学 位 論 文 内 容 の 要 旨

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学 位 論 文 題 名

A Study on Mixed Precision Iterative Refinement using Low Precision Krylov Methods
（低精度クリロフ部分空間法を用いた混合精度反復改良法に関する研究）

Linear algebra has various applications in many aspects of life and is at the heart of computational aspects of practical application problems. In practical applications, a problem is often transformed into a problem in linear algebra, e.g., a linear system, an eigenvalue problem, etc. Thus, numerical methods for solving these problems in linear algebra play a crucial role and have attracted much attention in engineering and information science. Among them, a linear solver, which solves a linear system $Ax = b$, is one of the vital building blocks in scientific computations and is the performance bottleneck of the computational process in many applications. An efficient linear solver helps improve the computational efficiency of practical applications. Therefore, more efficient linear solvers are strongly required to accelerate application programs.

Traditionally, double-precision floating-point number (FP64) has been widely used as a standard in scientific computations. However, due to the limitation of the power budget, it is getting difficult to improve the performance of FP64. Meanwhile, some new hardware, such as GPU with Tensor Cores, appears, in which powerful performance of low precision computing is provided. Besides, many new applications, such as AI/Machine Learning, accept and have already used low precision computing. These situations prompt the development of low precision computing techniques in the field of numerical linear algebra. In algorithms of numerical linear algebra, low precision computing should be fully utilized, while the output accuracy needs to be guaranteed at the same time. A key approach to develop such algorithms is mixed precision (MP) computing, which efficiently combines different precisions and achieves the same computational accuracy as the traditional method using only FP64.

In this study, I focus on solving a linear system of equations $Ax = b$, where A is large, sparse, and non-symmetric. For this problem, the Krylov subspace methods are widely used in the field of numerical linear algebra. Some studies on mixed precision computing for the Krylov subspace methods have already been reported, however, there are still many issues to be investigated. This study aims to provide new insights on mixed precision computing using the Krylov subspace methods, which will contribute to improving the computation performance of various applications that need liner solvers.

This study considers two Krylov subspace methods, which are commonly used to solve non-symmetric linear systems. One is the GMRES(m) method, and the other is the BiCGSTAB method. Based on these two algorithms, I develop mixed precision algorithms and conduct comprehensive numerical experiments using 26 test matrices selected from the SuiteSparse Matrix Collection. Through numerical experiments, I investigate the numerical characteristics of the developed mixed precision algorithms and evaluate their effectiveness in detail.

In Chapter 3, a mixed precision variant of the GMRES(m) method using FP64 and FP32, which is called MP-GMRES(m), is investigated. Through comprehensive experiments with different settings of the restart frequency m, I have studied its numerical behavior and compared it with the traditional GMRES(m) method using only FP64 from both theoretical and practical aspects. Detailed analysis and comparison of the obtained results are given from the following three aspects: the maximum attainable accuracy, the number of iterations, and the execution time. The obtained results indicate that MP-GMRES(m) has almost the same problem-solving ability as GMRES(m). Although MP-GMRES(m) needs more iterations than GMRES(m), the execution time of MP- GMRES(m) is shorter than that of GMRES(m) for many cases. In addition, I have found differences between these two methods; for example, as m increases, the number of iterations tends to decrease in GMRES(m), while it tends to increase in MP-GMRES(m).

In Chapter 4, a mixed precision variant of iterative refinement using the BiCGSTAB method, which is called MP-IR using BiCGSTAB, is developed and investigated in detail. In this method, the BiCGSTAB method with FP32 is employed as the inner solver in mixed precision iterative refinement, and two approaches are used for determining the restart timing: the number of inner iterations and the decrease of the residual 2-norm in the inner BiCGSTAB loop. Several sets of experiments are conducted, and the obtained results are analyzed and compared with the traditional BiCGSTAB method, as well as MP-GMRES(m), which is studied in Chapter 3. From the obtained results, I have found that MP-IR using BiCGSTAB sometimes outperforms MP-GMRES(m) and is the fastest method, especially for problems with small condition number, although MP-IR using BiCGSTAB is sensitive to a target problem. Similar to Chapter 3, detailed experimental results on the attainable accuracy, the number of iterations, and the execution time are also provided.

The rest of the thesis is organized as follows. In Chapter 1, I introduce the research background, purposes, and main contributions. The structure of the thesis is also briefly described. Chapter 2 summarizes the related work, including mixed precision computing, the Krylov subspace methods, and some recent research on applying mixed precision computing to the Krylov subspace methods. Chapter 5 provides an overall summary of my research and discusses future work.

In summary, the thesis aims to investigate the numerical characteristics and effectiveness of the mixed precision methods for solving a large, sparse, and non-symmetric linear system. Through the comprehensive numerical experiments and the detailed analyses on the obtained results, many new insights on the mixed precision methods based on iterative refinement using the low precision Krylov subspace methods have been provided. They will contribute to both accelerating many applications and promoting further development of mixed precision computing in the field of numerical linear algebra.