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Estimating Lifetime Models from Field Load Spectra using Damage Accumulation Modeling

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Accurate lifetime models are essential for reliability assessment and design validation in industrial development. However, deriving such models from laboratory testing is often costly and time-consuming as it typically requires experiments at several load levels and repeated tests to characterize statistical scatter. In practice, it is possible that comprehensive test data are unavailable, while failed components and the corresponding field load spectra are available. This paper presents a simple engineering approach for estimating lifetime model parameters directly from failure-related field load spectra. In contrast to more advanced statistical methods, the proposed approach does not require maximum likelihood estimation, repeated testing at multiple load levels, or knowledge of the exact load sequence. Instead, each observed Spectrum leading to failure is interpreted as representing a complete damage event. Based on linear damage accumulation, the spectra are normalized and the model parameters are reconstructed by minimizing the deviation of accumulated damage from unity across multiple spectra.

Keywords: Lifetime Modelling, Damage Accumulation, Reconstructing Cumulative Damage, Lifetime Model, Operational Spectra, Load Spectra, Field Data Analysis

1 Introduction and State of the Art

Lifetime models are a central element of reliability engineering, as they describe the relationship between component lifetime, reliability, and influencing variables such as mechanical stress, temperature, or humidity. In practice, two experimental strategies are commonly used to identify such models: load-horizon tests and step-stress testing. The objective of both approaches is not only to estimate the parameters of the lifetime model itself, but also to characterize the statistical scatter of failure, often by means of an additional distribution parameter such as the Weibull shape parameter.

In load-horizon testing, each specimen is exposed to a fixed load level that remains constant throughout the entire experiment. To assess the effect of the influencing variable on damage, further experiments are performed at different constant load levels (Bertsche and Dazer 2022). In contrast, step-stress testing deviates from this approach by successively increasing the load applied to the same specimen during the experiment, instead of maintaining one constant load level for each specimen throughout the test.

This changing exposure must be accounted for explicitly in the statistical evaluation, which is why step-stress analysis is commonly formulated within cumulative-exposure-based modelling frameworks (Nelson 1980). The theoretical basis for such evaluations is often provided by Nelson's Cumulative Exposure Model, which assumes that the remaining life of a specimen depends on its current cumulative damage state rather than on the detailed sequence of the previous stress history. Building on this concept, more advanced methods have been developed for the statistical evaluation of non-constant loading conditions. For instance, (Meeker and Escobar 1998) provide a framework for Maximum Likelihood Estimation (MLE) under non-constant loads, while (Johannesson et al. 2005) proposed an approach for deriving lifetime models directly from variable-amplitude load spectra by introducing equivalent load amplitudes into the likelihood formulation.

While these likelihood-based approaches possess statistical validity, their practical implementation in the context of field data remains challenging. Real operational load spectra frequently encompass a multitude of stress levels and individual loading histories, which complicates the formulation and numerical optimization of the likelihood function (Nelson 1980; Johannesson et al. 2005).

To address this challenge, the present study adopts a simplified solution for the estimation of lifetime-model parameters from failed field spectra by reformulating the problem as a minimization problem based on cumulative-damage reconstruction. The methodological basis of this approach is introduced in the following section.

2 Approach for Estimating Lifetime Models by Reconstruction Cumulative Damage

The proposed methodology aims to reconstruct the parameters of a lifetime model from a set of field load spectra using the concept of linear damage accumulation.

The fundamental idea is that load spectra recorded from failed components can be assumed to correspond to a cumulative damage state of $D \approx 1$, consistent with the Palmgren–Miner hypothesis that failure occurs when the cumulative damage reaches $D = 1$ (Miner 1945). Based on this assumption, synthetic load spectra are generated such that their cumulative damage, evaluated against a predefined reference fatigue model, is approximately equal to unity. These spectra are subsequently used to reconstruct the parameters of the underlying lifetime model. The overall workflow consists of four steps, which are described in the following sections.

