmark	Brazil	22	0.94	39,521	3,529	11.20
30	Australia	Italy	1,950	0.94	27,030	25,450	1.06

metric, unobserved heterogeneous relations between trading countries. Then, we can interpret the results by referring to various factors, including cultural variables.

3 Estimation Model

In the current study, we introduce an estimation model of trade flows between a pair of countries that incorporates the asymmetric effects of unobserved relations. The model is composed of generalized empirical gravity equations of bilateral trade and is applied to a situation with frequent zero trade flows. Our estimated model of trade flows from an exporting country j to an importing country i is as follows:

$$d_{ij}^{*} = \underbrace{\mathbf{z}_{ij}'\delta}_{\text{observed heterogeneity}} + \underbrace{\alpha_{d,i} + \gamma_{d,j} + \mathscr{A}_{d,i}'\mathscr{G}_{d,j}}_{\text{unobserved heterogeneity}} + v_{ij} \tag{1}$$

$$y_{ij}^{*} = \underbrace{\mathbf{x}_{ij}'\beta}_{\text{observed heterogeneity}} + \underbrace{\alpha_{y,i} + \gamma_{y,j} + \mathscr{A}_{y,i}'\mathscr{G}_{y,j}}_{\text{unobserved heterogeneity}} + u_{ij} \tag{2}$$

$$y_{ij} = y_{ij}^{*} \quad \text{if } d_{ij} \equiv \mathbb{1}\{d_{ij}^{*} \geq 0\} = 1. \tag{3}$$

The first equation considers the extensive margin (the decision of country j to export to country i); the second considers the intensive margin (the export volume from country j to country i given the export decision)[5]. The standard sample selection model is extended to one with additive and interactive fixed effects, where $\mathbf{x}_{ij}$ and $\mathbf{z}_{ij}$ are $K_y \times 1$ and $K_d \times 1$ vectors of observable explanatory variables, respectively, and β and δ are their coefficient vectors. $\alpha_{y,i}$ and $\alpha_{d,i}$ ($\gamma_{y,j}$ and $\gamma_{d,j}$) are additive fixed effects of importer i (exporter j) in the outcome equation (the volume-of-export equation, (2)) and the selection equation (the decision-of-export equation, (1)). Further, $\mathscr{A}_{y,i}$ and $\mathscr{G}_{y,j}$ ($\mathscr{A}_{d,i}$ and $\mathscr{G}_{d,j}$) are $R \times 1$ vectors included in the outcome (the selection) equation in their multiplicative forms:

$$\underbrace{\mathscr{A}_{y,i}'\mathscr{G}_{y,j}}_{(1\times R)(R\times 1)} = \sum_{r=1}^{R} \underbrace{\mathscr{A}_{y,i,r} \times \mathscr{G}_{y,j,r}}_{1\times 1}$$

where $\mathscr{A}_{y,i,r}$ ($\mathscr{G}_{y,j,r}$) is the r-th unobserved factor of the importer (exporter) in the outcome equation. We temporarily assume that the number of included factors, R, is known. We discuss the estimation from our data set in Section 4. Equation (3) is the observational rule of the volume of export if the countries are engaged in exporting. We assume that the joint distribution of the error terms is the bivariate normal[6] to implement the inference of the system (1), (2), and (3) :

$$\begin{pmatrix} u_{ij} \\ v_{ij} \end{pmatrix} \sim N(\mathbf{0}, \Sigma), \;\; \Sigma = \begin{pmatrix} \sigma^2 & \rho \cdot \sigma \\ \rho \cdot \sigma & 1 \end{pmatrix}. \tag{4}$$

The most distinctive feature of our model is the inclusion of the interactive fixed effects in addition to the usual additive fixed effects. The additive fixed effects incorporate country-specific heterogeneities such as economic and demographic characteristics (an explanation based on a theoretical model is given in Helpman et al. (2008), in particular, in equation (9)), whereas the interactive fixed effects are multiples of importer- and exporter-specific heterogeneity between two countries. Aside from the effects of the observed relations ($\mathbf{x}_{ij}$ or $\mathbf{z}_{ij}$) between the importer and the exporter and their unobserved but country-specific additive terms (α_i or γ_j), the interactive terms ($\mathscr{A}_i'\mathscr{G}_j$) capture the effects of unobserved interactions between them on the dependent variable, which, in our empirical context, is an important determinant. An example of an important but unobserved interaction in cultural goods trade is cultural relations, which is the main focus of this study.

There are several advantages to using interactive fixed effect terms. First, the interactive term is generically asymmetric because the product, $\mathscr{A}_{y,i,r} \times \mathscr{G}_{y,j,r}$, usually differs from $\mathscr{A}_{y,j,r} \times \mathscr{G}_{y,i,r}$: The fixed effect of country i as an importer, $\mathscr{A}_{y,i,r}$, is not restricted to be the same as the fixed effect of the country as an exporter, $\mathscr{G}_{y,i,r}$. Second, the degree of asymmetry (the difference between $\mathscr{A}_{y,i,r} \times \mathscr{G}_{y,j,r}$ and $\mathscr{A}_{y,j,r} \times \mathscr{G}_{y,i,r}$) can be adaptively estimated from the data. Thus, asymmetry helps to explain asymmetric trade flows, even between two equipollent countries. Third, if omitted heterogeneous

[5] We estimated the joint model to avoid selection bias. In our sample, only 18.2%(=6328/34782) country pairs are in the trade. This is a typical situation with frequent zero trade flows. If we drop country pairs not in trade from our sample or treat their trade flow as zero, the estimated parameters become biased.

[6] The normality assumption can be extended to the distribution with fat tails and/or skewness (e.g., Chen et al. (2014)) at the cost of computational simplicity.

Table 4 Definition of Explanatory Variables

Trade Volume of Music:	Amount of traded products that are coded HS8524.32, that is, discs for laser reading systems for reproducing sound only.
Language:	Binary variable that takes the value of 1 if both country i and country j use the same language, and 0 otherwise.
Religion:	Following the way in Helpman et al. (2008), Religion is calculated as follows:
$\text{Religion}_{ij} =$	(% of Protestant in country i) $\times$ (% of Protestant in country j) +(% of Catholics in country i) $\times$ (% of Catholics in country j) +(% of Muslims in country i) $\times$ (% of Muslims in country j)
Border:	Binary variable that takes the value of 1 if country i and country j are contiguous, and 0 otherwise.
Distance:	Distance between the capital cities of country i and country j (in km and expressed in logarithm).
Colony:	Binary variable that takes the value of 1 if country i and country j possess colonial ties, and 0 otherwise.
Lomax:	Binary variable that takes the value of 1 if both country i and country j are in the same cultural region according to the ethnomusicological classification based on Lomax (1959), and 0 otherwise.
Huntington:	Binary variable that takes the value of 1 if both country i and country j are in the same cultural region according to the civilization classification based on Huntington (1996), and 0 otherwise.

relations in the outcome and selection equations are driven by a small number of dominant factors, as in traditional factor analyses, the estimated number of included interactive terms also remains relatively small, which makes the interpretation of results easier. In addition, we can estimate the effects of standard explanatory variables more precisely because the effects of unobserved relations are controlled by the interactive terms. Certainly, the multiplicative form is restrictive for fully capturing double-indexed omitted factors. However, this specification enables us to make simplified inferences about the system. Moreover, the use of multiple terms gathers omitted heterogeneous relations in the model. We focus on the interpretation of interactive terms with regard to music traditions and trends. Further, in Section 5, we investigate the connections between the estimated interactive terms and the cultural relation variables used in the literature.

The explanatory variables are the *distance between two countries*, *a border sharing dummy*, *a linguistic proximity dummy*, *a past colonial relation dummy*, *religious proximity*, *a music tradition sharing dummy* and *the societal values sharing dummy*. These variables are taken from the data set of Takara (2018). The precise definitions of each explanatory variable are shown in Table 4. The summary statistics of all observations and in-trade observations are shown in Table 5.

Table 5 Summary Statistics

	All				In Trade			
	Mean	Std.	Min	Max	Mean	Std.	Min	Max
Trade Volume(Log)	-	-	-	-	5.520	3.938	-1.946	17.819
Language	0.151	0.358	0	1	0.177	0.381	0	1
Religion	0.139	0.223	0	1	0.151	0.237	0	1
Border	0.015	0.123	0	1	0.045	0.208	0	1
Distance(Log)	8.772	0.778	2.349	9.899	8.412	0.978	4.087	9.891
Colony	0.010	0.104	0	1	0.044	0.205	0	1
Lomax	0.227	0.419	0	1	0.408	0.491	0	1
Huntington	0.152	0.359	0	1	0.272	0.445	0	1

Note: The number of observations is 34,782, and 6,328 trading country pairs.

4 Estimation Results

In this section, we discuss the estimation results for the number of included factors, the structural parameters, and the additive and interactive fixed effect parameters. We confirm the implications of the estimates referring to Helpman et al. (2008) and Takara (2018). The differences between our implications and the implications of these two studies are because of differences in model specification. The models of Helpman et al. (2008) and Takara (2018) adopt the control function approach, which accounts for selective exporting behavior based on exporters' productivity of music CDs that are in high demand. The models include only the importer- and exporter-specific additive fixed effects to mitigate the effects of unobserved heterogeneities. However, we control for the selectivity from zero trade flows in accordance with the standard sample selection model (Heckman (1979)) and introduce the importer- and exporter-specific additive and interactive fixed effects to alleviate unobserved heterogeneous relations between countries.

4.1 Number of Factors R

First, we discuss the estimation of the number of factors, R, included in the model. There are several model selection criteria, such as information criteria (e.g., Bai and Ng (2002), Choi and Jeong (2018)) and the eigenvalue-based test (e.g., Ahn and Horenstein (2013)) for the standard factor model of large dimensions (e.g., Bai and Wang (2016)). However, these methods are only applicable to pure factor structures without regressors and without selectivity (e.g., Lu and Su (2016)). Thus, we use a traditional model selection procedure in the factor analyses: the contribution ratio. We calculate the eigenvalues of cross-product matrices of residuals to account for the presence of regressors in the first and second steps of conditional maximization. In this regard, the eigenvalues are obtained as by-products of interactive fixed effect parameter estimations because the eigenvectors are used for the factor estimates (Bai (2009)). The contribution ratio of a factor is defined as the ratio of the corresponding eigenvalue to the sum of all eigenvalues. We select the minimum number of factors to establish a cumulative contribution ratio. Table 6 shows the contribution ratios in the outcome equation. The results of the selection equation are similar; thus, they are omitted. A candidate for the estimated number of included factors is at least four ($R = 4$). When the included number is less than four, the cumulative contribution ratios up to R rapidly increase.

