A Model of the Growth of a "Managerial" Farm Firm

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This paper represents an attempt to make a contribution towards the theory of the growth of the farm firm by rigorous development of a growth model combining growth with managerial factors. There have been increasing battery of works alleged to depict the growth of the firm. Some of these analyze the importance of managerial factors in the growth of the firm (3, 5, 6, 7, 21), while some others incorporate the concept of adjustment cost and derive the optimum growth rate and growth path (4, 9, 10, 17, 26). No one of them, however, has succeeded in presenting a consistent answer to the essential questions which the model of growth must be able to answer; What is the source of growth? What limits the growth of the firm? What is the process of growth? Why there exist such big differences between firms in their growth? To make a frontal attack on the problem I prepare the factor of managerial ability and specify it as the augmentation speed of knowledge in the firm, and advance the theory here that the growth of the firm in general is to be ascribed to managerial factors and then draw from it some important implications.

In section I, I present the basic assumptions for the firm in my model. Section II and III describe four kinds of farm managerial role and corresponding abilities, and formulate them by adapting the concepts drawn on the stock of already accepted ideas. I proceed, in section IV, to weave them with a precise mathematical development into a general explanatory growth model of a farm firm, extending the range of the role of the farm manager[1] from an operator to an innovator. In section V, an investigation for the economies of scale in my model is presented.

I. The basic assumptions for the structure of the farm firm.

The firm in this paper is conceived as a collection of productive resources organized by an administrative entity to procure the required productive resources and to sell the farm products. The farm manager as the administrative entity possesses his own land and a little family labor, and purchases hired labor, fertilizer, seed and machines etc. to produce farm products for sale.

Output of the farm (Q) is related to the purchased input (V)—hired labor and fertilizer etc. and the self-supplied input (L)—land and family labor by a Cobb-Douglas type production function with diminishing marginal productivity of input.

$$Q = W\ V^{\alpha} L^{\beta},\ Q''_{V} < 0 \quad \cdots\cdots\cdots\cdots \quad (1)$$

The quantity of land and family labor is fixed and regarded as a given stock for the given year's production. The farm firm is assumed to decide the input quantity of purchased input and to distribute the self-supplied input to each production line so as to maximize the value-added of the self-supplied input of the year. The farm firm purchases and sells the purchased input and the output in perfectly competitive markets with constant prices over time. The latter assumption seems heroic. Leaving out of the assumption, however, will not change the basic implications of this model although some additional implications can be obtained as shown in some authors' works (9, 17).

II. Technological role of farm manager and corresponding abilities.

In microeconomics, the production technology of a firm is abstracted by a production function, and its change is by transformation of the function. This kind of abstraction for the technological side of a firm has been regarded as necessary and sufficient for the theory of the firm.[2] It proves, however, that a new view for the technology of a firm may provide very impressive ideas for the evolution of the growth theory of the firm. Drawn into a little more engineering or biological aspect for it, we realize that the technique of a firm can be specified by following three factors:[3]

a. what inputs, and what type of the inputs, employed for the production.
b. how the inputs operated.
c. how well the inputs operated.

Expressing it with a vector for simplicity.

$$t = \begin{pmatrix} i \\ m \\ s \end{pmatrix}$$

where t denotes a production technique,

i is the set of input,

m is the set of mode of operation of the set i,

s is the set of skill of operation of the set i,

We can infer from the above specification that change of the production technique of a firm will be induced by, change of the input set (i), improvement of the mode(m) and the skill(s) of operation.

Meanwhile, production function is defined as a function to show the maximum output attainable from a specified set of input. Here "maximum output —" implicates it presupposes the best mode of operation and the best skill the farm manager can do. Therefore, the production function is to be transformed when the mode of operation is bettered and when the skill of operation is improved. And of course by change of the specified set of input itself.

We have to note that these three kinds of source of technological change in a firm are just availability and their appearence in an individual firm is made in practice only by the managerial decision and behavior. This consideration tells us that the technological roles of the farm manager are accomplished and revealed by the way of the three routes.

I call the effect of the last two factors, m and s, on production as operator effect,[4)] since these two factors are concretely associated with the activity of operation of the farm manager. This operator effect (28, 29) which is associated with the purchased input in the physical production process, means to produce different quantities of output from a given bundle of input according to the operator, and the effect is supposed to be be augmented over time [5)] of operation by the process of learning, that is, integrating the personal experiences acquired during operation and the transferred knowledge about the given input and output the firm uses and prodcuces.