2.1 *Demonstration Using the Basquin/Wöhler Model*

To illustrate the proposed methodology, the Basquin lifetime model (Basquin 1910) is used as a representative lifetime model.
The model describes the relationship between the stress amplitude and the number of load cycles to failure under cyclic loading conditions. The model can be described as:

$$N = C \cdot \sigma^{-k} \tag{1}$$

Where N denotes the number of cycles to failure, σ the stress amplitude, C the fatigue life coefficient, and k the slope parameter of the S–N curve. By applying a logarithmic transformation, the equation can be expressed in linear form as:

$$log(N) = log(C) - \mathrm{k} \cdot log(\sigma) \tag{2}$$

Within the proposed estimation framework, the parameters C and k are treated as unknown and must be reconstructed exclusively from the available field load spectra. However, for validation purposes in this study, an artificial "true" model is defined beforehand. Based on this reference model, synthetic field load spectra are generated, which are subsequently used to evaluate the performance of the proposed reconstruction methodology.

2.2 *Reconstruction of the fatigue parameters via a minimization problem*

Assume that $Y_{Spectra}$ failed load spectra are available. Each spectrum j consists of m_j discrete stress classes with stress amplitudes σ_{ij} and corresponding cycle counts n_{ij}.
Using Palmgren-Miner linear damage accumulation, the cumulative damage of spectrum j, where N_{ij} denotes the fatigue life associated with stress level σ_{ij}. The Substitution yields

$$D_j(C, k) = \sum_{i=1}^{m_j} \frac{n_{ij}}{N_{ij}} = \sum_{i=1}^{m_j} \frac{n_{ij}}{C \cdot {\sigma_{ij}}^{-k}} \tag{3}$$

Under the central assumption of the method, each failed spectrum corresponds approximately to a cumulative damage of one (Miner 1945). The unknown parameters of the lifetime model are therefore obtained by minimizing the summed squared deviation of the reconstructed damages from unity.

$$\min_{C,k} \sum_{j=1}^{Y_{Spectra}} (D_j(C,k) - 1)^2 \tag{4}$$

For the numerical solution of Eq. (4), the optimization was carried out in logarithmic parameter space using a quasi-Newton method, specifically the L-BFGS-B algorithm. Since the quality of the solution depends strongly on the initial values provided to the optimizer, a robust initialization strategy is described in the following section.

2.3 *Initial Parameter Estimation Using Log-Linear Regression*

To initialize the nonlinear optimization of Eq. (4), reliable starting values for C and k are required. In practical applications it is possible that no prior fatigue model or knowledge about the parameters of the lifetime model is available, so the initial estimates must be derived exclusively from the load spectra themselves.
To obtain such initial estimates, all stress levels σ_{ij} and corresponding cycle counts n_{ij} from the available load spectra are aggregated over all spectra and all stress classes.

For initialization only, the observed cycle counts are interpreted as a first approximation of the corresponding lifetimes $N_{ij} \approx n_{ij}$. This approximation serves to construct a physically plausible starting point for the nonlinear optimization. This leads to the approximate relation:

$$log_{10}(n_{ij}) \approx log_{10}(C) - \ \mathrm{k}\ \cdot log_{10}(\sigma_{ij}) \tag{5}$$

Thus, the initial parameter values can be obtained from an ordinary least-squares fit in double-logarithmic space using all stress levels and corresponding cycle counts from all available spectra. Since the slope of the fitted straight line corresponds to $-k$, the initial estimate of the Basquin slope parameter is given by

$$k_{init} = -\frac{\sum_{i,j}\left(log_{10}(\sigma_{ij}) - \overline{log_{10}(\sigma)}\right) \cdot \left(log_{10}(n_{ij}) - \overline{log_{10}(n)}\right)}{\sum\left(log_{10}(\sigma_{ij}) - \overline{log_{10}(\sigma)}\right)^2} \tag{6}$$

The corresponding initial estimate of the fatigue coefficient follows from the intercept of the fitted line as

$$log_{10}(C_{init}) = \overline{log_{10}(n)} + k_{init} \cdot \overline{log_{10}(\sigma)} \tag{7}$$

These estimates provide a physically consistent starting point that is derived solely from the available spectrum data and requires no prior knowledge of the fatigue model.

