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Skyrmion generation via Laguerre–Gaussian beam irradiation in frustrated magnets

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Since their discovery, the study of magnetic skyrmions has been on the rise. In this paper, we discuss our investigations on the light-induced mechanisms for skyrmion generation in a centrosymmetric triangular magnetic lattice with competing J_1 – J_3 interactions, and easy-axis anisotropy. We solve the stochastic Landau–Lifshitz–Gilbert equation for the lattice spin dynamics under Laguerre–Gaussian beam irradiation. Numerical results show that skyrmions are nucleated in two thermodynamic regions, each favoring different phases: the ferromagnetic phase and the skyrmion-lattice phase. In the ferromagnetic region, isolated skyrmions are generated mainly through stochastic thermal nucleation. In this regime, higher temperatures and larger beam widths are required to overcome the nucleation barrier. In contrast, in the skyrmion-lattice region, skyrmion nucleation occurs via thermal annealing, where the system relaxes toward its true ground state. These findings establish a comprehensive theoretical framework for optimizing optical control to generate light-induced skyrmionic textures in frustrated magnets.

I. INTRODUCTION

A magnetic skyrmion is a topological excitation with a noncoplanar vortex geometry [1–4]. Having been conceptualized in 1961 [1], magnetic skyrmions were discovered within helimagnets in 2009 [5, 6]. The discovery of skyrmions highlighted the importance of topology in condensed matter systems, where nontrivial spin textures are characterized by quantized topological invariants [7, 8]. The spin structure of a skyrmion induces an effective magnetic flux on spin-polarized electrons and magnons, leading to the topological Hall effect [9–15]. The motion of a skyrmion can be manipulated by electric current, thermal gradient, and strain [13, 15–20]. In particular, skyrmions that move in low electric currents have sparked the development of skyrmionics, with the aim of developing next-generation data devices [4, 21–25].

Most of the earlier research on skyrmions has focused on the role of the Dzyaloshinskii–Moriya (DM) interaction in skyrmion formation [26–29]. Although the DM interaction only directly promotes the spiral spin configuration, a skyrmion lattice can arise if both the DM interaction and an optimal magnetic field are applied to a triangular lattice [3, 30–32]. Increasing the magnetic field further above a critical field results in the transition to the ferromagnetic state, where isolated skyrmions can exist as topologically protected defects [31, 32]. However, more recent research has shifted focus towards other mechanisms of skyrmion formation in centrosymmetric magnets without the DM interaction [33–37]. Several studies have shown that easy-axis anisotropy in frustrated triangular lattices can stabilize skyrmions in a manner similar to the DM interaction in noncentrosymmetric magnets [38–45].

Beyond skyrmion generation and stability, a recent

topic of interest in skyrmionics is the manipulation of skyrmionic textures through external stimuli such as electric [46–53] and optical fields [54]. In the context of optical fields, Fujita and Sato have demonstrated that structured optical fields provide an effective route to imprint beam profiles onto a chiral magnetic field and thereby generate skyrmionic defects [54, 55]. Through irradiation under different modes of the Laguerre–Gaussian (LG) beam, which is characterized by its vanishing intensity at the center, they succeeded in creating conventional skyrmions, skyrmioniums (2π vortices), and higher-order skyrmion multiplexes ($n\pi$ vortices) on magnets with strong DM interaction.

Skyrmion generation via optical means has been demonstrated experimentally in several magnetic systems. In particular, the creation–annihilation of skyrmions in FeGe via ultrafast light irradiation was achieved by Berruto *et al.*, where they used transmission electron microscopy [50]. In addition, laser-induced skyrmion bubble generation in the ultrathin magnetic film Ta/Fe₇₂Co₈B₂₀/TaO_x has been reported [49]. These studies demonstrate the feasibility of optical manipulation of topological spin textures and have stimulated growing interest in optically controlled skyrmionics.

As for frustration-induced skyrmions, several centrosymmetric materials exhibit the ability to host skyrmion lattices. Most of these materials are Gd- and Eu-based compounds [45]. Namely, skyrmion lattices have been observed in hexagonal materials: Gd₂PdSi₃ [34, 56, 57] and Gd₃Ru₄Al₁₂ [35, 58], as well as in tetragonal materials including GdRu₂Si₂ [59], GdRu₂Ge₂ [60], and EuAl₄ [61]. Motivated by these results, we investigate whether optical imprinting mechanisms can also generate skyrmionic textures in frustrated centrosymmetric magnets, where skyrmions have additional degrees of freedom: vorticity and helicity.

To address this question, we consider the irradiation of LG beams onto a frustrated magnetic sheet without DM interaction and examine the success of generating

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topological textures. In our simulations, the LG beam irradiation is modeled as a stochastic, spatially non-uniform thermal field [54]. We then investigate the resulting spin dynamics by solving the stochastic Landau–Lifshitz–Gilbert (sLLG) equation. Our numerical results demonstrate that LG-beam irradiation can indeed induce two distinct types of skyrmionic spin textures depending on the underlying magnetic ground-state phase: isolated skyrmions in the high-field regime and skyrmion lattices in the intermediate-field regime. These spin textures emerge through different physical mechanisms. In the high-field regime, isolated skyrmions are created via stochastic nucleation, where randomly excited local spots evolve and stabilize into topologically nontrivial defects. In contrast, the intermediate-field region favors the generation of skyrmion lattices. As such, the LG beam-induced heating acts as an annealer, driving the system from the metastable ferromagnetic state towards the true ground state, i.e., the skyrmion lattice. Furthermore, we find that introducing bond-dependent planar anisotropy in conjunction with LG-beam irradiation significantly suppresses the occurrence of antiskyrmions with the negative vorticity, thereby enhancing the selective generation of skyrmions with the positive vorticity.

The rest of the paper is organized as follows. In Sec. II, we discuss the data-generation methods used in the study. Specifically, we explain the triangular Heisenberg model, the LG beam profile, and the variational calculations that allowed us to attain and interpret our results. Sections III and IV dive deeper into the generation mechanism of two skyrmionic spin textures: isolated skyrmion and skyrmion lattice. Section V discusses the influence of bond-dependent planar anisotropy on the distinction between skyrmions and antiskyrmions. Finally, a conclusion is reached in Sec. VI, where we summarize our findings.

II. METHODS

This section explains the methodologies used in the research. Section II A describes the Hamiltonian model used in the simulations. Section II B discusses the LG beam and its properties. Finally, the phase diagram attained through variational calculations can be found in Sec. II C along with the ansatz used in the calculations.

A. Model

We use a frustrated triangular-lattice model with easy-axis anisotropy.

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - H \sum_i S_i^z - A \sum_i (S_i^z)^2. \quad (1)$$

Each spin vector $\mathbf{S}_i$ is normalized with a magnitude of 1. The first term denotes the isotropic exchange in-

teraction, which includes the nearest-neighbor ferromagnetic coupling $J_1 < 0$ and third-nearest-neighbor antiferromagnetic coupling $J_3 > 0$. We set $J_1 = -1$ to be the unit of energy and the lattice constant a is the unit of length in the simulations. The second term denotes the Zeeman coupling due to an external magnetic field H , while the third term denotes the easy-axis anisotropy with anisotropic constant $A > 0$. It should be noted that the long-range dipolar interaction is not explicitly included in the present model for simplicity. Nevertheless, the bond-dependent planar anisotropy introduced in Eq. (16) in Sec. V can partly mimic the anisotropic effect of the dipolar interaction at the nearest-neighbor level.

Due to the C_6 symmetry of the triangular lattice, there are six possible ordering wave vectors $\mathbf{q}_v = \pm Q(\cos 2u\pi/3, \sin 2u\pi/3)$ where $Q = 2\cos^{-1}[(1 + \sqrt{1 - 2J_1/J_3})/2]$ and $v = u + 1 = 1, 2, 3$. These wave vectors are parallel to the nearest-neighbor bond directions. To maintain the boundary conditions of a lattice with $N = L^2$ sites, the wave number must be in the form $Q = 2n\pi/L$ with $n = 1, 2, \dots, L$, where L is the length of one side of the triangular lattice. Preliminary simulations on various lattice sizes and $J_3/|J_1|$ ratios showed that the conclusive results were independent of these parameters. Consequently, the lattice size N and $J_3/|J_1|$ ratio were fixed for the findings presented in this paper. We used a lattice with $N = 60^2 = 3600$ sites, resulting in a commensurate ordering wave number $Q = 2\pi/5$ ($n = 12$) for $J_3/|J_1| = 0.5$.