I assume the operator effect, W, augments with a diminishing rate over time as expressed in (2), in which λ is the very parameter of the farm manager's ability as an operator, The assumption of diminishing augmentation rate is founded on the allegation that the rate of change in useful knowledge is related to the size of the gap between the possible maximum level and the one in practice (2, 20)

$$\frac{\dot{W}}{W}=\frac{\lambda}{t+1} \quad \cdots\cdots (2)$$

$$W_t=W_0(1+t)^{\lambda} \quad \cdots\cdots (3)$$

$$W_0=W_0(\lambda) \quad \cdots\cdots (4)$$

W_0, which is the initial level of operator effect at the moment of adaption of the given input, output and method, is considered as an increasing function of λ, since the initial level itself is the consequence of past learning (22, p. 54).

I call the effect of the factor i on production as innovator effect, since it is associated with the activity of decision making for new input of the farm manager. Innovator effect (18, 20, 29) is to search and decode the informations for new input and output, and to evaluate the benefit and uncertainty associated with them, and then to decide the adaptability of them to his farm. This ability is expressed with a parameter, rate of increasing the knowledge for new things and adapting them. This effect, however, grows in a very different pattern from the above operator effect, because the gap between the available maximum knowledge and the one in practice is not shrinked over time as far as advanced inputs and outputs are developed in the economy without pause, which is the real situation of a dynamic economy. Formular specification for the augmentation of this effect is delayed by the section IV.

III. Allocational and organizational role of farm manager and corresponding abilities.

I present in this section two kinds of non-technological role of farm manager associated with production activity. The one is the role of allocation and the other is that of organization.

Allocator effect (9, 13, 28) is to determine the input quantity of purchased input for each production line, gathering and decoding the informations transferred and acquired with personal experiences for the purchased input and the output. The farm manager without perfect knowledge for either his input and output or for the market takes the process of dynamic adjustment toward the optimum over time, reflecting the newly acquired knowledge. I specify the process of approaching to the optimum quantity of purchased input with the continuous partial adjustment form.

$$V = \frac{\delta}{D+\delta} V^{*} \quad \cdots\cdots (5)$$

V^{*} is the optimum quantity, and δ is the adjustment speed. D is the differential operator. This speed is the paramenter of the manager's allocation ability.

Organizer effect and coordination cost (22, 24) are concerned with allocating self-supplied input among multiple production lines, and making an operation system.[6] As for the coordination cost, there are excellent papers to have applied the concept of adjustment cost to the problem of investment and growth (9, 17, 26). Co-ordination cost in this paper is corresponding to the adjustment cost, but it is not necessarily identical with it in its source and explanation. This cost, foregone output in more precise meaning, is associated with both expansion of self-supplied input and innovation. Both of them should require a revised organization but the farm manager without perfect knowledge can not accomplish the reorganization immediately (19, 22). A part of product (or revenue) which otherwise is supposed to be produced with the quantity of input is to be sacrificed and foregone due to the above imperfect reorganization. Consequently, the apparant production function is obtained by revising the 'supposed production function' (1) in section I with the complementary term which corresponds to the sacrificed quantity due to expansion and innovation. This co-ordination cost (ϕ) is assumed to be a function of expansion size, E, and investment for innovation, I.[7] It is twice differentiable to fulfill the following conditions:

$$\phi=\phi\ (E,\ I) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (6)$$

$$\phi'_E,\ \phi'_I>0 \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (7)$$

$$\phi''_E,\ \phi''_I>0 \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (8)$$

Therefore the 'apparant production function' will be,

$$\hat{Q}=Q(V,\ L)-\phi(E,\ I) \quad \cdots\cdots\cdots\cdots \quad (9)$$

Due to the same reasoning in section II, the co-ordination cost is assumed to diminish at a rate ε, which is the parameter of managerial ability to organize the fixed input and the operation system.