2.4 *Generation of synthetic field load spectra*

To emulate realistic field data, synthetic load spectra are generated using a dual-distribution approach based on the Beta distribution, following (Benz and Dazer 2024). In this procedure, one Beta distribution (α_1, β_1) defines the relative stress levels, while a second independent Beta distribution (α_2, β_2) determines the relative cycle contributions across the same load classes. The absolute number of cycles per class is then obtained by inverting Eq. (1) for each stress amplitude and scaling the cycle fractions such that the accumulated damage reaches a prescribed target value D. To introduce scatter representative of field failure data, the target damage is not fixed to $D = 1$, but is sampled from a Weibull distribution, $D \sim Weibull(b, \lambda)$ where the shape parameter b is varied in the parameter study and the scale parameter is fixed at $\lambda = 1$. To represent a broad range of possible usage profiles, the Beta distribution parameters $(\alpha_1, \beta_1, \alpha_2, \beta_2)$ are drawn independently from the interval [2.0, 8.0] for each spectrum. No clustering of spectra is imposed, such that the study also includes strongly different loading histories. This approach allows the generation of spectra ranging from damage-dominated to cycle-dominated profiles.

2.5 *Parameter Study for the Evaluation of the Proposed Reconstruction Approach*

To assess the validity and robustness of the proposed approach, a parameter study is conducted. The objective is to analyse how the reconstruction quality depends on three factors: the number of available load spectra, the spectral resolution, and the scatter in the cumulative damage at failure. The number of spectra, representing the amount of failure related field data available for parameter estimation, is varied between 5 and 42. The spectral resolution, defined by the number of bins per load spectrum, is varied between 5 and 19. The scatter in the cumulative damage at failure is controlled by the Weibull shape parameter b, which is varied from 0.5 to 8.0 (Section 2.4). The selected parameter ranges are chosen to cover a broad set of conditions considered relevant and plausible within the scope of this study, thereby providing an initial assessment of the performance limits and robustness of the proposed reconstruction approach. All parameter combinations are generated using a full-factorial design. The number of spectra and the spectral resolution (number of bins) are varied in steps of two, while the Weibull shape parameter b is varied in steps of 0.5. At each design point, 1000 repeated Monte Carlo simulations are performed. This design space enables a systematic evaluation of the conditions under which the proposed reconstruction approach yields reliable estimates of the lifetime-model parameters.

3 Evaluation of reconstructed Parameters of the lifetime model

The objective of the proposed method is to provide a simple method for parameterizing a lifetime model from failed operational spectra. Figure 1 compares the reconstructed Basquin parameters with the true reference parameters used to generate the synthetic spectra. The left heatmap shows the mean relative error of the slope parameter k, while the right heatmap shows the mean relative error of the fatigue coefficient C in logarithmic representation. In both plots, the error is shown over the Weibull shape parameter b, which controls the scatter in the damage at failure, and the number of available load spectra. The number of bins was kept constant at 17. Overall, the results reveal a clear and consistent trend. For both reconstructed parameters, the mean error decreases as the number of available spectra increases and as the variability in the cumulative damage at failure decreases.