Table 6 Contribution Ratio of Each Factor in the Outcome Equation

	R=1	R=2	R=3	R=4	R=5	R=6	R=7	R=8	R=9	R=10
up to R	0.61	0.83	0.90	0.94	0.94	0.95	0.96	0.96	0.97	0.98
1st	0.61	0.50	0.32	0.28	0.23	0.24	0.23	0.23	0.22	0.21
2nd	0.02	0.33	0.31	0.24	0.22	0.17	0.15	0.14	0.13	0.12
3rd	0.02	0.01	0.26	0.22	0.17	0.15	0.14	0.12	0.12	0.11
4th	0.02	0.01	0.01	0.19	0.16	0.14	0.12	0.11	0.11	0.10
5th	0.01	0.01	0.00	0.00	0.15	0.13	0.12	0.10	0.09	0.09
6th	0.01	0.01	0.00	0.00	0.00	0.12	0.11	0.09	0.08	0.08
7th	0.01	0.01	0.00	0.00	0.00	0.00	0.09	0.09	0.08	0.07
8th	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.08	0.08	0.07
9th	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.07
10th	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
others	0.26	0.12	0.08	0.05	0.05	0.04	0.04	0.03	0.03	0.02

We examine to what extent asymmetries in trade flows are eliminated. The asymmetry in trade flows is defined in the following way: First, we define the observed asymmetry in trade flows

$$Asym_{ij} = Y_{ij} - Y_{ji}, i, j = 1, 2, \cdots, N, j \neq i, \tag{5}$$

where Y_{ij} is a level value of the export from country i to country j. Second, the predicted asymmetry is defined as follows,

$$\hat{Asym_{ij}}^{(R)} = \hat{Y}_{ij}^{(R)} - \hat{Y}_{ji}^{(R)}, R = 0, 1, \cdots, 10, \tag{6}$$

where R is the dimension of the interactive fixed effects. Finally, for each R, we define the measure of eliminated asymmetries (MEA) in trade flows as,

$$MEA^{(R)} = \ln \sum_i \sum_j (Asym_{ij} - \hat{Asym_{ij}}^{(R)})^2. \tag{7}$$

Note that the predicted values of the dependent variables from non-trading pairs are assigned a value of zero when computing the asymmetric measure, $\hat{Asym_{ij}}^{(R)}$, since either Y_{ij} or Y_{ji} is not observed. Therefore, a smaller (7) is evidence of superior goodness-of-fit from the estimated model.

Table7 summarizes the estimated (7) for each R in our sample. Compared with the result with $R = 0$ (only additive fixed effects are included), the result with $R = 3$ (interactive fixed effects are also included) indicates thorough elimination of asymmetries in our sample. The measure is monotonically decreasing until $R = 5$. Although we see a non-monotone decrease or increase until $R = 10$, we find a monotonic decrease thereafter. In summary, some parts of asymmetries in trade flows are well explained by introducing the interactive fixed effects into the estimated model. Table 7 indicates that the estimated model with $R = 5$ may be the best model in terms of the two aspects of goodness-of-fit and the parsimonious principle: the explanatory power of asymmetric trade flows (see Table 7) and of the interactive fixed effect terms represented by the cumulative contribution ratio of factors (see Table 6).

Table 7 The Measure of Eliminated Asymmetries

R=0	R=1	R=2	R=3	R=4	R=5
19.09	19.75	19.14	18.91	18.65	18.58
R=6	R=7	R=8	R=9	R=10	
18.88	18.66	18.49	18.63	18.58	

4.2 Parameter Estimates

Next, we discuss the parameter estimates of the sample selection model. The results are presented in Tables 8 and 9. In gravity models, the total trade between two countries becomes more active in a way that is proportional to the product of the trading countries' economic scale [7]. However, trading costs such as transportation costs, business administration costs, and other frictions may reduce opportunities for international trade as well as the volumes traded. The observed explanatory variables included in our estimation, *sharing a common language* (Language), *distance between two countries* (Distance), *sharing borders* (Border), *past colonial relations* (Colony), *religious proximity* (Religion), *music tradition sharing based on Lomax (1959)* (Lomax), and *societal value sharing based on Huntington (1996)* (Huntington) act as trade friction proxy variables (trading costs) and indicate cultural affinity between the exporter and the importer.

To estimate the joint model, we need to exclude at least one variable from outcome equation. In general, the variable which appears in selection equation and is not contained in outcome equation (such as fixed cost) are excluded. Following Takara (2018), we employ the exclusion restriction of the cultural proximity variable by Lomax (1959) from the outcome equation. The variable shows that some music traditions are shared by two countries, and tradition is a proxy variable of their cultural proximity in music. In general, it is costly to access unfamiliar music, and non-proximity often acts as a fixed cost when becoming acquainted with such music. The fixed costs due to unfamiliarity have substantial influences on the establishment of new trade relations. However, once such relations are built, these fixed costs hardly affect the volume of trade. For these reasons, we exclude the cultural proximity variable Lomax from the outcome equation (see also Takara (2018)).

[7] Theoretically, economic scale is explained by additive fixed effect terms. In the Appendix, we confirm that the estimated values of additive terms are proportional to the economic scale.

The sign of the coefficient estimate of *distance between two countries* is expected to be negative when it is regarded as a proxy variable for transportation costs. The sign is actually negative in both Tables 8 and 9: Countries that are more distant are less involved in trade relations and export less. Compared with those in Takara (2018) (-0.698 in the selection equation, -1.167 in the outcome equation), the estimated parameters in the current study are larger. If, as discussed in the prior sections, one of the interactive terms shows strong demand for the novelty of cross-cultural goods, the interactive term would be positive in the equations. Given the tendency for the distances between two countries of different cultural backgrounds to be greater, the coefficient estimation of transportation costs is partially offset by the reverse effects of demand for distant, cross-cultural goods if the model does not include the interactive fixed effect terms. Note that the coefficient of “Distance” in Takara (2018) may be estimated with bias toward zero. Thus, we have more precise estimates of the effect of transportation costs on trade by including the interactive fixed effect terms.

Similarly, the tendency of cross-cultural novelty is captured by one of the interactive fixed effect terms. We also find that the coefficient estimates of “Border” have smaller values in the outcome equation and those of “Language,” “Colony,” and “Religion” have larger values in both equations. The dummy variable “Border” accounts for two countries sharing the same national border. With regard to the economizing indicator of trade costs, such as transportation costs, the coefficient is expected to be positive in the selection and outcome equations. However, this expectation implies that countries often share a border and various aspects of their cultural background. Thus, the effect of being neighbors has negative effects on cross-culture-oriented trade. Using the estimation model without accounting for cross-cultural preferences provides an underestimated coefficient of the variable “Border” in the selection equation. However, the effect is not large enough to increase the probability of entering a new cross-cultural trade relation (see Table 8).

The variables “Religion,” “Language,” and “Colony” reflect aspects of cultural backgrounds common to trading countries. While using a common language reduces trading costs by facilitating direct communication, two countries that do not share a common language and common cultural background often do not have an overlapping variety of cultural goods. With such a relation, the volume of trade in cultural goods increases based on strong cross-cultural factors. Thus, “Language” has negative and positive effects on trade. Tables 8 and 9 show that the inclusion of interactive fixed effect terms gives positive values for the coefficients of sharing a common language and past colonial relations, which are offset by the effects of preferences for cross-cultural goods. The variable “Religion” also has larger, positive effects on trading volume but insignificant effects on trade probabilities after accounting for cross-cultural engagement in accordance with the interactive terms. Based on our findings, we conclude that interactive fixed effect terms separate the effects of cross-cultural demand from trade frictions or trade costs for which the explanatory variables in gravity models act as proxies.

Table 8 Estimation Result: Selection Equation (1)

	The number of factors					
Variables	R=1	R=2	R=3	R=4	R=5	Takara (2018)
Language	0.551	0.561	0.567	0.541	0.660	0.491
	(0.063)	(0.076)	(0.092)	(0.116)	(0.147)	(0.046)
Religion	-0.124	-0.258	-0.208	-0.404	-0.378	-0.112
	(0.107)	(0.138)	(0.180)	(0.204)	(0.267)	(0.079)
Border	0.094	0.062	0.095	0.092	0.137	0.100
	(0.116)	(0.137)	(0.167)	(0.194)	(0.259)	(0.103)
Distance	-0.695	-0.757	-0.821	-0.831	-0.935	-0.698
	(0.032)	(0.038)	(0.043)	(0.052)	(0.054)	(0.026)
Colony	0.705	0.884	1.037	1.036	1.127	0.635
	(0.147)	(0.192)	(0.237)	(0.308)	(0.507)	(0.100)
Lomax	0.214	0.194	0.293	0.295	0.325	0.202
	(0.051)	(0.060)	(0.076)	(0.090)	(0.113)	(0.039)
Huntington	0.269	0.327	0.318	0.306	0.407	0.310
	(0.063)	(0.077)	(0.096)	(0.115)	(0.141)	(0.047)

Note: Standard errors are in parentheses.

Table 9 Estimation Result: Outcome Equation (2)

	The number of factors					
Variables	R=1	R=2	R=3	R=4	R=5	Takara (2018)
Language	0.973	1.000	1.048	0.941	0.907	0.831
	(0.091)	(0.092)	(0.091)	(0.091)	(0.093)	(0.248)
Religion	0.422	0.496	0.876	0.780	0.958	0.333
	(0.153)	(0.154)	(0.150)	(0.148)	(0.152)	(0.206)
Border	0.553	0.529	0.526	0.709	0.924	0.962
	(0.146)	(0.149)	(0.146)	(0.143)	(0.149)	(0.203)
Distance	-1.441	-1.439	-1.442	-1.290	-1.217	-1.167
	(0.037)	(0.037)	(0.037)	(0.037)	(0.037)	(0.340)
Colony	1.713	1.721	1.703	1.775	1.822	0.976
	(0.166)	(0.162)	(0.157)	(0.153)	(0.147)	(0.316)
Huntington	0.709	0.673	0.519	0.641	0.492	0.585
	(0.092)	(0.092)	(0.089)	(0.089)	(0.089)	(0.181)
ρ	0.177	0.155	0.208	0.204	0.275	-
	(0.035)	(0.040)	(0.058)	(0.072)	(0.103)	-
σ	2.286	2.181	2.069	1.990	1.914	-
	(0.021)	(0.021)	(0.020)	(0.020)	(0.020)	-

Note: Standard errors are in parentheses.

5 Interactive Fixed Effects

5.1 Classification of Exporter/Importer Effects

In our estimation model, we incorporate additive fixed effect terms to control for unobserved heterogeneities specific to each country. We also include interactive fixed effect terms to control for unobserved heterogeneous relations between trading countries. In this subsection, we discuss interpretations of the estimation results of the interactive fixed effect parameters with reference to relations based on music culture.

To summarize and interpret the estimates of interactive fixed effects, we use the k-means method to classify countries into several groups using estimated fixed effects from the outcome equation, $\mathscr{A}_{y,i}$ and $\mathscr{G}_{y,j}$. It is well known that classification results from naïve k-means methods are sensitive to the initial location of group centers and that the number of clusters must be known prior to the analysis. We use the "k-means++" algorithm by Arthur and Vassilvitskii (2007) to avoid sensitivity of the initial setup. For the latter, we investigate the number of clusters in an exploratory way; we took the number of clusters from $K = 3$ to $K = 8$. We found that in the cases of $K \geq 5$, the classification results with $k+1$ clusters are mostly incorporated in the result with a smaller number of clusters. Therefore, we select $K = 5$ as the minimum required number of country classifications.