$$\phi_t = \phi(E, I)\, e^{-\varepsilon t} \quad \cdots\cdots\cdots\cdots (10)$$

IV. Growth path.

1. In this section I will develop the growth path formulating the above concepts. Growth of the farm firm is defined as growth in the farm firm's activity represented by its total revenue. Let us suppose the farm to expand its land as much E and invest the amount of I for a new input at time $t=0$. The growth path of the farm firm is obtained by substituting (1), (10) into (9),

$$R_t = P_q\{W_t V_t^{\alpha} L^{\beta} - \phi_t(E, I)\} \quad \cdots\cdots\cdots\cdots (11)$$

Where, $L = L_o + E$, and P_q is the price of the output. The optimum quantity of the purchased input, V^*, under the fixed quantity of land, is given by a simple calculus [8]

$$V^* = ML^a \quad \cdots\cdots\cdots\cdots (12)$$

where, $M = (W\, P_v^{-1} P_q)^{\frac{1}{1-\alpha}}$

$$a = \frac{\beta}{1-\alpha}$$

P_v is the price of the purchased input.

Substituting (11) into (4) and solving the differential equation, we will find the actual input,

$$V_t = ML^a - (ML^a - V_0)e^{-\delta t} \quad \cdots\cdots\cdots\cdots (13)$$

$$V_0 = \rho V^* = \rho ML^a \quad \cdots\cdots\cdots\cdots (14)$$

$$\rho = \rho(\delta) \quad \cdots\cdots\cdots\cdots (15)$$

where ρ is the appropriate number to express the ratio between the optimum and the initial.

This ρ is also considered as an increasing function of the adjustment speed, δ, due to the same reason as for the operator effect. Substituting (3), (10) and (13) into (11), we are given the complete growth *path*;

$$R_t = P_q\{W_0(1+t)^{\lambda}(1-e^{-\delta t}+\rho e^{-\delta t})^{\alpha}M^{\alpha}L^{a}-\phi(E,I)e^{-\varepsilon t}\} \cdots (16)$$

We proceed to formulate the growth rate over time. Transpose the co-ordination cost term, ϕ, in (9) and take logarithm,

$$\mathrm{In}(\hat{Q}_t+\phi_t)=\mathrm{In}Q_t \cdots\cdots (17)$$

Differentiating it with time, we obtain,

$$\frac{\dot{\hat{Q}}_t+\dot{\phi}_t}{\hat{Q}_t+\phi_t}=\frac{\dot{Q}_t}{Q_t}$$

$$=\frac{\dot{W}}{W}+\alpha\frac{\dot{V}}{V}+\beta\frac{E}{L}$$

$$=\frac{\lambda}{t+1}+\frac{\delta(1-\rho)}{e^{\delta t}+\rho-1} \cdots\cdots (18)$$

Meanwhile, with a simple development we can obtain, (see appendix 2)

$$\frac{\dot{\hat{Q}}_t}{\hat{Q}_t}=\frac{\dot{\hat{Q}}_t+\dot{\phi}_t}{\hat{Q}_t+\phi_t}+(\frac{\dot{\hat{Q}}_t+\dot{\phi}_t}{\hat{Q}_t+\phi_t}+\varepsilon)\frac{\phi_t}{Q_t} \cdots\cdots (19)$$

Therefore,

$$\frac{\dot{R}_t}{R_t}=(\frac{\lambda}{t+1}+\frac{\delta(1-\rho)}{e^{\delta t}+\rho-1})+(\frac{\lambda}{t+1}+\frac{\delta(1-\rho)}{e^{\delta t}+\rho-1}+\varepsilon)\frac{\phi e^{-\varepsilon t}}{Q_t} \cdots\cdots (20)$$

The growth rate at time $t=0$, $t=\infty$,

$$\left.\frac{\dot{R}_t}{R_t}\right|_{t=0}=(\lambda-\delta+\frac{\delta}{\rho})+(\lambda-\delta+\frac{\delta}{\rho}+\varepsilon)\frac{\phi}{Q_0} \cdots\cdots (21)$$

$$\left.\frac{\dot{R}_t}{R_t}\right|_{t=\infty}\to 0 \cdots\cdots (22)$$

We find the growth rate is a function of the three managerial parameters and diminishing down to zero as time passes, which means the farm firm falls into stagnation.

