HOKKAIDO UNIVERSITY

Title	Reliability analysis of passive safety system of small reactor under different Monte Carlo methods
Author(s)	Zengfanyue; Jianghong; Pengchanghon
Relation	Proceedings of the 10th International Symposium on Reliability Engineering and Risk Management (ISRERM 2026), June 28 - July 1, 2026, Sapporo, Japan
Citation	Proceedings of the 10th International Symposium on Reliability Engineering and Risk Management (ISRERM 2026)
Issue Date	2026-07-01
Doc URL	https://hdl.handle.net/2115/100422
Type	conference paper
File Information	C000014_1.pdf

Hokkaido University Collection of Scholarly and Academic Papers : HUSCAP

10th International Symposium on Reliability Engineering and Risk Management (*ISRERM 2026*),
28 June-01 July 2026, Sapporo, Japan
Kitahara, T., Miyamori, Y. & Beer, M. (editors)

Reliability analysis of passive safety system of small reactor under different Monte Carlo methods

Zengfanyue[1], Jianghong[2], and Pengchanghong[1,*]

[1] *School of Nuclear Science and Technology, University of Science and Technology of China, No. 96, Jinzhai Road, Baohe District, Hefei 230026, China.*
E-mail: zgfnye@mail.ustc.edu.cn
E-mail: pengch@ustc.edu.cn

[2] *State Key Laboratory of Nuclear Power Safety Technology and Equipment, China Nuclear Power Engineering Co., Ltd.,Shenzhen518172, China.*
E-mail: jh1997@mail.ustc.edu.cn
**Corresponding Author*

Due to the inherent safety of passive systems, the failure probability of reactor passive systems is extremely low. Calculating the failure probability of passive systems using Monte Carlo sampling would consume a significant amount of computational resources for running thermal-hydraulic codes. To alleviate the computational burden, this paper first compares Subset Simulation, Subset Simulation Importance Sampling, and other methods across multiple numerical cases. It then contrasts Monte Carlo methods accelerated by surrogate models, specifically the Active Learning Reliability Method Combining Kriging and Monte Carlo Simulation (AK-MCS) and the Adaptive Metamodel Subset Simulation Importance Sampling (AM-SIS). Finally, the paper employs the AK-MCS and AM-SIS methods to calculate the failure probability of the Emergency Heat Removal System (EHRS) of the International Reactor Innovative and Secure (IRIS) reactor.

Keywords: Reliability analysis, Monte Carlo Simulation, Subset Simulation Importance Sampling, Surrogate Model Acceleration, IRIS Reactor.

1 Introduction

Reliability aims at determining the probability of failure of a system by considering its inputs as random. The performance function $G(\boldsymbol{x})$ characterizes the response of the system: a negative value means that the failure occurs, a positive implies that the system is safe in this configuration. The border between the negative and positive domains is called the limit state ($G(\boldsymbol{x}) = 0$). To assess the probability of failure, several methods exist. The most common of them remains the Monte Carlo Simulation. It consists of a random population of size n_{MC} for which the performance function is evaluated. An estimation of the probability of failure P_f is then obtained as the ratio between $n_{G\leq 0}$, the number of samples giving a negative or null G values and the size of the sampling n_{MC}:

$$\hat{P}_f = \frac{n_{G\leq 0}}{n_{MC}}$$

The number of evaluations n_{MC} can be extremely important as it depends on the expected probability of failure. Indeed, a very weak probability of failure requires a large random population to ascertain the estimation. To quantify the uncertainty on the probability of failure, its coefficient of variation is calculated as:

$$\text{C.O.V}_{P_f} = \sqrt{\frac{1-\hat{P}_f}{\hat{P}_f n_{MC}}}$$

From this equation, it is evident that the sample size n_{MC} required to achieve a specific estimation accuracy (i.e., a low C.O.V.) is proportional to $1/P_f$. Consequently, for systems characterized by extremely low failure probabilities, such as the passive safety systems crucial to the nuclear energy industry, accurate assessment using crude Monte Carlo methods would necessitate an excessively large number of samples, leading to prohibitive computational costs.

While the Monte Carlo method is straightforward in principle, easy to implement, and widely applicable, its fundamental limitation lies in the requirement to evaluate the performance function for the entire random population. This is feasible for analytical systems with negligible computational cost; however, for engineering problems involving complex computational models (e.g., thermal-hydraulic system codes or finite element models), a single evaluation of the performance function can be time-consuming, taking minutes or even hours. Despite continuous advancements in computing technology, simulations of such engineering models remain computationally expensive, often rendering the large-scale sampling required by crude Monte Carlo simulation (MCS) practically infeasible for real-world applications.