The interactive fixed effect of the i-th country in the outcome equation, $\mathscr{A}_{y,i}$, is given as a row vector of $1 \times R$, $\mathscr{A}_{y,i} = \{\mathscr{A}_{y,i}^{(1)}, \mathscr{A}_{y,i}^{(2)}, \ldots, \mathscr{A}_{y,i}^{(R)}\}$. We interpret this vector as the importer effect of country i with R unobserved features as an importer. As estimates, we obtain such vectors for all countries in our sample. We multiply them with the exporter effects from all countries and obtain the interactive effects on the trade relations between the exporting and the importing country. The interactive terms give us various information on unobserved factors in trade relations: if estimated fixed effects would capture some features of cultural classifications known in the literature, or if estimated interactive terms would capture some features of relations in music culture between countries, the estimated fixed effects and the interactive effects would be interpreted as effects of music cultural relations on the trade of music goods.

In this paper, we use the country classification results from the outcome equation to interpret the estimated fixed effects, $\mathscr{A}_{y,i}$ as the importer effect of country i and $\mathscr{G}_{y,j}$ as the exporter effect of country j [8]. The same classification results are applied to the country classification of both effects in the selection equation [9].

According to the estimated importer effects, $\mathscr{A}_{y,\bullet}$, countries are classified into the following five groups by the k-means method:

- Group 1 (47 countries): countries with large domestic music markets, such as the United States, some western European countries, and Japan.

[8] Country lists that show the countries in each group are given in the Appendix.

[9] There are several ways to use the country classification results from cluster analyses: applying the k-means method to each equation and classifying countries' effects based on each result, or applying the k-means method only to the selection equation and classifying countries based on the result. However, a unified classification rule to both equations simplifies the interpretations. That is the reason we apply the country classification rules from the outcome equation to both equations.

- Group 2 (39 countries): countries in south, eastern-south, middle, and mid-east Asia.
- Group 3 (38 countries): countries in South America, African countries, some south-eastern Asian countries.
- Group 4 (29 countries): countries mainly in Africa.
- Group 5 (34 countries): countries with smaller populations or less land in various areas.

We depict the country classification result on a world map in the Figure 1. The countries classified in terms of the importer effect, $\mathscr{A}_{y,\bullet}$, are colored red for *group 1*, orange for *group 2*, yellow for *group 3*, green for *group 4*, and violet for *group 5* in Figure 1.

Similarly, according to the estimated exporter effects, $\mathscr{G}_{y,\bullet}$, countries are classified into the following five groups:

- Group a (43 countries): countries with large domestic music markets, such as the United States, some western European countries, and Japan.
- Group b (31 countries): countries in eastern-south, mid-east Asia, some African countries, Finland, and Poland.
- Group c (37 countries): countries in eastern Europe and Russia, some African countries, Columbia, Chili, and Timor.
- Group d (23 countries): countries in middle and south America, several eastern Europe countries, and North Korea.
- Group e (53 countries): countries in Africa and several American and Asian countries.

The country classification result on a world map is in the Figure 2. Regarding the exporter effect, $\mathscr{G}_{y,\bullet}$, countries are colored red for *group a*, orange for *group b*, yellow for *group c*, green for *group d*, and violet for *group e* in Figure 2.

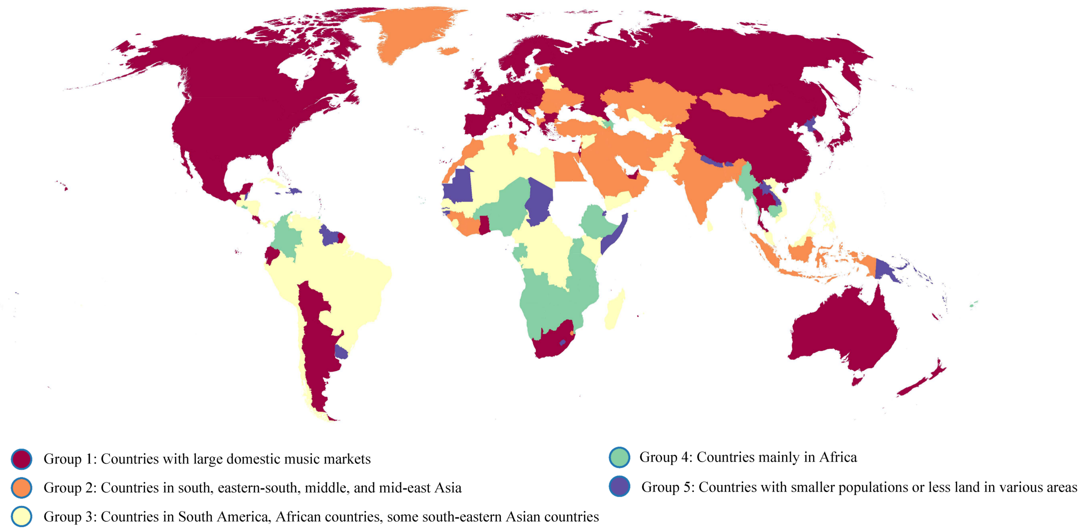

Fig. 1 Importing Groups on the World Map ($k = 5$)

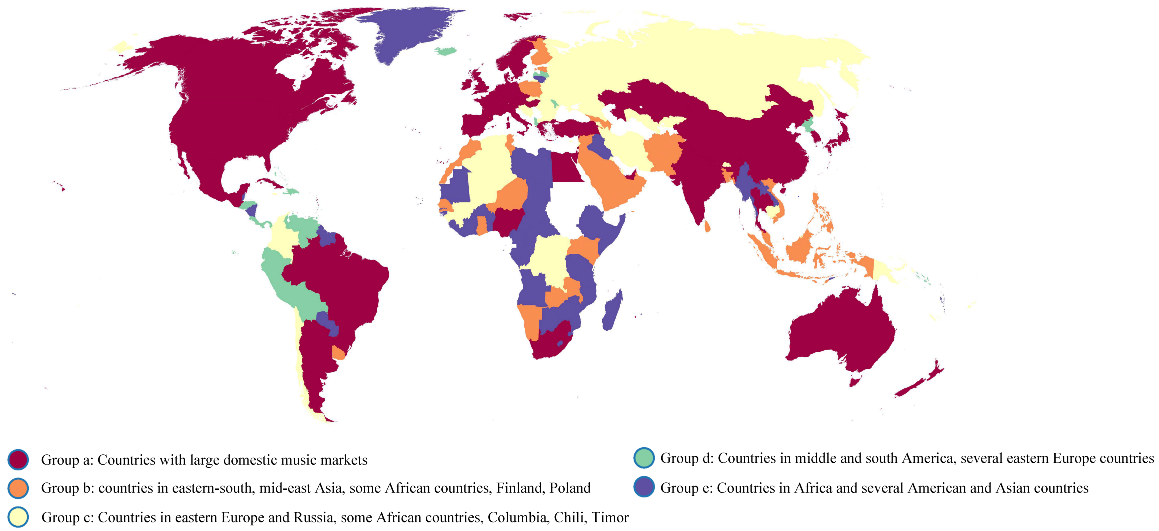

Fig. 2 Exporting Groups on the World Map ($k = 5$)

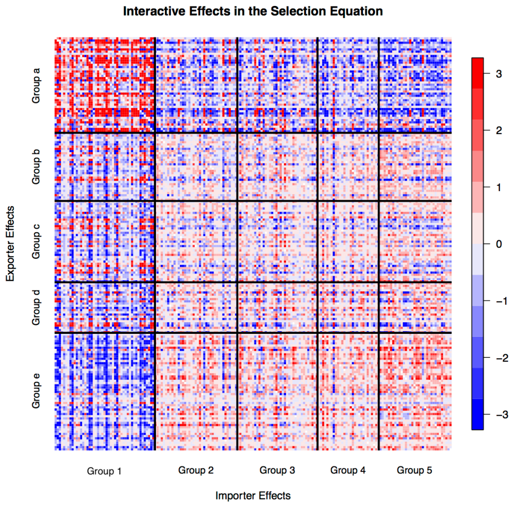

Fig. 3 Heatmap of $\mathscr{A}_{d,i}'\mathscr{G}_{d,j}$

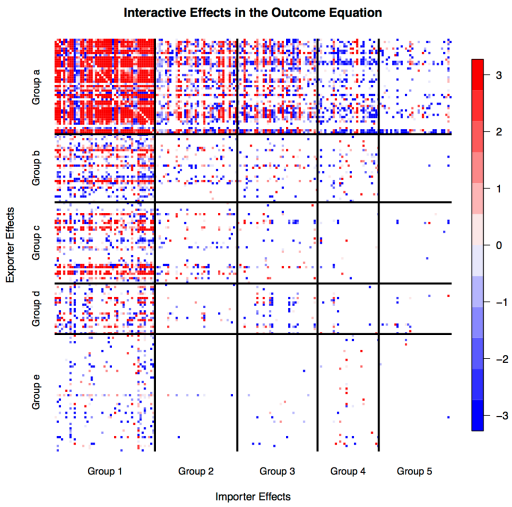

Fig. 4 Heatmap of $\mathscr{A}_{y,i}'\mathscr{G}_{y,j}$

In Figures 3 and 4, we indicate the values of the interactive terms. The vertical line shows exporting countries, and the horizontal line shows importing countries. Each cell shows the value of the interactive terms; for example, Figure 3, $(1,1)$ shows the value of $\mathscr{A}'_{d,1}\mathscr{G}_{d,1}$. The values of each $\mathscr{A}'_{d,i}\mathscr{G}_{d,j}$ and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ are depicted in Figure 3 and 4 as heat maps. Positive values of $\mathscr{A}'_{d,i}\mathscr{G}_{d,j}$, and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ are represented by red, and negative values are represented by blue [10]. From Figure 3, 4, we can confirm that in most of the active paths, the interactive terms have a positive value.

Then, we examine the features of each country group. First, we focus on Group 1 for the importer effect and Group a for the exporter effect. Those countries import 114,863 CDs and export 125,453 CDs on average per year, much more than other groups of countries. In Figure5, we depict a choropleth map that visualizes the domestic music market size in 2004. Darker red indicates that the country has larger music market. As shown in Figure5, countries classified into Group 1 and Group a also have larger domestic music markets than others. The ratio of active trade paths is 87%, which implies most traded paths are active between Group 1 and Group a. Most influential countries in the international music market, such as the United Kingdom, France, Germany, and the United States, are included in these categories. Therefore, we identify these countries as major music powers (MMP). In terms of traditional music cultural background, as explained by Lomax (1959), most MMP countries have been classified mainly into "Eurasian," "Modern European," and "Old European" cultural groups.

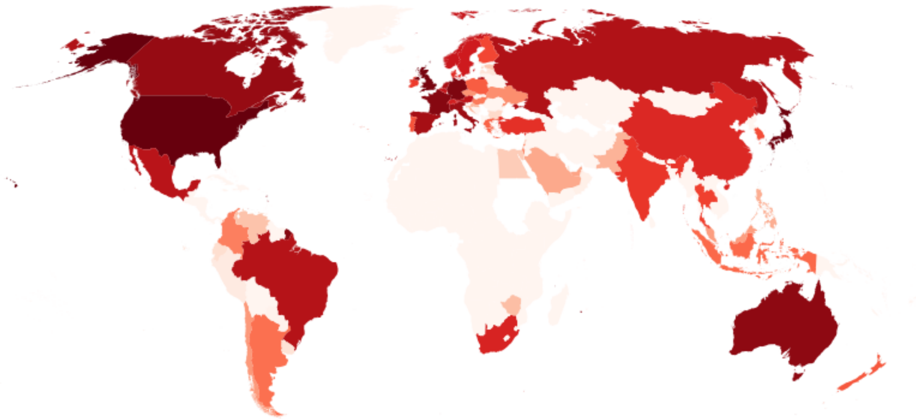

Fig. 5 Domestic Music Market Size in 2004

Next, we focus on Groups 2 and 3 for the importer effect and Groups b and c for the exporter effect. In terms of trading volume, the domestic music markets and trade activities are second to MMP: Group 2 (3) imports 2,307 (1,811) CDs on average per year while Group b (c) exports 4,087 (1,454) CDs on average per year.