B. Beam Profile

The LG beams carry intrinsic orbital angular momentum associated with the azimuthal phase factor, $e^{im\phi}$, with azimuthal quantum number m [62, 63]. For $m \neq 0$, this phase structure leads to a phase singularity on the beam axis ($\rho = 0$). To keep the electromagnetic field finite and single-valued, the amplitude necessarily vanishes at the origin, resulting in a characteristic doughnut-shaped intensity profile. We use the following LG beam equation in cylindrical coordinates:

$$u^{\text{LG}}(\rho, \phi) = \frac{1}{\sqrt{|w|}} \left(\frac{\rho}{w}\right)^{|m|} e^{-\frac{2\rho^2}{w^2} + im\phi} \left[L_p^{|m|} \left(\frac{2\rho^2}{w^2} \right) \right], \quad (2)$$

where $L_p^{|m|}$ is the generalized Laguerre function and p is the radial mode index. The LG equation is evaluated at $z = 0$, as it is where the beam waist w is at its minimum ($w = w_0$). Away from $z = 0$, the beam radius expands according to $w(z) = w_0\sqrt{1 + |z|/Z}$, where Z is the Rayleigh range, defined as the propagation distance at which the cross-sectional area of the beam increases by a factor of two, i.e., from πw_0^2 to $2\pi w_0^2$. Figure 1 displays the intensity profiles of LG beams for several representative modes characterized by different values of p and w . LG-beams can be experimentally generated through spatial light modulators and spiral phase plates [64–66].

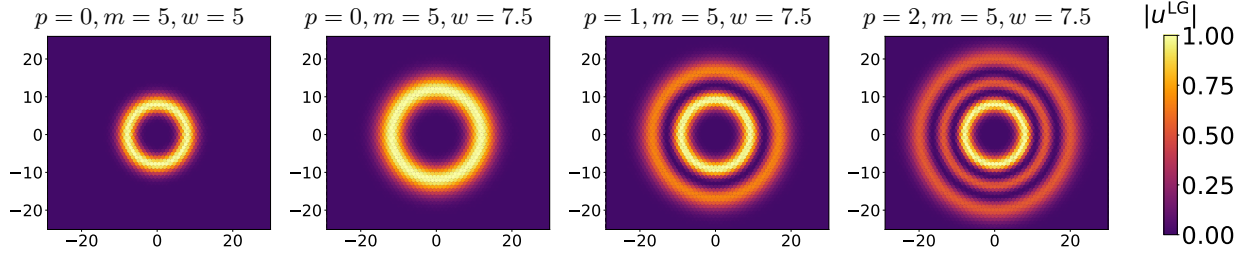


FIG. 1. Beam profiles at different (p, m, w) . The colors denote the local intensity at that site in the triangular lattice.

In the present study, the effect of LG-beam irradiation is modeled purely as a thermal energy transfer process that generates a spatially and temporally dependent temperature profile; explicit spin-photon coupling effects are not included. Other non-thermal effects include the inverse Faraday effect, orbital angular momentum transfer, and ultrafast effects. The inverse Faraday effect can induce an effective localized magnetic field that directly couples to spins and may provide a promising route toward quantum printing or skyrmionium generation [29]. By allowing orbital angular momentum transfer, we can expect control over skyrmion chirality through m in Eq. (2). In addition, realistic optical excitations may involve magnon-photon coupling and spin-orbit torques generated by circularly polarized light, which could further influence the nucleation dynamics and helicity selection of skyrmionic textures. Finally, ultrafast effects include laser-induced demagnetization and anisotropy quenching, both of which allow domain-locking in simulations that start in one thermodynamic regime but end in another.

The thermal effect of the LG-beam is defined as a spatially and temporally dependent temperature function $T(\mathbf{r}, t)$ [54]

$$T(\mathbf{r}, t) = u^{\text{LG}}(\rho, \phi) T_0 \left(1 - \frac{t}{t_0} \right) \theta(t_0 - t) \theta(t), \quad (3)$$

where $\theta(t)$ denotes the Heaviside step function, T_0 is the beam temperature, and t_0 is the duration of beam irradiation. The spatially nonuniform temperature profile plays an important role in localizing the skyrmion nucleation process. While uniform heating can also generate skyrmions through stochastic thermal fluctuations, the nucleation events occur randomly throughout the system. In contrast, the LG beam confines the thermal excitation within a finite spatial region, enabling directed local generation of isolated skyrmions and skyrmion textures. The thermal function can be used in the fluctuation-dissipation theorem to attain the thermal profile $\mathbf{h}_T(t)$, which is given by

$$\mathbf{h}_T(t) = \frac{2k_B T(\mathbf{r}, t) \alpha}{\hbar \gamma^2}, \quad (4)$$

where k_B is the Boltzmann constant, α is the Gilbert damping constant, and γ is the gyromagnetic ratio. The

Hamiltonian in Eq. (1) and the thermal profile is used in the sLLG equation,

$$\frac{d\mathbf{S}}{dt} = -\gamma \mathbf{S} \times \left(-\frac{\partial \mathcal{H}}{\partial \mathbf{S}} + \mathbf{h}_T(t) \right) + \alpha \mathbf{S} \times \frac{d\mathbf{S}}{dt}, \quad (5)$$

where Heun's method is applied to give the spin dynamics. The sLLG equation and the simulations assume natural units $\hbar = 1$, $k_B = 1$, and $\gamma = 1$. Furthermore, our simulations use a fixed $\alpha = 0.1$, allowing convergence to a stable state. This is also consistent with other research using a similar model [39, 67].

Each simulation can be divided into two processes: irradiation (heating) and annealing. Based on initial simulations with various irradiation and annealing times, the irradiation time $t_0 = 500$ and annealing time $t_1 = 500$ were found to be optimal. Succeeding simulations were then done at $t_0 = 500$ and $t_1 = 500$ for a total simulation time $t_{\text{sim}} = t_0 + t_1 = 1000$. The conversion factor $\hbar/J$ is applied to the simulation time to get the real time $t_{\text{real}} = t_{\text{sim}}(\hbar/J)$. Correspondingly, the temperature also scales with J , $T_{\text{real}} = T_{\text{sim}}(J/k_B)$. At $J = 1$ meV, the simulation time unit would correspond to $t_{\text{real}} \approx 6.58$ ps.

C. Variational Calculations

Before presenting the results under LG-beam irradiation, we first discuss the ground-state phase diagram of the model Hamiltonian in Eq. (1) in the absence of the beam [38, 39]. Figure 2 presents the phase diagram obtained by performing the variational calculations at $T = 0$ with $L = 60$ ($N = 3600$) spins and commensurate ordering wave number $Q = 2\pi/5$. The phase diagram was attained through fixed- q variational calculations. By fixing the wave number to $Q = 2\pi/5$ for $J_3/|J_1| = 0.5$, the resulting ground-state phases become commensurate with the underlying lattice. We consider a total of 6 phases [38, 39], with each spin configuration defined by $\mathbf{S} = \mathbf{M}/|\mathbf{M}|$.

1. Ferromagnetic:

$$M_i^z = 1, \quad M_i^x = M_i^y = 0 \quad (6)$$

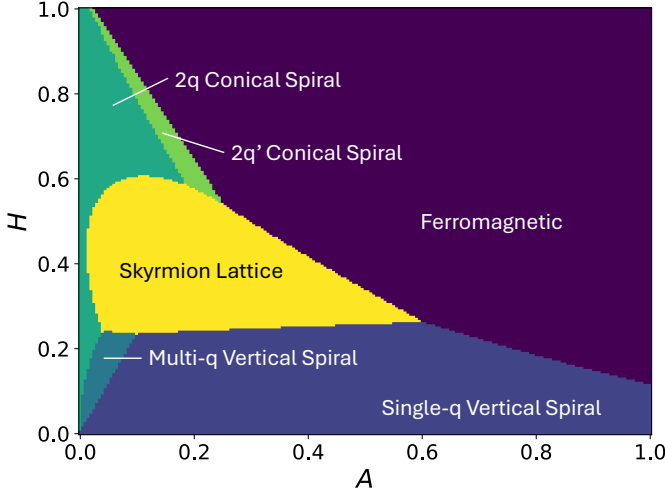


FIG. 2. Phase diagram at $T = 0$, obtained through variational calculations [38, 39]. A total of 201×201 data points were generated, each comparing the energies of 6 phases.