On one side, fast-running surrogate models (or metamodels) can be properly built to mimic the response of the original T-H model code. The core idea of these methods is to construct a computationally inexpensive approximation of the performance function through a strategic Design of Experiments (DoE). Commonly used surrogate models

include the Response Surface Method (fast but with limited global interpolation accuracy), Polynomial Chaos Expansion (which performs well within a specific basis but involves complexities in order selection and DoE) (Ghanem and Spanos 1991, Sudret and Der Kiureghian 2002, Blatman and Sudret 2010), and, more recently, machine learning models such as Support Vector Machines, which have gained considerable attention (Hurtado 2004, Deheeger and Lemaire 2007). Kriging (or Gaussian Process modeling) has emerged as a particularly powerful surrogate model for reliability analysis. It provides not only a prediction of the performance function at any untried point but also an estimate of the prediction variance, which quantifies the local model uncertainty. This dual output makes Kriging especially suitable for active learning approaches, where the model's uncertainty guides the selection of new training points to iteratively improve accuracy near the limit state (Echard, Gayton and Lemaire 2011).

On the other side, advanced Monte Carlo Simulation methods can be adopted to realize robust estimations by means of a limited amount of random samples. Among these, Latin Hypercube Sampling (LHS) is a stratified sampling technique that ensures a more uniform coverage of the input parameter space compared to crude random sampling, thereby improving the representativeness of the sample set with fewer evaluations. Importance Sampling (IS) shifts the sampling distribution toward the failure region, concentrating computational effort on the most relevant parts of the input space and significantly reducing the variance of the failure probability estimator. Subset Simulation (SS) further advances this paradigm by expressing a rare failure event as a product of a sequence of more frequent intermediate events, efficiently exploring the parameter space through conditional sampling based on Markov Chain Monte Carlo (Papaioannou, Betz, Zwirglmaier and Straub 2015).

In the present paper, the AK-MCS (Echard, Gayton and Lemaire 2011) method and the AM-SIS (Pedroni and Zio 2015) method are adopted for the IRIS EHRS case. The selection of these two methods is motivated by several considerations. First, both methods are specifically designed to handle extremely low failure probabilities characteristic of passive safety systems, where crude MCS would be prohibitively expensive. And in this case, the thermal-hydraulic system codes need about an hour for a single evaluation. Second, they represent two complementary philosophies in surrogate-assisted reliability analysis: AK-MCS builds a globally accurate Kriging model of the limit state and performs MCS on this surrogate, while AM-SIS uses an Artificial Neural Network (ANN) (Cheng et al. 2008, Cardoso et al. 2008, Pedroni et al. 2010, Zio et al. 2010) to guide an adaptive Subset Simulation process and constructs the Importance Sampling Distribution via Kernel Density Estimation. This complementary nature allows for a meaningful comparative assessment of their respective strengths and limitations. Third, both methods have demonstrated superior computational efficiency in benchmark studies, typically requiring orders of magnitude fewer calls to the expensive performance function compared to traditional approaches.

2 Principles of sampling methods

This section will introduce the principles and advantages and disadvantages of various Monte Carlo sampling methods, including importance sampling (IS), subset simulation sampling (SS), subset simulation importance sampling, AK-MCS, and AM-SIS methods.

2.1 *Importance sampling*

IS is a powerful variance reduction technique designed to improve the computational efficiency of Monte Carlo simulations, particularly when estimating rare event probabilities such as the failure of a system.

The core idea of IS is to perform the sampling from a new, strategically chosen probability distribution $h(\boldsymbol{x})$, known as the importance sampling distribution or biasing distribution, rather than from the original nominal distribution $f(\boldsymbol{x})$ of the input random vector $\boldsymbol{x}$. The failure probability P_f is expressed as an expectation under the original distribution:

$$P_f = \int I_F(\boldsymbol{x}) f(\boldsymbol{x}) d\boldsymbol{x} = E_f[I_F(\boldsymbol{x})]. \tag{1}$$

where $I_F(\boldsymbol{x})$ is the indicator function for the failure domain F:

$$I_F(\mathbf{x}) = \begin{cases} 1, & \text{if } G(\boldsymbol{x}) \le 0 (\text{failure}), \\ 0, & \text{otherwise(safe)}. \end{cases} \tag{2}$$

By introducing the biasing distribution $h(\boldsymbol{x})$ ($h(\boldsymbol{x})$>0 whenever $I_F(\boldsymbol{x}) f(\boldsymbol{x})$>0, the integral is reformulated:

$$P_f = \int I_F(\boldsymbol{x}) \frac{f(\boldsymbol{x})}{h(\boldsymbol{x})} h(\boldsymbol{x}) d\boldsymbol{x} = E_h\left[I_F(\boldsymbol{x}) \frac{f(\boldsymbol{x})}{h(\boldsymbol{x})}\right] = E_h[I_F(\boldsymbol{x}) w(\boldsymbol{x})]. \tag{3}$$

where $w(\boldsymbol{x}) = \frac{f(\boldsymbol{x})}{h(\boldsymbol{x})}$ is the likelihood ratio or weight function. The IS estimator for P_f using N independent and identically distributed (i.i.d.) samples $\{\mathbf{x}^{(i)}\}_{i=1}^{N}$ drawn from $h(\boldsymbol{x})$ is:

$$\hat{P}_f^{IS} = \frac{1}{N} \sum_{i=1}^{N} I_F(\mathbf{x}^{(i)}) w(\mathbf{x}^{(i)}). \tag{4}$$

