HOKKAIDO UNIVERSITY

Title	Deterioration distribution of cold region structures considering risk
Author(s)	Sutoh, Atsushi; Kanekiyo, Hiroaki; Sato, Takashi
Relation	Proceedings of the 10th International Symposium on Reliability Engineering and Risk Management (ISRERM 2026), June 28 - July 1, 2026, Sapporo, Japan
Citation	Proceedings of the 10th International Symposium on Reliability Engineering and Risk Management (ISRERM 2026)
Issue Date	2026-07-01
Doc URL	https://hdl.handle.net/2115/100429
Type	conference paper
File Information	C000029_1.pdf

Hokkaido University Collection of Scholarly and Academic Papers : HUSCAP

10th International Symposium on Reliability Engineering and Risk Management (ISRERM 2026),
28 June-01 July 2026, Sapporo, Japan
Kitahara, T., Miyamori, Y. & Beer, M. (editors)

Deterioration distribution of cold region structures considering risk

Atsushi Sutoh[1, *], Hiroaki Kanekiyo[2], and Takashi Sato[3]

[1] *Institute of Technology Department of Civil Engineering and Management, Sendai, Japan.*
E-mail: atsu-sutoh@tohtech.ac.jp
[2]*Department of Civil, Environmental and Applied Systems Engineering, Kansai University, Oosaka, Japan.*
E-mail: hiro.t.k@kansai-u.ac.jp
[3]*Civil Engineering Research Institute for Cold Region, Sapporo, Japan.*
E-mail: taka4@ceri.go.jp
**Corresponding Author*

The aging of social infrastructure constructed during the period of rapid economic growth has become apparent, and the safety of these structures is being shaken. As structures deteriorate over time, there is a strong demand for efficient maintenance and life cycle management, which involves understanding their current state and predicting their future condition through periodic inspections, in order to create a sustainable society. However, because structures are exposed to a variety of natural environments, the variability in the state of deterioration, particularly the tail of the probability distribution, converges very slowly, making it extremely difficult to identify the distribution shape. Such tail shapes are often observed in probability distributions that describe the variability in the strength of highly brittle materials. With distributions that have these characteristics, known as heavy-tailed distributions, there is a risk that even if a certain amount of data is obtained, the distribution shape may be interpreted as being on the safety side as not having a heavy tail. Therefore, in this study, the deterioration process of structures is expressed using the probabilistic Black-Scholes model and by applying the periodic inspection values of actual mountain tunnels, The characteristics of the deterioration distribution of structures with heavy tails are clarified in order to understand the probability distribution shape with heavy tails and the influence of the tail on the assessment of the risk.

Keywords: Deterioration process, Deterioration rate, Black-Scholes model, power-law distribution, Pareto distribution.

1 Introduction

Hokkaido is located in the northernmost part of Japan; Hokkaido's citizens' lives depend more on roads. In addition, structures are infrastructures that require a long service life and can with-stand harsh natural environments. Therefore, accurate understanding and prediction of deterioration conditions are essential for the development and operation of long-life measures based on preventive maintenance based on the results of periodic inspections of structures. The authors then calculated continuous deterioration evaluation values from damage maps created during regular inspections of mountain tunnels in Hokkaido and found that the distribution had a special tail structure, with even larger deterioration evaluation values appearing with a non-negligible probability.

Therefore, we focused on the fact that the characteristics of the variability of continuous deterioration assessment values quantified from periodic inspection values of tunnel lining surfaces are similar to log-normal distributions or Pareto distributions, e.g. Pareto (1898), and pro-posed a model for the irregular time growth of deterioration assessment values using a probability model similar to the Black-Scholes stock price fluctuations used in financial derivatives theory, e.g. F. Black and M. Scholes (1973).