2. Single- q vertical spiral: the variational parameters are a_1 , a_2 , and $\bar{m}$.

$$\begin{aligned} M_i^x &= a_1 \sin(\mathbf{q}_1 \cdot \mathbf{r}_i) \\ M_i^y &= 0 \\ M_i^z &= a_2 \cos(\mathbf{q}_1 \cdot \mathbf{r}_i) + \bar{m} \end{aligned} \quad (7)$$

3. Multiple- q vertical spiral: the variational parameters are a_1 , a_2 , ϕ and $\bar{m}$.

$$\begin{aligned} M_i^x &= a_1 \cos(\mathbf{q}_2 \cdot \mathbf{r}_i + \phi) - a_1 \cos(\mathbf{q}_3 \cdot \mathbf{r}_i - \phi) \\ M_i^y &= -a_2 \sin(\mathbf{q}_1 \cdot \mathbf{r}_i) \\ M_i^z &= a_2 \cos(\mathbf{q}_1 \cdot \mathbf{r}_i) + \bar{m} \end{aligned} \quad (8)$$

4. 2 q conical spiral: a multiple- q spiral configuration wherein $a_1 \neq a_2$. The variational parameters are a_1 , a_2 , a_3 , and $\bar{m}$.

$$\begin{aligned} M_i^x &= a_1 \cos(\mathbf{q}_1 \cdot \mathbf{r}_i) + a_2 \cos(\mathbf{q}_2 \cdot \mathbf{r}_i) \\ M_i^y &= -a_1 \sin(\mathbf{q}_1 \cdot \mathbf{r}_i) + a_2 \sin(\mathbf{q}_2 \cdot \mathbf{r}_i) \\ M_i^z &= a_3 \cos(\mathbf{q}_3 \cdot \mathbf{r}_i) + \bar{m} \end{aligned} \quad (9)$$

5. 2 q' conical spiral: a multiple- q spiral configuration wherein $a_1 = a_2$. The variational parameters are a_1 , a_3 , and $\bar{m}$.

$$\begin{aligned} M_i^x &= a_1 \cos(\mathbf{q}_1 \cdot \mathbf{r}_i) + a_1 \cos(\mathbf{q}_2 \cdot \mathbf{r}_i) \\ M_i^y &= -a_1 \sin(\mathbf{q}_1 \cdot \mathbf{r}_i) + a_1 \sin(\mathbf{q}_2 \cdot \mathbf{r}_i) \\ M_i^z &= a_3 \cos(\mathbf{q}_3 \cdot \mathbf{r}_i) + \bar{m} \end{aligned} \quad (10)$$

6. Skymion lattice: the variational parameters are a_1 , a_2 , and $\bar{m}$.

$$\begin{aligned} M_i^x &= a_1 \sum_{v=1}^3 [\sin(\mathbf{q}_v \cdot \mathbf{r}_i)] \mathbf{e}_v \cdot \hat{\mathbf{x}} \\ M_i^y &= a_1 \sum_{v=1}^3 [\sin(\mathbf{q}_v \cdot \mathbf{r}_i)] \mathbf{e}_v \cdot \hat{\mathbf{y}} \\ M_i^z &= a_2 \sum_{v=1}^3 [\cos(\mathbf{q}_v \cdot \mathbf{r}_i)] \mathbf{e}_v \cdot \hat{\mathbf{z}} + \bar{m} \end{aligned} \quad (11)$$

where $\mathbf{e}_1 = \hat{\mathbf{y}} - \hat{\mathbf{z}}$, $\mathbf{e}_2 = -\sqrt{3}/2\hat{\mathbf{x}} - 1/2\hat{\mathbf{y}} - \hat{\mathbf{z}}$, and $\mathbf{e}_3 = \sqrt{3}/2\hat{\mathbf{x}} - 1/2\hat{\mathbf{y}} - \hat{\mathbf{z}}$. Figure 3 shows an example of a skymion lattice configuration from this ansatz.

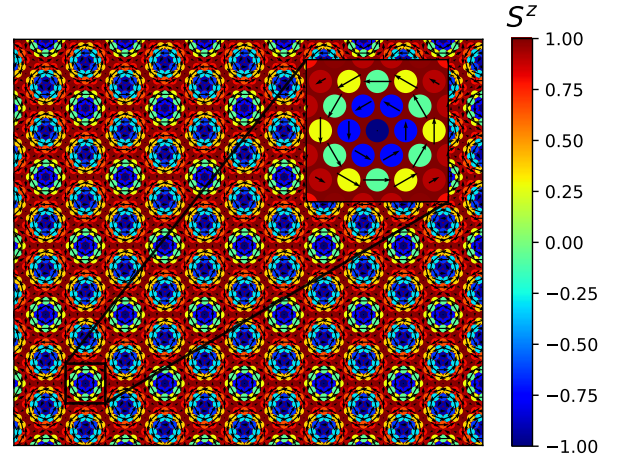


FIG. 3. An example of a skymion lattice from variational calculations at $a_1 = a_2 = 0.8$ and $\bar{m} = 0.2$. The upper right area of the figure shows the skymion centered at $(x, y) = (-20, -10\sqrt{3})$.

III. ISOLATED SKYRMIONS

In this section, we discuss the generation of isolated skyrmions. Figure 4 illustrates the time evolution of an isolated skyrmion generated by LG beam irradiation in the ferromagnetic regime. The simulation is performed at $A = 0.7$ and $H = 0.3$, where the ferromagnetic state is the ground state in the absence of external perturbations. The beam parameters are chosen as $T_0 = 5$, $p = 0$, $m = 5$, and $w = 10$. Throughout this work, we set $p = 0$ and $m = 5$, since different choices of these parameters do not qualitatively alter the results. Before beam irradiation, the initial spin configuration is the ferromagnetic spin state.

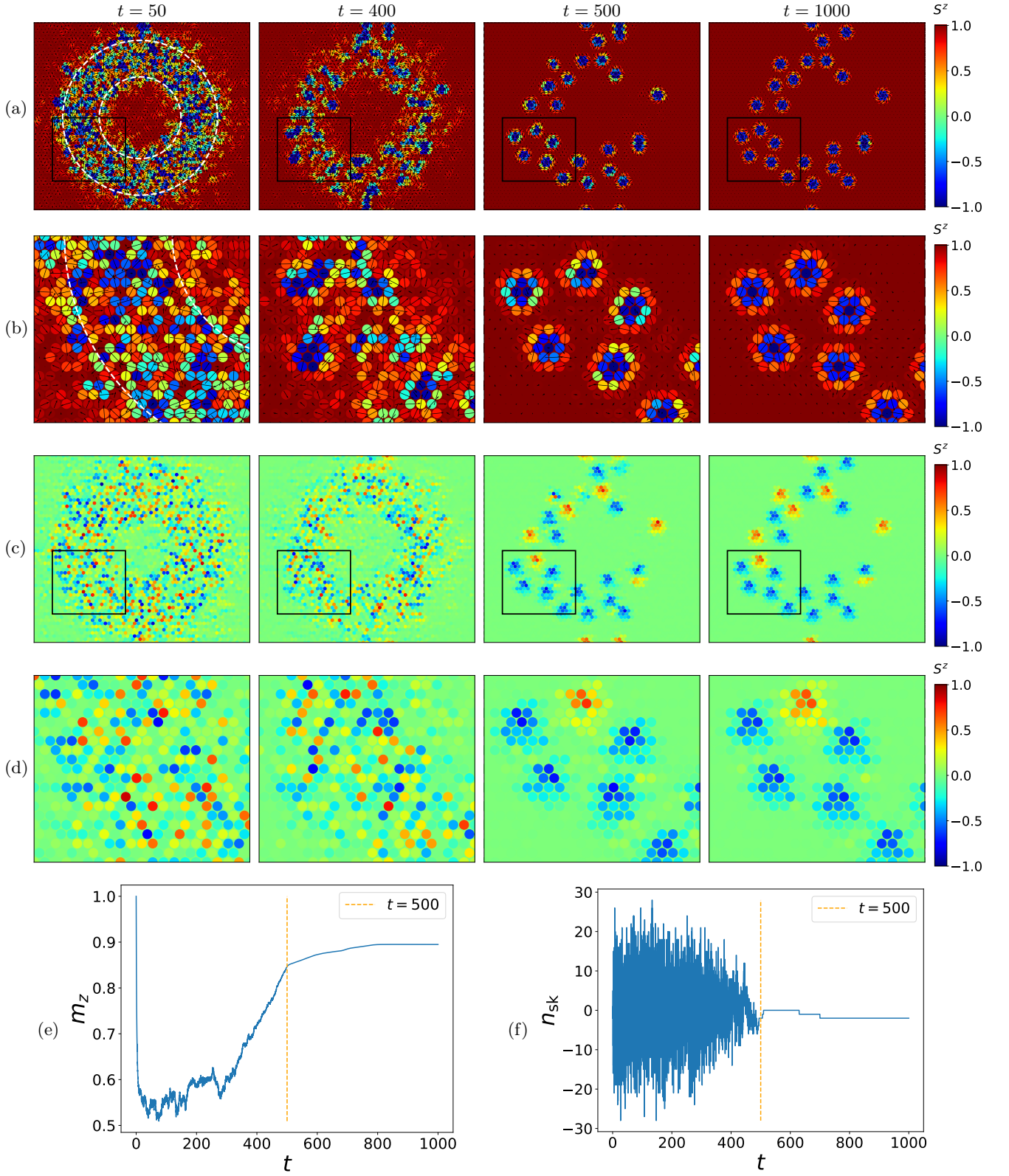


FIG. 4. (a) Time evolution of isolated skyrmions generated at $A = 0.7$ and $H = 0.3$ with beam parameters $T_0 = 5$, $p = 0$, $m = 5$, and $w = 10$. The white dashed circles at $t = 50$ represent the upper and lower boundaries of the applied beam. The colorbar represents the out-of-plane spin component S^z . (b) A 10x zoomed-in version of (a) centered at $(x, y) = (-14, -5\sqrt{3})$. (c) The scalar spin chirality taken at each timestamp for (a), i.e. $t = 50, 400, 500, 1000$. (d) A 10x zoomed-in version of (c) centered at $(x, y) = (-14, -5\sqrt{3})$. The time dependence of z -magnetization and skyrmion number n_{sk} is shown on (e) and (f), respectively. Nucleation videos are available in the Supplemental Materials [68]