2. The remaining problem in this model is how the size of expansion is determined. Fig 1 represents it. The necessary conditions for profit maximization are; [9]

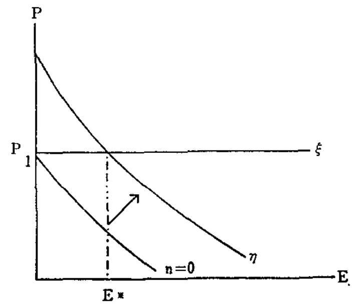

Fig1. Optimum expansion.

i) $\eta=\xi$ .... (23)

ii) $\frac{\partial\eta}{\partial E}<0$ .... (24)

where, η is the present value of marginal revenue of expansion, ξ is the marginal cost of expansion. Meanwhile,

$$\eta=\int_0^\infty \frac{\partial R}{\partial E}e^{-rt}dt$$

$$=\int_0^\infty P_q\{W_0(1+t)^\lambda\ (1-e^{-\delta t}+\rho e^{-\delta t})^\alpha M^\alpha a\ L^{a-1}-$$

$$\phi'_E(E,\ I)e^{-\varepsilon t}\}e^{-rt}\ dt \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\ (25)$$

where, r is the interest rate.

Using the average theorem 1, (see appendix 3)

$$\eta=\frac{P_q}{r}W_0\ aM^\alpha L^{a-1}(1+\tau)^\lambda\ (1-e^{-\delta\tau}+\rho e^{-\delta\tau})^\alpha-$$

$$\frac{P_q}{\varepsilon+r}\phi'_E \cdots\cdots\cdots\cdots\ (26)$$

where, τ is a proper positive number.

And,

$\xi=P_1$ .... (27)

where P_1 is the price of land.

$\dot{E}^*$, where η is crossed by ξ, is the optimum expansion. Second order condition, (24), is satisfied if $\alpha+\beta$ is not enough bigger than unity. (see the appendix 1)

3. Owing to the process of learning explained in section II, knowledge in the firm is augmented continuously over time after expansion. Production technique is growing, allocation and organization is progressing to the optimum condition, and accordingly the capacity to expand again is to be accumulated. 'n' years after expansion, the marginal revenue curve of expansion will become.

$$\eta_n = \frac{P_q}{L^2 r} Q_\tau - P_q \frac{\phi'(\bar{E}+E)}{\varepsilon + r}$$

where, $\phi'(E^*)e^{-\varepsilon n} = \phi'(\bar{E})$ and $\frac{\partial \bar{E}}{\partial n} < 0$ …… (28)

τ is the same positive number with the one in (26). Its slope is given, if we take the simplest case of $\alpha+\beta=1$,

$$\frac{\partial \eta_n}{\partial E} = -P_q \frac{\phi''(\bar{E}+E)}{\varepsilon + r} < 0 \quad \cdots\cdots\cdots\cdots \quad (29)$$

It means the curve of the present value of marginal revenue of expansion shifts upwards from the starting position as time passes after expansion as shown in fig 1, while the marginal cost curve of expansion is fixed at P_1. Therefore the new optimum expansion size will grow year by year and it will play a role to stimulate the farmer to expand again. Although I am not preparing the precise formula to show when the farm will come into implementation,[10] the farm will expand its land again in time and starts a new expansion cycle.

It will be very useful to investigate the relation between the optimum expansion and the current size of land at a given time by constructing a phase diagram in the (L_0, E^*) plane as shown in fig 2. (See appendix 4)

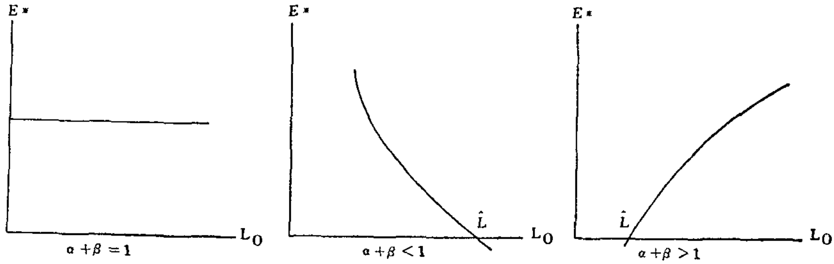

Fig2. Current size and optimum expansion

By definition

$$\eta(E^*)=\xi(E^*) \quad \cdots\cdots (30)$$

Taking total derivative and dividing with dL_0

$$\frac{d\eta}{dL_0}=\frac{\partial\eta}{\partial L_0}+\frac{\partial\eta}{\partial E^*}\ \frac{dE^*}{dL_0}=0 \quad \cdots\cdots (31)$$

