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It has become increasingly clear that the early use of decomposition for addition is associated with later mathematical achievement. This study examined how younger children execute a base-10 decomposition strategy to solve complex arithmetic (e.g., two-digit addition). 24 addition problems in two modalities (WA: Written Arithmetic; OA: Oral Arithmetic) with sums less than 100 were administered to 22 Japanese and 22 Singaporean 6-year-old kindergarteners. Our findings reveal that they were able to solve complex addition. For instance, Japanese kindergarteners tended to solve complex arithmetic using base-10 decomposition across the modality, whereas Singaporean kindergarteners used standard algorithms and basic counting to solve complex WA and OA problems, respectively. We speculate that Japanese kindergarteners might have a clearer understanding of the base-10 concept and were able to use this knowledge more readily than Singaporean kindergarteners. Mathematical experiences in kindergarten and number-naming systems have been put forward as two of the crucial contributors for such cross-cultural differences. This study also provides new directions for future research on the understanding of the base-10 concept and its application among young children.

Keywords: base-10 concept; decomposition strategy; kindergarten; mathematical experiences; number-naming system

Introduction

Studies have consistently demonstrated that early mathematical knowledge provides a firm foundation for later mathematical competencies at school (e.g., Duncan et al. 2007), particularly the early use of decomposition strategy (e.g., Geary et al. 2013). For kindergarteners, simple arithmetic (e.g., single-digit addition) (e.g., Jordan, Huttenlocher, and Levine 1992), such as the use of counting and retrieval strategies are the two typical assessments of their mathematical proficiency (Bisanz et al. 2005). While it may be influenced by the Western emphasis on the developmental appropriated approach, little attention has been given to complex arithmetic (e.g., two-digit addition) and the use of more advanced and efficient strategies, such as base-10 decomposition strategy in young children. Yet, cross-cultural studies have found that due to the high transparency of base-10 system in East Asian number system (e.g., Chinese, Japanese and Korean), East Asian children show a better understanding of base-10 concept and place value than their western counterparts even before formal schooling (Miura et al. 1994; Miura and Okamoto 2003).

Besides, having a base-10 structured number system, developing an understanding of the base-10 concept before formal schooling could be a plausible contributor for later superior mathematical achievement among East Asian children. Even though it is evident that young Asian children possess a clear understanding of the base-10 concept before formal schooling, it remains unclear how they extend this understanding to handle mathematical tasks, for example, solving complex arithmetic. Considering the underdeveloped base-10 aspect within the literature on early mathematics development, it is beneficial to go beyond merely investigating young Asian children's understanding of the base-10 concept and explore their arithmetic processing, especially complex arithmetic and factors contributing to this process.

Strategy choice and problem difficulty

Before schooling, children acquire nascent mathematical skills and understanding in their everyday lives by participating in a variety of mathematical-related activities provided by their cultures. According to Fuson and Kwon (1992), there are five arithmetic problem-solving strategies (including counting-all, counting-on, decomposition, retrieval, and guessing or no answer). Other studies affirmed three major mental calculation methods that children commonly use: counting, decomposition, and retrieval (Geary et al. 1996; Shrager and Sielger 1998).

(1) Counting strategy - includes both counting all (e.g., 5+7, the child counts from the first addend all the way to the second one, 1,2, . . ., 5, . . ., 10, 11, 12) and counting on (e.g., 4+3, the child keeps 4 in mind and starts counting from 5 to 7).
(2) Decomposition strategy - the child simplifies the calculation by decomposing the addends into smaller numbers and regrouping them (e.g., 11+15 = 10+10+1+5 = 20+6 = 26).
(3) Retrieval does not involve any mental computation or counting, and the answer is directly retrieved from the memory.

Since children use a variety of addition strategies, different factors are influencing their strategy choice. And problem difficulty is one of the most plausible ones. For example, children tended to use retrieval to solve easier or familiar problems (e.g., single-digit addition) and switched to strategies that involve computation when solving complex arithmetic because retrieving solutions from long-term memory (LTM) becomes less possible (Lemaire and Callies 2009; Siegler and Shipley 1995).

Besides revolving around the issue of whether one retrieves the solutions from LTM, Shrager and Sielger (1998) also saw strategy choice as a competition between efficiency and accuracy. For example, Siegler and Jenkins (1989) examined the use of counting strategies among pre-schoolers. They reported that children tended to use more advanced count-on strategy when solving those problems with addends above 20 than simple problems. In another study of addition, LeFevre, Sadesky, and Bisanz (1996) revealed that subjects used decomposition strategy more often when solving difficult problems with sums above 10 than those with sums less than 10. These studies suggested that for young children, the usage of advanced strategies increases along with problem difficulty.