Because the time transition of the deterioration assessment value varied greatly compared to the growth rate, and the generated time growth samples repeatedly increased and de-creased, the average time growth was not changed, and the model was modified to one in which the probability of sample decrease was zero. At the same time, the tail part of the probability distribution of the deterioration assessment value was considered, and optimal maintenance and repair measures for the tunnel lining were sought using stochastic control theory, e.g. A. Sutoh, O. Maruyama, H. Tanaka (Kanekiyo), T. Satoh and H. Nishi (2011).

However, the probability distribution of the deterioration evaluation values, which is converted from the damage assessment category obtained from regular inspections, does not sufficiently consider the characteristics of the region of severe deterioration, the so-called tail part of the probability distribution. Furthermore, even in the case of bridge structures, no consideration has been given to the variance in the probability distribution of the deterioration index, the distribution with a heavy tail part, and the dual mode nature of the rate of deterioration growth. Therefore, this study aims to utilize appropriate periodic inspection values in preventive maintenance-based maintenance management, and specifically conducts the following investigations:

1) The advantages and disadvantages of continuous and discrete expressions when characterizing deterioration phenomena in structures, as well as the characteristics and features of their probability distributions, are considered.

2) The characteristics and features of the probability distribution of deterioration evaluation values converted from damage assessment categories obtained from periodic inspection values of bridges in Hokkaido are verified and

considered, and the tail structure of the probability distribution for tunnel lining deterioration phenomena is also considered.

2 Probability Distribution Characteristics of Deterioration Evaluation Values

2.1 *General Explanation of Probability Distributions*

The normal distribution is often used as a model for the distribution of general characterized physical quantities, but many natural phenomena are distributed in a manner that follows a power law. Furthermore, because the power-law distribution is a long-tail distribution in which the tail of the probability distribution converges very slowly, it is often extremely difficult to specify or identify the distribution shape. In recent years, there has been growing interest in probability distributions with such characteristics as heavy tails and fat tails in risk analysis and communication network traffic analysis.
Furthermore, in highly brittle materials, the probability distribution and variation of strength often have a heavy-tail or fat-tail shape, and when estimating such a distribution, even if a certain amount of data is obtained, it is often judged that the distribution does not have a heavy tail, especially in the case of highly brittle materials such as concrete.

2.2 *Distribution Characteristics of Evaluation in Tunnel Linings*

The early stage of tunnel lining parameters of the probability distribution and Pareto or power distribution of deterioration evaluation values obtained from the results of regular inspections of 436 data points, from 167 number of tunnels constructed using the sheet pile method in Hokkaido were identified using the maximum likelihood method from the actual measured values. The results are shown in Equation (1) and Figure 1.

$$f(x) = 4.0 \cdot x^{-0.80} \tag{1}$$

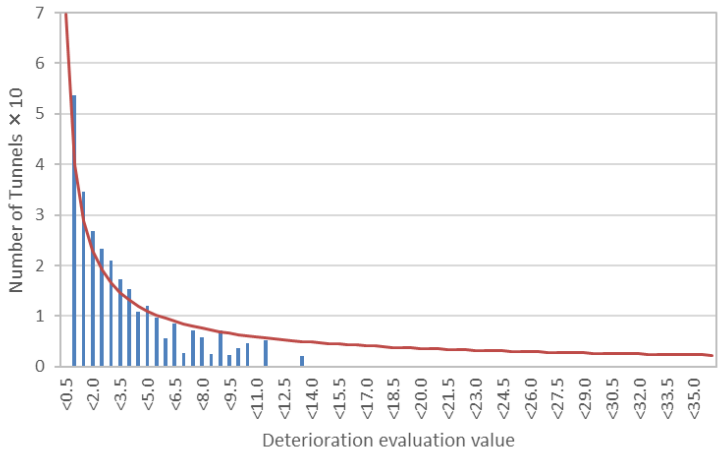


Figure 1. Probability distribution of deterioration evaluation values for tunnel linings.