As $\frac{\partial\eta}{\partial E^*}$ is non-negative within the relevant range,

i. if $\alpha+\beta>1$, $\frac{\partial\eta}{\partial L_0}>0$ and $\frac{dE^*}{dL_0}>0$

ii. if $\alpha+\beta\leqq1$, $\frac{\partial\eta}{\partial L_0}\leqq0$ and $\frac{dE^*}{dL_0}\leqq0$

It is not difficult to prove that

i. if $\alpha+\beta\leqq1$, $\frac{d^2E^*}{dL_0^2}\geqq0$

ii, if $\alpha+\beta>1$, $2\alpha+\beta<2, \frac{d^2E}{dL_0^2}<0$

iii, if $2\alpha+\beta>2$, $\frac{d^2E}{dL_0^2}>0$

What is made clear in this diagram is that the optimum expansion is decreasing, constant and increasing as the return to scale is decreasing, constant and increasing. In most probable cases, the rate of expansion should diminish, and we can conclude that even though continuous expansion is available and optimum expansion is continuously implemented, the growth rate will converge to zero since the expansion rate, $\frac{E}{L}$, is diminishing. To our surprise, the farm firm falls into stagnation again. But it is very implicative to note that there is a very interesting case where the growth rate does not slow down, the case iii, $2\alpha+\beta>2$. Fig 3 shows the region to satisfy the condition. In other words, if the scale elasticity is fairly big, and if there exists a special relation between production coefficients of the self-supplied input and the purchased input, optimum expansion rate does not diminish.

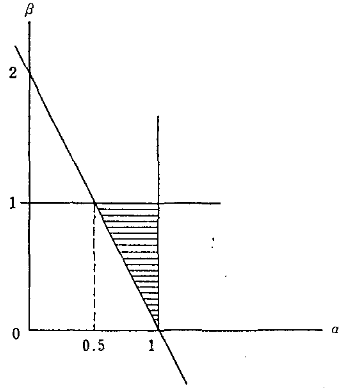

Fig3. The region of continuous growth without innovation.

4. As shown so far, the growth rate of the farm can not but fall into stagnation after all regardless of its ability to produce, allocate and organize except a very special case. What is the source to make the firm grow without stagnation? It is innovation. If we assume technological progress advances at a rate in the economy, we can also assume a rate of technical progress, g, in an individual farm firm. The rate is the parameter of innovator ability of the farm manager. It means immediately that the alternative revenue of the firm, $\tilde{R}_t$, is increasing with the same parameter,

$$W_0=W_0e^{gt} \quad \cdots\cdots (32)$$

$$\tilde{R}_t=P_q\{e^{gt}Q_0-\phi(\bar{E}\ ,I)\} \quad \cdots\cdots (33)$$

The alternative revenue is the one subjectively evaluated by the farm manager. Therefore this value will be higher for the manager who possesses more informations for new things than who have less and for the manager who can more concretely evaluate the risk and uncertainty associated with the new things than who can do less.

At T_1 in the fig 4, the alternative revenue of the firm is equalized to the revenue in practice and earlier or later innovation is taken.[11] Afterward T_1, a new phase of learning starts, and according new actual and alternative revenue curves ($R_{t2}, \tilde{R}_{t2}$) appear. Actual curve, R_{t2}, is according to (16) without new comment, but the new alternative curve, $\tilde{R}_{t2}$, requires a new explanation. Adaptability of a technique for a farm firm is constrained by its technique used hitherto and now. It means that the technique and its progress adaptable to a farm is confined to a part of all existing techniques and all newly developed techniques. Accordingly, once a particular technique is choosed at time T_1, technological progress relevant for the farm firm is confined, and alternative revenue will be $\tilde{R}_{t2}$, not the extension of $\tilde{R}_{t1}$ which will be effective under the situation of perfect technological flexibility. This is the process that the particular technological system of each farm firm is and the particular technological distribution in an economy is, formed.

