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# Design of PLC $E_{31}$ - $E_{13}$ and $E$ -LP tapered mode converters using fast quasiadiabatic dynamics

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**Abstract** The previous PLC  $E_{31}$  and  $E_{13}$  mode converters employ linear tapered structures to fulfill the adiabatic coupling condition and achieve a high mode conversion efficiency, which requires long taper lengths. In this paper, we propose two types of PLC tapered structures that can be applied in different scenarios using fast quasiadiabatic dynamics. These structures enable mode conversions of  $E_{31}$ - $E_{13}$  modes, as well as  $E_{31}$ -LP<sub>02</sub> and  $E_{13}$ -LP<sub>21b</sub> modes, respectively. The  $E_{31}$ - $E_{13}$  mode converter can be reduced from the previous three-stage linear taper of 12000  $\mu\text{m}$  to approximately 4000  $\mu\text{m}$  in length. Similarly, the  $E_{31}$ -LP<sub>02</sub>,  $E_{13}$ -LP<sub>21b</sub> mode converter can be reduced from the previous two-stage linear taper of over 6000  $\mu\text{m}$  to approximately 4000  $\mu\text{m}$  in length.

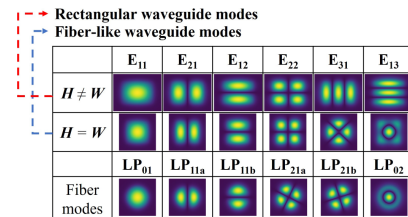
**Keywords:** mode converter, space division multiplexing, mode division multiplexing

**Classification:** Optical hardware

## 1. Introduction

With the rapid increase in Internet traffic, the demand for communication capacity expansion of optical fiber is increasing [1]. Since the 1980s, optical fiber communication technology has experienced several technical innovations, and the development of multiplexing technologies such as wavelength division multiplexing (WDM), time division multiplexing (TDM), and polarization division multiplexing (PDM) have increased the communication capacity of single-mode fiber. However, the communication capacity of conventional optical fiber is approaching its limit. In order to further expand the communication capacity, mode division multiplexing (MDM) transmission, which is one of the spatial division multiplexing (SDM) technologies, has received attention [2]. MDM can further expand the transmission capacity by using multiple spatial modes propagating in one optical waveguide as transmission channels.

The mode multiplexers (MUXs) or demultiplexers (DMUXs) are used to connect single-mode and multi-mode systems to realize the combination or separation of spatial modes in the MDM system. At present, on-board mode MUXs are primarily composed of Si waveguides [3, 4, 5, 6], polymer materials [7, 8, 9], and SiO<sub>2</sub>-based planar lightwave circuits (PLC) [10, 11, 12, 13, 14, 15, 16]. This study focused



**Fig. 1** Corresponding relationship between rectangular waveguide modes and optical fiber modes.

on the SiO<sub>2</sub>-based PLC which has advantages such as low loss and easy connection with optical fibers [12]. In the past few years, we have proposed two types of 6-mode MUXs based on asymmetric directional coupler (ADC) structures and photonic lantern (PL) structures, respectively [12, 16]. In the ADC-based 6-mode MUX, to achieve 6-mode multiplexing, a linear taper waveguide structure is utilized to realize the conversion from the  $E_{31}$  mode to the  $E_{13}$  mode, enabling the excitation of the  $E_{13}$  mode [12]. And in the 6-mode MUX based on PL, the output waveguide is rectangular in shape (which means that waveguide width  $W$  and waveguide height  $H$  are not equal), resulting in waveguide modes being emitted [16]. To facilitate better connectivity with optical fibers, it is necessary to convert the waveguide modes into fiber-like waveguide modes as shown in Fig. 1. This can be achieved by using a taper waveguide structure to transition from a rectangular waveguide ( $W \neq H$ ) to a square waveguide ( $W = H$ ) [17].