There is a so-called "optimal ISD" $h^*(x) = I_F(x)f(x)/P(F)$, i.e., the ISD that minimizes the variance of the MC estimator $\hat{P}_f^{IS}$ [Au and Beck 2003, Au 2004, Asmussen 2011, Dubourg 2013].

2.2 *Subset simulation*

2.2.1 *Core Principle: Nested Subsets and Conditional Probability*

The central idea of SS is to represent the rare failure event F as a sequence of intermediate, nested failure events:

$$F \equiv F_m \subset F_{m-1} \subset \cdots \subset F_2 \subset F_1. \tag{5}$$

Here, F_1 is a more frequent (i.e., less rare) event chosen such that its probability $P(F_1)$ is relatively large and can be estimated efficiently by standard MCS. Each subsequent event F_k is defined by a stricter performance threshold, progressively approaching the target failure domain.

The target failure probability $P_f = P(F_m)$ is then computed as a product of conditional probabilities:

$$P_f = P(F_m) = P(F_1) \cdot \prod_{k=2}^{m} P(F_k|F_{k-1}). \tag{6}$$

The algorithm is designed so that each conditional probability $P(F_k|F_{k-1})$ is also a reasonably large value (e.g., p_0=0.1), making its estimation via simulation far more efficient than directly estimating $P(F_m)$.

2.2.2 *Algorithmic Steps*

A standard SS procedure consists of the following stages:

1) Direct Monte Carlo (Level 0): Generate N independent samples $\{\mathbf{x}_0^{(i)}\}_{i=1}^N$ from the original probability density function $f(\boldsymbol{x})$. Evaluate their performance function values $G(\mathbf{x}_0^{(i)})$. Define the first intermediate threshold b_1 as the (1-p_0)N-th smallest value among the $G(\mathbf{x}_0^{(i)})$, where p_0 is a fixed conditional probability level (e.g., 0.1). All samples with $G(\mathbf{x})\leq b_1$ belong to subset F_1.

2) Markov Chain Monte Carlo Simulation (Levels k ≥ 1): For each subsequent level k (k = 1, 2, …, m − 1): The samples from the previous level k-1 that lie in F_k (i.e., $G(\mathbf{x})\leq b_\mathrm{k}$) serve as "seed" samples. From each seed sample, a Markov Chain (e.g., using the Modified Metropolis-Hastings algorithm) is used to generate new conditional samples. These chains are simulated such that their stationary distribution is the original PDF $f(\boldsymbol{x})$ conditioned on the event F_k.This process yields N new conditional samples $\{\mathbf{x}_k^{(i)}\}_{i=1}^N$, all satisfying $G(\mathbf{x})\leq b_\mathrm{k}$. Evaluate $G(\mathbf{x}_k^{(i)})$ for these new samples. Define the next threshold b_k+1 as the (1-p_0)N-th smallest value among these $G(\mathbf{x}_k^{(i)})$. This defines the next subset F_{k+1}.

3) Termination and Probability Estimation: The simulation proceeds level-by-level until the target failure region F_m (defined by the actual limit state $G(\boldsymbol{x}) \leq 0$) is reached. The failure probability is then estimated as:

$$\hat{P}_f^{SS} = p_0^{(m-1)} \cdot \frac{N_f^{(m)}}{N}. \tag{7}$$

Where $p_0^{(m-1)}$ is the product of the estimated conditional probabilities for the first $(m-1)$ levels (each $\approx p_0$), $\frac{N_f^{(m)}}{N}$ is the estimated conditional probability for the final level, with $N_f^{(m)}$ being the number of samples in the final level that satisfy $G(\boldsymbol{x}) \leq 0$.

2.3 *AK-MCS*

2.3.1 *Core Principles: Kriging and Active Learning*

The method's efficiency stems from two pillars: First, Kriging provides not only a prediction of the performance function $\hat{G}(\boldsymbol{x})$ at any untried point but also an estimate of the prediction variance $\hat{\sigma}^2(\boldsymbol{x})$. This variance is a direct measure of local model uncertainty, typically high in regions with sparse data. For reliability analysis, this allows for predicting the probability of misclassification at any point. Second, instead of evaluating a large, predefined set of Monte Carlo samples, AK-MCS employs an active learning function to identify and select, from a large candidate pool generated by MCS, the single most promising sample for the next true model evaluation. This process iteratively improves the surrogate's accuracy specifically near the limit state $\hat{G}(\boldsymbol{x}) = 0$, which is most critical for failure probability estimation.

