Abstract of Doctoral Dissertation

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Free multiarrangements and integral expressions of their derivations
(自由多重超平面配置とその導分の積分表示)

Given a reflection group generated by reflections, we can obtain a set of fixed subspaces of the reflections which forms a reflection arrangement. The fixed point set of each reflection is called reflecting hyperplane. In Orlik-Terao's book, they developed general theory of hyperplane arrangements. Many aspects of hyperplane arrangements, e.g. combinatorial, algebraic, and topological properties, have been studied.

Studying the freeness of a central arrangement $\mathcal{A}$ is to consider its derivation module $D(\mathcal{A})$. When $D(\mathcal{A})$ is free, we say $\mathcal{A}$ is free. It is well known that every reflection arrangement is free. However, most arrangements are not free. On the other hand, the algebraic property of the module $D(\mathcal{A})$ reflects the combinatorial property of $\mathcal{A}$. Terao conjectured the free arrangement $\mathcal{A}$ implies the freeness of the arrangement $\mathcal{B}$ with the same intersection lattices. This conjecture has not been solved even in the three-dimensional case. In a word, studying freeness of arrangements is a major topic.

In the process of studying free arrangements, many researchers have found that multiarrangements play an important role. Many results applied to hyperplane arrangements are generalized to multiarrangements.

Many researchers have studied the freeness of special multiarrangements. But it is quite difficult to construct an explicit basis for multiarrangements, even for two-dimensional cases. The doctoral thesis is inspired by the integral expressions of quasi-invariants and based on the joint work with Profs. Feigin and Yoshinaga. We construct the basis for certain multiarrangements by integral expressions.

In the first chapter, we give some definitions and classical theorems of arrangements, multiarrangements and reflection arrangements, we also introduce the monomial group. In the second chapter, we introduce the integral expressions and then construct a basis for multiarrangements defined in the first chapter. In the third chapter, we recall the primitive derivations, and observe its action on the integral expressions. In the fourth chapter, we give more applications for integral expressions.