From Figure 1, it appears that the deterioration assessment values and frequency distribution for tunnel linings constructed using the sheet pile method in Hokkaido show a distribution pattern similar to a Pareto distribution or power-law distribution. The deterioration assessment values shown here are a comprehensive numerical value that takes into account not only the size of cracks and damage in the Damage diagram of CAD data on the tunnel lining surface, but also other factors such as the relative positions of cracks and fissures and peeling and also incorporates the results of hammering tests.

2.3 *Transition of Deterioration Evaluation Value of Tunnel Linings*

Next, we consider the transition of the probability distribution of the deterioration assessment values for conventional construction methods shown in Figure 1. Figure 2 shows the average deterioration assessment values for tunnel linings for each year of service using conventional construction methods and the transition of their distribution of (1) 15 to 30 years, (2) 30 to 40 years, (3) 40 to 50 years.
As can be seen from Figure 2, the average and distribution of deterioration assessment values for tunnel linings constructed using conventional methods in Hokkaido is clearly not a normal distribution but an asymmetric distribution, with the median and most frequent values appearing at the left end of the distribution. Furthermore, as the number of years pass, the average deterioration value moves in the direction of progression, and the tails of the deterioration are also seen to become longer and fatter.

Therefore, the most important thing in maintenance is to carry out repairs and reinforcements on the tunnel lining as soon as possible, as this long tail is becoming more and more severe. However, the current deterioration assessment does not include an evaluation of this long fat tail portion, so it is necessary to be aware that this can be dangerous when carrying out maintenance.

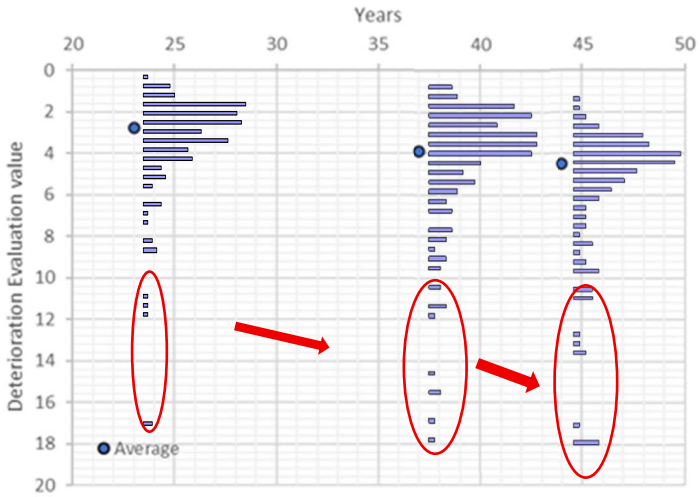


Figure 2. Probability distribution of deterioration evaluation values for tunnel linings.

2.4 *Relationship between Frequency Distribution of Deterioration Evaluation Values*

If the relationship between the frequency of the degradation assessment values shown in Figure 2 and the degradation assessment values is plotted on a double logarithmic scale, it will be a straight line with a Z shape as shown in Figure 3 (in Figure 3, there are two change points, but the gradient of the large and small degradation assessment values is almost the same, so these and the intermediate line represent a dual mode). For example, in the failure of fiber-reinforced composites, the failure mode caused by the rupture of the fiber part and the failure mode caused by damage to the matrix part coexist, so they often show dual-mode characteristics.

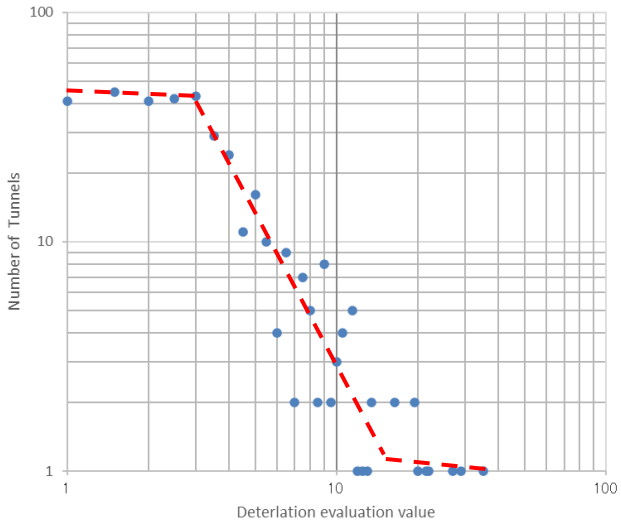


Figure 3. Frequency distribution of deterioration evaluation values (log-log scale).