2.3.2 *Algorithmic Steps*

The standard AK-MCS algorithm follows these key steps:

1) Generation of Candidate Pool & Initial DoE: Generate a large set of NMC candidate samples $\mathcal{X}_{pool}$ according to the input distribution $f(\boldsymbol{x})$. This pool remains fixed throughout the process. Then, create a small initial Design of Experiments (DoE) of size n0 by randomly selecting points from the pool and evaluating the true performance function $G(\boldsymbol{x})$ for them.

2) Kriging Model Construction & U-function Evaluation: Build/Update a Kriging model using all currently available evaluated points. For every candidate sample $\mathbf{x}^{(i)}$ in the pool, use the Kriging model to predict its performance $G(\mathbf{x}^{(i)})$ and the corresponding standard deviation $\hat{\sigma}^2(\mathbf{x}^{(i)})$. The U-learning function is then computed as:

$$U(\mathbf{x}^{(i)}) = \frac{|G(\mathbf{x}^{(i)})|}{\hat{\sigma}(\mathbf{x}^{(i)})}. \tag{8}$$

A small value of U indicates the point is close to the predicted limit state ($\hat{G}(\boldsymbol{x}) \approx 0$) and/or the prediction uncertainty is high ($\hat{\sigma}(\mathbf{x}^{(i)})$ is large). Such a point is a prime candidate for evaluation to reduce classification error.

3) Sample Selection & Stopping Criterion: Identify the candidate point with the minimum U-value:

$$\mathbf{x}_{next} = \underset{\mathbf{x}\in\mathcal{X}_{pool}}{argmin} U(\mathbf{x}). \tag{9}$$

Evaluate the true performance function $G(\mathbf{x}_{next})$, add this new point to the DoE, and update the Kriging model. The iterative enrichment continues until a stopping criterion is met. A common and robust criterion is:

$$\underset{\mathbf{x}\in\mathcal{X}_{pool}}{min}\ U(\mathbf{x}) \geq 2. \tag{10}$$

A U-value greater than or equal to 2 theoretically implies a probability of misclassification less than $\Phi(-2) \approx 0.023$ for all candidates, ensuring the surrogate's predictions are sufficiently reliable for sign estimation.

4) Final Failure Probability Estimation: Once the stopping criterion is satisfied, the final Kriging model is used to predict the sign of $G(\boldsymbol{x})$ for all N_{MCS} samples in the pool. The failure probability is estimated by counting the samples predicted to be in the failure domain:

$$\hat{P}_f^{AK-MCS} = \frac{1}{N_{MCS}} \sum_{i=1}^{N_{MCS}} I(\hat{G}(\mathbf{x}^{(i)}) \leq 0). \tag{11}$$

2.4 *AM-SIS*

2.4.1 *Core Concept: Surrogate-Assisted Adaptive Importance Sampling*

The fundamental innovation of AM-SIS lies in its hybrid architecture: First, it replaces the computationally intensive first stage of standard SIS—where a full SS is run with the expensive original model—with an ANN-guided adaptive exploration process. Second, this adaptive process directly aims to generate a population of samples that accurately represent the optimal ISD for the failure region, without the need for a full, costly SS run on the original model. The ISD is an approximately optimal distribution constructed from samples obtained via subset simulation. Finally, the approximately optimal ISD is then used to perform a final, high-efficiency IS analysis with a limited number of validating calls to the original expensive model.

2.4.2 *Algorithmic Steps*

The implementation of AM-SIS involves the following key steps:

1) Initialization: Start with a small initial Design of Experiments (DoE). Train an initial ANN surrogate model $\hat{G}(\boldsymbol{x})$.

2) Exploration Sampling: Execute a standard Subset Simulation as described in Section 2.2 for m levels. Then, preserve the samples from all levels, particularly the last levels (e.g., F_m, F_{m-1}). This collection of samples $\{\mathbf{x}_{SS}^{(j)}\}$, located in the high-probability failure region, serves as the ideal input for constructing an efficient ISD.

3) Active Learning for Model Update: From the exploration samples $\{\mathbf{x}_{SS}^{(j)}\}$, select the most informative points for refining the ANN. A common criterion focuses on failure samples x (i.e., $\hat{G}(\mathbf{x}) < 0$). Evaluate the true expensive model $G(\boldsymbol{x})$ at these selected points. Augment the training set and retrain the ANN model.

4) Iteration: Repeat steps 2 and 3. This process iteratively improves the ANN's accuracy in the failure region and pushes the exploration further towards the true failure domain, analogous to progressing through the levels of Subset Simulation but at a much lower cost.