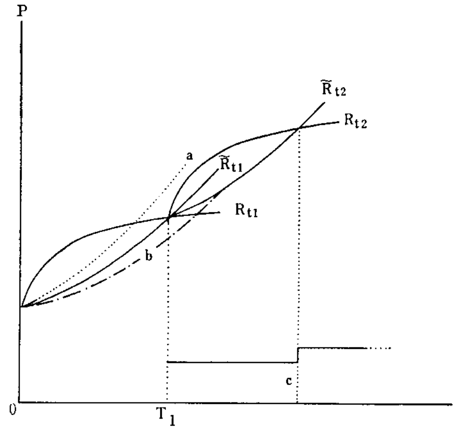

Fig4. Innovation cycle.
* a in case of no co-ordination cost.
b in case of no organization improvement.
c the portion of technological inflexibility.

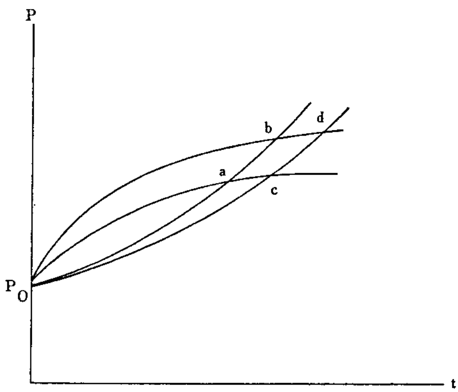

Fig5. Different innovation cycle according to different managerial ability
* a high ξ, ζ, low $\lambda, \delta, \varepsilon$.
b high ε, ζ, high $\lambda, \delta, \varepsilon$.
c low ε, ζ, low $\lambda, \zeta, \varepsilon$.
d low ε, ζ, high $\lambda, \zeta, \varepsilon$.

We note the slope of the alternative revenue curve is dependent on the innovation and organization ability parameters for a given type of Q and ϕ, but the time of innovating is and therefore the long-run growth path also is, associated with all other abilities as much as with innovation ability. Some illustrations are shown in fig 5.

The final complete growth path of the firm,

$$R_t = P_q e^{mgT} W_0 (1+t_1)^{\lambda} (1-e^{-\delta t_1} + \rho e^{-\delta t_1})^{\alpha} M^{\alpha} L^{a} - P_q \phi e^{-\varepsilon t_2} \quad \cdots\cdots (34)$$

where, T is the interval between innovation and innovation, t_1 is the time after latest innovation, t_2 is the time after the later one between the latest innovation and the latest expansion, m is the number of innovation.

Accordingly the growth rate over time,

$$\frac{\dot{R}_t}{R_t} = \left(\frac{\lambda}{t_1+1} + \frac{\delta(1-\rho)}{e^{\delta t_1}+\rho-1}\right) + \left(\frac{\lambda}{t_1+1} + \frac{\delta(1-\rho)}{e^{\delta t_1}+\rho-1} + \varepsilon\right)\frac{\phi e^{-\varepsilon t_2}}{Q_t} \quad \cdots\cdots (35)$$

If $t_1=0$, $t_2=0$,

$$\frac{\dot{R}_t}{R_t} = \left(e^{gt} + \lambda + \frac{\delta}{\rho} - \delta + \nu_t\right) + \left(e^{gt} + \lambda + \frac{\delta}{\rho} - \delta + \nu_t + \varepsilon\right)\frac{\phi}{Q_t} \cdots\cdots (36)$$

where, ν_t is the rate of land expansion at time t determined in fig 1.

V. **Economies of scale** is one of the subjects which have been debated most by economists (11, 19, 21) from early time and the definition of the concept is not always agreed by all. However, I will confine the concept of it to only production function, and discuss about two points; return to scale as defined with scale elasticity(8), relation between current size and optimum expansion.

According to this model, ‘return to scale’ should be distinguished from ‘return to expansion’. The first is static and ex-ante facto for the firm, while the second is dynamic and ex-post. It will be made very distingiushable to express in a mathematical form.

$$s=\frac{dQ}{dm}\cdot\frac{m}{Q} \quad : \text{ return to scale} \cdots\cdots\cdots\cdots\cdots\cdots (37)$$

$$e=\frac{d\hat{Q}}{dm}\cdot\frac{m}{Q}=\left(\frac{dQ}{dm}-\frac{d\phi}{dm}\right)\frac{m}{Q} : \text{ return to expansion} \cdots\cdots (38)$$

$$=\frac{dQ}{dm}\cdot\frac{m}{Q}-\frac{d\phi}{dm}\cdot\frac{m}{Q}$$

= technological return to expansion − organizational return to expansion.