Complex arithmetic and base-10 decomposition strategy

Solving complex arithmetic can be a laborious process if we use those counting strategies by counting from the first addend to the second one. Furthermore, it is also highly likely to miscount the addends, which later leads to an inaccurate solution. Therefore, more advanced strategies, such as base-10 decomposition strategy, become a handy tool for children when solving complex arithmetic (e.g., Laski, Ermakova, and Vasilyeva 2014; Laski et al. 2016). In this strategy, both addends are split into tens and units, and then sum up separately before combining them together (e.g., 24 + 13: 20 + 10 = 30, 4 + 3= 7, 30 + 7= 37). To execute this strategy, one needs to have a clear understanding of the base-10 concept. Therefore, a good grasp of place-value notation and additive composition of numbers are required since they are the two central mathematical principles of base-10 concept.

However, these principles are never easy for young children to acquire and comprehend. For place-value notation, one must understand that numbers can be expressed in different units, such as ones, tens, hundreds, and the position of each digit in a number determines its numeric magnitude. As for additive composition of numbers, one requires to know that all positive integer ‘n’ can be composed and decomposed into two or more others that are smaller than ‘n’ in terms of their numeric magnitude, and these others sum up to exactly to ‘n’. Therefore, given the complexity of the base-10 concept, many researchers maintain that children’s understanding of this concept often does not develop until a later age (Kerbs, Squire, and Bryant 2003) and would only emerge with formal schooling (Geary 1995; Naito and Miura 2001). It could explain why little attention has been given to how young children acquire the base-10 concept and its application.

In recent years, base-10 understanding has widely believed as a crucial aspect of early mathematics (Geary 2006; NCTM 2000), which facilitates the computation of complex arithmetic (Fuson and Briars 1990; National Research Council 2001). It is also associated with the use of decomposition strategies, and the early use of these strategies promotes later mathematical achievement (Laski, Ermakova, and Vasilyeva 2014). To avoid discussion of the relation between base-10 understanding and arithmetic problem solving that focused mainly on the theoretical aspects (Geary et al. 2013; Laski et al. 2016), researchers attempted to examine how kindergarteners solve complex arithmetic using base-10 decomposition.

For instance, Laski, Ermakova, and Vasilyeva (2014) investigated the use of decomposition for addition in kindergarteners from the USA, Russia, and Taiwan. In their study, besides single-digit addition, they included a mixed-digit addition comprising of a single-digit number and a double-digit number (e.g., 26+8), to explore whether these problems induce kindergarteners to use decomposition strategy. Like other prior studies, their findings revealed that counting remained the predominant arithmetic strategy among the participants, even for solving mixed-digit addition. However, the findings of their block-task revealed that the usage rate of canonical base-10 strategies (e.g., they used 2 ten-blocks and 8 unit-blocks represent the number 28) to represent double-digit numbers was high. This finding has made us ponder why they did not use base-10 decomposition to solve those mixed-digit addition? One plausible reason could be due to the nature of mixed-digit addition. For example, to solve 18 + 3, it is easier and more efficient to use count-on (keeps 18 in mind and starts counting from 19 to 21) than to decompose the addends into groups of tens and units (18 + 2 = 20, 20 + 3 = 23, 23– 2 = 21). Thus, these problems might end up inducing children to use a counting strategy rather than a decomposition strategy.

To avoid the same problem, we attempt to raise the problem difficulty a little by including the two-digit addition (e.g., 12 + 13, 23 + 19) in this study. It might increase the usage of base-10 decomposition among the participants and allow us to fully capture the process of how they execute the strategy to handle complex arithmetic.

Similarities and differences between Japanese and Singaporean kindergarteners

In this study, kindergarteners from two countries – Japan and Singapore, were selected for two main reasons. Firstly, superior mathematical abilities possessed by both Japanese and Singaporean children have been widely acknowledged in numerous cross-cultural comparison studies (Ee, Wong, and Aunio 2006; Miura et al. 1994) and in several international academic assessment tests, such as Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) (e.g., Mullis et al. 2012; OECD 2016). In our recent cross-national study (Authors 2016), we found out that on average, both the Japanese and Singaporean kindergarteners solved up to two-digit addition (e.g., 16 + 12, 23 +14) in oral and written forms. Secondly, both countries share a common linguistic structure of their counting systems. For instance, Japanese kindergarteners count using the Japanese number system.