The previous PLC  $E_{31}$  and  $E_{13}$  mode converters used linear taper structures to fulfill the adiabatic coupling condition and achieve a high mode conversion efficiency, which requires long taper lengths. In recent years, a quantum-mechanics-inspired technique known as shortcut to adiabaticity (STA) [18, 19], has been increasingly applied in the field of optics [20, 21]. Recently, a novel STA protocol called fast quasiadiabatic dynamics (FAQUAD) has been proposed [22]. This method aims to accelerate the adiabatic coupling process by controlling the adiabatic parameters to achieve a uniform distribution during the coupling process. Devices such as Y-branch waveguides [26, 27, 28] and ADCs [23, 24, 25] designed using the FAQUAD approach have been reported. These devices achieve adiabatic coupling even with relatively short device lengths. However, current research on the FAQUAD protocol is primarily limited to the design of Si and polymer platforms. Although the SiO<sub>2</sub>-based PLC mode converter for  $E_{12}$  and  $E_{21}$  modes has been proposed [29], only the 2LP mode operation is considered

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with complicated structures, and higher order modes have not been investigated.

In this paper, we propose two types of PLC taper structures that can be applied in different scenarios using FAQUAD. These structures enable mode conversions of  $E_{31}$ - $E_{13}$ , as well as  $E_{31}$ - $LP_{02}$  and  $E_{13}$ - $LP_{21b}$ , respectively. Compared to the conventional linear structures, the proposed designs significantly reduce the taper length. The  $E_{31}$ - $E_{13}$  mode converter can be reduced from the previous three-stage linear taper of  $12000\ \mu\text{m}$  [12] to approximately  $4000\ \mu\text{m}$  in length. Similarly, the  $E_{31}$ - $LP_{02}$ ,  $E_{13}$ - $LP_{21b}$  mode converter can be reduced from the previous two-stage linear taper of over  $6000\ \mu\text{m}$  [12] to approximately  $4000\ \mu\text{m}$  in length.

## 2. Operation principle and device design

### 2.1 $E_{31}$ - $E_{13}$ mode converter

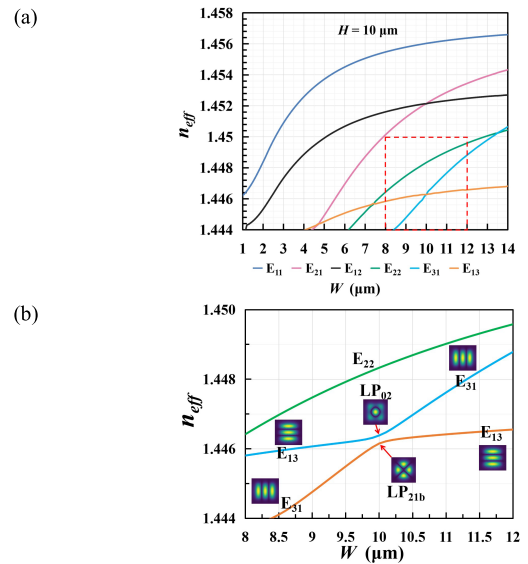
In this study, we used PLC waveguides with  $\text{SiO}_2$  cladding, and the refractive index of the cladding is assumed to be 1.444. The relative refractive index difference between core and cladding is 1.0%. According to references [12], we have fixed the height of the waveguide to  $H = 10\ \mu\text{m}$  and will focus our discussion solely on the waveguide width, denoted as  $W$ . Figure 2(a) shows the calculated waveguide width dependency of the effective refractive index when  $H = 10\ \mu\text{m}$  at the wavelength of  $1.55\ \mu\text{m}$ . In Fig. 2(b), we have presented a zoomed-in view of the region corresponding to  $W$  ranging from  $8\ \mu\text{m}$  to  $12\ \mu\text{m}$ , as shown in the region enclosed by the red dashed line in Fig. 2(a). From Fig. 2(b), it can be seen that we can use a taper waveguide structure with a width ranging from  $9\ \mu\text{m}$  to  $11\ \mu\text{m}$  to achieve  $E_{31}$ - $E_{13}$  mode conversions. Previously, such taper waveguides were constructed using a linear taper or a multi-stage linear taper structure [12]. However, to achieve a gradual adiabatic coupling process, an extremely long taper length  $L$  was required, resulting in a significant increase in the device size.