5) Construct ISD and Perform Importance Sampling: Using the failure-region samples $\{\mathbf{x}_{SS}^{(j)}\}$ obtained from the surrogate-based SS, construct the Importance Sampling Distribution:

$$h(\boldsymbol{x}) = \frac{1}{M} \sum_{m=1}^{M} \kappa(\boldsymbol{x}|\mathbf{x}_{SS}^{\mathrm{m}}). \tag{12}$$

Following the suggestion of Botev, L'Ecuyer and Tuffin (2013), in this paper we use the transition density $\kappa(\boldsymbol{x}|\mathbf{x}_{SS}^{\mathrm{m}})$ adopted by the Gibbs Sampler (Neal 1993) (see the Appendix for thorough technical details).

6) Importance Sampling: Draw N_{IS} independent samples $\mathbf{x}_{IS}^{(j)}$ from the constructed ISD $h(\boldsymbol{x})$, for each sample, compute its performance function value $G(\mathbf{x}_{IS}^{(j)})$ and its importance weight $w^{(j)} = f(\mathbf{x}_{IS}^{(j)})/h(\mathbf{x}_{IS}^{(j)})$. Then we can compute the IS estimator for the failure probability:

$$\hat{P}_f^{AM-SIS} = \frac{1}{N_{IS}} \sum_{j=1}^{N_{IS}} I(G(\mathbf{x}_{IS}^{(j)}) \leq 0) \cdot w^{(j)}. \tag{13}$$

3 Case study: IRIS EHRS

This section presents the IRIS EHRS case study and shows the failure probability results calculated using the AK-MCS and AM-SIS methods.

3.1 *Introduction to IRIS reactor*

The International Reactor Innovative and Secure (IRIS) is an advanced integral pressurized water reactor led by Westinghouse Electric Corporation of the United States, with joint research and development by more than ten countries and over twenty research institutions worldwide. The conceptual design of the reactor was initiated in 2001, and the pre-licensing application process with the U.S. Nuclear Regulatory Commission (NRC) was launched in 2002. Starting from 2006, to meet the requirements of design certification, Westinghouse carried out a series of experimental verification works. After 2010, Westinghouse withdrew from the project leadership, and the subsequent R&D efforts have been mainly continued by relevant institutions in Italy, Croatia and Japan (Ricotte 2011).

The IRIS reactor is equipped with eight built-in once-through steam generators with helical tube bundles, which serve as the core components of the passive EHRS. Compared with the traditional straight tube design, the helical tube structure can better absorb thermal expansion stress and suppress flow-induced vibration, improving the reliability under transient accident conditions (Cinotti, Bruzzone, Meda, et al 2002).

IRIS features a unique integral design that integrates all major primary loop equipment, including the reactor core, control rod drive mechanisms, steam generators, main coolant pumps and pressurizer, into a single pressure vessel. This design eliminates the possibility of large break loss of coolant accidents (LBLOCA) and significantly reduces the risk of small break loss of coolant accidents (SBLOCA). Its safety systems inherit the passive safety philosophy of the AP1000, primarily relying on physical principles such as natural circulation, gravity and condensation to achieve long-term core cooling without external power supply or active intervention, thus greatly reducing the accident probability. According to the preliminary Probabilistic Risk Assessment (PRA) (Ricotte 2011), the Core Damage Frequency (CDF) of the IRIS reactor can be reduced to 1.21E-8.

3.2 *Analysis tools and models*

This study adopted the internationally recognized reactor system safety analysis code RELAP5, developed by the Idaho National Laboratory of the United States, as the high-fidelity thermal-hydraulic simulation tool. RELAP5 is capable of simulating a wide range of thermal-hydraulic transients and accidents in light water reactor systems, including various break accidents and loss of off-site power, and its models and algorithms have been extensively verified and recognized by regulatory and research institutions at home and abroad.

The RELAP5 nodalization diagram includes automatic depressurization system (ADS), EHRS, pressurizer, emergency boration tank (EBT), steam generator (SG), reactor core and so on, as shown in Figure 1.

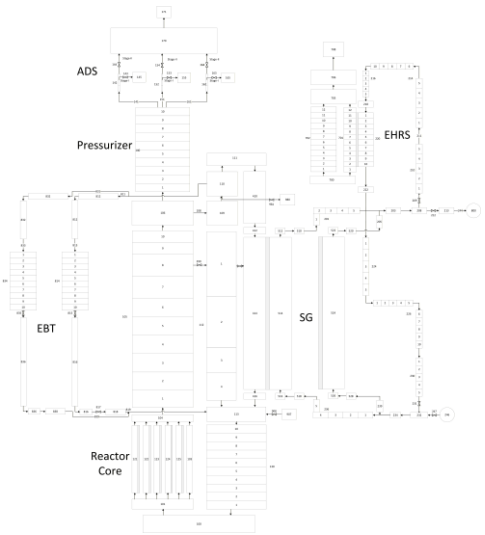

Figure 1. RELAP5 nodalization diagram of the IRIS reactor.