On the other hand, English might be the medium of instruction in Singaporean kindergartens (Ee, Wong, and Aunio 2006), but learning of the mother tongue, especially the Chinese language (Chinese ethnic group makes up 75% of Singapore's population) is compulsory. Therefore, a large portion of Singaporean kindergarteners also learn to count in Chinese. Since base-10 structured Japanese and Chinese number systems promote the understanding of the base-10 concept which is crucial for executing base-10 decomposition strategy, by including participants from these two countries would increase the likelihood of capturing a fuller picture of kindergarteners not only solving complex arithmetic but also the use of base-10 decomposition strategy.

Similarities: Effects of number system

Both Japanese and Singaporean cultures share a common linguistic structure in their number systems, which is the high transparency of the base-10 system. Such characteristic plays an important role in facilitating children's mathematical development and learning which later also accounted for cross-cultural differences in ability and achievement (e.g., Miura et al. 1994; Paik et al. 2011). For example, the East Asian number system (e.g., Chinese, Korean, and Japanese) are structured in terms of base-10. In this system, the value of a given digit in a multi-digit numeral depends on the face value of the digit (0-9) and its position, or place, in the numeral, with the value of each position increasing by powers of 10 (十) from right to left. For instance, 11 in Japanese is (十一) which can be translated into English as "ten-one", and it goes the same for the rest, 12 (十二) as "ten-two" and 13 (十三) as "ten-three". Therefore, number-words can be easily remembered as tens and ones. Contrarily, in English, number names such as "eleven" and "twelve" remain arbitrary with no clue about number composition. Similarly, for 13, the "ten" becomes "teen", and "three" becomes "thir-", and for 15, "ten" becomes "teen", and "five" becomes “fif-”. Besides these complexities, another problem is that numbers in the first ten are reversed in spoken English but not in the Japanese system. For example, instead of “one-ten-four” or “teen-four”, “14” is read as “fourteen”. Young children are likely to be confused as smaller numbers are named before large numbers.

Differences: Written numeral vs. oral numeral context

Our previous study (Authors 2016) reported that on average, 6-year-old Japanese and Singaporean kindergarteners were able to solve up to two-digit addition, but they tended to fare better in one of the modalities than the other, and their performances were contrary to each other. Specifically, Japanese kindergarteners performed well in those addition problems presented in oral numeral than those in written form, whereas their Singapore counterparts displayed a contrary pattern. In that study, experiences in kindergarten were put forward as one of the main contributors to this cross-cultural difference.

For example, Singapore’s educational system places a great emphasis on the assessment of learning and written assessment, which has resulted in the pedagogy to gear towards written arithmetical teaching and learning. Therefore, formal mathematics lessons are conducted even in Singaporean kindergartens (Aunio et al. 2004). Such consistent experiences in processing written numerals could have resulted in better performance besides problems presented in written numerals than those in oral numerals.

On the other hand, systematic teaching/formal lessons are not conducted in Japanese kindergartens. However, mathematical concepts are often embedded into activities such as taking attendance, cleaning up, playing games, etc. (Sakakibara 2014), and numbers are commonly represented and manipulated by teachers or children in its oral form (e.g., Sakakibara 2014). Therefore, their frequent experience and practising of numbers in oral form might lead them to perform better in OA problems than WA ones.

Despite that, our previous study did not provide a detailed insight into the arithmetic processing of both countries’ kindergarteners. Therefore, we attempted to replicate the study to explore further not only their arithmetic processing but also factors contributing to the process.

Differences: Learning base-10 through play, games and everyday activities vs. formal lesson

Even though systematic teaching/formal lessons are not conducted in Japanese kindergartens, it is not uncommon to see the ‘presence’ of base-10 concept in their activities. For instance, teachers counting out loud by tens during the skipping games, children, and items being grouped into tens and tasks/games come with 10 different levels of difficulty to accomplish (Author 2018). Furthermore, Miura et al. (1994) also suggested that those informal learning in kindergartens promotes Japanese first graders’ abilities to construct numbers in canonical base-10 structure and their understanding of place-value. Thus, in Japanese kindergartens, free play, games and everyday activities are likely to provide an effective platform for young children to develop their base-10 concept since prior studies have also reported that informal learning, such as playing games, promotes young children’s mathematical learning and competencies (Cohrssen and Niklas 2019).

Contrarily, young children attend regular mathematics formal lessons in Singaporean kindergartens (Aunio et al. 2004). During the lessons, they often learn and practise standard algorithm by doing both single-digit (e.g., 5+3) and two-digit (e.g., 14+15) arithmetic problems on their workbook/worksheet. Even though learning of this strategy is not stated in the general guidelines provided by the Ministry of Education (A framework for a kindergarten curriculum in Singapore 2012), Singaporean kindergartens tend to place great emphasis on it. One plausible reason is that they are preparing the kindergarteners for their primary school mathematics because learning standard algorithms for addition and subtraction of 2-digit numbers are part of Primary one mathematics syllabus (Mathematics syllabus Primary one to six, implementing starting with 2013 Primary one cohort 2012). And since the mechanism of a standard algorithm is dependent on the base-10 concept, Singaporean kindergarteners could have also learnt this concept and its related principles during the lessons.