Since concrete is a composite material, the distribution of deterioration evaluation values for tunnel linings constructed using the constructed using conventional method also exhibits a double-modal distribution as shown in the following equations (2) to (4), e.g. H. Kanekiyo, O. Maruyama, A. Sutoh and T. Satoh (2016).

$$f_X(\mathrm{x},\mathrm{y}) = \sum_{i=1}^{2} \zeta_i \, f_X(x,t;\mu_i) \tag{2}$$

$$f_X(x,t;\mu) = \frac{e^{-\lambda t}}{x} \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} f_W^{n*}\left(log\frac{x}{x_0} - \mu^t\right) \tag{3}$$

$$f_W^{n*}(x) = \frac{(\rho w)^{n-1}}{(n-1)!}\rho e^{-\rho w} \quad (n = 1,2,\cdots) \tag{4}$$

Here, μ1 and μ2 are the average progressive resistance in each mode, x_0 is the initial average damage level, λ is the average number of jumps per unit time in the compound Poisson process introduced as noise, ρ is a parameter related to the probability distribution of jumps in the compound Poisson process, ζ1 and ζ2 are the distribution ratios of each mode, ζ1 +ζ2 = 1, and f_W^{0*} is a delta function.

3 Overview of Poisson Type Damage Random Growth Model

3.1 Basic Formula

In models such as those using a compound Poisson process as the driving noise the average time growth rate $dX(t)\ /dt$ is a diffusion-type stochastic model proportional to the magnitude of the structural deterioration level X(t) (deterioration evaluation value), and the irregular time growth of the deterioration level is given by the following Ito-type stochastic differential equation.

$$dX(t) = \mu X(t)dt + X(t-)dC(t) \tag{5}$$

Here, μ is a coefficient that corresponds to the crack propagation resistance in fracture mechanics, and the stochastic process C(t) in the second term on the right-hand side is a compound Poisson process shown in the following equation. Here, since the deterioration index is not a strict physical quantity, the average time progression fluctuation is described using the Kachanov model, an empirical deterioration model in material mechanics, e.g. Kachanov, R. (1999).

$$dX(t)/dt = -\beta_0 X(t)^{-\gamma} \tag{6}$$

where β_0 is the average progressive resistance value for degradation, and γ is the shape parameter for degradation.
As a result, the stochastic differential equation (9) with irregular time degradation (sudden changes such as earthquakes) as the driving noise of the compound Poisson process is as follows, e.g. T. Satoh, H. Nishi, A. Sutoh and O. Maruyama (2014).

$$dX(t) = -\beta X(t)dt - gX(t-)dC(t) \tag{7}$$

Figure 4 shows the results of identifying β (trend) and γ (volatility) in equation (7) of the Kachanov model using the maximum likelihood method based on the deterioration evaluation values from the periodic inspection results of mountain tunnels. In addition, the variation in deterioration assessment values in conventional construction methods is much larger than in NATM.