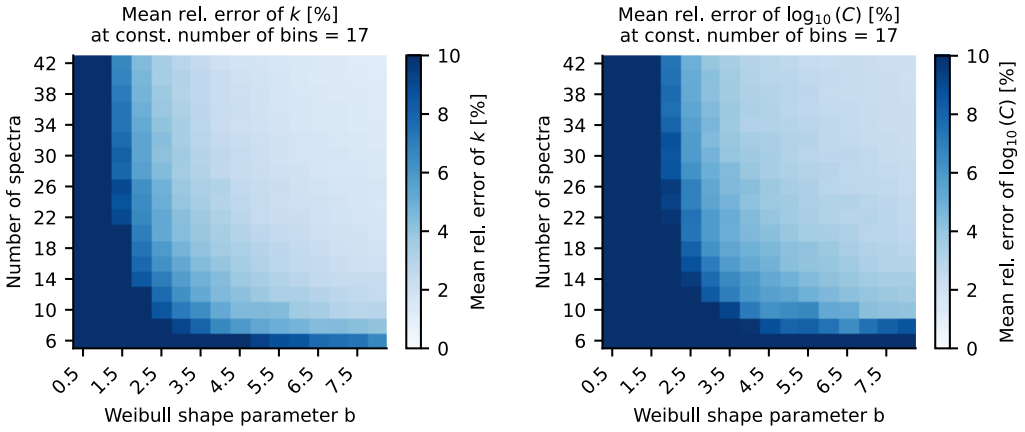

Figure 1. Mean relative error of k and $log(C)$ at a constant number of bins as a function of the Weibull shape parameter b and the number of available spectra

The mean values in Figure 1 provide a first overview of the reconstruction performance. However, mean errors alone are not sufficient for judging the practical usefulness of the method, because they do not reflect the spread of the estimation results across repeated realizations.

To address this aspect, Figure 2 shows the distribution of the relative errors of both reconstructed parameters for a fixed number of spectra and a fixed number of bins, while varying the Weibull shape parameter b. The results show that the spread of the estimates is particularly large for low values of b. In this regime, relative errors above 100% for k and above 200% for C can occur. For larger shape parameters, especially for $b > 2$, the spread decreases markedly for both parameters.

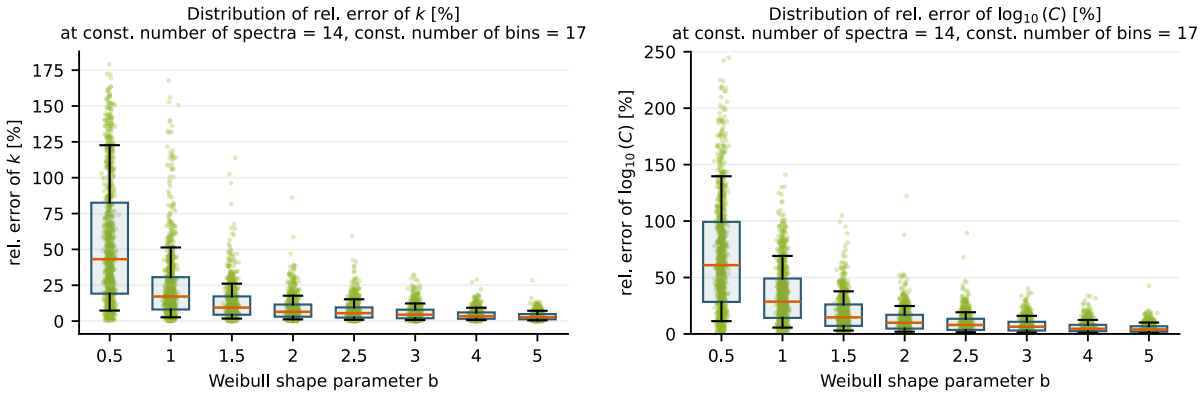

Figure 2. Distribution of rel. error of k and $log(C)$ at a constant number of spectra and a constant number of bins

A similar trend is observed when the number of available spectra is varied, as shown in Figure 3 (left). Here the spread of the estimation error is largest when only a few spectra are available. With increasing numbers of failed spectra, both the mean relative error and its variability decrease. At the same time, the benefit of each additional spectrum gradually diminishes, indicating a decreasing marginal gain. In the investigated examples, a substantial improvement is obtained up to approximately 20 spectra, while further increases still improve the result, but less strongly.

On the right-hand side of Figure 3, the influence of the number of bins is shown for the case of only six available spectra. The results indicate that the dispersion of the relative error decreases as the number of bins increases. However, a relatively large number of outliers is still observed, even at higher numbers of bins. This effect is particularly pronounced in scenarios where the number of available spectra is limited. The findings indicate that coarse discretization should be avoided, particularly when only a limited number of failed spectra is available.