Based on the above comparison, it is clear that there is a sharp contrast in the pedagogical approaches between the two countries. In other words, Japanese kindergarten teachers embed mathematical components into their kindergarten activities, but interestingly, they often do not have the intention of teaching mathematics to kindergarteners. On the other hand, Singaporean kindergarten teachers plan their mathematics curriculum and adopt systematic teaching approach. Therefore, they tend to place great emphasis on the accuracy of the kindergarteners' answers to mathematical problems and make every effort to correct their mathematical errors, whereas their Japanese counterparts focus more on helping kindergarteners to develop their social and emotional skills than mathematical skills.

In order to have a better understanding of kindergarteners' arithmetic processing, this study had two main goals:

(1) To examine and compare the arithmetical abilities of Japanese and Singaporean kindergarteners across the modality (oral and written arithmetic)
(2) To explore and contrast the arithmetical processing of Japanese and Singaporean kindergarteners, especially solving of complex arithmetic (e.g., two-digit addition problems) and use of base-10 decomposition strategy.

We then sought to discuss how their experiences in kindergartens and their number systems can contribute to those differences.

Method

Participants

22 Japanese (11 boys and 11 girls) (*M* = 75 months; *SD* = 3) and 22 Singaporean (11 boys and 11 girls) (*M* = 73 months; *SD* = 3) six-year-olds attending kindergartens participated in this study. All Japanese participants spoke Japanese only, whereas all Singaporean participants were bilingual (i.e., speaking both English and Chinese). These participants were selected through convenience sampling from four typical kindergartens in Singapore and Japan. Informed consent was obtained from all the participants' parents before the experiment.

Pre-test

The goal of the pre-test was to ensure that all participants were able to recognize numbers and perform addition. The pre-test comprised two types of problems (number recognition and simple addition) in two different modalities (oral and written). Both modalities were selected because they were also two of the most common modalities through which numbers were represented. The experimenter readout oral modality's task one at a time, whereas each written modality's problem was printed on an A4-sized card and displayed to the participants (see Table 1). Participants were first asked to complete the number recognition task, followed by the simple addition task (sum less than 5). And they filled in all their answers on the response sheet provided. Only participants who correctly answered all the problems proceeded to the main addition task.
[Table 1 near here]

Main Addition task

24 addition problems in two types of modalities (WA: Written Arithmetic; OA: Oral Arithmetic) and four different levels with sums less than 100 were selected (See Table 2). Half of the problems were presented in WA and the rest in OA. Each WA problem was printed on an A4-sized card and displayed to the participants, whereas OA problems were read out to them. These problems were the same as those used in our previous study (Authors 2016). Papers, pencils, and 100 circular cards (3cm in diameter) were provided as an aid in computation. All participants were asked to fill in their answers on the response sheet provided. [Table 2 near here]

Half of the participants from each country were randomly assigned to solve WA problems, followed by OA problems, while the rest solved the problems in the opposite order. Participants were encouraged to solve the problems using whichever strategy they preferred. The experimenter observed and recorded those overt behaviours displayed by the participants during the task, for example, counting the fingers or circular cards, counting aloud, etc. In the case of no overt behaviour, the experimenter asked the participant how he/she solved the problem upon providing an answer. These two ways of identifying children's strategy use were reported to be crucial for valid strategy classification (Laski, Ermakova, and Vasilyeva 2014). To keep any form of distress to the minimum, the task stopped once the participant wrongly answered two items or refused to proceed on.

Based on the notes and video-recordings, participants' strategies on each problem were coded and classified into five main types: counting, retrieval, standard algorithm, base-10 decomposition and others (see Table 3). The classification of the strategies was based on Laski, Ermakova, and Vasilyeva (2014) but the slight modification was made to capture the strategy use among the participants. For example, the inclusion of a standard algorithm since this strategy is commonly taught in Singapore kindergartens, and the removal of fact-based decomposition because no participant used this strategy. The data from 10 participants (5 Japanese and 5 Singaporean: 22% of the sample) were analysed independently by two raters, and the agreement rate was 92%.

Finding and discussion

This study revealed three main findings,

(1) Test scores: Japanese children (OA>WA), Singaporean children (WA>OA)
(2) Problems comprising of 10 were easier for Japanese kindergarteners than Singaporean kindergarteners
(3) Japanese children tended to use more base-10 decomposition strategy to solve complex arithmetic than their Singaporean counterparts across the modality.