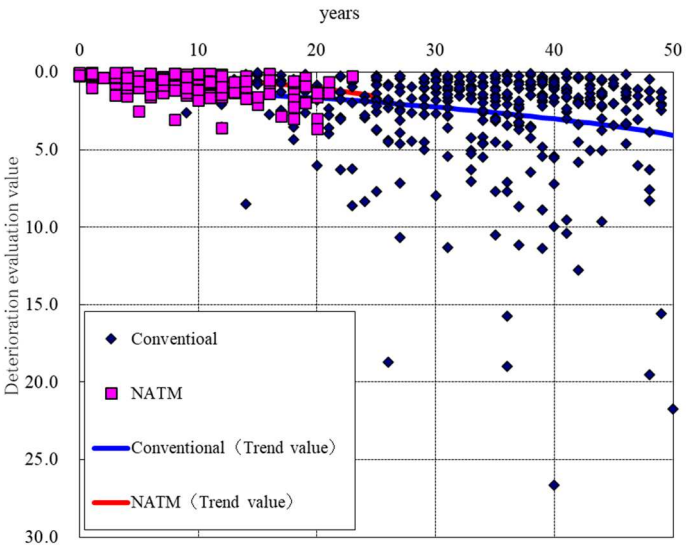


Figure 4. Deterioration identification using Kachanov models.

3.2 Dual-mode Characteristics of the Rate of Progression in the Probability Distribution of Degradation Evaluation Values

It is reasonable to consider that deterioration phenomena on the surface of composite materials, such as tunnel linings, include a certain proportion of deterioration phenomena that grow larger in a short period of time compared to normal deterioration phenomena, e.g. T. Mikosch, (1999).
To clarify this point, we will investigate the frequency of the growth rate of deterioration assessment values in the conventional construction methods. Note that the periodic inspection values used were extracted and verified from inspection values of tunnels where multiple inspections were carried out on the lining surface. Note that the periodic inspection values used were extracted and verified from inspection values of tunnels where multiple inspections were carried out on the lining surface. Figure 5 shows the relationship between the deterioration evaluation value and the rate of progression in conventional construction methods.
Figure 5 shows that the deterioration evaluation values for conventional construction methods show a dual mode of slow growth rate group (1) and fast growth rate group (2). In particular, the tail portion of the frequency distribution of growth rate corresponds to fast growth rate group (2), and there is also a large variation in the deterioration evaluation values.

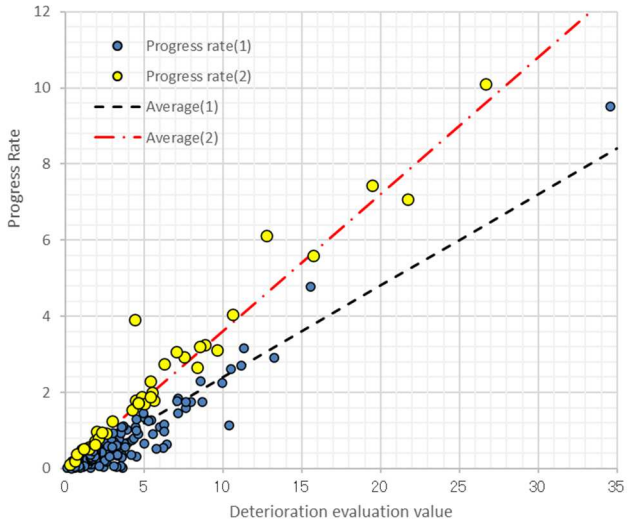


Figure 5. Deterioration assessment value and progression rate of conventional construction methods.

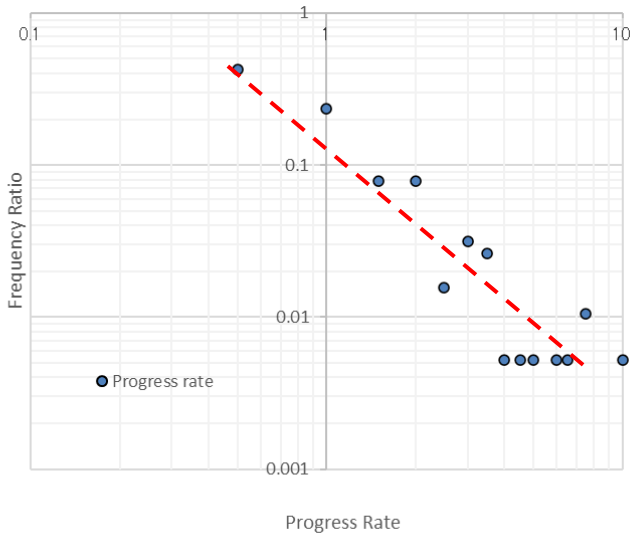


Figure 6. Relationship between the rate of deterioration progression and the frequency ratio (log-log scale).