[10] In Figure 4, we treat values as zero if the trade path is not active.

The groups' domestic music markets are relatively large (see Figure5) and the ratio of active trade paths is close to that of MMP. We call the country group secondary music majors (SMM). Based on Lomax (1959), SMM are classified as "Eurasian," "Modern European," "Old European," and "African."

Finally, the remaining countries are the importer groups 4 and 5, and the exporter groups d and e. Both trade frequencies and trade volumes are less active: the average import is 808 (Group 4) and 231 (Group 5) CDs per year, and the average export is 302 (Group d) and 127 (Group e) CDs per year. Their domestic music markets are relatively small compared with those of other groups (see Figure5). These groups are called "small music countries" (SMCs) hereafter. Most countries in these categories are included in "African" and "American Indian" in the classification of Lomax (1959).

To summarize, all countries are classified into three clusters—MMP, SMM, and SMC—based on estimated unobserved heterogeneous relations between trading countries. The same tendency is confirmed when we change the number of clusters, K. Therefore, we focus on these three clusters in the latter part of this paper.

5.2 Interpretation of Interactive Fixed Effects

We discuss the relation between each cluster using the values of $\mathscr{A}_{d,i}'\mathscr{G}_{d,j}$ and $\mathscr{A}_{y,i}'\mathscr{G}_{y,j}$. Table10 summarizes the mean, standard deviation, and other summary statistics of $\mathscr{A}_{d,i}'\mathscr{G}_{d,j}$ and $\mathscr{A}_{y,i}'\mathscr{G}_{y,j}$. The columns "Importer" and "Exporter" show the pairs of importing and exporting country clusters. The column "Active" denotes the number of country pairs in trade. "All" is the total number of country pairs.

Table 10 Mean and Std. of $\mathscr{A}_{d,i}'\mathscr{G}_{d,j}$, $\mathscr{A}_{y,i}'\mathscr{G}_{y,j}$ and Trade Volume

		$\mathscr{A}_{d,i}'\mathscr{G}_{d,j}$		$\mathscr{A}_{y,i}'\mathscr{G}_{y,j}$		Path		Trade
Importer	**Exporter**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Active**	**All**	**Mean(log)**
MMP	**MMP**	2.411	4.151	4.427	1.858	1,731 (87%)	1,989	13.102
MMP	**SMM**	-0.299	2.209	-0.218	2.590	1,164 (37%)	3,186	9.118
MMP	**SMC**	-1.016	1.650	-2.339	2.705	525 (15%)	3,567	5.544
SMM	**MMP**	-0.218	2.459	-0.751	2.206	1,600 (48%)	3,303	8.891
SMM	**SMM**	-0.011	1.021	-1.253	4.103	384 (7%)	5,193	6.668
SMM	**SMC**	0.124	1.051	1.546	4.209	119 (2%)	5,826	5.513
SMC	**MMP**	-1.425	2.183	-2.418	2.826	628 (23%)	2,706	7.558
SMC	**SMM**	0.222	1.104	1.683	4.858	102 (2%)	4,269	4.644
SMC	**SMC**	0.576	0.998	-0.117	4.435	75 (2%)	4,743	3.684

First, we focus on the case where MMP exports to MMP. In this case, the average values of $\mathscr{A}_{d,i}'\mathscr{G}_{d,j}$(2.411) and $\mathscr{A}_{y,i}'\mathscr{G}_{y,j}$(4.427) are high. In addition, the "Active Ratio" is also high (87%), and the average value of music traded is the largest (13.102). This result shows that there are unobserved positive relations between MMP countries. Other things being equal, MMP countries build positive trade relations with MMP countries and trade more music.

Second, we focus on the case where SMM exports to SMM. In this case, the average value of $\mathscr{A}'_{d,i}\mathscr{G}_{d,j}$ is nearly zero. Moreover, $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ tends to have negative values. This result implies that SMM countries, nevertheless, have a large domestic music market and a negative attitude toward music imports from SMM countries, other things being equal. The imported volume of music to SMM countries is less than the level explained by factors in the gravity model. We consider that because SMM countries have a large domestic music market, they may actively meet the music demand of consumers through domestic goods rather than imported goods. This negative attitude is closely related to the discussion on home consumption bias in Ferreira and Waldfogel (2013).

Third, we focus on the case where SMC exports to SMC. In this case, the average value of $\mathscr{A}'_{d,i}\mathscr{G}_{d,j}$ is positive, but the average value of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ is negative. We consider this result to show that although there are relatively positive relations between SMC countries, they do not import music from SMC countries because of the existence of home consumption bias.

Then, we focus on trade between each cluster. For the case of MMP countries exporting to SMM and SMC, the average values of $\mathscr{A}'_{d,i}\mathscr{G}_{d,j}$ and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ are negative. This result implies that the relatively higher values of the active ratio and average volume of traded music could be explained by the effect of observed factors such as distance between two countries, language proximity, and unobserved economic status including GDP and population. After controlling for these factors, there are unobserved negative relations between MMP and SMM or SMC countries. The same negative relations can be found in the case of SMM or SMC countries exporting to MMP countries.

On the other hand, the average values of $\mathscr{A}'_{d,i}\mathscr{G}_{d,j}$ and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ are positive when SMM countries export to SMC countries and vice versa. SMM and SMC do not trade a significant amount of music goods, however, there are unobserved positive relations between these two clusters. We interpret such positive effects as a result of the positive effect of cultural proximity and/or the novelty of music goods. We discuss this point in the next subsection.

5.3 Musical Classification and Interactive Fixed Effect Parameters

We determine what effect the estimated interactive terms capture, referring to the traditional cultural classification of music based on Lomax (1959). Takara (2018) shows that cultural proximity in traditional music has a positive effect on music trade. However, there could exist a negative effect of cultural proximity such as demand for culturally novel goods or home consumption bias. Moreover, in some trade paths between MMP countries, there exist cases where two countries trade more although they do not share the traditional music culture. To distinguish such possible cases, we focus on the traditional classification and the signs of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ simultaneously. Specifically, we divide all paths into four cases based on whether $Lomax = 1$ (two countries share music tradition) and whether the sign of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ is positive. Table11 shows the noteworthy country pairs for possible four cases. In each case, country pairs are arranged in decreasing order using the absolute values of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$.

Table 11 Traditional Cultural Proximity and Unobserved Factors

		The sign of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ is positive or not.							
		Positive (sorted in decreasing order)				Negative (sorted in increasing order)			
Share the traditional music culture or not.	Share (Lomax=1)	Exporter	Importer	$\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$	Trade	Exporter	Importer	$\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$	Trade
		China, Hong Kong SAR	Singapore	9.197	672,278	Bahamas	Trinidad and Tobago	-12.745	0.143
		Austria	Sweden	8.989	3,584,074	El Salvador	Cuba	-12.575	0.143
		Bulgaria	Czech Rep.	8.833	226,050	Ecuador	Dominican Rep.	-12.034	0.143
		Netherlands	Sweden	8.655	6,516,654	Botswana	Malawi	-11.719	0.429
		Austria	United Kingdom	8.636	8,560,628	Br. Virgin Isds	Guyana	-11.630	0.143
		Czech Rep.	Netherlands	8.539	5,140,373	Poland	Belarus	-10.928	0.571
		Austria	Portugal	8.508	1,254,995	Somalia	Ethiopia	-10.894	0.429
		Brazil	Mexico	8.472	967,803	Costa Rica	Dominican Rep.	-10.839	61.857
		Czech Rep.	United Kingdom	8.396	14,900,000	Bosnia Herzegovina	Albania	-10.768	0.286
		Netherlands	Portugal	8.395	940,912	Malawi	Zimbabwe	-10.583	0.286
		Austria	Netherlands	8.380	9,445,854	Panama	Barbados	-10.537	0.857
		Denmark	Sweden	8.357	3,515,643	Ghana	Togo	-10.232	2.714
		Brazil	USA	8.307	445,015	Costa Rica	Jamaica	-10.156	63.286
		Spain	Sweden	8.180	16,442	Armenia	Georgia	-10.130	1.571
		Austria	Greece	8.137	120,186	Honduras	El Salvador	-10.128	153.714
	Do not share (Lomax=0)	Exporter	Importer	$\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$	Trade	Exporter	Importer	$\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$	Trade
		Pakistan	Netherlands	8.666	29,748	Netherlands	Saint Lucia	-9.996	0.143
		Austria	Rep. of Korea	8.620	5,014	Singapore	El Salvador	-9.317	0.143
		Austria	Mexico	8.543	8,991	Czech Rep.	Bermuda	-9.229	0.143
		Austria	South Africa	8.508	29,528	Austria	Jamaica	-9.147	0.143
		China, Hong Kong SAR	Australia	8.371	919,975	China, Hong Kong SAR	Myanmar	-8.975	0.286
		China, Hong Kong SAR	South Africa	8.317	105,795	Botswana	Switzerland	-8.924	0.571
		Austria	Japan	8.307	339,606	Germany	Nepal	-8.852	95.000
		Brazil	Netherlands	8.204	4,964	Malta	Estonia	-8.851	1.429
		Czech Rep.	Rep. of Korea	8.148	1,289	Trinidad and Tobago	Singapore	-8.814	0.286
		Netherlands	Mexico	8.120	343,109	Fiji	Tonga	-8.791	4.143
		China, Hong Kong SAR	Netherlands	8.086	251,874	Uganda	United Arab Emirates	-8.732	0.143
		Netherlands	South Africa	8.004	380,615	Albania	TFYR of Macedonia	-8.624	0.857
		Netherlands	Rep. of Korea	7.978	225,382	United Kingdom	Bhutan	-8.615	0.143
		Japan	Mexico	7.900	18,607	Costa Rica	Estonia	-8.568	0.857
		China, Hong Kong SAR	Mexico	7.855	78,562	Solomon Isds	Uruguay	-8.509	1.429

In the first case, two countries share music tradition, and the unobserved factor has a positive effect ($Lomax = 1$ and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j} > 0$). In this case, both traditional cultural proximity and unobserved factors have a positive effect on trade. These country pairs have strong relations that are not explained by traditional cultural factors and observed trade determinants. Trade paths of European countries tend to be categorized in this case. One noteworthy path in this case is Hong Kong to Singapore. In the sampling period, the ratio of the Chinese population in Singapore was 76 %[11]. Such a large number of Chinese people in the population could create strong relations between Singapore and Hong Kong, and our interactive terms could capture the effect of this relation on trade.

In the second case, two countries share music tradition, and the unobservable factor has a negative effect ($Lomax = 1$ and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j} < 0$). In this case, there is a positive effect of cultural proximity between two countries, but the trade volume is less than expected. Countries that have an adequate domestic music market could substitute the demand for foreign music by culturally resembling domestic music. If this is the case, the negative effect of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ is a result of home consumption bias.