Test scores: Japanese kindergarteners (OA>WA), Singaporean kindergarteners (WA>OA)

The first goal was to examine and compare the arithmetical abilities of Japanese and Singaporean kindergarteners across two common modalities (oral and written). Figure 1 shows kindergarteners' mean test scores on addition across the modality. A two-way ANOVA was conducted on both countries' kindergarteners' test scores on addition. The ANOVA revealed a significant interaction effect between country and modality (F (1, 42) = 32.64, p<.001, ηp^2 = .44, see Figure1). Follow-up post-hoc analysis indicated that Japanese kindergarteners performed better in OA problems than WA problems (p<.001), whereas Singaporean kindergarteners displayed a contrary pattern (p<.001). Furthermore, Singaporean kindergarteners (9.0 out of 12) fared better than Japanese kindergarteners (6.4 out of 12) in terms of WA problems (p<.001). Even though there was no significant effect on OA problems, Japanese kindergarteners' mean test score (7.8 out of 12) was higher than those of their Singapore counterparts (6.9 out of 12). Overall, such patterns of performance displayed by both countries' kindergarteners were identical to those of our previous study. Therefore, we continue to maintain that their experiences in kindergarten could be one of the main contributors to their performances. [Figure 1 near here]

For example, Japanese kindergarten teachers frequently embed mathematics into kindergarteners' activities, such as singing, exercising, taking attendance, practicing performances (Sakakibara 2014), and numbers are commonly represented and manipulated by teachers or children in its oral form. Seemingly, frequent handling of oral numerals is likely to enhance children's sensitivity to numerals through the sense of hearing, which later develops not only familiarity but also expertise in processing numerals that are presented in oral form. This could explain why Japanese kindergarteners fared better in OA problems than WA problems.

On the contrary, Singapore's educational system places a great emphasis on the assessment of learning and written assessment, which has resulted in the pedagogy to gear towards written arithmetical teaching and learning. Formal mathematics lessons are therefore conducted in Singaporean kindergartens (Aunio et al. 2004). Their consistent experiences in context that comprises of written numerals could lead them to develop familiarity and expertise in processing numerals that are presented in written form. This could explain why they were better at solving WA problems than OA problems.

Problems comprising of 10 are easier for Japanese kindergarteners than Singaporean kindergarteners

A similar solving pattern was found between both countries' kindergarteners as they solved Level A problems across the modality. More specifically, retrieval was their predominant strategy (See Table 4 and 5), which suggests that Level A problems were easy for them. Therefore, they could produce the solutions instantly without even counting. [Table 4 and Table 5 near here]

However, as they proceeded to problems comprising of 10 (Level B), the difference between the two countries became more evident. More concretely, retrieval continued to be the predominant strategy among Japanese kindergarteners but not for their Singapore counterparts. A chi-square test for this difference was conducted. The test indicated that Japanese kindergarteners were more likely to solve problems comprising of 10 using retrieval than Singaporean kindergarteners across the modality (OA($\chi^2(3) = 15.24$, $p<.001$) and WA($\chi^2(4) = 18.66$, $p<.001$)).

According to Siegler and his colleagues (Shrager and Siegler 1998; Siegler and Shipley 1995), a child will first attempt to retrieve the solution from his/her memory when solving arithmetic problems. The associative strength between the problem and the solution will determine whether the solution can be retrieved correctly. Therefore, it is more likely to retrieve solutions for those easier and more familiar problems, whereas computation is required to solve more complex or unfamiliar problems (e.g., two-digit addition). Thus, Level B problems tended to be easier for the Japanese kindergarteners than Singaporean kindergarteners. But why is it so?

We suggest that the number system could be one of the main contributors to this cross-cultural difference. Base-10 structured Japanese number system encourages Japanese children to think and represent multi-digit numbers in terms of base-10 (e.g., 18 as 1 ten and 8 ones instead of 18 ones) (e.g., Miura et al. 1994) which could lead them to develop cognitive number structures that reflect base-10 system. Therefore, when a problem comprising of 10 is presented to them, they activate this cognitive structure and produce the solution immediately without calculation.

On the other hand, though under the bilingual education system in Singapore, English remains the medium of instruction (first language) in Singapore schools (Ee, Wong, and Aunio 2006), this might lead to the problem of language dominance among Singaporean kindergarteners. Language dominance refers to the situation in which bilingual speakers exhibit better control of one language over the other (Nicholadis and Genesee 1997). In other words, Singaporean kindergarteners might be more proficient in English than in Chinese. Like those English-Chinese bilingual preschool children in other studies (e.g., Rasmussen et al. 2006), Singaporean kindergarteners might not benefit from the transparency of the Chinese number system due to the insufficient exposure to counting in Chinese or learning to count in English obscures the transparency of the Chinese language. This could explain why Singaporean kindergarteners tended to take some effort (e.g., counting) when solving Level B problems even though they also learn to count in Chinese, which number names are congruent with base-10 system just like the Japanese number system.