In the analysis, the simulation model of the IRIS reactor under SBLOCA was constructed by writing and modifying specific input cards of RELAP5 (such as pipe cards, thermal structure cards, valve cards and control system cards). During the probabilistic sampling process, random assignment of each key uncertainty parameter was realized through programmatic modification of the input card text, thus automatically generating a large number of calculation conditions reflecting parameter uncertainties, which provides a foundation for subsequent reliability analysis based on the Monte Carlo method and advanced sampling methods.

3.3 *EHRS accident analysis*

According to the preliminary probabilistic safety assessment (PSA) of IRIS (Mizuno, Ninokata and Finnicum 2005), among numerous design basis accidents, SBLOCA contributes the highest proportion to the Core Damage Frequency (CDF). In this accident sequence, the EHRS undertakes the critical safety functions of removing decay heat from the pressure vessel and depressurizing the system, and its reliability directly determines the severity of accident consequences. Therefore, it is reasonable and necessary to select SBLOCA as the reference accident for evaluating the reliability of the EHRS.

The SBLOCA scenario can be summarized as follows (Kujawski, Kitch and Conway 2002). Three main phases can be identified:

1) Discharge Phase (Phase 1): Following the initiation of LOCA, the RV depressurizes, while the Containment Vessel (CV) pressure begins to rise due to mass and energy release from the Reactor Vessel (RV). After the delay time, reactor scram and pump trip occur, and the EBT is activated to inject borated water. The EHRS is started, depressurizing the primary side by condensing steam on the secondary side of the steam generators (depressurization without mass loss), and finally the ADS is actuated to assist the EHRS in depressurizing the RV. Containment pressure is limited by the Pressure Suppression System (PSS), and the break flow is reduced due to heat removal by the EHRS (via condensation) and depressurization of the RV by ADS discharge. At the end of the discharge phase, the RV and CV pressures equalize (pressure equilibrium), with the CV pressure peak at approximately 8 barg.

2) RV/CV Depressurization Phase (Phase 2): After pressure equalization, the EHRS depressurizes the coupled RV/CV system by condensing steam within the RV. During this phase, the break flow reverses because heat is being removed from inside the vessel (via the EHRS) rather than from the containment atmosphere. The water level in the RV rises due to condensation of containment steam inside the reactor vessel and makeup water provided by the PSS. As the CV pressure decreases, a portion of the suppression pool water is expelled through the vents, helping to flood the cavity. Eventually, both RV and CV pressures are reduced to below 2 barg in less than 12 hours.

3) Long-Term Cooling Phase (Phase 3): As the core decay heat decreases, the RV and CV pressures decrease slowly. During this recovery phase of the accident, gravity makeup of borated water from the suppression pool and the reactor cavity is available as needed. Since the decay heat is removed directly from inside the vessel (via the EHRS), and the vessel and containment are thermodynamically coupled, the long-term break flow is not dependent on the core decay heat, but is in practice limited primarily by heat losses from the containment to the environment.

3.4 *Failure criteria and key uncertainty parameters*

To quantitatively evaluate the reliability of the EHRS, the system's failure criteria must be clearly defined. Based on the principle of conservatism and to ensure the computability of the analysis, the failure criterion in this study is defined as follows: the minimum hot leg collapsed liquid level in the core falls below the height of the core active region (6.08 m) during the accident progression (Ye 2019). This criterion is directly related to the functional integrity of the core fuel assemblies and is a key indicator for measuring whether the EHRS can effectively prevent core uncovering and overheating.

To ensure the quality of the reliability analysis, this study systematically identified and quantified the key uncertainty parameters affecting the SBLOCA progression and EHRS performance based on the Phenomena Identification and Ranking Table (PIRT) methodology. The PIRT analysis was completed by a diverse expert committee independent of the project, covering key fields such as nuclear safety, thermal-hydraulics and engineering design (US Nuclear Regulatory Commission (NRC) 1997). Through expert judgment, key phenomena with high importance to the accident progression but with existing uncertainties in current understanding were screened out and mapped to specific input parameters.

Based on the test data and analysis results from relevant literatures (Sallehhudin and Diab 2021), this study finally determined the key input parameters for probabilistic analysis and their probability distributions. The input uncertain parameters include core power (P_{core}), decay heat (Q_d), fuel thermal conductivity (k_f), in-containment refueling water storage tank (IRWST) water temperature (T_{IRWST}), EHRS fouling factor (ff), pressurizer pressure (P_{pzr}), ADS discharge coefficient (C_d), and ADS friction coefficient (C_f). The statistical properties of each input parameter include the mean (μ), standard deviation (σ), lower bound (LB), and the upper bound (UB). The distributional characteristics of the different input parameters are provided in Table 1.