At an early stage of irradiation ($t = 50$), the LG beam already induces a local reversal of the out-of-plane spin component S^z within the beam-irradiated region around $w(\sqrt{m}/2) \leq \rho \leq w(\sqrt{m}/2 + 1)$. This initial spin-flip event serves as the seed for the subsequent formation of the skyrmion core. As the heating effect of the LG beam further develops ($t = 400$), additional local fluctuations appear within the annular region of the beam profile. Subsequently, around the end of the irradiation process ($t = 500$), vortex spin textures begin to nucleate, signaling the emergence of skyrmionic defects. At this point, the skyrmion boundaries still contain mid-range out-of-plane spin components ($S^z \approx 0$), as represented by the color scale. This is in contrast to after the annealing stage ($t = 1000$), where the system relaxes into a stable isolated skyrmion embedded in the ferromagnetic background and the out-of-plane spins of the skyrmion boundaries ($S^z \approx 0.8$) and skyrmion core ($S^z \approx -1$) are at extremes.

This extremity in the out-of-plane spins is not found in the skyrmion lattice configurations generated through Eq. (11) as shown in Fig. 3. This signals the viability of isolated skyrmion application to next-generation data devices as the extremity allows for a more accurate skyrmion detection via the topological Hall effect [69]. We attribute this extremity in isolated skyrmions to easy-axis anisotropy, as the two regimes discussed in this section and in Sec. IV differ mainly in easy-axis anisotropy. High easy-axis anisotropy pushes the out-of-plane spins on the skyrmion boundary to align with either extreme, as seen from the $(S^z)^2$ factor in Eq. (1). As the high-field regime discussed in this section favors the ferromagnetic spin state, the mid-range out-of-plane spin components are pushed towards one end ($S^z \rightarrow 1$).

Overall, these results demonstrate that, even in the absence of the DM interaction, stochastic thermal excitation induced by the LG beam can overcome the nucleation barrier and lead to the formation of isolated skyrmions in frustrated magnets.

To identify the optimal parameters for skyrmion generation, we show the skyrmion creation probability as a heatmap over various beam parameters in Fig. 5. The skyrmion generation probability P was calculated as the number of successful trials s over the total number of trials $n_{\text{trial}} = 50$, converted into a percentage,

$$P = \frac{s}{50} \times 100\%. \quad (12)$$

Each trial operates on a given parameter set but with randomly varying noise for the thermal field. To simulate random variation between trials, a seed to the random generator was set for each Heun step dt in each simulation trial. A successful trial is defined by a nonzero absolute topological charge on the entire lattice, also known as the

absolute skyrmion number $n_{|\text{sk}|}$ [70],

$$\begin{aligned} n_{\text{sk}} &= \frac{1}{4\pi} \sum_{\langle i,j,k \rangle} \arctan \Omega_{ijk} \\ n_{|\text{sk}|} &= \frac{1}{4\pi} \sum_{\langle i,j,k \rangle} \arctan |\Omega_{ijk}| \\ \Omega_{ijk} &= \frac{\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k)}{1 + \mathbf{S}_i \cdot \mathbf{S}_j + \mathbf{S}_j \cdot \mathbf{S}_k + \mathbf{S}_k \cdot \mathbf{S}_i} \end{aligned} \quad (13)$$

where the summation runs over elementary triangular plaquettes in a counterclockwise direction. The topological charge signifies the existence of isolated skyrmions and other topological textures on the lattice; the skyrmion gives $n_{\text{sk}} = -1$ and the antiskyrmion gives $n_{\text{sk}} = 1$. Due to the absence of topological textures in a purely ferromagnetic spin configuration, it has no topological charge. Since both skyrmions and antiskyrmions can be generated with nearly equal probability in the present model, trials with an equal number of skyrmion and antiskyrmions will give a net topological charge of zero, $n_{\text{sk}} = 0$, despite the presence of topological defects. To distinguish such cases from topologically trivial ferromagnetic states, we introduce the absolute topological charge $n_{|\text{sk}|}$, which counts the total number of skyrmions and antiskyrmions irrespective of their sign. A trial with a net zero absolute topological charge over the entire lattice, i.e., $n_{|\text{sk}|} = 0$, is considered as a failed trial. The data are obtained in the ferromagnetic parameter regime, where skyrmions appear only as metastable excitations.

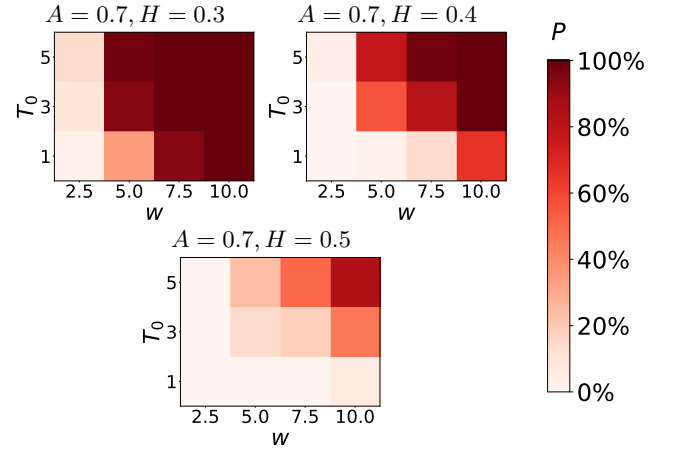


FIG. 5. Probability maps at $A = 0.7$ and $H = 0.3, 0.4$, and 0.5 . The temperature is in units J_1/k_B , while the beam width w is in lattice unit a .

As a reference, it is useful to contrast our frustrated system with DM interaction-driven chiral systems. The DM interaction is defined by,

$$\mathcal{H}_{\text{DM}} = \sum_{\langle i,j \rangle} \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j), \quad (14)$$

where $\mathbf{D}_{ij}$ represents the DM vector. As a cross-product, the DM interaction energetically favors twisted spin tex-

tures, and the characteristic skyrmion size is typically controlled by the ratio between the DM interaction constant $D = |\mathbf{D}|$ and the ferromagnetic exchange interaction J_1 or simply J .

Within the parameter range examined, the skyrmion generation probability increases with the beam width and is maximized for the largest value considered. This behavior indicates that the present mechanism does not require the LG beam to be focused down to the intrinsic skyrmion size. Indeed, successful skyrmion generation mainly occurs for relatively large beam waists $w \gtrsim 10a$, where a is the lattice constant. Instead, the beam primarily determines the spatial region of thermal excitation, while the skyrmion size itself is governed by the intrinsic frustrated magnetic interactions.

This behavior can be attributed to the fact that the skyrmion formation in the current frustrated model is heavily governed by the competing isotropic exchange interactions, J_1 and J_3 . Let us consider the spin at a conventional skyrmion center $\mathbf{S}_c$, its nearest neighbor $\mathbf{S}_{nn}$, and third-nearest neighbor $\mathbf{S}_{3nn}$. Due to the ferromagnetic nearest-neighbor coupling ($J_1 < 0$), it is energetically favorable to have both $\mathbf{S}_c$ and $\mathbf{S}_{nn}$ in the same direction. In contrast, the antiferromagnetic third-nearest-neighbor coupling ($J_3 > 0$) allows $\mathbf{S}_{3nn}$ to be anti-parallel to $\mathbf{S}_c$. Applying this over all the nearest neighbors and third-nearest neighbors of $\mathbf{S}_c$, we arrive at the isolated skyrmionic defect. The size of this defect is given by $\sim 2a$ as inferred from the distance of the third-nearest neighbor from the skyrmion center, and observed from the final spin configuration ($t = 1000$) on Fig. 4. This fixed skyrmion size is not present in chiral ferromagnets due to the lack of frustrated interactions, resulting in an optimal beam width [54]. In our present study, the wider beams effectively induce more stochastic nucleation events, which subsequently anneals into isolated skyrmions. Moreover, because the stabilization mechanism here does not rely on the continuous twisting favored by the small DM interaction, complex skyrmionic textures such as skyrmioniums (2π vortices) and higher-order skyrmion multiplexes ($n\pi$ vortices) are not realized in the present frustrated system.

Another important distinction between skyrmions in chiral magnets and those in frustrated centrosymmetric systems concerns their internal degrees of freedom, such as vorticity and helicity [33]. In DM interaction-driven skyrmions, the sense of rotation and the helicity are essentially fixed by the form of the DM interaction [1, 2, 71, 72], which selects a unique spin chirality and thereby locks the skyrmion texture into a specific vorticity–helicity combination. In contrast, skyrmions stabilized by competing symmetric interactions in frustrated magnets do not possess such intrinsic chiral preference. As a result, both the vorticity and helicity can, in principle, take arbitrary values, leading to a larger degeneracy of skyrmionic configurations. Indeed, various types of skyrmions with distinct helicities and vorticities are found in Fig. 4.