Japanese kindergarteners used base-10 decomposition strategy to solve complex arithmetic more than Singaporean kindergarteners

As the participant proceeded to complex arithmetic (e.g., Level C and D problems), the difficulty level of the problems increases. It has become almost impossible for 6-year-old kindergarteners to solve those problems without some forms of calculation. Therefore, none of the participants used retrieval to solve the problems at Level C and D. Instead, some forms of calculation are required to solve those questions. For example, base-10 decomposition was predominantly used by Japanese kindergarteners across the modality but not Singaporean kindergarteners (See Table 4 and 5). A chi-square test[1] for this difference was conducted. The test indicated that Japanese kindergarteners tended to solve complex arithmetic using base-10 decomposition more than Singaporean kindergarteners across the modality. (OA (Level C, $\chi^2(3) = 22.98$, $p<.001$; Level D, $\chi^2(1) = 11.05$, $p<.001$), WA (Level C, $\chi^2(2) = 29.50$, $p<.001$; Level D, $\chi^2(2) = 14.49$, $p<.001$)). In addition, the usage of base-10 decomposition among Japanese kindergarteners was also consistent across the modality (Level C, $\chi^2(2) = 0.75$, $p<.69$; Level D, $\chi^2(1) = 0.57$, $p<.45$). On the other hand, for Singaporean kindergarteners, besides their low usage of base-10 decomposition, their predominant strategies also varied across the modality (Level C, $\chi^2(3) = 22.81$, $p<.002$; Level D, $\chi^2(2) = 12.81$, $p<.001$). More concretely, standard algorithm and counting were their predominant strategies to solve WA and OA problems, respectively, at Level C and D.

Considered together, it suggests that Japanese kindergarteners possessed a clearer understanding of the base-10 concept than Singaporean kindergarteners since the usage of base-10 decomposition among Japanese kindergarteners was consistently high across the modality. In other words, the types of modality through which two-digit addition problems were presented did not seem to affect Japanese kindergarteners' ability to use base-10 decomposition. The finding further suggests that the base-10 concept might have been "deeply embedded" in Japanese kindergarteners even though it is not formally taught in kindergartens. So how did they manage to acquire base-10 understanding and extend this understanding to solve complex and novel arithmetic like those problems at Level C and D?

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[1] In the case in which both cells were zero for both countries (e.g., Table 4' s cells for retrieval at Level C), those data were excluded for Chi-square test.

Besides the linguistic advantage that Japanese children enjoy, kindergarten might be an important setting for developing a base-10 concept. In fact, it is not uncommon to see the inclusion of multiples of tens in Japanese kindergarten activities. All these could serve as a platform for the children to experience and learn about base-10 concept and later develop cognitive number structures that reflect base-10 structure even without formal schooling.

On the contrary, for Singaporean kindergarteners, besides the low usage of base-10 decomposition, their predominant strategies varied across the modality. As mentioned earlier, they tended to solve Level C and D's WA problems using standard algorithms. It could be due to the learning and practicing of standard algorithms by solving addition problems at kindergarten, and those problems share the same modality as WA problems. However, the standard algorithms were replaced by counting strategy as their predominant strategy for Level C and D's OA problems. In fact, they could have also used standard algorithms to solve OA problems, but why were they using counting which is less efficient and more prone to error? We speculated that they might have memorized the procedural aspect of the standard algorithms without understanding the base-10 concept and place-value notation. For example, two-digit problems can also be solved by standard algorithm by simply following the steps, such as aligning all the addends of a given addition problem vertically, adding the columns, beginning from the right column and an extra digit will be "carried" into the next column if that column exceeds nine. Together with the earlier discussion on problems comprising of 10, it is plausible that Singapore kindergarteners might be representing multi-digit numbers in one-to-one collection, whereas the Japanese kindergarteners were representing those numbers in structures of tens and ones.

Conclusion

In this study, we have presented that 6-year-old kindergarteners could solve not only simple addition (e.g., 3+6), but also complex addition that comprises of two-digit addends (e.g., 16+12). The result suggests that they were much more competent than what has been commonly reported in many previous studies that focused mainly on simple arithmetic. There were no significant gender differences among both countries' participants. This implies that the arithmetical abilities of both countries' kindergarteners were not influenced by gender. Similar to our previous study (Authors 2016), we continue to maintain that their experiences in kindergarten could be one of the main contributors to their performances. Thus, frequent use of oral numerals in Japanese kindergarten activities is likely to result in better performance in solving OA problems than WA problems among the Japanese kindergarteners. While learning mathematics through formal lessons which often involve the use of written numerals has seemed to train Singapore kindergarteners to become more proficient in handling WA problem than those of OA.