[11] Source: Department of Statistics Singapore (*https://www.singstat.gov.sg*).

The trade paths of mid and South American countries tend to be categorized in this case. Such countries share the same music culture, but music trade is suppressed.

In the third case, two countries do not share music tradition, and the unobserved factor has a positive effect ($Lomax = 0$ and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j} > 0$). In this case, music trade is greater than expected although there is no cultural proximity in the sense of traditional music. The paths from Pakistan to the Netherlands, Austria to South Africa, and Japan to Mexico are examples with large positive values for $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$. This positive effect could be interpreted in two ways. One is that the interactive terms could capture the positive effect of demand for culturally novel goods. The other is that the interactive terms could capture the positive effect of cultural proximity, which is not in the traditional sense. In this third case, there are many MMP country trade paths. This feature implies that MMP countries construct new cultural classes that differ from the traditional classes, and our interactive terms could capture the positive effect of new cultural proximity between MMP countries on trade.

Finally, in the last case, two countries do not share music tradition, and the unobservable factor has a negative effect ($Lomax = 0$ and $\mathscr{A}'_{y,i}\mathscr{G}_{y,j} < 0$). In this case, it could be considered that $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ captures the negative effect of the absence of cultural proximity.

These results could be related to the recent problem of "cultural convergence" (Bisin and Verdier (2014)). For the case of MMP countries, we observe a type of "cultural convergence" because the values of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ tend to be positive between MMP countries. In addition, the values of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ also tend to be positive when MMP countries import music from culturally different countries in the sense of music tradition. This could be interpreted as the existence of preferences for culturally novel goods in MMP countries, and our interactive terms capture these positive effects.

On the other hand, we can observe the case that SMM and SMC countries tend to have negative values of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ even if they share the music tradition. Deloumeaux (2018) point out that for developing countries, which include several SMM and SMC countries, the limitation of human and financial capacity prevents the export of cultural goods. However, our estimation result shows that the values of $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ tend to be negative after controlling such economic conditions using additive fixed effect terms. This implies that, especially in the case of developing countries, we must consider other effects of values, cultural relations, barriers, and other unobserved factors which are captured by $\mathscr{A}'_{y,i}\mathscr{G}_{y,j}$ besides commonly used determinants of trade including economic conditions.

6 Conclusions

This study's aim is to measure the effect of unobserved cultural relations on cultural goods trade. In this regard, we introduced additive and interactive fixed effects to the standard sample selection model. Additionally, we presented an estimation procedure for the model using an expectation-conditional-maximization algorithm.

We showed that commonly observed trade asymmetries are explained by our interactive fixed effects. Some standard explanatory variables in gravity models are interpreted as proxies of trade frictions and cultural relations between two countries.

By incorporating the interactive terms, the coefficients are more precisely estimated as proxies of trade frictions because of cultural aspects, whereby the variables are incorporated into the interactive terms. The estimated values of the additive terms are also consistent with trade theory: the additive terms successfully capture economic factors such as GDP, population, and the price index.

In addition, based on the result of post-estimation analysis using multivariate analysis, the estimated values of the interactive terms are interpreted as the effect of cultural relations on music trade. We applied the k-means method to the estimated values of interactive terms and showed that countries are classified into three groups, MMP, SMP, and SMC. Comparing traditional cultural classification (Lomax (1959)) and our classification results, our interactive terms capture the positive effect of cultural proximity on trade, the negative effect on trade such as home consumption bias (Ferreira and Waldfogel (2013)), and the positive effect that could not be explained by cultural proximity defined according to traditional cultural classification.

We also found that the effect tend to be positive between MMP countries, however, tend to be negative in the case when developing countries trade music. These results imply that besides the economic factors, unobserved cultural factors including values and relations play important role when countries trade cultural goods and we must consider the effect of such unobserved factors to promote cultural goods trade.

While this study provides novel knowledge of the effect of unobserved cultural relations on trade, it suffers from limitations, which could be addressed by further research. Primarily, we created cultural maps and discussed the properties of cultural relations using these maps. However, more information on the music market at the country level is needed to investigate the maps in detail. In addition, more objective criteria for deciding the number of included factors R and the number of groups K in the k-means method is required.

Lastly, we introduce an important further research topic to be covered in cultural goods trade, preparation for digital trade data. In the cultural goods market including music, the share of digital goods and services increase rapidly. Deloumeaux (2018) point out that, for developing countries, the progress in digital goods and services platform can foster the export of their cultural goods. However, it is hard to analyze the digital market of cultural goods at the moment because of a lack of data. Therefore, preparation for data of digital goods and services market in cultural goods must be needed immediately.

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Appendix

Interpretation of Additive Fixed Effect Parameters

Base on their theoretical setting, Helpman et al. (2008) derive the empirical gravity model with exporter- and importer-specific additive fixed effects, and the economic status of each country in terms of GDP, population, price level and other (possibly unobserved) country-specific trade frictions are incorporated into this model. These factors are assumed to be country-specific and independent of trading partners. Figures 6, 7, 8 and 9 show the choropleth maps where countries are differentiated based on the additive fixed effect estimates from the outcome and the selection equation. $\alpha_{y,i}$ ($\alpha_{d,i}$) is the fixed effect parameter of the i-th as an importer in the outcome equation (the selection equation), and $\gamma_{y,j}$ ($\gamma_{d,j}$) is the fixed effect parameter of the j-th as an exporter in the outcome equation (the selection equation). Darker red colors indicate larger estimate values.

In these maps, the additive fixed estimates seem to be proportional to their economic scale. This tendency is shared by both estimates of exporter- and importer-specific fixed effect parameters in both equations. These findings are consistent with the theoretical implications concerning the additive fixed effect parameters obtained in Helpman et al. (2008). Some country-specific, partner-independent factors, such as economic scale, are important determinants but non-identifiable determinants of trade in gravity models when we only use the dyadic, cross-sectional dataset. It is true that the additive fixed effects can control for these factors, but other factors remain that are not accounted for by the explanatory variables and the additive fixed effects in the gravity equation. Then, those factors could be interpreted as unobserved, heterogeneous institutional relations or cultural relations between the importing and exporting country.

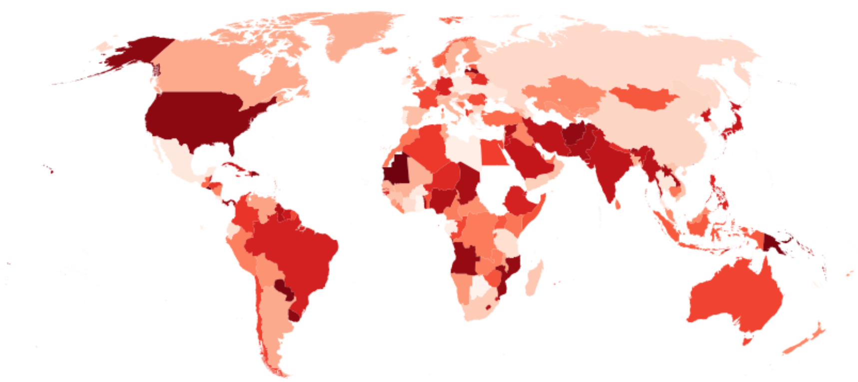

Fig. 6 Values of $\alpha_{y,i}$ on the World Map

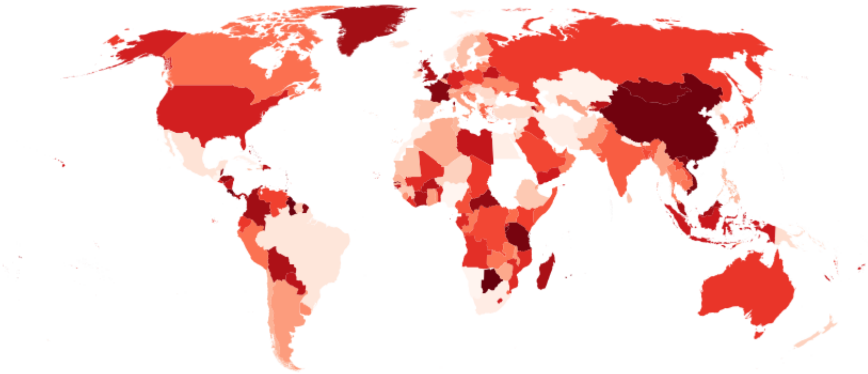

Fig. 7 Values of $\gamma_{y,j}$ on the World Map

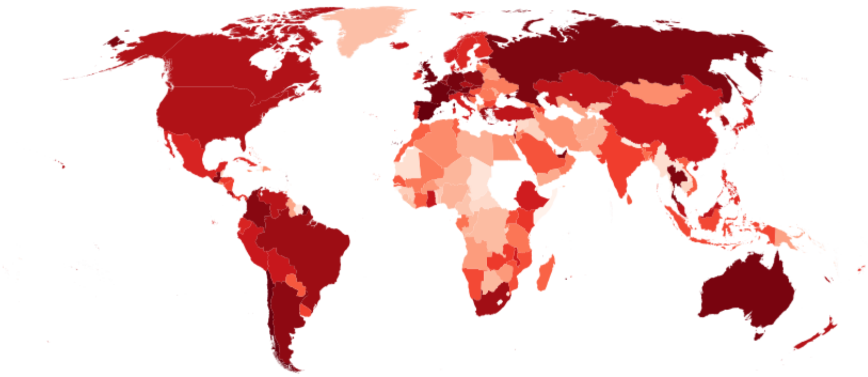

Fig. 8 Values of $\alpha_{d,i}$ on the World Map

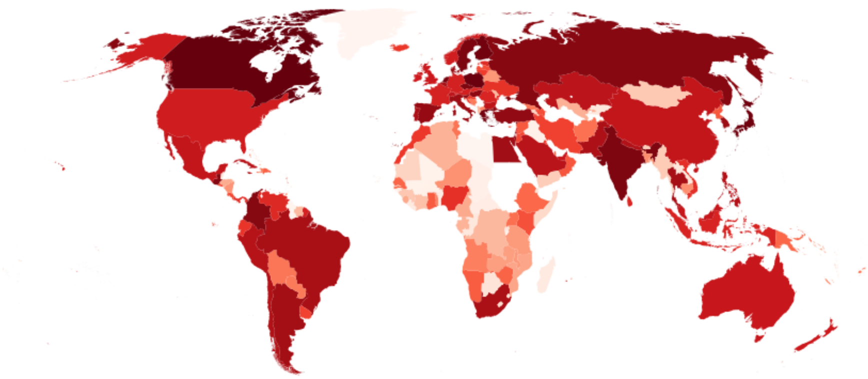

Fig. 9 Values of $\gamma_{d,j}$ on the World Map

Country List by Importing/Exporting Groups

The lists of importing/exporting groups are in Table 12 and 13

Table 12 Country List by Importing Groups ($\mathscr{A}_{y,i}$)