The adiabatic mode conversion process in optical waveguide devices requires the following relationship.

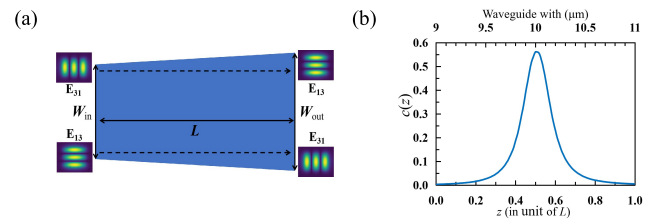
$$c(z) \equiv \left| \frac{\iint \left[ \mathbf{E}_m \times \frac{\partial}{\partial z} \mathbf{H}_n^* \right] \cdot \mathbf{i}_z dx dy}{\beta_m - \beta_n} \right| \ll 1. \quad (1)$$

Here,  $c(z)$  is called the adiabaticity parameter.  $\mathbf{E}_m$  and  $\mathbf{H}_n$  represent the electric field distribution of the  $m$ th eigenmode and the magnetic field distribution of the  $n$ th eigenmode of the waveguide, respectively.  $\beta_m$  and  $\beta_n$  represent the propagation constants of the  $m$ th and  $n$ th modes, respectively. In this case, the  $m$ th mode corresponds to the  $E_{13} \sim LP_{02} \sim E_{31}$  mode (cyan line), and the  $n$ th mode corresponds to the  $E_{31} \sim LP_{21b} \sim E_{13}$  mode (orange line) in Fig. 2(b).

Figure 3(a) shows the linear taper structure with input waveguide width  $W_{\text{in}} = 9\ \mu\text{m}$  and output waveguide width  $W_{\text{out}} = 11\ \mu\text{m}$ . The waveguide height is fixed to  $H = 10\ \mu\text{m}$ . Figure 3(b) shows the adiabaticity parameter  $c(z)$  for the  $E_{31}$  and  $E_{13}$  or these hybrid modes in the linear taper. (The top axis represents the width of the linear taper waveguide, while the bottom axis represents the position  $z$  normalized to the taper length  $L$  in the propagation direction.) From Fig. 3, we can observe that the  $c(z)$  of the linear taper waveguide



**Fig. 2** (a) Waveguide width dependency of the effective refractive index when  $H = 10\ \mu\text{m}$ . (b) The detail image of the region of  $W = 8\text{--}12\ \mu\text{m}$ , as shown in the region enclosed by the red dashed line in Fig. 2(a).



**Fig. 3** (a) Linear taper structure with input waveguide width  $W_{\text{in}} = 9\ \mu\text{m}$  and output waveguide width  $W_{\text{out}} = 11\ \mu\text{m}$ . (b) The adiabaticity parameter  $c(z)$  for the  $E_{31}$  and  $E_{13}$  or these hybrid modes in the linear taper waveguide from  $W_{\text{in}} = 9\ \mu\text{m}$  to  $W_{\text{out}} = 11\ \mu\text{m}$ .

sharply increases within the range of  $9.5\ \mu\text{m}$  to  $10.5\ \mu\text{m}$  in width. This implies that to satisfy the adiabatic coupling condition, the linear taper waveguide requires a sufficiently long waveguide length  $L$  to compensate for the large  $c(z)$ .