Nevertheless, this degeneracy is generally lifted once additional anisotropic interactions are introduced as discussed in Sec. V. In particular, anisotropic in-plane magnetic interactions, such as bond-dependent planar anisotropy considered in Sec. V, can energetically favor specific orientations of the spin rotation, and thus determine the preferred vorticity and helicity in frustrated systems [73, 74]. Therefore, while frustrated magnets allow a broader variety of skyrmion textures compared to chiral DM systems, the actual realization of their internal structure is strongly influenced by magnetic anisotropies.

Next, we discuss the dependence of the skyrmion generation probability on the external magnetic field and the easy-axis anisotropy. In the ferromagnetic regime, the Zeeman coupling favors a uniformly polarized spin state, thereby suppressing topological defects such as skyrmions. Accordingly, as the magnetic field is increased, the system is driven deeper into the fully polarized state, and the probability of skyrmion generation is reduced, as shown in Fig. 5.

Figure 6 presents the skyrmion generation probability at varying values of the easy-axis anisotropy parameter A . We find that the probability remains almost insensitive to A within the examined range. This can be understood from the fact that easy-axis anisotropy generally supports the formation and stability of skyrmion cores by favoring out-of-plane spin components, in contrast to a strong magnetic field, which tends to eliminate anti-parallel textures. Therefore, while increasing the field suppresses skyrmion nucleation, moderate variations in A do not significantly affect the generation efficiency in the present parameter regime.

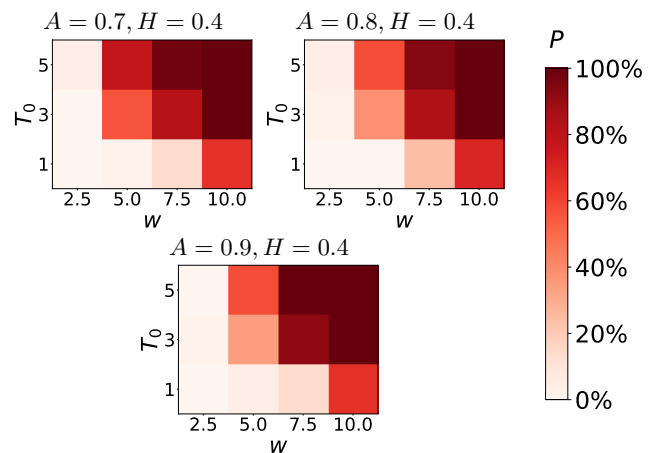


FIG. 6. Probability maps at $H = 0.4$ and $A = 0.7, 0.8$, and 0.9 . The temperature is in units J_1/k_B , while the beam width w is in lattice unit a .

IV. SKYRMION LATTICE

Figure 7 shows the time evolution of a skyrmion lattice generated by LG beam irradiation in the skyrmion-lattice

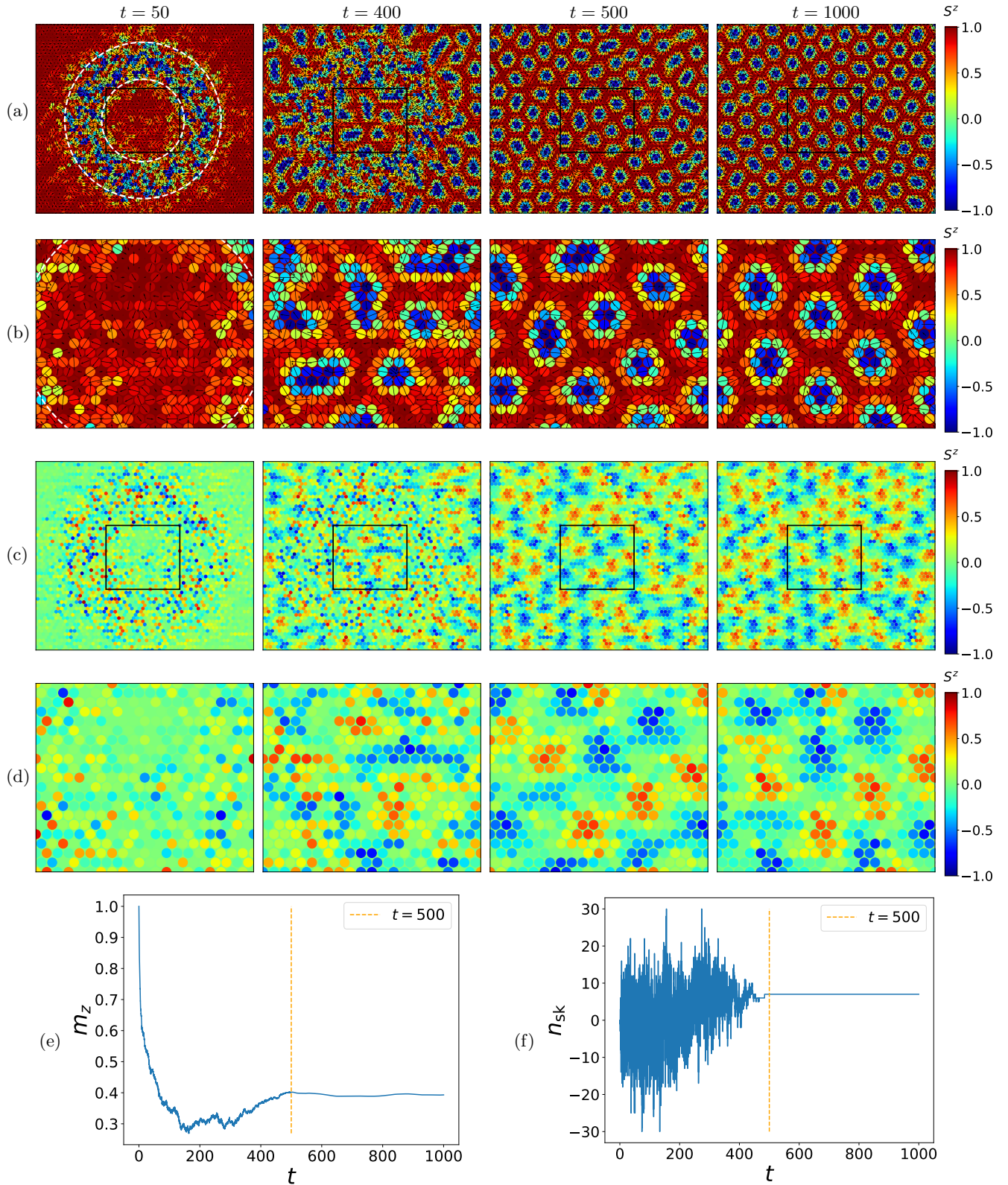


FIG. 7. (a) Time evolution of a skyrmion lattice at $A = 0.13$ and $H = 0.35$ with beam parameters $T_0 = 3$, $p = 0$, $m = 5$, and $w = 10$. The white dashed circles at $t = 10$ represent the upper and lower boundaries of the applied beam. The colorbar represents the out-of-plane spin component S^z . (b) A 10x zoomed-in version of (a) centered at the origin $(x, y) = (0, 0)$. (c) The scalar spin chirality taken at each timestamp for (a), i.e. $t = 10, 200, 500, 1000$. (d) A 10x zoomed-in version of (c) centered at the origin $(x, y) = (0, 0)$. The time dependence of z -magnetization and skyrmion number n_{sk} is shown on (e) and (f), respectively. Nucleation videos are available in the Supplemental Materials [75]

regime and in its vicinity. The simulation is performed at $A = 0.13$ and $H = 0.35$, where the skyrmion lattice is thermodynamically favored as the ground state. The beam parameters are chosen as $T_0 = 3$, $p = 0$, $m = 5$, and $w = 10$. The simulations start from a ferromagnetic spin configuration, mimicking a rapid field quench from the ferromagnetic regime into the skyrmion-lattice region. This setup is intended to model an experimentally relevant situation in which a sufficiently strong magnetic field first polarizes the system before optical irradiation.

At the initial stage of irradiation ($t = 10$), the spin configuration remains close to the ferromagnetic state, although local thermal fluctuations begin to develop within the beam-irradiated region. As the irradiation proceeds ($t = 200$), nonuniform spin modulations emerge, indicating the onset of multiple- q instabilities characteristic of the skyrmion-lattice phase. Around the end of the heating process ($t = 500$), skyrmion cores become clearly visible, and the system starts forming a crystalline arrangement of topological textures. After the subsequent annealing stage ($t = 1000$), the spin configuration relaxes into an ordered skyrmion lattice, representing the stable state in this parameter regime. It should be noted that the obtained skyrmion-lattice state may form domains characterized by different vorticities, i.e., the coexistence of skyrmions and antiskyrmions. This is because the LG beam creates skyrmions and antiskyrmions with equal probability, as shown in Sec. III.