However, within the present study, the effects of the scatter in the cumulative damage at failure and the number of available spectra appear to be more dominant overall. A more detailed assessment of the specific influence of the number of bins therefore requires further investigation.

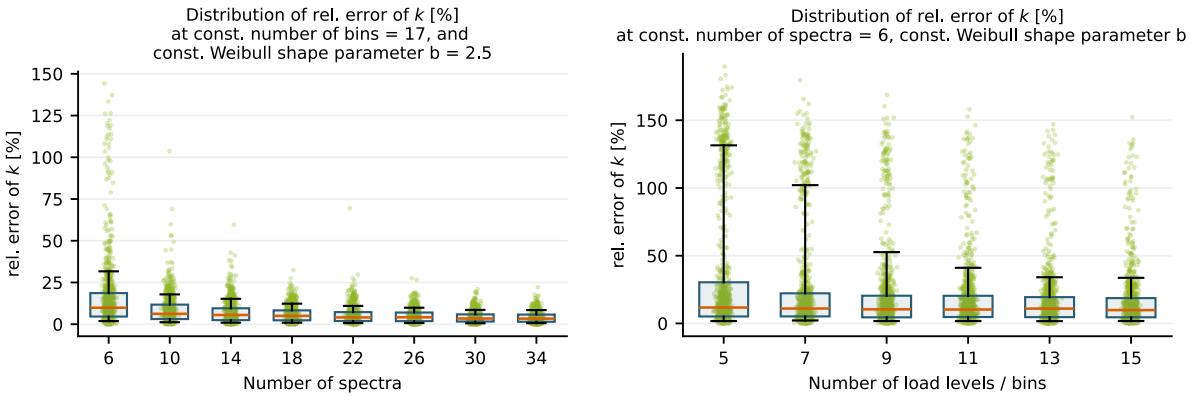

Figure 3. Distribution of rel. error of k at b=2.5: influence of the number of spectra (left) and the number of bins (right)

Given that the method is intended for practical lifetime-model estimation, a conservative error measure is more informative than the mean. For this reason, the analysis was repeated using the 90th percentile of the relative error. The corresponding heatmaps are displayed in Figure 4.

The 90th-percentile results confirm the trends already visible in the mean values, but they reveal more clearly how sensitive the method is to unfavorable parameter constellations. Even when the average reconstruction error appears acceptable, the upper range of the estimation error can remain substantial if only a few spectra are available or if the damage at failure shows strong scatter. The 90th-percentile analysis therefore provides a more realistic basis for judging whether the method is sufficiently robust for a given application.

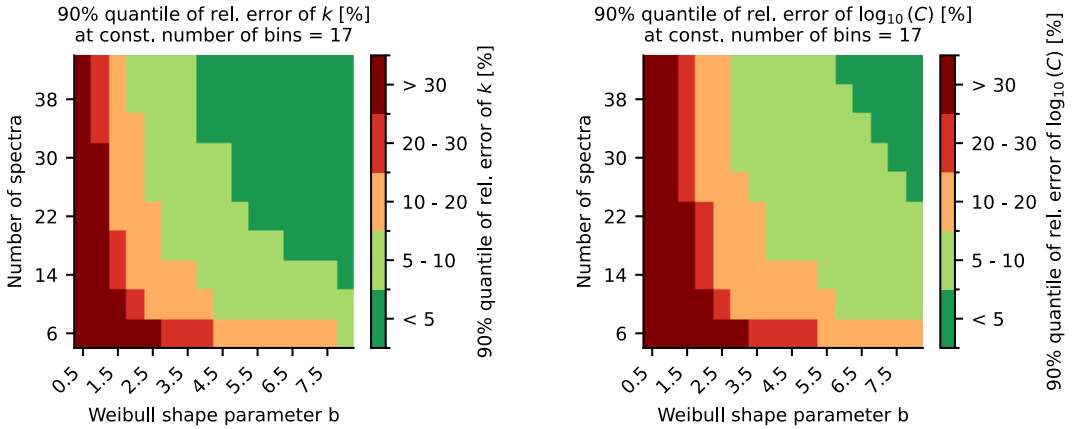

Figure 4. 90% percentile of the relative error of k and $log(C)$ at a constant number of bins