The FAQUAD method accelerates the adiabatic coupling evolution process by controlling the adiabatic parameters to achieve a uniform distribution during the adiabatic coupling process. The key to the FAQUAD method is to achieve a uniform distribution of the adiabatic parameter along the  $z$ -direction by fixing the adiabaticity parameter in Eq. (1) to a constant value  $\varepsilon$ . In other words, according to the following procedure [24], we should find  $z_{\text{opt}}(W)$  satisfying  $c(z) = \varepsilon$ , where  $\varepsilon$  is a certain constant value. At the beginning, using the chain rule for  $z$  and  $W$ , Eq. (1) is rewritten as

$$c(z) = \left| \frac{dW}{dz} \right| \cdot c_W(W) = \varepsilon, \quad (2)$$

$$c_W(W) = \left| \frac{\iint \left[ \mathbf{E}_m \times \frac{\partial}{\partial W} \mathbf{H}_n^* \right] \cdot \mathbf{i}_z dx dy}{\beta_m - \beta_n} \right|. \quad (3)$$

Since  $dW/dz$  is a constant in the linear taper waveguide, the adiabatic parameter for the linear taper waveguide as a function of  $W$  can be expressed as

$$c_{\text{lin}}(W) = \frac{|W_{\text{out}} - W_{\text{in}}|}{L} c_W(W). \quad (4)$$

Substituting Eq. (2) into Eq. (4) and inverting  $z$  and  $W$ , we can obtain the following relationship.

$$\frac{dz}{dW} = \frac{L}{W_{\text{out}} - W_{\text{in}}} \cdot \frac{c_{\text{lin}}(W)}{\varepsilon}, \quad (5)$$

noting that the sign of  $dz/dw$  should be coincident with that of  $W_{\text{out}} - W_{\text{in}}$ . Integrating Eq. (5) with the condition of  $z(W_{\text{in}}) = 0$ , we can obtain  $z_{\text{opt}}(W)$  as

$$z_{\text{opt}}(W) = \frac{L}{W_{\text{out}} - W_{\text{in}}} \frac{1}{\varepsilon} \int_{W_{\text{in}}}^W c_{\text{lin}}(t) dt, \quad (6)$$

where  $t$  is an integration variable. Noting that the length of the optimized taper given by  $z_{\text{opt}}$ , which is  $L_{\text{opt}} = z_{\text{opt}}(W_{\text{out}})$ , depends on  $\varepsilon$ . If we fix  $\varepsilon$  to satisfy  $L_{\text{opt}} = L$ ,  $\varepsilon$  is derived as

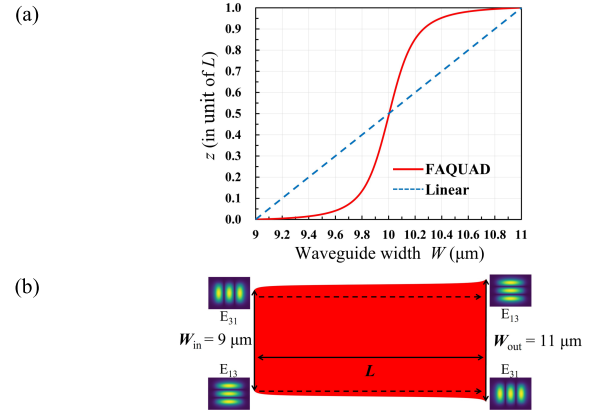
$$\varepsilon = \frac{\int_{W_{\text{in}}}^{W_{\text{out}}} c_{\text{lin}}(t) dt}{W_{\text{out}} - W_{\text{in}}}. \quad (7)$$

Substituting Eq. (7) into Eq. (6), we can finally obtain the structure of the FAQUAD taper waveguide with the taper length of  $L$  as the following expression.