The color scale denotes the out-of-plane spin component S^z , which highlights the periodic array of skyrmion cores. These results demonstrate that, in contrast to the ferromagnetic regime where isolated skyrmions are created through stochastic nucleation, skyrmion lattice generation occurs through a thermal annealing process that drives the system toward its equilibrium skyrmion-lattice ground state.

To quantify how efficiently such a skyrmion-lattice state can be generated from the ferromagnetic spin configuration under different irradiation conditions, we next examine the success probability across various beam parameters. As discussed in Sec. III, the generation probability is defined by Eq. (12). However, the crystalline configuration of the skyrmion lattice requires a stricter success condition. To determine successful trials, we first filter the simulations according to the absolute skyrmion number $n_{|\text{sk}|}$. Since the lattice ansatz shown in Fig. 3 possesses a skyrmion number $n_{\text{sk}} = 108$, we only retain simulations satisfying $n_{|\text{sk}|} \geq 100$. We use the absolute skyrmion number instead of the net topological charge because skyrmions and antiskyrmions are degenerate in the present model and may coexist in the generated configurations. We then further classify the remaining configurations using the spin structure factor $S(\mathbf{q})$, defined by,

$$S(\mathbf{q}) = \frac{1}{N} \left| \sum_i e^{i\mathbf{q} \cdot \mathbf{r}_i} (S_i^z - \langle S^z \rangle) \right|^2, \quad (15)$$

where $\mathbf{r}_i$ denotes the distance from the origin to site i and the sum is taken over every site. Figure 8 shows the spin structure factor graph of a successful and an unsuccessful trial. Specifically, the classification is performed numerically rather than by visual inspection. To avoid ambiguities originating from neighboring pixels with identical intensity, we first smoothen the spin structure factor through a Gaussian filter. We then identify the maximum intensity I_{max} and define a peak as a local maximum satisfying $I > 0.7I_{\text{max}}$. Since an ideal skyrmion lattice exhibits six symmetry-related peaks, a trial is regarded as successful when it satisfies both $n_{|\text{sk}|} \geq 100$ and the presence of six peaks in $S(\mathbf{q})$.

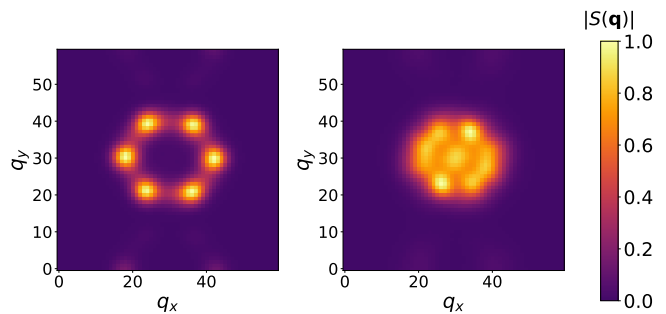


FIG. 8. Spin structure factor $S(\mathbf{q})$ of two different simulation trials. The right figure denotes an unsuccessful trial due to the mixing of wave vector $\mathbf{q}$, whereas the left figure denotes a successful trial due to the clean separation of 6 peaks, which is characteristic for a skyrmion lattice.

The success probability is evaluated from 50 independent stochastic simulation trials for each set of beam parameters, and is defined as the percentage of runs in which an ordered skyrmion-lattice state is obtained after the irradiation and annealing processes. Figure 9 presents the probability maps for skyrmion lattice generation under LG beam irradiation in the skyrmion-lattice regime. These maps follow three values of the magnetic field, $H = 0.3, 0.4$, and 0.5 , a fixed easy-axis anisotropy $A = 0.2$, and varying beam parameters such as $T_0 = 1, 3, 5$ and $w = 1.25, 2.5, 5, 10$. We use a non-linear scale for w to showcase the non-zero probability at beam widths near the skyrmion defect size discussed in Sec. III, $w \sim 2a$. While small beam widths were not able to generate isolated skyrmions in the ferromagnetic regime, some simulations in the skyrmion-lattice regime were successful in generating skyrmion-lattices even with a small beam width.

A notable feature of Fig. 9 is that the success probability exhibits a nonlinear and rather sensitive dependence on H . In particular, while high magnetic fields have completely prevented skyrmion lattice annealing, low magnetic fields do not necessarily enhance the probability of skyrmion lattice formation. This behavior contrasts with the isolated skyrmion case discussed in Sec. III, where the generation probability linearly increased with a decrease in the applied magnetic field. This reflects the fact that

the formation of a crystalline skyrmion lattice requires a collective rearrangement of spins over the entire system, which can be hindered when the applied magnetic field excessively constrains spin fluctuations.

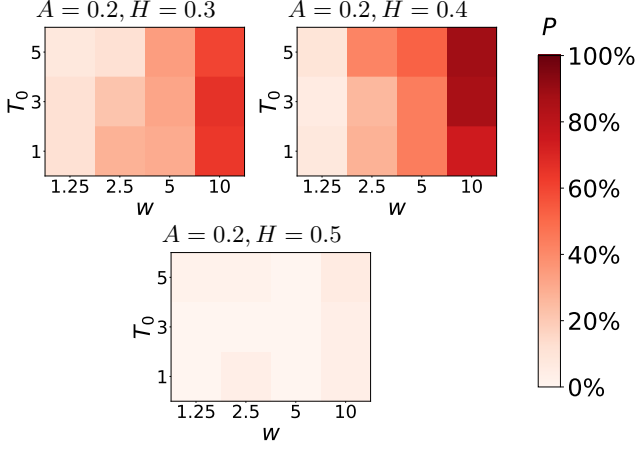


FIG. 9. Probability maps at $H = 0.3, 0.4$, and 0.5 for $A = 0.2$. The temperature is in units J_1/k_B , while the beam width w is in lattice unit a .

Furthermore, Fig. 9 indicates that the skyrmion lattice generation is relatively less dependent on the detailed beam parameters such as T_0 and w , compared to the isolated skyrmion regime. Instead, the success of skyrmion lattice formation is primarily governed by whether the system parameters (A, H) lie sufficiently close to the equilibrium skyrmion-lattice phase. From these probability maps, we identify $T_0 = 3$ and $w = 10$ as optimal beam parameters for subsequent simulations exploring the broader skyrmion-lattice region.

Having identified $T_0 = 3$ and $w = 10$ as optimal irradiation parameters from Fig. 9, we next explore how the success probability of skyrmion lattice generation depends on the intrinsic system parameters. Figure 10 shows the probability map in the A - H plane, obtained from 20 independent stochastic trials at each parameter set. The region surrounded by the dashed black lines indicates the equilibrium skyrmion-lattice phase in the ground-state phase diagram in Fig. 2, providing a reference for comparison. As seen in Fig. 10, high success probabilities are indeed concentrated within the skyrmion-lattice regime, demonstrating that LG beam irradiation can effectively drive the system from an initial ferromagnetic configuration into the topologically non-trivial skyrmion-lattice state when the parameters lie sufficiently close to the equilibrium phase. At the same time, the region of relatively high success probability is noticeably narrower than the skyrmion-lattice domain expected from the $T = 0$ phase diagram. This reduction reflects the fact that, for larger easy-axis anisotropy, spins become strongly constrained along the out-of-plane direction, which suppresses the collective spin rearrangements required to establish a crystalline skyrmion texture. Consequently, transitioning from a uniformly polarized back-

ground into the skyrmion-lattice phase becomes increasingly difficult under strong easy-axis anisotropy, even under thermal excitation through LG beam irradiation.

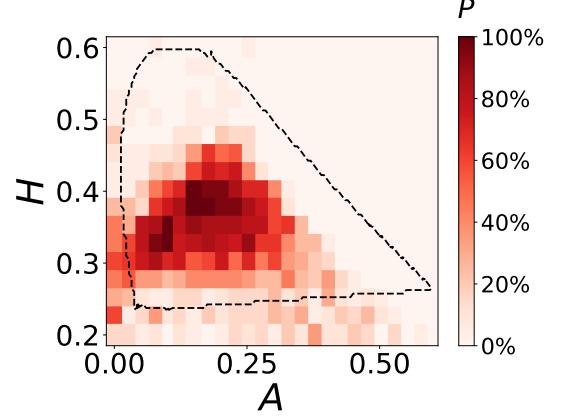


FIG. 10. Probability map over $A \in [0, 0.6]$ and $H \in [0.2, 0.6]$ obtained from 20 independent trials. The black region is the skyrmion lattice regime.