Another key strength of this study is that it provides the window into the world of how young children extend their nascent knowledge to solve complex arithmetic. Here, we examined and compared the strategy use of Japanese and Singaporean kindergarteners. Even though it might be difficult to determine the precise influence of cultural and contextual factors on children's mathematical development, by examining the differences between their arithmetical performances and solving strategies, we suggest that their mathematical experiences in kindergarten and the number system they used for counting are two important contributors for those cross-cultural differences. And understanding those differences gives some new directions for future research, especially those related to the understanding of the base-10 concept and its application among young children. For example, as we look at the strategies used by Japanese and Singaporean kindergarteners in this study, it has become increasingly clear that the former group tended to have a clearer understanding of base-10 concept and place value than the latter group. Specifically, Singaporean kindergarteners, who have English as first language, might not have a good grasp of base-10 concept and place value. So as Rasmussen et al. (2006) reported that counting in English might obscure the transparency of East Asian languages, resulting in bilingual children faring worse than monolingual East Asian children in terms of their counting competency.

This study goes beyond prior studies and suggests that young children's understanding of base-10 concept, place value, and the ability to use more advanced strategies, such as base-10 decomposition to solve complex arithmetic, might also be hampered to a certain extent, as reflected in the case of Singaporean kindergarteners. However, to better understand the influence of language on children's arithmetical abilities, future studies will include kindergarteners from other contexts into the present comparison between Japanese and Singaporean kindergarteners. For example, kindergarteners who are exposed to less formal, play-based curriculum where English is the only instructional language (e.g., Australia, New Zealand). And another group of kindergarteners who attend formal mathematics lesson in kindergarten where East Asian language, such as Chinese is the medium of instruction (e.g., parts of mainland China).

Next, although many researchers maintain that children's understanding of base-10 number concept and its related principles, such as place value often do not develop until a later age (e.g., Kerbs, Squire, and Bryant 2003) and would only emerge with formal schooling (Geary 1995; Naito and Miura 2001), Singaporean kindergarteners did not appear to benefit from their formal mathematics lessons. Instead, Japanese kindergarteners who did not receive any form of a mathematical lesson tended to have a better grasp of the base-10 concept and readily extend their prior experience to solve those novel complex addition with base-10 decomposition strategy. This finding makes us question whether young children's abilities to understand base-10 concepts would emerge and enhance only by formal schooling and direct instruction. Therefore, future studies are suggested to focus on the mechanism of how informal support by teachers that connect meaningfully to children's lives and promote young children's understanding of the base-10 concept and its application without even relying on those conventional systematic teaching. Exploring a free play-based Japanese kindergarten curriculum could be a good embarking point.

Note

1. In the case in which both cells were zero for both countries (e.g. Table 4's cells for retrieval at Level C), those data were excluded for Chi-square test.

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Figure 1. Test scores over the two modalities

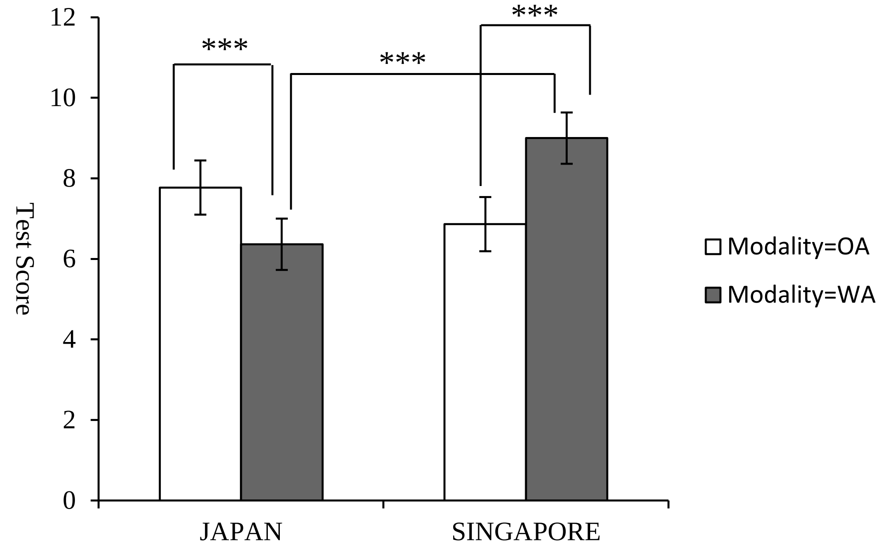

Table 1. Description of pre-test

<u>*Number recognition task:*</u>
(1) Oral, 5, 9, 15, 28, 55
(2) Written, 4, 8, 13, 24, 47
<u>*Simple addition task:*</u>
(1) Oral, 1 + 1, 2 + 2, 1 + 2
(2) Written, 1 + 1, 2 + 1, 2 + 2