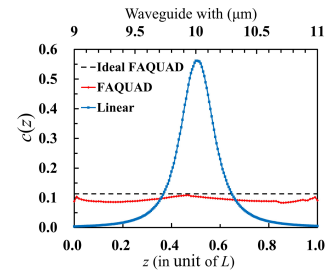
$$z_{\text{opt}}(W) = L \frac{\int_{W_{\text{in}}}^W c_{\text{lin}}(t) dt}{\int_{W_{\text{in}}}^{W_{\text{out}}} c_{\text{lin}}(t) dt}. \quad (8)$$

By achieving a uniform distribution of  $c(z)$  across the entire device, the length of the stronger coupling region can be adjusted to satisfy the adiabatic coupling condition, while the length of the weaker coupling region is minimized, thereby the length of the adiabatic taper waveguide can be greatly reduced. Figure 4(a) shows the shape parameters  $z_{\text{opt}}(W)$  of the linear and FAQUAD taper waveguides with waveguide widths ranging from  $W_{\text{in}} = 9 \mu\text{m}$  to  $W_{\text{out}} = 11 \mu\text{m}$ . Figure 4(b) shows the structure of the FAQUAD taper obtained using the shape parameters from Fig. 4(a). Figure 5 shows the adiabaticity parameter  $c(z)$  of the ideal FAQUAD taper using Eq. (7) (black dashed line), linear taper (blue line), and shape designed FAQUAD taper waveguide (structure in Fig. 4(b)) using Eq. (1) (red line). From Fig. 5, the adiabaticity parameter of the FAQUAD taper waveguide approaches that of the ideal FAQUAD design.

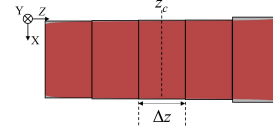
Since the waveguide shape parameters have been determined, the next step is to optimize the waveguide length by scanning the taper length  $L$ . Here, we use two calculation methods, BPM (Beam Propagation Method) and CLMT (Coupled Local Mode Theory), for the optimization of  $L$ . CLMT is a computationally accurate method for analyzing waveguide structures with varying widths. Figure 6 shows the approximated structure of a FAQUAD taper used in CLMT, which approximates the analytical structure as a continuous cuboid interval of length  $\Delta z$ . The field distribution at any arbitrary propagation position is approximated using the mode of the XY cross-section at the corresponding interval center  $z = z_c$ . The local coupled-mode equations



**Fig. 4** (a) Shape parameters  $z(W)$  for the linear and FAQUAD taper waveguides with waveguide widths ranging from  $W_{\text{in}} = 9 \mu\text{m}$  to  $W_{\text{out}} = 11 \mu\text{m}$ . (b) The structure of FAQUAD taper obtained using the shape parameters from Fig. 4(a).



**Fig. 5** Adiabaticity parameter  $c(z)$  for the linear taper (blue line), ideal FAQUAD taper (black dashed line), and shape designed FAQUAD taper waveguide (red line).



**Fig. 6** Approximated structure of a FAQUAD taper used in CLMT.

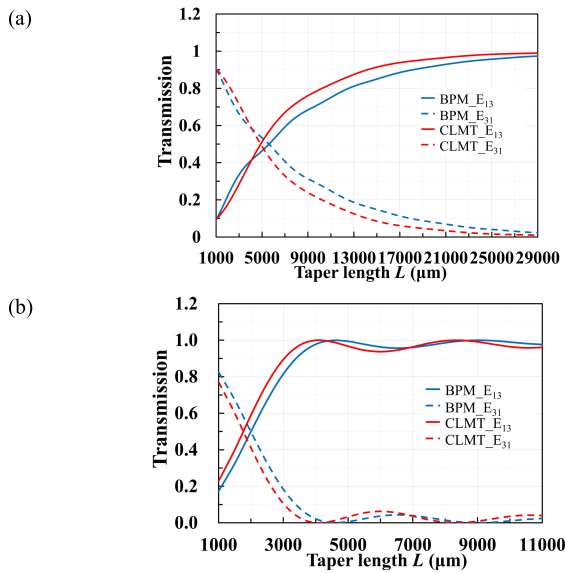
of CLMT are expressed as follows [30].

$$\frac{da_m}{dz} = -\kappa_{mn}(z) \exp \left\{ j \int_0^z [\beta_m(z) - \beta_n(z)] dz \right\} a_n \quad (9)$$