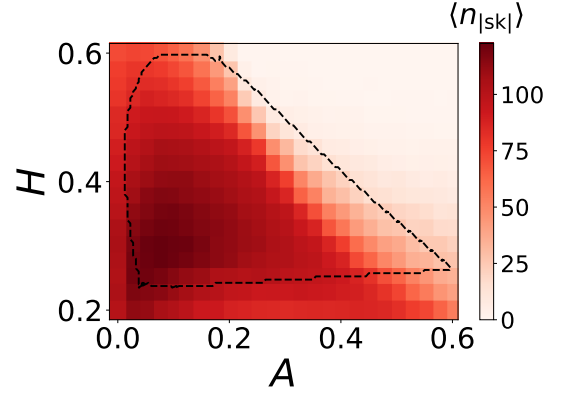


FIG. 11. Field dependence of the averaged absolute skyrmion number $\langle n_{\text{sk}} \rangle$ and the skyrmion crystal formation probability after annealing, obtained from 20 independent trials. This figure follows the same A - H plane as Fig. 10 with $A \in [0.0, 0.6]$ and $H \in [0.2, 0.6]$.

Figure 11 shows the density map of the averaged absolute skyrmion number $\langle n_{\text{sk}} \rangle$ in the A - H plane after LG-beam irradiation, obtained by averaging over 20 independent runs for each parameter set. We use the absolute skyrmion number to distinguish between the ferromagnetic state ($n_{\text{sk}} = 0$) and non-ferromagnetic states ($n_{\text{sk}} \neq 0$), which include both skyrmions and antiskyrmions. Furthermore, due to the xy -symmetry in Eq. (1), skyrmions and antiskyrmions are degenerate. Breaking this degeneracy will be discussed in Sec. V. The color scale quantifies the absolute topological charge generated by LG beam irradiation, averaged over 20 trials. The parameter region with high $\langle n_{\text{sk}} \rangle$ largely overlaps with the high-probability region for skyrmion lat-

tice formation shown in Fig. 10, indicating that efficient topological-charge generation occurs near the equilibrium skyrmion-lattice regime.

However, the two figures are not identical. While Fig. 10 measures the probability of forming an ordered skyrmion lattice, Fig. 11 captures the total amount of topological charge, regardless of spatial order. Consequently, even in regions where the skyrmion-lattice formation probability is low, $\langle n_{\text{sk}} \rangle$ can remain finite, suggesting the emergence of topological textures in a more disordered form, such as coexistence states of skyrmions and helical modulations, rather than a well-developed crystal. These results demonstrate that LG-beam irradiation can robustly create topological defects over a broad parameter range, whereas the emergence of a crystalline skyrmion lattice requires more restrictive conditions.

V. EFFECTS OF BOND-DEPENDENT PLANAR ANISOTROPY

In this section, we discuss the effects of bond-dependent planar anisotropy on the distinction of skyrmions and antiskyrmions, and on skyrmion helicity. In frustrated centrosymmetric magnets, skyrmionic textures are not uniquely constrained by an intrinsic chiral interaction such as the DM coupling. As a consequence, their internal degrees of freedom, including vorticity and helicity, can remain highly degenerate. In particular, due to the xy -planar symmetry of the Hamiltonian in Eq. (1), both skyrmions and antiskyrmions are energetically allowed and may appear with comparable stability in the present model.

However, from the viewpoint of discrete rotational symmetry of the lattice structure, additional bond-dependent anisotropic interactions are generally permitted in materials with strong spin-orbit coupling [76–79], such as CoI_2 [80] and YbMgGaO_4 [81]. Such anisotropies can play an important role in selecting specific spin textures and significantly affect the stability and dynamics of skyrmions [82–86]. Experimentally, the effective strength and symmetry of such bond-dependent anisotropies may also be tuned through strain engineering [87], substrate-induced symmetry breaking [88], or electric-field control of the local crystal environment [53, 89, 90]. Moreover, introducing planar anisotropy provides a natural mechanism to lift the degeneracy between skyrmions and antiskyrmions [73], as well as to fix the preferred vorticity and helicity of the generated skyrmionic states.

To break the planar symmetry and incorporate these symmetry-allowed effects, we add a bond-dependent planar anisotropic term to the model in Eq. (1), which is given by

$$\mathcal{H}^{\text{BA}} = J_a \sum_{\langle i,j \rangle} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-), \quad (16)$$

where J_a is the magnitude of the bond-dependent

anisotropy, $\gamma_{ij} = \gamma_{ji} = 1, e^{i2\pi/3}, e^{-i2\pi/3}$ is the phase shift for each bond direction for any two given nearest-neighbor sites i and j , and $S_i^\pm = (S_i^x \pm iS_i^y)/\sqrt{2}$. The numerical procedure in this section follows that in Secs. III and IV. To focus on the role of the bond-dependent planar anisotropy without overwhelming the competing exchange interactions or the beam-induced heating effects, we restrict its magnitude to a relatively small range, $J_a \in [-0.1, 0.1]$.

To distinguish skyrmions from antiskyrmions in the resulting spin configurations, we evaluate the total topological charge n_{sk} of the lattice as shown in Eq. (13). As antiskyrmions carry a topological charge opposite to skyrmions, with skyrmions having $n_{\text{sk}} = -1$ and antiskyrmions having $n_{\text{sk}} = 1$, the average value of n_{sk} provides a useful quantitative measure of the relative skyrmion population.

Figure 12 shows the dependence of the average skyrmion number $\langle n_{\text{sk}} \rangle$ on the planar anisotropy strength J_a , obtained from 50 independent stochastic trials for each parameter set. The shaded region represents the statistical uncertainty with a 90% confidence interval. Notably, at $J_a = 0$, skyrmions and antiskyrmions are nearly degenerate due to the xy -planar symmetry of the Hamiltonian, and they are generated with comparable probability. As a result, their opposite topological charges tend to cancel out, leading to an average value $\langle n_{\text{sk}} \rangle \simeq 0$.

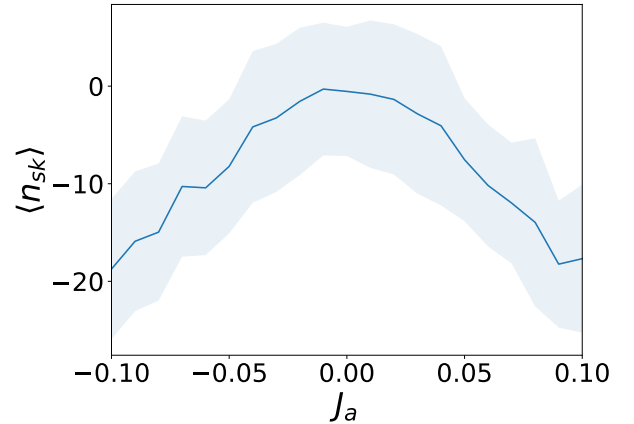


FIG. 12. Planar anisotropy J_a dependence of the average skyrmion number $\langle n_{\text{sk}} \rangle$ across 50 trials. The shaded area denotes the n_{sk} error per J_a with 90% confidence.

Once a finite bond-dependent planar anisotropy is introduced, this degeneracy is lifted; the skyrmion with negative n_{sk} is favored compared to the antiskyrmion with positive n_{sk} , irrespective of the sign of J_a . An increase in skyrmion population, signaled by the decrease in $\langle n_{\text{sk}} \rangle$, is observed for $J_a \neq 0$. This indicates that the anisotropy suppresses antiskyrmion formation and favors skyrmions with a particular sense of spin rotation. Furthermore, Fig. 12 demonstrates that even weak symmetry-allowed planar anisotropy ($|J_a| \ll 1$) can im-

prove the selectivity of skyrmion generation under optical irradiation by resolving the near-degeneracy between skyrmions and antiskyrmions in frustrated magnets.

We further investigate a quenching-like situation, where the bond-dependent planar anisotropy is not present during the LG-beam irradiation process, but is instead switched on only after the irradiation has finished. This setup mimics the case in which the planar anisotropy term is effectively induced or activated during the subsequent relaxation stage, rather than directly influencing the stochastic nucleation dynamics under the beam.

Figure 13 shows representative spin configurations obtained in such setting. The bottom panel corresponds to the state immediately after irradiation, where $J_a = 0$ and both skyrmions and antiskyrmions can still appear due to their degeneracy. When the system is subsequently annealed with a finite planar anisotropy, however, the resulting skyrmions exhibit a clear helicity selection. In particular, for $J_a < 0$, the skyrmions relax into a Bloch-type configuration with helicity $\gamma = \pm\pi/2$, whereas for $J_a > 0$, a Néel-type texture with $\gamma = 0$ or π becomes energetically preferred.

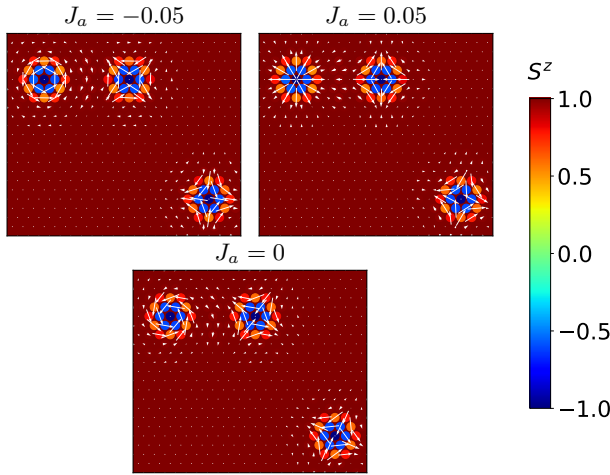


FIG. 13. Simulations wherein planar anisotropy J_a was applied only at annealing and not during beam irradiation. The bottom graph shows the spin configuration before annealing ($J_a = 0$). When annealing at $J_a = -0.05$ (upper left), the system relaxes into Bloch-type skyrmions. Meanwhile, annealing at $J_a = 0.05$ (upper right) results into Néel-type skyrmions.