Table 1. distributional characteristics of key input parameters.

parameters	Distribution	μ	σ	LB / μ	UB / μ
P_{core} / W	Truncated normal	1E9	1E7	0.98	1.02
Q_d / W	Truncated normal	8207426	270845	0.934	1.066
k_f / (W·m^{-1}·K^{-1})	Uniform	5.988342		0.847	1.153
T_{IRWST} / K	Truncated normal	322	10.304	0.98	1.02
ff	Truncated normal	1	0.1965	0.936	1.064
P_{pzr} / Pa	Uniform	1.5512E7		0.607	1.393
C_d	Uniform	1		0.84	1.16
C_f	Uniform	6.3E-6		0.8	1.2

3.5 *Result of different Monte Carlo methods*

Each evaluation of the IRIS EHRS failure status using the high-fidelity RELAP5 model requires approximately one hour on average. Since no reference failure probability for the IRIS EHRS case is currently available in the literature, this study employs the AM-SIS and AK-MCS methods to compute the failure probability. Two methods to calculate the probability $\hat{P}_f$, the number of calls to the thermal-hydraulic code N_{call} and the computation time are summarized in Table 2.

Table 2. Failure Probability of IRIS EHRS by different Monte Carlo methods.

Method	$\hat{P}_f$	N_{call}	Time
AM-SIS	0.02955	448	448h
AK-MCS	0.001755	432	432h

The AM-SIS method converged after 13 iterations, yielding a failure probability of $\hat{P}_f$=0.02955. In contrast, the AK-MCS method required only 11 iterations to converge but produced a failure probability of $\hat{P}_f$=0.001755, which differs significantly from the AM-SIS result. Regarding computational efficiency, AK-MCS appears substantially faster than AM-SIS in terms of iteration count.

4 Conclusions

This paper compares the performance of the AM-SIS and AK-MCS methods in the IRIS EHRS case study. The computational results show that the failure probability obtained by the AM-SIS method is 0.02955, while that obtained by the AK-MCS method is 0.001335, indicating a significant discrepancy between the two. In terms of computational cost, the AM-SIS method required 448 evaluations of the original model, totaling 448 hours, whereas the AK-MCS method required 336 evaluations, totaling 336 hours.

Appendix A. *Nonparametric estimator for the optimal, zero-variance ISD*

We using a set of points $\{\mathbf{x}_{SS}^{\mathrm{m}} : \mathrm{m} = 1, 2, ..., M\}$ approximately distributed as the ideal, zero- variance ISD, an estimator $h(\boldsymbol{x})$ is here constructed as follows:

$$h(\boldsymbol{x}) = \frac{1}{M}\sum_{m=1}^{M} \kappa(\boldsymbol{x}|\mathbf{x}_{SS}^{\mathrm{m}}). \tag{A.1}$$

Since our target density is the zero-variance ISD $h(\boldsymbol{x})$, then the transition density $\kappa(\boldsymbol{x}|\mathbf{x}_{SS}^{\mathrm{m}})$of the corresponding well-known Gibbs Sampler assumes the following form (Neal 1993):

$$\kappa(\boldsymbol{x}|\mathbf{x}_{SS}^{\mathrm{m}}) = \prod_{j=1}^{n} h^*(x_j|x_1, x_2, \ldots, x_{j-1}, \mathrm{x}_{SS,j+1}^{m}, \mathrm{x}_{SS,j+2}^{m}, \ldots, \mathrm{x}_{SS,n}^{m}). \tag{A.2}$$

where $h^*(x_j|x_1, x_2, \ldots, x_{j-1}, \mathrm{x}_{SS,j+1}^{m}, \mathrm{x}_{SS,j+2}^{m}, \ldots, \mathrm{x}_{SS,n}^{m})$ is the conditional zero-variance ISD of variable x_j, given that all the other components are fixed to $x_1, x_2, \ldots, x_{j-1}, \mathrm{x}_{SS,j+1}^{m}, \mathrm{x}_{SS,j+2}^{m}, \ldots, \mathrm{x}_{SS,n}^{m}$.

Tailoring (A.2) to the particular case of failure probability estimation of interest to the present paper and remembering that $h^*(x) = I_F(x)f(x)/P(F)$, then $h(\boldsymbol{x})$ (A.1) can be rewritten as:

$$h(\boldsymbol{x}) = \frac{1}{M}\sum_{m=1}^{M} \prod_{j=1}^{n} \frac{f(x_j|x_1,x_2,\ldots,x_{j-1},x_{SS,j+1}^{m},\ldots,x_{SS,n}^{m}) I_F(x_j|x_1,x_2,\ldots,x_{j-1},x_{SS,j+1}^{m},\ldots,x_{SS,n}^{m})}{P[Y(x)>\alpha_Y|x_1,x_2,\ldots,x_{j-1},x_{SS,j+1}^{m},\ldots,x_{SS,n}^{m}]}. \tag{A.3}$$

References

Au S.K., Beck, J.L., Importance sampling in high dimensions, Struct. Saf. 25 (2), 139–163, 2003.