Group 1			
Argentina	Cyprus	Italy	Singapore
Australia	Czech Rep.	Japan	Slovakia
Austria	Denmark	Malta	Slovenia
Bahamas	Ecuador	Mexico	South Africa
Belgium	Finland	Netherlands	Spain
Bolivia	France	New Caledonia	Sweden
Bulgaria	Germany	New Zealand	Switzerland
Canada	Ghana	Norway	Thailand
China	Greece	Poland	United Arab Emirates
China, Hong Kong SAR	Hungary	Portugal	United Kingdom
Costa Rica	Ireland	Rep. of Korea	USA
Croatia	Israel	Russian Federation	
Group 2			
Afghanistan	Greenland	Kyrgyzstan	Rep. of Moldova
Albania	Guinea	Latvia	Romania
Andorra	Iceland	Lebanon	Saudi Arabia
Bahrain	India	Liberia	Swaziland
Bangladesh	Indonesia	Lithuania	TFYR of Macedonia
Bosnia Herzegovina	Iran	Luxembourg	Timor-Leste
Côte d'Ivoire	Iraq	Mongolia	Tunisia
Egypt	Jordan	Morocco	Turkey
Estonia	Kazakhstan	Oman	Ukraine
Faeroe Isds	Kuwait	Qatar	
Group 3			
Algeria	Congo	Malaysia	Senegal
Armenia	Cuba	Mali	Sierra Leone
Belarus	Democratic Republic of the Congo	Mauritius	Sri Lanka
Bermuda	French Polynesia	Nicaragua	Syria
Brazil	Georgia	Pakistan	Uzbekistan
Brunei Darussalam	Guatemala	Panama	Venezuela
Cameroon	Honduras	Paraguay	Viet Nam
Central African Rep.	Kenya	Peru	Yemen
Chile	Libya	Philippines	
China, Macao SAR	Madagascar	San Marino	

Group 4			
Angola	Colombia	Myanmar	Trinidad and Tobago
Azerbaijan	El Salvador	N. Mariana Isds	Uganda
Benin	Ethiopia	Namibia	United Rep. of Tanzania
Botswana	Fiji	Niger	Zambia
Burkina Faso	Gabon	Nigeria	Zimbabwe
Burundi	Malawi	Rwanda	
Cambodia	Maldives	Seychelles	
Cape Verde	Mozambique	Togo	
Group 5			
Antigua and Barbuda	Dominica	Lesotho	Solomon Isds
Aruba	Dominican Rep.	Mauritania	Somalia
Barbados	Gambia	Nauru	Suriname
Belize	Grenada	Nepal	Tonga
Bhutan	Guinea-Bissau	Papua New Guinea	Tuvalu
Br. Virgin Isds	Guyana	Saint Lucia	Uruguay
Chad	Haiti	Saint Vincent and the Grenadines	Vanuatu
Dem. People's Rep. of Korea	Jamaica	Samoa	
Djibouti	Lao People's Dem. Rep.	Sao Tome and Principe	

Table 13 Country List by Exporting Groups ($\mathscr{G}_{y,j}$)

Group a			
Argentina	Egypt	Kazakhstan	South Africa
Australia	France	Luxembourg	Spain
Austria	Germany	Mexico	Sweden
Belgium	Greece	Nepal	Switzerland
Brazil	Guatemala	Netherlands	Thailand
Canada	Hungary	New Zealand	Turkey
China	India	Nigeria	Tuvalu
China, Hong Kong SAR	Ireland	Norway	United Arab Emirates
Cuba	Israel	Portugal	United Kingdom
Czech Rep.	Italy	Rep. of Korea	USA
Denmark	Japan	Singapore	
Group b			
Afghanistan	Ghana	Niger	Syria
Andorra	Indonesia	Oman	Tunisia
Azerbaijan	Jordan	Pakistan	Uganda
Bahrain	Kenya	Philippines	Uruguay
Bangladesh	Malawi	Poland	Viet Nam
Estonia	Malaysia	Saudi Arabia	Yemen
Finland	Morocco	Senegal	Zambia
Georgia	Namibia	Sri Lanka	

Group c			
Algeria	Colombia	Kyrgyzstan	San Marino
Armenia	Croatia	Lebanon	Slovakia
Belarus	Cyprus	Mali	Slovenia
Bhutan	Democratic Republic of the Congo	Mongolia	TFYR of Macedonia
Bosnia Herzegovina	Fiji	Nauru	Tonga
Brunei Darussalam	French Polynesia	New Caledonia	Ukraine
Bulgaria	Guinea	Papua New Guinea	Uzbekistan
Cambodia	Iran	Qatar	
Chile	Jamaica	Romania	
China, Macao SAR	Kuwait	Russian Federation	
Group d			
Albania	Dem. People's Rep. of Korea	Honduras	Rep. of Moldova
Bahamas	Dominican Rep.	Iceland	Saint Lucia
Barbados	Ecuador	Latvia	Solomon Isds
Bermuda	El Salvador	Malta	Trinidad and Tobago
Bolivia	Faeroe Isds	Panama	Venezuela
Costa Rica	Haiti	Peru	
Group e			
Angola	Congo	Libya	Sao Tome and Principe
Antigua and Barbuda	Djibouti	Lithuania	Seychelles
Aruba	Dominica	Madagascar	Sierra Leone
Belize	Ethiopia	Maldives	Somalia
Benin	Gabon	Mauritania	Suriname
Botswana	Gambia	Mauritius	Swaziland
Br. Virgin Isds	Greenland	Mozambique	Timor-Leste
Burkina Faso	Grenada	Myanmar	Togo
Burundi	Guinea-Bissau	N. Mariana Isds	United Rep. of Tanzania
Côte d'Ivoire	Guyana	Nicaragua	Vanuatu
Cameroon	Iraq	Paraguay	Zimbabwe
Cape Verde	Lao People's Dem. Rep.	Rwanda	
Central African Rep.	Lesotho	Saint Vincent and the Grenadines	
Chad	Liberia	Samoa	

Estimation Procedure

Denote the structural parameters (the coefficient vectors in (1) and (2), the variance and the correlation coefficient parameter) as $\theta' = (\beta', \delta', \sigma, \rho)$ and the incidental parameters (the additive and interactive fixed effect parameters) as

$$\underbrace{\pi'_{ij}}_{1\times 2(2+2r)} = (\underbrace{a'_i}_{1\times(2+2r)}, \underbrace{g'_j}_{1\times(2+2r)}),\ i = 1,2,\ldots,N,\ j = 1,2,\ldots,N, j \neq i. \tag{8}$$

where

$$a'_i = (a'_{y,i}, a'_{d,i}) = (\underbrace{\alpha_{y,i}, \mathscr{A}_{y,i}}_{1+r}, \underbrace{\alpha_{d,y}, \mathscr{A}_{d,i}}_{1+r})$$
$$g'_j = (g'_{y,j}, g'_{d,t}) = (\underbrace{\gamma_{y,t}, \mathscr{G}_{y,t}}_{1+r}, \underbrace{\gamma_{d,y}, \mathscr{G}_{d,t}}_{1+r}).$$

For identification, we employ the same restrictions as Bai (2009):

$$\sum_{i=1}^{N}\alpha_{y,i}=\sum_{i=1}^{N}\alpha_{d,i}=\sum_{j=1}^{N}\gamma_{y,j}=\sum_{j=1}^{N}\gamma_{d,j}=0,\ \ \sum_{i=1}^{N}\mathscr{A}'_{y,i}=\sum_{i=1}^{N}\mathscr{A}'_{d,i}=\sum_{j=1}^{N}\mathscr{G}'_{y,j}=\sum_{j=1}^{N}\mathscr{G}'_{d,j}=\underbrace{0}_{r\times 1},$$

$$\mathscr{A}'_y\mathscr{A}_y=\mathscr{A}'_d\mathscr{A}_d=I_r,\ \underbrace{\mathscr{A}'_y}_{r\times N}=(\mathscr{A}'_{y,1},\mathscr{A}'_{y,2},\dots,\mathscr{A}'_{y,N}),\ \underbrace{\mathscr{A}'_d}_{r\times N}=(\mathscr{A}'_{d,1},\mathscr{A}'_{d,2},\dots,\mathscr{A}'_{d,N})$$

$$\mathscr{G}'_y\mathscr{G}_y=\Lambda_y,\ \mathscr{G}'_d\mathscr{G}_d=\Lambda_d,\ \underbrace{\mathscr{G}'_y}_{r\times N}=(\mathscr{G}'_{y,1},\mathscr{G}'_{y,2},\dots,\mathscr{G}'_{y,N}),\ \underbrace{\mathscr{G}'_d}_{r\times N}=(\mathscr{G}'_{d,1},\mathscr{G}'_{d,2},\dots,\mathscr{G}'_{d,N}),$$

where I_r is the $r\times r$ identity matrix, and Λ_y and Λ_d are $r\times r$ diagonal matrices.

We employ an expectation-maximization (EM) algorithm (e.g., Dempster et al. (1977)) to obtain the maximum likelihood estimate of the unknown parameters in (1), (2), (3), and (4). The procedure follows two steps: expectation (the conditional expectation of the complete likelihood function given the observations, E-step) and maximization (the maximization of the expected likelihood with respect to unknown parameters, M-step). The actual procedure to obtain the maximum likelihood estimator (MLE) by the EM algorithm is as follows[12]:

E-step: Given the initial values of the parameters $\{\beta^{(s)}, a_y^{(s)}, g_y^{(s)}\}$, $\{\delta^{(s)}, a_d^{(s)}, g_d^{(s)}\}$, $\rho^{(s)}$, $\sigma^{(s)}$, define the conditional means and variances of the latent dependent variables, y^*_{ij} and d^*_{ij},

$$\mathbb{E}_{y^*_{ij}|d=0}[y^*_{ij}],\ \mathbb{E}_{d^*_{ij}|y_{ij},d_{ij}=1}[d^*_{ij}],\ \mathbb{E}_{d^*_{ij}|d_{ij}=0}[d^*_{ij}],\ \mathbb{V}_{y^*_{ij}|d_{ij}=0}[y^*_{ij}],\ \mathbb{V}_{d^*_{ij}|y_{ij},d_{ij}=1}[d^*_{ij}],\ \text{and}\ \mathbb{V}_{d^*_{ij}|d_{ij}=0}[d^*_{ij}]$$

and define

$$\tilde{y}_{ij}=d_{ij}\cdot y_{ij}+(1-d_{ij})\cdot\mathbb{E}_{y^*_{ij}|d_{ij}=0}[y^*_{ij}]$$
$$\tilde{d}_{ij}=d_{ij}\cdot\mathbb{E}_{d^*_{ij}|y_{ij},d_{ij}=1}[d^*_{ij}]+(1-d_{ij})\cdot\mathbb{E}_{d^*_{ij}|d_{ij}=0}[d^*_{ij}]$$

M-step: This step uses three conditional maximization sub-steps (e.g., Meng and Rubin (1993); McLachlan and Krishnan (2008)):

1. Given $\{\beta^{(s)}, a_y^{(s)}, g_y^{(s)}\}$,$\{\delta^{(s)}, a_d^{(s)}, g_d^{(s)}\}$, $\rho^{(s)}$, $\sigma^{(s)}$, define

$$\hat{\tilde{y}}^{(s)}_{ij}\equiv\tilde{y}^{(s)}_{ij}-\rho^{(s)}\sigma^{(s)}(\tilde{d}^{(s)}_{ij}-\mathbf{z}'_{ij}\delta^{(s)}-\alpha^{(s)}_{d,i}-\gamma^{(s)}_{d,j}-\mathscr{A}^{(s)}_{d,i}(\mathscr{G}^{(s)}_{d,j})').$$

Then, update $\{\beta, a_y, g_y\}$ by the minimizer of the following criterion function:

$$(\beta^{(s+1)},a_y^{(s+1)},g_y^{(s+1)})=\arg\min_{\beta,a_y,g_y}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\hat{\tilde{y}}^{(s)}_{ij}-\mathbf{x}'_{ij}\beta-\alpha_{y,i}-\gamma_{y,j}-\mathscr{A}_{y,i}\mathscr{G}'_{y,j}\right)^2.$$

[12] Ruud (1991) proposes another estimation scheme via EM algorithm for the standard sample selection model. The proposed estimation procedure here is an extension of the expectation conditional maximization (ECM) algorithm proposed by Chen (2016) for the Probit model with interactive fixed effect terms.