Figure 6 shows the relationship between the rate of deterioration and the frequency ratio. Figure 6 shows that the rate of deterioration and the logarithm of the frequency ratio show a linear relationship, which suggests that the deterioration of infrastructure is also related to a power-law distribution.

If a proportional relationship is found between (dX(t))/dt and X(t) in Figure 5, then X(t) will show exponential growth, which explains the characteristics of the time-dependent change in the actual deterioration index shown in Figure 4. The slope in this case corresponds to the progressive resistance value μ.
Based on the above data reduction, if the probability density function of the deterioration index with dual modes is $\widetilde{f_X}(x,t|x_0)$, it can be expressed as in equation (8).

$$\widetilde{f_X}(x,t|x_0) = \sum_{i=1}^{2} \zeta_i \, f_X(x,t|x_0;\mu_0^{(i)}) \tag{8}$$

Here, $f_X(x,t|x_0;\mu_0^{(i)})$ is $\mu_0(=\mu+\lambda q_1)$ and $\mu_0^{(i)}$, and ζ_i, is represents the probability that μ_0, $\mu_0^{(i)}$ becomes $(i = 1,2,\cdots)$, $\zeta_1+\zeta_2=1$.
Therefore, in order to effectively maintain mountain tunnels, it is desirable to understand the dual-mode characteristics of the rate of progression of deterioration evaluation values obtained from periodic inspection results.

4 Conclusions

1. It was found that the long-heavy-tailed probability distribution of deterioration evaluation values obtained from periodic inspections of mountain tunnels in Hokkaido is caused by the growth rate showing a dual mode.

2. Treating deterioration phenomena in civil engineering structures, etc., as a normal distribution underestimates the probability distribution in areas of severe damage, resulting in an assessment that is on the dangerous side. Furthermore, it is believed that understanding the dual mode nature of growth rates in the maintenance and management of mountain tunnels will lead to greater efficiency.

3. Probability distributions that exhibit a long heavy tail due to dual modes of degradation events have a significant impact on level exceedance probability, etc., so it is important to understand whether a probability distribution has long heavy tail characteristics in reliability analysis and risk analysis.

Acknowledgments

We would like to thank the Hokkaido Civil Engineering Society (Tunnel Research Committee) and the Tunnel Management System Committee for providing us with valuable observation data.

References

Pareto, V., Cours d'economie politique, *Journal of Political Economy.*, 6(4), 549-662, 1896.
Black, F. and Scholes M., The pricing of options and corporate liabilities, *Journal of Political Economy*, 83, 637-654, 1973.
Sutoh, A., Maruyama, O., Tanaka (Kanekiyo), H., Satoh, T. and Nishi, H., Identification of degrading process for tunnel lining using observation data, *Proc. Of 7th Japan Conference on Structural Safety and Reliability (JCOSSAR 2011)*, 783-788, 2011.
Kanekiyo, H., Maruyama, O., Sutoh, A. and Satoh, T., Tail Properties of Probability Distribution of Damage Degree for Tunnel Concrete Liners in Cold Region, *Journal of JSMS*, 65(4), 275-281, 2016.
Kachanov, R., Rupture time under creep conditions, *International Journal of Fracture*, 97(4), 11-18, 1999.
Satoh, T., Nishi, H., Sutoh, A. and Maruyama, O., Investigation of evaluation and estimation of tunnel lining deterioration based on inspection data, *XIVth International Winter Road Congress Andorra 2014*, Session D, 96, 2014.
Mikosch, T., Regular Variation, Subexponentiality and Their Applications in Probability Theory, *Technical Report, Univ. of Groningen*, 1999.