Au S.K., Probabilistic failure analysis by importance sampling Markov chain simulation, J. Eng. Mech. 130 (3), 303–311, 2004.

Asmussen, S., et al., Efficient simulation of tail probabilities of sums of correlated lognormals, Ann. Oper. Res. 189 (1), 5–23, 2011.

Blatman, G., Sudret, B., An adaptive algorithm to build up sparse polynomial chaos expansions for stochastic finite element analysis, Probabilist Eng Mech, 25(2), 183-197, 2010.

Botev, Z.I., L'Ecuyer, P., Tuffin, B., Markov chain importance sampling with applications to rare event probability estimation, Stat. Comput., 23(2), 271-285, 2013.

Cinotti, L., Bruzzone, M., Meda, N., et al., Steam generator of the international reactor innovative and secure, Proc. 10th Int. Conf. Nucl. Eng., 983-990, 2002.

Cardoso, J.B., De Almeida, J.R., Dias, J.M., Coelho, P.G., Structural reliability analysis using Monte Carlo simulation and neural networks, Adv. Eng. Softw., 39, 505-513, 2008.

Cheng, J., Li, Q.S., Xiao, R.C., A new artificial neural network-based response surface method for structural reliability analysis, Probab. Eng. Mech., 23, 51-63, 2008.

Deheeger, F., Lemaire, M., Support vector machine for efficient subset simulations: 2SMART method, Proc. ICASP 10–Applications of Statistics and Probability in Civil Engineering, Taylor and Francis, 2007.

Dubourg, V., Sudret, B., Deheeger, F., Metamodel-based importance samplingfor structural reliability analysis, Probab. Eng. Mech. 33, 47–57, 2013.

Echard, B., Gayton, N., Lemaire, M., AK-MCS: An active learning reliability method combining Kriging and Monte Carlo Simulation, Struct Safety, 33(2), 145-154, 2011.

Ghanem, R.G., Spanos, P.D., Stochastic finite elements: a spectral approach, Springer, Berlin, 1991.

Hurtado, J.E., An examination of methods for approximating implicit limit state functions from the viewpoint of statistical learning theory, Struct Safety, 26(3), 271-293, 2004.

Kujawski, J.M., Kitch, D.M., Conway, L.E., The IRIS spool-type reactor coolant pump, Proc. 10th Int. Conf. Nucl. Eng., 999-1002, 2002.

Mizuno, Y., Ninokata, H., Finnicum, D.J., Risk-informed design of IRIS using a level 1 probabilistic risk assessment from its conceptual design phase, Reliab. Eng. Syst. Saf., 87(2), 201-209, 2005.

Neal, R.M., Probabilistic Inference Using Markov Chain Monte Carlo Methods, Technical Report CRG-TR-93-1, Dept. Comput. Sci., Univ. Toronto, 1993.

Papaioannou, I., Betz, W., Zwirglmaier, K., Straub, D., MCMC algorithms for Subset Simulation, Probabilist Eng Mech, 41, 89-103, 2015.

Pedroni, N., Zio, E., Estimating the small failure probability of a nuclear passive safety system by means of an efficient adaptive metamodel-based subset importance sampling method, Proc. Eur. Saf. Reliab. Conf. (ESREL 2015), Taylor and Francis Group, London, UK, 1853-1861, 2015, doi: 10.1201/b19094-241.

Pedroni, N., Zio, E., Apostolakis, G.E., Comparison of bootstrapped artificial neural networks and quadratic response surfaces for the estimation of the functional failure probability of a thermal-hydraulic passive system, Reliab. Eng. Syst. Saf., 95(4), 386-395, 2010.

Ricotte, M.E., Technology Development, Design and Safety Features of IRIS and its Deployment Scheme, Tech. Meet./Workshop on Technol. Assess. of SMRs for Near Term Deployment IAEA, Vienna, 2011.

Sallehhudin, W., Diab, A., Using Machine Learning to Predict the Fuel Peak Cladding Temperature for a Large Break Loss of Coolant Accident, Front. Energy Res., 609, 2021.

Sudret, B., Der Kiureghian, A., Comparison of finite element reliability methods, Probabilist Eng Mech, 17(4), 337-348, 2002.

US Nuclear Regulatory Commission (NRC), Phenomena identification and ranking tables for Westinghouse AP600 small break loss-of-coolant accident, main steam line break, and steam generator tube rupture scenarios, NUREG/CR-6541, 1997.

Ye, Q., Best Estimate Plus Uncertainty Analysis for SBLOCA on a Small Modular Reactor, Master's thesis, Huazhong Univ. Sci. Technol., Wuhan, China, 2019.

Zio, E., Apostolakis, G.E., Pedroni, N., Quantitative functional failure analysis of a thermal–hydraulic passive system by means of bootstrapped artificial neural networks, Ann. Nucl. Energy, 37(5), 639-649, 2010.