2. Given $\{\beta^{(s+1)}, a_y^{(s+1)}, g_y^{(s+1)}\}$, $\{\delta^{(s)}, a_d^{(s)}, g_d^{(s)}\}$, $\rho^{(s)}, \sigma^{(s)}$, define

$$\hat{\tilde{d}}_{ij}^{(s)} \equiv \tilde{d}_{ij}^{(s)} - \frac{\rho^{(s)}}{\sigma^{(s)}}(\tilde{y}_{ij}^{(s)} - \mathbf{x}'_{ij}\beta^{(s+1)} - \alpha_{y,i}^{(s)} - \gamma_{y,j}^{(s)} - \mathscr{A}_{y,i}^{(s)}(\mathscr{G}_{y,j}^{(s)})').$$

Then, update $\{\delta^{(s)}, a_d^{(s)}, g_d^{(s)}\}$ by the minimizer of the following criterion function:

$$(\delta^{(s+1)}, a_d^{(s+1)}, g_d^{(s+1)}) = \arg\min_{\delta, a_d, g_d} \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\hat{\tilde{d}}_{ij}^{(s)} - \mathbf{z}'_{ij}\delta - \alpha_{d,i} - \gamma_{d,j} - \mathscr{A}_{d,i}\mathscr{G}'_{d,j}\right)^2.$$

3. Given $\{\beta^{(s+1)}, a_y^{(s+1)}, g_y^{(s+1)}\}, \{\delta^{(s+1)}, a_d^{(s+1)}, g_d^{(s+1)}\}, \rho^{(s)}, \sigma^{(s)}$, update ρ and σ by

$$\sigma^{(s+1)} = \left(T_{yy}^{(s)} + \frac{(T_{yd}^{(s)})^2}{T_{dd}^{(s)}} \cdot (1 - T_{dd}^{(s)})\right)^{1/2}, \quad \rho^{(s+1)} = \frac{1}{\sigma^{(s+1)}} \cdot \frac{T_{yd}^{(s)}}{T_{dd}^{(s)}}$$

where ($\pi_{y,ij} \equiv \alpha_{y,i} - \gamma_{y,j} - \mathscr{A}_{y,i}\mathscr{G}'_{y,j}$, $\pi_{d,ij} \equiv \alpha_{d,i} - \gamma_{d,j} - \mathscr{A}_{d,i}\mathscr{G}'_{d,j}$), and

$$T_{yy}^{(s)} = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\left\{(\tilde{y}_{ij}^{(s)} - \mathbf{x}'_{ij}\beta^{(s+1)} - \pi_{y,ij}^{(s+1)})^2 + (1 - d_{ij}) \cdot \mathbb{V}_{y^*|d=0}^{(s)}[y_{ij}^*]\right\}$$

$$T_{yd}^{(s)} = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\Big\{(\tilde{y}_{ij}^{(s)} - \mathbf{x}'_{ij}\beta^{(s+1)} - \pi_{y,ij}^{(s+1)})(\tilde{d}_{ij}^{(s)} - \mathbf{z}'_{ij}\delta^{(s+1)} - \pi_{d,ij}^{(s+1)}) + (1 - d_{ij}) \cdot \rho^{(s)}\sigma^{(s)} \cdot \mathbb{V}_{d^*|d=0}^{(s)}[d_{ij}^*]\Big\}$$

$$T_{dd}^{(s)} = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\Big\{(\tilde{d}_{ij}^{(s)} - \mathbf{z}'_{ij}\delta^{(s+1)} - \pi_{d,ij}^{(s+1)})^2 + d_{ij} \cdot \mathbb{V}_{d^*|y,d=1}^{(s)}[d_{ij}^*] + (1 - d_{ij}) \cdot \mathbb{V}_{d^*|d=0}^{(s)}[d_{ij}^*]\Big\}$$

The procedure proposed here is easy to implement: the minimization problems in the first two conditional maximization sub-steps can be solved in the same way as Bai (2009), and the third sub-step has closed-form solutions.

The asymptotic distribution of the MLE has a non-zero mean vector since we are facing the situation where the sample sizes of exporting countries and importing countries simultaneously go to infinity at the same rate. The asymptotic properties of the bias-corrected estimator can be derived along the same lines as Hahn and Kuersteiner (2002), Arellano and Hahn (2016), and Fernández-Val and Weidner (2018): the bias-corrected estimator is defined as:

$$\tilde{\theta} = \hat{\theta} - \mathbf{b}_N - \mathbf{d}_N$$

where $\hat{\theta}$ is the MLE, and $\mathbf{b}_N$ and $\mathbf{d}_N$ are bias correction terms defined in (10) and (11) in the Appendix. The asymptotic distribution of the estimator is given as follows:

$$\sqrt{N(N-1)}\left(\tilde{\theta} - \theta_0\right) \xrightarrow{d} N(\mathbf{0},\ \mathscr{W}_{NN}^{-1}\Psi_{NN}\mathscr{W}_{NN}^{-1})$$

Conditional Expectation of the Latent Variables

The conditional means and variances of the latent variables, y_{ij}^* and d_{ij}^*, given observations, y_{ij} and $d_{ij}=1$, or $d_{ij}=0$ are,

$$\mathbb{E}_{y_{ij}^*|d_{ij}=0}[y_{ij}^*] = \mathbf{x}_{ij}'\beta + \pi_{y,ij} - \rho\sigma_u \cdot \frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}$$

$$\mathbb{E}_{d_{ij}^*|y_{ij},d_{ij}=1}[d_{ij}^*] = (1-\rho^2)^{1/2}\cdot\left\{\frac{\eta_{ij}}{(1-\rho^2)^{1/2}} + \frac{\phi\left(\frac{\eta_{ij}}{(1-\rho^2)^{1/2}}\right)}{\Phi\left(\frac{\eta_{ij}}{(1-\rho^2)^{1/2}}\right)}\right\}$$

$$\mathbb{E}_{d_{ij}^*|d_{ij}=0}[d_{ij}^*] = \mathbf{z}_{ij}'\gamma + \pi_{d,ij} - \frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}$$

where $\pi_{y,ij} \equiv \alpha_{y,i} - \gamma_{y,j} - \mathscr{A}_{y,i}\mathscr{G}_{y,j}'$, $\pi_{d,ij} \equiv \alpha_{d,i} - \gamma_{d,j} - \mathscr{A}_{d,i}\mathscr{G}_{d,j}'$, and

$$\mathbb{V}_{y_{ij}^*|d_{ij}=0}[y_{ij}^*] = \sigma_u^2\left\{1+\rho^2\frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}\left\{\mathbf{z}_{ij}'\gamma + \pi_{d,ij} - \frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}\right\}\right\}$$

$$\mathbb{V}_{d_{ij}^*|y_{ij},d_{ij}=1}[d_{ij}^*] = (1-\rho^2)\cdot\left\{1-\frac{\phi\left(\frac{\eta_{ij}}{(1-\rho^2)^{1/2}}\right)}{\Phi\left(\frac{\eta_{ij}}{(1-\rho^2)^{1/2}}\right)}\left(\frac{\eta_{ij}}{(1-\rho^2)^{1/2}} + \frac{\phi\left(\frac{\eta_{ij}}{(1-\rho^2)^{1/2}}\right)}{\Phi\left(\frac{\eta_{ij}}{(1-\rho^2)^{1/2}}\right)}\right)\right\}$$

$$\mathbb{V}_{d_{ij}^*|d_{ij}=0}[d_i^*] = 1+\frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij} - \frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}\right)$$

$$\begin{aligned}\mathrm{cov}(y_{ij}^*, d_{ij}^*) &= \rho\sigma_u\left\{1+\frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}\cdot\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij} - \frac{\phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}{1-\Phi\left(\mathbf{z}_{ij}'\gamma + \pi_{d,ij}\right)}\right)\right\} \\ &= \rho\sigma_u\cdot\mathbb{V}_{d_{ij}^*|d=0}[d_{ij}^*]\end{aligned}$$

where

$$\eta_{ij} = \mathbf{z}_{ij}'\gamma + \pi_{d,ij} + \rho\cdot\frac{y_{ij}^* - \mathbf{x}_{ij}'\beta - \pi_{y,ij}}{\sigma_u}.$$

Asymptotic Properties of the Bias-corrected Estimator

Denote

$$\mu_{y,ij} \equiv \mathbf{x}_{ij}'\beta + \alpha_{y,i} + \gamma_{y,j} + \mathscr{A}_{y,i}'\mathscr{G}_{y,j} \equiv \mathbf{x}_{ij}'\beta + \pi_{y,ij},$$

and

$$\mu_{d,ij} \equiv \mathbf{z}_{ij}'\delta + \alpha_{d,i} + \gamma_{d,j} + \mathscr{A}_{d,i}'\mathscr{G}_{d,j} \equiv \mathbf{z}_{ij}'\delta + \pi_{d,ij}.$$

The likelihood function of the sample $\{\,\{\,y_{ij},d_{ij}\,\}_{j=1,j\neq i}^{N}\,\}_{i=1}^{N}$ is

$$\log L(\theta,a_y,g_y,a_d,g_d)=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\ell_{ij}(\theta,a_{y,i},g_{y,j},a_{d,i},g_{d,j})$$

where ℓ_{ij} is the log-likelihood contribution of the (i,j)-th observation:

$$\ell_{ij}=d_{ij}\cdot\log\frac{1}{\sigma}\phi\left(\frac{y_{ij}-\mu_{y,ij}}{\sigma}\right)+d_{ij}\cdot\log\Phi\left(\frac{\mu_{z,ij}+\rho\cdot(y_{ij}-\mu_{y,ij})/\sigma}{(1-\rho^2)^{1/2}}\right)+(1-d_{ij})\cdot\log\Phi\left(\mu_{z,ij}\right),$$

The parameter vectors are summarized into the following vectors,

$$a_{d,i}\equiv(\alpha_{d,i},\mathscr{A}_{d,i})',\ a_{y,i}\equiv(\alpha_{y,i},\mathscr{A}_{y,i})',\ g_{d,i}\equiv(\gamma_{d,j},\mathscr{G}_{d,j})',\ g_{y,i}\equiv(\gamma_{y,j},\mathscr{G}_{y,j})',$$

and

$$a_i\equiv(a_{d,i}',a_{y,i}')',\ g_j\equiv(g_{d,i}',g_{y,i}')',\theta\equiv(\beta',\delta',\sigma_u,\rho)'.$$

The bias corrected estimator of θ is asymptotically regarded as the solution of the following corrected score function (Fernández-Val and Weidner (2018) and Arellano and Hahn (2016)),

$$\frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\psi_{ij}(\theta)\equiv\frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\left(\frac{\partial\ell_{ij}}{\partial\theta}+\Xi_{i\bullet}'\frac{\partial\ell_{ij}}{\partial a_i}+\Xi_{\bullet j}'\frac{\partial\ell_{ij}}{\partial g_j}\right)=\mathbf{0},\tag{9}$$

where the incidental parameters, a_i and g_j, are evaluated at a given value of θ in the maximization process, and

$$\underbrace{\Xi_{i\bullet}}_{(2+2r)\times K}=\operatorname*{plim}_{N\to\infty}\left(-\sum_{j=1,j\neq i}^{N}\frac{\partial^2\ell_{ij}}{\partial a_i\partial a_i'}\right)^{-1}\left(\sum_{j=1,j\neq i}^{N}\frac{\partial^2\ell_{ij}}{\partial a_i\partial\theta'}\right)$$

$$\underbrace{\Xi_{\bullet j}}_{(2+2r)\times K}=\operatorname*{plim}_{N\to\infty}\left(-\sum_{i=1,i\neq j}^{N}\frac{\partial^2\ell_{ij}}{\partial g_j\partial g_j'}\right)^{-1}\left(\sum_{i=1,i\neq j}^{N}\frac{\partial^2\ell_{ij}}{\partial g_j\partial\theta'}\right)$$

are terms which make the log-likelihood function informationally orthogonal between θ and the incidental parameters, a_i and g_j. See Section 4.2 of Fernández-Val and Weidner (2018).