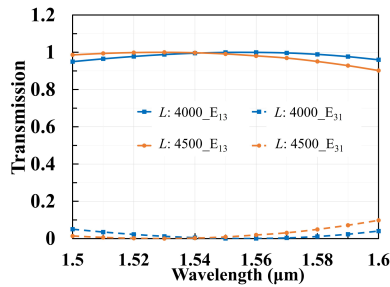
$$\kappa_{mn}(z) = \iint \left[ \mathbf{E}_m \times \frac{\partial}{\partial z} \mathbf{H}_n^* \right] \cdot \mathbf{i}_z dx dy \quad (10)$$

Here,  $a_m$ ,  $a_n$  are the complex amplitude coefficients of the  $m$ th and  $n$ th modes, respectively. And  $\kappa_{mn}$  represents the coupling coefficient between the  $m$ th and  $n$ th modes.

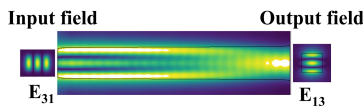
Figures 7(a) and (b) show the relationship between the transmission for the input  $E_{31}$  mode and the waveguide length  $L$  in the case of (a) linear taper and (b) FAQUAD taper, when  $W_{\text{in}} = 9 \mu\text{m}$  and  $W_{\text{out}} = 11 \mu\text{m}$ , respectively. From Fig. 7, it can be seen that the results of CLMT are basically consistent with those of BPM. According to the CLMT results, it is determined that a linear taper requires an  $L$  of  $14200 \mu\text{m}$  to achieve a transmission over 90%, while an  $L$  of  $30000 \mu\text{m}$  is needed to achieve a transmission over 99%. The FAQUAD taper only needs a length of about  $4000 \mu\text{m}$  to achieve a transmission of over 99%. Figure 8



**Fig. 7** Relationship between the transmission for the input  $E_{31}$  mode and  $L$  in the case of (a) linear taper and (b) FAQUAD taper, when  $W_{in} = 9 \mu\text{m}$  and  $W_{out} = 11 \mu\text{m}$ .



**Fig. 8** Relationship between the transmission for the input  $E_{31}$  mode and the wavelength for FAQUAD  $E_{31}$ - $E_{13}$  mode converter at various waveguide lengths.

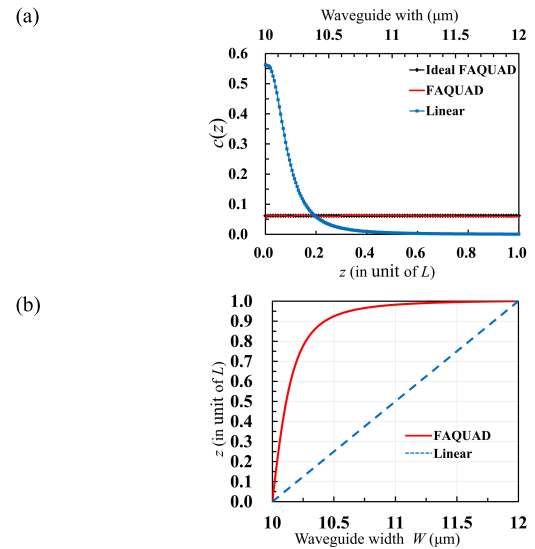


**Fig. 9** The field distribution when  $E_{31}$  mode is input from the input port. The taper length  $L = 4000 \mu\text{m}$ , at the wavelength of  $1.55 \mu\text{m}$ .

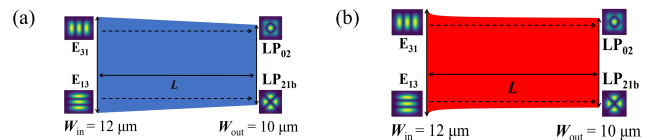
shows the relationship between the transmission for the input  $E_{31}$  mode and the wavelength of the FAQUAD  $E_{31}$ - $E_{13}$  mode converter at various waveguide lengths (calculated by CLMT). According to Fig. 8, when  $L = 4000 \mu\text{m}$  or  $L = 4500 \mu\text{m}$ , the device can maintain transmission of over 90% within the wavelength range of  $1.5 \mu\text{m}$  to  $1.6 \mu\text{m}$ . Figure 9 shows the field distribution of the  $E_{31}$  mode input when  $L = 4000 \mu\text{m}$ , at the wavelength of  $1.55 \mu\text{m}$ , which proves that the device can realize the  $E_{31}$ - $E_{13}$  mode conversion with a shorter waveguide length.