Table 2. Description of main addition task

Level	Written Arithmetic (OA)	Oral Arithmetic (OA)	Details
A	3+4, 7+1, 2+6	2+3, 5+1, 3+6	Single-digit addends, sum less than 10
B	5+10, 9+10, 10+12	2+10, 10+8, 14+10	At least one addend which is 10
C	16+12, 13+11, 21+17	12+13, 17+11, 23+14	Sum more than 20, absence of carrying operations
D	17+16, 23+19, 36+28	15+17, 25+16, 33+29	Sum more than 20, presence of carrying operations

Table 3. Types of addition strategies used by kindergarteners

Strategy	Explanation	Examples
Counting	Counts from number "1" all the way to the second addend (count-all) or from one of the addends to the other addend (Count-on)	Problem: *5+4* Count-all, fingers: Child displays 5 fingers on one hand and 4 fingers on the other hand, then started counting all his fingers from 1 to 9 (1,2,3,4,5,6,7,8,9). Count all, round cards: Child first placed 5 round cards on the table, before placing another 4 more. He/she then started counting all the cards from 1 to 9 (1,2,3,4,5,6,7,8,9). Count-on verbal: child says "6,7,8,9"
Standard algorithm	Aligns all the addends vertically, adding the columns, beginning from the right column and an extra digit will be "carried" into the next column if that column exceeds nine	*Problem: 12+13* 12 +13 25
Base-10 Decomposition	Transforms the original problem into two or more simpler problems using base-10	Problem: *16 + 12* Child explains, "10 + 10=20, 6 + 2=8, 20 and 8 gives you 28"
Retrieval	Solves problems within 3 seconds with no overt counting behaviour	Problem: *2 + 3* Child explains, "the answer 5 is in my head"
Others	Reports guessing or not knowing, or the strategy cannot be discerned	

Table 4 Usage of addition strategies among Japanese and Singaporean in OA task

	Level A		Level B		Level C		Level D	
Singaporean	Freq.	%	Freq.	%	Freq.	%	Freq.	%
Counting	15	24.6%	32	60.4%	19	63.3%	6	85.7%
Standard Algorithm	0	0%	0	0%	5	16.7%	0	0%
Decomposition	0	0%	0	0%	4	13.3%	1	14.3%
Retrieval	46	75.4%	19	35.8%	0	0%	0	0%
Others	0	0%	2	3.8%	2	6.7%	0	0%
Correctly answered (Total)	61	100%	53	100%	30	100%	7	100%

Japanese	Freq.	%	Freq.	%	Freq.	%	Freq.	%
Counting	20	32.3%	15	27.8%	10	27.8%	3	15.8%
Standard Algorithm	0	0%	0	0%	0	0%	0	0%
Decomposition	0	0%	5	9.3%	25	69.4%	16	84.2%
Retrieval	40	64.5%	33	61.1%	0	0%	0	0%
Others	2	3.2%	1	1.9%	1	2.8%	0	0%
Correctly answered (Total)	62	100%	54	100%	36	100%	19	100%

Note ％ round to 1 decimal point

Table 5 Usage of addition strategies among Japanese and Singaporean in WA task

	Level A		Level B		Level C		Level D	
Singaporean	Freq.	%	Freq.	%	Freq.	%	Freq.	%
Counting	19	28.8%	26	44.4%	7	17.1%	6	18.8%
Standard Algorithm	0	0%	8	13.6%	28	68.3%	20	62.5%
Decomposition	0	0%	1	1.7%	6	14.6%	6	18.7%
Retrieval	47	71.2%	21	35.6%	0	0%	0	0%
Others	0	0%	3	5.1%	0	0%	0	0%
Correctly answered (Total)	66	100%	59	100%	41	100%	32	100%

Japanese	Freq.	%	Freq.	%	Freq.	%	Freq.	%
Counting	18	30.0%	13	27.1%	5	23.8%	3	27.3%
Standard Algorithm	0	0%	0	0%	0	0%	0	0%
Decomposition	0	0%	5	10.4%	16	76.2%	8	72.7%
Retrieval	40	66.7%	30	62.5%	0	0%	0	0%
Others	2	3.3%	0	0%	0	0%	0	0%
Correctly answered (Total)	60	100%	48	100%	21	100%	11	100%

Note ％round to 1 decimal point

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