The asymptotic variance and the analytical expression of the bias correction term are analogously derived as Example 10 in Fernández-Val and Weidner (2018). The asymptotic variance of the bias-corrected estimator is given as $\mathscr{W}_{NN}^{-1}\Psi_{NN}\mathscr{W}_{NN}^{-1}$ where

$$\Psi_{NN}=\frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\left(\frac{\partial\ell_{ij}}{\partial\theta}+\Xi_{N,i\bullet}'\frac{\partial\ell_{ij}}{\partial a_i}+\Xi_{N,\bullet j}'\frac{\partial\ell_{ij}}{\partial g_j}\right)\cdot\left(\frac{\partial\ell_{ij}}{\partial\theta}+\Xi_{N,i\bullet}'\frac{\partial\ell_{ij}}{\partial a_i}+\Xi_{N,\bullet j}'\frac{\partial\ell_{ij}}{\partial g_j}\right)'$$

$$\mathscr{W}_{NN}=\frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\left\{\frac{\partial^2\ell_{ij}}{\partial\theta\partial\theta'}+\Xi_{N,i\bullet}'\frac{\partial^2\ell_{ij}}{\partial a_i\partial a_i'}\Xi_{N,i\bullet}+\Xi_{N,\bullet j}'\frac{\partial^2\ell_{ij}}{\partial g_j\partial g_j'}\Xi_{N,\bullet j}\right\}.$$

where $\Xi_{N,i\bullet}$ and $\Xi_{N,\bullet j}$ are sample analogs of $\Xi_{i\bullet}$ and $\Xi_{\bullet j}$, respectively. According to the formula in page 129 in Fernández-Val and Weidner (2018), the k-th element of the bias correction terms are derived as follows (note that we use the following abbreviations, $\sum_{j\neq i}^{N}$ and $\sum_{i\neq j}^{N}$ for $\sum_{j=1,j\neq i}^{N}$ and $\sum_{i=1,i\neq j}^{N}$, respectively),

$$b_{k,N} = -\frac{1}{N}\sum_{i=1}^{N}\text{trace}\left[\left(\sum_{j\neq i}^{N}\frac{\partial^2 \ell_{ij}}{\partial a_i \partial a_i'}\right)^{-1}\left(\sum_{j\neq i}^{N}\frac{\partial \ell_{ij}}{\partial a_i}\frac{\partial \psi_{k,ij}}{\partial a_i'}\right)\right] + \frac{1}{2N}\sum_{i=1}^{N}\text{trace}\left[\left(\sum_{j\neq i}^{N}\frac{\partial^2 \psi_{k,ij}}{\partial a_i \partial a_i'}\right)\left(\sum_{j\neq i}^{N}\frac{\partial^2 \ell_{ij}}{\partial a_i \partial a_i'}\right)^{-1}\left(\sum_{j\neq i}^{N}\frac{\partial \ell_{ij}}{\partial a_i}\frac{\partial \ell_{ij}}{\partial a_i'}\right)\left(\sum_{j\neq i}^{N}\frac{\partial^2 \ell_{ij}}{\partial a_i \partial a_i'}\right)^{-1}\right] \tag{10}$$

$$d_{k,N} = -\frac{1}{N}\sum_{j=1}^{N}\text{trace}\left[\left(\sum_{i\neq j}^{N}\frac{\partial^2 \ell_{ij}}{\partial g_j \partial g_j'}\right)^{-1}\left(\sum_{i\neq j}^{N}\frac{\partial \ell_{ij}}{\partial g_j}\frac{\partial \psi_{k,ij}}{\partial g_j'}\right)\right] + \frac{1}{2N}\sum_{j=1}^{N}\text{trace}\left[\left(\sum_{i\neq j}^{N}\frac{\partial \psi_{k,ij}}{\partial g_j \partial g_j'}\right)\left(\sum_{i\neq j}^{N}\frac{\partial^2 \ell_{ij}}{\partial g_j \partial g_j'}\right)^{-1}\left(\sum_{i\neq j}^{N}\frac{\partial \ell_{ij}}{\partial g_j}\frac{\partial \ell_{ij}}{\partial g_j'}\right)\left(\sum_{i\neq j}^{N}\frac{\partial^2 \ell_{ij}}{\partial g_j \partial g_j'}\right)^{-1}\right] \tag{11}$$

$k = 1, 2, \ldots, K.$

where $\psi_{k,ij}$ is the k-th element of ψ_{ij} defined in (9). The partial derivatives in these formula are given as follows ($\eta_{ij} \equiv \mu_{d,ij} - \rho(y_{ij} - \mu_{yij})/\sigma$, $\zeta_{ij} \equiv \mu_{d,ij}$),

$$\underbrace{\frac{\partial \ell_{ij}}{\partial a_i}}_{(2+2r)\times 1} = \begin{pmatrix} d_{ij}\cdot\frac{1}{\sigma}\cdot\left\{\frac{y_{ij}-\mu_{y,ij}}{\sigma} - \lambda(\eta_{ij})\cdot\frac{\rho}{(1-\rho^2)^{1/2}}\right\}\cdot\begin{pmatrix}1\\ \mathscr{G}_{y,j}\end{pmatrix} \\ \left\{d_{ij}\cdot\lambda(\eta_{ij})\cdot\frac{1}{(1-\rho^2)^{1/2}} - (1-d_{ij})\cdot\lambda(-\zeta_{ij})\right\}\cdot\begin{pmatrix}1\\ \mathscr{G}_{d,j}\end{pmatrix}\end{pmatrix}$$

$$\underbrace{\frac{\partial^2 \ell_{ij}}{\partial a_i \partial a_i'}}_{(2+2r)\times(2+2r)} = \begin{pmatrix} \left\{-1-\cdot\frac{\rho^2\cdot\xi_+(\eta_{ij})}{1-\rho^2}\right\}\cdot\frac{d_{ij}}{\sigma^2}\cdot\Delta^{\mathscr{G}}_{yy,j} & \frac{\rho\cdot\xi_+(\eta_{ij})}{1-\rho^2}\cdot\frac{d_{ij}}{\sigma}\cdot\Delta^{\mathscr{G}}_{yd,j} \\ \frac{\rho\cdot\xi_+(\eta_{ij})}{1-\rho^2}\cdot\frac{d_{ij}}{\sigma}\cdot\Delta^{\mathscr{G}}_{dy,j} & -\left\{\frac{d_{it}\cdot\xi_+(\eta_{ij})}{1-\rho^2} - (1-d_{ij})\cdot\xi_-(\zeta_{ij})\right\}\Delta^{\mathscr{G}}_{dd,j}\end{pmatrix},$$

$$\frac{\partial \ell_{ij}}{\partial g_j} = \begin{pmatrix} d_{ij}\cdot\frac{1}{\sigma}\cdot\left\{\frac{y_{ij}-\mu_{y,ij}}{\sigma} - \lambda(\eta_{ij})\cdot\frac{\rho}{(1-\rho^2)^{1/2}}\right\}\cdot\begin{pmatrix}1\\ \mathscr{A}_{y,i}\end{pmatrix} \\ \left\{d_{ij}\cdot\lambda(\eta_{ij})\cdot\frac{1}{(1-\rho^2)^{1/2}} - (1-d_{ij})\cdot\lambda(-\zeta_{ij})\right\}\cdot\begin{pmatrix}1\\ \mathscr{A}_{d,i}\end{pmatrix}\end{pmatrix}$$

$$\frac{\partial^2 \ell_{ij}}{\partial g_j \partial g_j'} = \begin{pmatrix} \left\{-1-\frac{\rho^2\cdot\xi_+(\eta_{ij})}{1-\rho^2}\right\}\cdot\frac{d_{ij}}{\sigma^2}\cdot\Delta^{\mathscr{A}}_{yy,i} & \xi_+(\eta_{ij})\cdot\frac{\rho}{1-\rho^2}\cdot\frac{d_{ij}}{\sigma}\cdot\Delta^{\mathscr{A}}_{yy,i} \\ \frac{\rho\cdot\xi_+(\eta_{ij})}{1-\rho^2}\cdot\frac{d_{ij}}{\sigma}\cdot\Delta^{\mathscr{A}}_{dy,i} & -\left\{d_{ij}\cdot\frac{\xi_+(\eta_{ij})}{1-\rho^2} - (1-d_{ij})\cdot\xi_-(\zeta_{ij})\right\}\Delta^{\mathscr{A}}_{dd,i}\end{pmatrix}$$

where $\Delta^{\mathscr{G}}_{uv,j} = \begin{pmatrix}1\\ \mathscr{G}_{u,j}\end{pmatrix}\begin{pmatrix}1\\ \mathscr{G}_{v,j}\end{pmatrix}'$ and $\Delta^{\mathscr{A}}_{uv,i} = \begin{pmatrix}1\\ \mathscr{A}_{u,i}\end{pmatrix}\begin{pmatrix}1\\ \mathscr{A}_{v,i}\end{pmatrix}'$, $u, v = y, d$, and

$$\lambda(\eta_{ij}) = \frac{\phi(\eta_{ij})}{\Phi(\eta_{ij})},\ \xi_+(\eta_{ij}) = \lambda(\eta_{ij})\left\{\eta_{ij} + \lambda(\eta_{ij})\right\},\ \xi_-(\zeta_{ij}) = \lambda(-\zeta_{ij})\left\{\zeta_{ij} - \lambda(-\zeta_{ij})\right\}.$$

Other partial derivatives, $\frac{\partial^3 \ell_{ij}}{\partial \theta_k \partial a_i \partial a_i'}$, $\frac{\partial^2 \ell_{ij}}{\partial \theta_k \partial a_i'}$, $\frac{\partial^2 \ell_{ij}}{\partial \theta_k \partial g_j'}$, and $\frac{\partial^3 \ell_{ij}}{\partial \theta_k \partial g_j \partial g_j'}$ are also derived in a similar way.

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