Notably, unlike the case where J_a is applied throughout the entire irradiation process (Fig. 12), quenching does not significantly reduce the number of antiskyrmions. This indicates that the suppression of antiskyrmion formation requires the planar anisotropy to act already at the nucleation stage. On the other hand, once a skyrmion core is formed, the planar anisotropy can still play a crucial role in fixing its internal degree of freedom, namely the helicity, during the relaxation process. Thus, bond-dependent planar anisotropy can serve as a possible control parameter not only for lifting the

skyrmion–antiskyrmion degeneracy, but also for selecting the rotational character of optically generated skyrmions.

VI. CONCLUSIONS

We investigated the generation of skyrmionic textures in a frustrated J_1 – J_3 ferromagnetic system with easy-axis anisotropy through LG beam irradiation. The LG beam irradiation was modeled as a stochastic and spatially non-uniform thermal field, and the ensuing spin dynamics were analyzed by solving the sLLG equation. We demonstrated that LG beam irradiation can generate two distinct classes of skyrmionic states depending on the underlying magnetic regime: isolated skyrmions in the high-field ferromagnetic phase and skyrmion lattices in the intermediate-field region where the skyrmion lattice is thermodynamically favored.

Our results highlight qualitative differences between skyrmions in frustrated magnets and DM interaction-driven skyrmions in chiral systems. In chiral magnets, the DM interaction fixes a sense of spin twisting, enabling efficient optical imprinting when the beam width matches the skyrmion radius [54]. In contrast, in frustrated centrosymmetric magnets, skyrmions are stabilized by competing symmetric exchanges and their vorticity and helicity are less constrained. In the ferromagnetic regime, we find that isolated skyrmions are generated via stochastic nucleation: beam-induced thermal fluctuations overcome the nucleation barrier and the defect is stabilized during relaxation. The probability maps show that the creation probability decreases with increasing magnetic field, while its dependence on the easy-axis anisotropy is weak in the range studied.

We also explored skyrmion-lattice generation by initializing the system in a ferromagnetic configuration, mimicking a rapid quench from the ferromagnetic regime into the skyrmion-lattice region. In this case, LG beam irradiation effectively acts as a thermal annealing process that drives the system toward the equilibrium skyrmion lattice. The probability maps show that the success of skyrmion-lattice formation is primarily governed by the intrinsic parameters (A, H) and by the proximity to the equilibrium skyrmion-lattice phase, whereas the dependence on the beam parameters (T_0, w) is relatively weaker.

Finally, we clarified the role of planar anisotropy J_a in controlling the topological character of the optically generated textures. At $J_a = 0$, the xy -planar symmetry renders skyrmions and antiskyrmions nearly degenerate, leading to $\langle n_{\text{sk}} \rangle \simeq 0$ due to cancellation between opposite topological charges. Introducing a finite bond-dependent planar anisotropy lifts this degeneracy and suppresses antiskyrmion formation during nucleation, thereby enhancing selective skyrmion generation. Moreover, considering planar anisotropy quenching allows for helicity control even after skyrmions have already nucleated: Bloch-type skyrmions are stabilized for $J_a < 0$ and Néel-type

skyrmions are stabilized for $J_a > 0$. We also note that the present mechanism may be extended to easy-plane frustrated magnets hosting bimerons under an in-plane magnetic field [44, 91]. Since the Laguerre–Gaussian beam mainly acts as a spatially modulated heating profile in the present simulations, similar optical imprinting dynamics are expected for frustrated bimerons and bimeron crystals as well. These findings establish structured optical heating as a viable route to create and control skyrmionic textures in frustrated magnets without relying on the DM interaction, and highlight planar anisotropy as an additional ingredient for tailoring the vorticity and helicity of the generated states.

DATA AVAILABILITY STATEMENT

The code used to generate the raw data is made available for public access in the following link: https://github.com/rjlaxamana/sllg_skyrmions.

ACKNOWLEDGMENTS

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Appendix A: Data Validation

Most probability maps listed in this study are based on 50 independent trials per data point. The exceptions are Figs. 10 and 11, which are based on 20 independent trials. To validate the number of trials used for these figures, we conducted convergence tests for each data point. Figure 14 describes the probability P at $A = 0.7$ and $H = 0.3$ as the number of independent trials n_{trial} increases. These A and H values correspond to simulations in the ferromagnetic regime, where isolated skyrmions can be generated. Similarly, for the skyrmion-lattice regime, Fig. 15 shows P at different n_{trial} for $A = 0.3$ and $H = 0.3$.

For the isolated-skyrmion case, it can be observed that the probability starts to approach its final value as soon as $n_{\text{trial}} = 7$. After this, the probability fluctuates around the final value. In the skyrmion-lattice case, P approaches its final value at around $n_{\text{trial}} = 19$ but starts to stabilize at $n_{\text{trial}} = 35$.

In the case of Figs. 10 and 11, the number of trials was lowered from 50 to 20 due to computational limitations. However, successful convergence can still be inferred from Fig. 16, which shows the same P vs. n_{trial} graph as Figs. 14 and 15, except that the total number of trials is now 20. Figure 16 shows that the probability

converged near its final value at $n_{\text{trial}} = 10$ but continued to fluctuate around that value as n_{trial} increased.

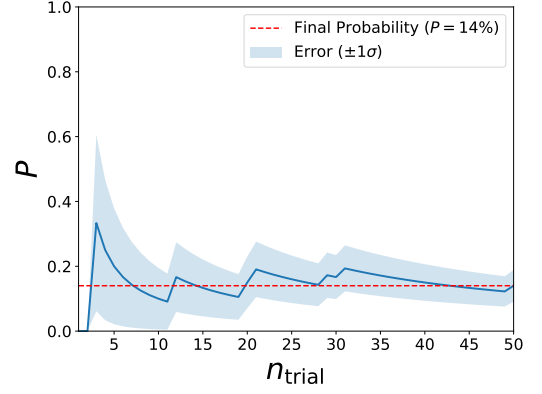


FIG. 14. Probability P versus the number of trials n_{trial} for $A = 0.7$, $H = 0.3$, $T_0 = 5$, and $w = 2.5$ (ferromagnetic regime). In this graph, we consider a total of 50 trials.

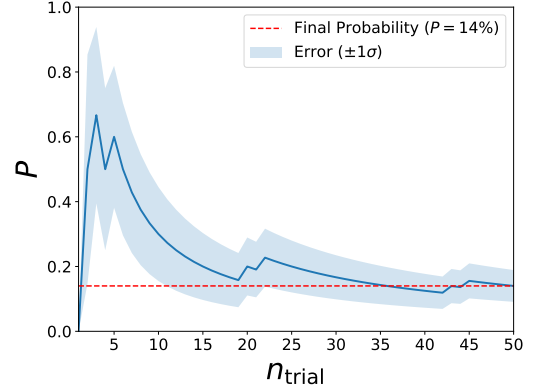


FIG. 15. Probability P versus the number of trials n_{trial} for $A = 0.3$, $H = 0.3$, $T_0 = 3$, and $w = 10$ (skyrmion-lattice regime). In this graph, we consider a total of 50 trials.

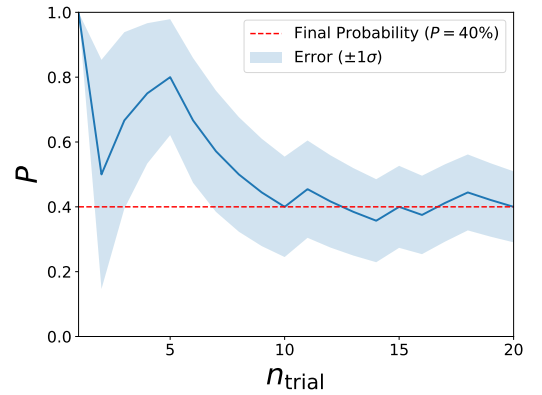


FIG. 16. Probability P versus the number of trials n_{trial} for $A = 0.05$, $H = 0.275$, $T_0 = 3$, and $w = 10$ (skyrmion-lattice regime). In this graph, we consider a total of 20 trials.

Other convergence tests and the simulation code can be found in the same repository given in the Data Availability Statement. Higher fluctuations can be observed, especially in cases where the number of trials was lim-

ited to 20. We suggest running additional simulations and tests on selected parameters before using these data for experimental observations.

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