## 2.2 $E_{31}$ - $LP_{02}$ , $E_{13}$ - $LP_{21b}$ mode converter

In the 6-mode MUX based on PL, the output waveguide is rectangular in shape, resulting in waveguide modes being emitted. To facilitate better connectivity with optical fibers, a tapered waveguide structure is needed at the output port to convert the rectangular waveguide into a square waveguide [17]. This enables the conversion of waveguide modes  $E_{31}$



**Fig. 10** (a) Adiabaticity parameter  $c(z)$  for the linear taper (blue line), ideal FAQUAD taper (black line), and shape designed FAQUAD taper waveguide (red line). (b) Shape parameters  $z(W)$  for the linear and FAQUAD taper waveguides with waveguide widths ranging from  $W_{in} = 12 \mu\text{m}$  to  $W_{out} = 10 \mu\text{m}$ .

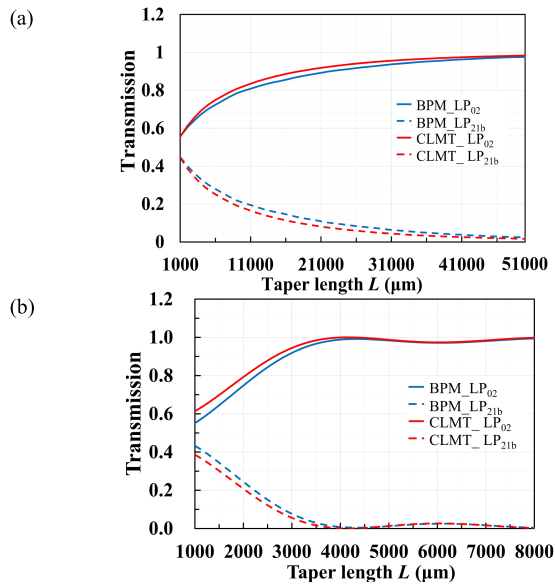


**Fig. 11** (a) The structure of the linear E-LP taper. (b) The structure of the FAQUAD E-LP taper.

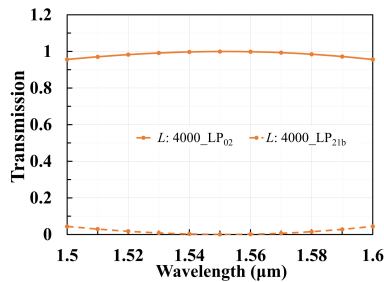
and  $E_{13}$  to fiber-like modes  $LP_{02}$  and  $LP_{21b}$ , respectively. To connect with the PL 6-mode MUX, we set the waveguide width at the input port of the mode converter to  $W_{in} = 12 \mu\text{m}$ , with the waveguide height fixed at  $H = 10 \mu\text{m}$  [16]. As mentioned in the previous section and indicated in Fig. 2(b), the  $E_{31}$ - $LP_{02}$ ,  $E_{13}$ - $LP_{21b}$  mode converter (hereafter referred to as E-LP taper) can be achieved through a taper waveguide with the waveguide width from  $W_{in} = 12 \mu\text{m}$  at the input port to  $W_{out} = 10 \mu\text{m}$  at the output port.

Figure 10(a) shows the adiabatic parameter  $c(z)$  of the ideal FAQUAD taper using Eq. (7) (black line), linear taper (blue line), and shape designed FAQUAD taper waveguide (structure in Fig. 11(b)) using Eq. (1) (red line). Figure 10(b) shows the shape parameters  $z(W)$  of the linear and the FAQUAD E-LP taper with waveguide widths ranging from  $W_{in} = 12 \mu\text{m}$  to  $W_{out} = 10 \mu\text{m}$ . Figures 11(a) and (b) show the structure of the linear E-LP taper and the FAQUAD E-LP taper, respectively.