Overall, the results demonstrate that the proposed reconstruction approach is feasible and can provide useful first estimates of lifetime-model parameters from failed operational spectra. However, the results also show clearly that its reliability depends strongly on the number of available spectra and on the variability of the cumulative damage at failure. The method should therefore be interpreted as a pragmatic first-estimation tool rather than a replacement for statistically rigorous lifetime-model identification. Whether the resulting uncertainty and scatter of the reconstructed lifetime model are acceptable depends on the specific application and requires case-by-case evaluation.

4 Limitations of the Presented Method

Despite its simplicity and practical appeal, the presented method has limitations that must be considered when interpreting the reconstructed lifetime models.

First, the method uses only failed units and their associated load spectra. Surviving or still-functional units are not included in the estimation. As a consequence, censored information is ignored, although such data contain substantial information. Accordingly, the reconstruction is based exclusively on failure-related information and therefore relies on the assumption that the observed failures are representative of the underlying lifetime behavior.

Second, the method does not estimate a complete probabilistic lifetime model. In contrast to likelihood-based approaches, it does not identify the statistical shape parameter together with the parameters of the lifetime equation. Instead, all failed spectra are forced, as far as possible, to correspond to a cumulative damage close to $D = 1$. This considerably simplifies implementation and data processing, which is a practical advantage. At the same time, however, the method provides no estimate of the underlying scatter, which remains unknown.

Third, the reconstruction quality depends strongly on the validity of the assumption that failure occurs at approximately $D = 1$. The parameter study has shown that the method becomes unreliable when the damage at failure exhibits strong scatter. In particular, low Weibull shape parameters ($b < 1$) in the synthetic study lead to very large deviations in the reconstructed parameters and, consequently, in the reconstructed S–N curves.

Fourth, the method is sensitive to data scarcity. When only a small number of failed spectra is available, the spread and the error of the reconstructed parameters increases substantially. A similar, though weaker, effect occurs when the spectra are represented with few number of bins.

Finally, the present study is limited to one lifetime model and one synthetic-data-generation framework. The method was demonstrated only for a normalized Basquin model, and the operational spectra were generated from randomly parameterized Beta distributions.

For these reasons, the proposed approach should not be understood as a general replacement for established statistical lifetime-modelling methods. Its main value lies in situations where only failed field spectra are available and a fast first estimate is needed, for example to support early design assessments or to define follow-up testing.

5 Summary and Future Work

This paper presents a pragmatic method for estimating lifetime-model parameters from failed operational spectra by reconstructing cumulative damage. The method is intentionally simple: instead of formulating a full likelihood model, it estimates the deterministic parameters of a Basquin lifetime equation by minimizing the deviation of the reconstructed Miner damage from unity across multiple failed spectra.

The results show that such a reconstruction is, in principle, possible using operational data alone. The method can provide a useful first estimate of the lifetime model when a sufficient number of failed spectra is available and when the scatter in the damage at failure is not too large. In this sense, the approach can serve as a practical engineering tool for early lifetime assessment, for obtaining initial parameter values, or for planning more targeted follow-up experiments. At the same time, the study also highlights clear limits of applicability, as the reconstruction error increases markedly for small numbers of failed spectra and for high variability in the cumulative damage at failure, which can lead to substantial deviations in the reconstructed S–N curve.

As a first step, future work should focus on refining the present proof of concept with regard to the generation of operational spectra. In the present study, the parameters of the Beta distributions used to generate the spectra are drawn randomly, which leads to a very high variability among the resulting load collectives. In real applications, however, this level of heterogeneity is often not realistic, since identical or comparable products frequently experience similar operating conditions and therefore similar load patterns. Future studies should therefore consider introducing a grouping, clustering, or restriction of the spectrum generation process in order to better represent families of comparable operational loads. Finally, the methodology should be evaluated for other lifetime models beyond the Basquin equation in order to assess its general applicability.

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