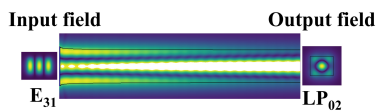
Figures 12(a) and (b) show the relationship between the transmission for the input  $E_{31}$  mode and the waveguide length  $L$  in the case of (a) linear E-LP taper and (b) FAQUAD E-LP taper, when  $W_{in} = 12 \mu\text{m}$  and  $W_{out} = 10 \mu\text{m}$ , respectively. From Fig. 12, it can be seen that the results of CLMT are basically consistent with those of BPM. According to the CLMT results, it is determined that a linear taper requires an  $L$  of  $18000 \mu\text{m}$  to achieve a transmission over 90%, while over  $50000 \mu\text{m}$  is needed to achieve a transmission over 99%. The FAQUAD taper only needs a length of about  $4000 \mu\text{m}$  to achieve a transmission of over 99%. Figure 13 shows



**Fig. 12** Relationship between the transmission for the input  $E_{31}$  mode and  $L$  for the case of (a) linear E-LP taper and (b) FAQUAD E-LP taper, with  $W_{\text{in}} = 12 \mu\text{m}$  and  $W_{\text{out}} = 10 \mu\text{m}$ .



**Fig. 13** Relationship between the transmission for the input  $E_{31}$  mode and the wavelength for FAQUAD  $E_{31}$ - $\text{LP}_{02}$ , and  $E_{13}$ - $\text{LP}_{21b}$  mode converter at various waveguide lengths.



**Fig. 14** The field distribution when  $E_{31}$  mode is input from the input port. The taper length  $L = 4000 \mu\text{m}$ , at the wavelength of  $1.55 \mu\text{m}$ .

the relationship between the transmission for the input  $E_{31}$  mode and the wavelength of the FAQUAD  $E_{31}$ - $\text{LP}_{02}$ , and  $E_{13}$ - $\text{LP}_{21b}$  mode converter when the waveguide length is  $L = 4000 \mu\text{m}$  (calculated by CLMT). According to Fig. 13, when  $L = 4000 \mu\text{m}$ , the device can maintain transmission of over 95% within the wavelength range of  $1.5 \mu\text{m}$  to  $1.6 \mu\text{m}$ . Figure 14 shows the field distribution of the  $E_{31}$  mode input when  $L = 4000 \mu\text{m}$ , at the wavelength of  $1.55 \mu\text{m}$ , which proves that the device can realize the  $E_{31}$ - $\text{LP}_{02}$  mode conversion with a shorter waveguide length.

### 3. Conclusion

We proposed two types of PLC tapered structures that can be applied in different scenarios using FAQUAD. These structures enable mode conversions of  $E_{31}$ - $E_{13}$ , as well as  $E_{31}$ - $\text{LP}_{02}$  and  $E_{13}$ - $\text{LP}_{21b}$ , respectively. In the case of the

$E_{31}$ - $E_{13}$  mode converter, compared with the one-stage linear taper greater than  $30000 \mu\text{m}$ , and the previous three-stage linear taper of greater than  $12000 \mu\text{m}$  [12], the FAQUAD taper can achieve a transmission over 99% with a length of  $4000 \mu\text{m}$ . In the case of the  $E_{31}$ - $\text{LP}_{02}$ , and  $E_{13}$ - $\text{LP}_{21b}$  mode converter, compared with the one-stage linear taper greater than  $50000 \mu\text{m}$ , and the previous two-stage linear taper of greater than  $6000 \mu\text{m}$  [12], the FAQUAD taper can achieve a transmission over 99% with a length of  $4000 \mu\text{m}$ .

### Acknowledgments

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