Return to expansion consists of two faces; technological return to expansion (= return to scale of traditional concept) and organizational return to expansion.

We should note the important fact that constant or increasing return to scale does not always mean rationality of indefinite expansion, since the advantage of larger scale is offset by the organizational disadvantage, although to disappear in the end. As the organizational return to expansion increases with expansion scale, we can speculate here that the more expansive and innovative the farm firm is, the more underestimated return to scale will be. [12]

Here I like to point one thing related with the called optimum size in the diagram 4. As noted in the diagram, there exists a stationary size of land, $\hat{L}$, at which no expansion is rationalized. However, it does not mean there is a limit of expansion for the farm firm. We have to note that the stationary size is dependent on the innovation effect, and the effect is growing at the speed of innovation. Accordingly, this stationary point is progressing forward so much as the farm firm's innovation speed is high. In a dynamic economy, therefore, the farm is under shadow-chasing relation with the stationary size, as far as over-expansion is not taken. [13]

Summary and Conclusions.

I specified four kinds of managerial effect; allocator effect, organizer effect and innovator effect. I assumed imperfect knowledge of the farm manager and learning-adjustment process taken in the firm. Incorporating these ideas, I developed a rigorous growth model and drew out of it following implications. Growth rate of an individual farm firm is determined by its managerial parameters, the rate of learning, so to speak, its ability to improve its operator skill and to advance allocation, organization and innovation. In spite of such importance of managerial ability on growth, marginal efficiency of managerial ability disappears in a stand-still economy without technological progress and change. The more dynamic the economy is, the more important managerial effect in growth of the firm will be. Because we found that the farm firm without high innovation ability or under stand-still economy could not but fall into stagnation, even if continuous expansion of land were available, regardless of the managerial ability as an operator and allocator and organizer. However, it never means innovation is the only important effect of all, because the innovating *time, and therefore, the long-run growth path is dependent* on *all* other managerial effects as much as on innovation effect.

Land expansion is described as a learning-expansion cycle, while innovation is a learning-innovation cycle. The capacity to expand again is augmented continuously after expansion as the operator effect is growing and co-ordination is progressing, and this accumulation stimulates the firm to expand again. Very similarly, since the knowledge for new things is accumulated in the farm after innovation, alternative revenue is increasing and finally the firm comes into implementation of innovation in time after the alternative passes over the actual.

We found in a dynamic economy there was no optimum size, but only stationary size in case of non-constant return to scale. It does not mean, however, there is any limit of its size because the stationary size is never caught up and there exists always only an optimum expansion regardless of return to scale. We found 'return to expansion' was more useful concept than 'return to scale' for growth study. Return to expansion consists of return to scale in traditional concept, I called it technological return to expansion, and organizational return to expansion. Since return to scale is offset by organizational return to expansion, estimation of usual return to scale with scale elasticity is likely more underestimated if the firm is more expansive and innovative or the firm is in a more dynamic economy.

APPENDIX 1.

$$\eta=\frac{aP_qQ_\tau}{rL}-\frac{P_q}{\varepsilon+r}\phi'_E$$

where, $Q_\tau=W_0(1+\tau)^\lambda M^\alpha L^a(1-e^{-\delta t}+\rho e^{-\delta t})^\alpha$

$$\frac{\partial\eta}{\partial E}=a(a-1)\frac{P_qQ_\tau}{rL^2}-\frac{P_q}{\varepsilon+r}\phi''_E \cdots\cdots\cdots\cdots\cdots\cdots \text{(A)}$$

While, $a(a-1)=(\frac{\beta}{1-\alpha})(\frac{\alpha+\beta-1}{1-\alpha})=\frac{\beta(\alpha+\beta-1)}{(1-\alpha)^2}\gtreqless 0$ as $\alpha+\beta\gtreqless 1$

and, $\phi''_E>0$.

The others are all positive.

Therefore, $\frac{\partial\eta}{\partial E}\leq 0$, as far as $\alpha+\beta$ is not so big that the absolute value of the first term of (A) exceeds the second.

APPENDIX 2.

$$\frac{\dot{\hat{Q}}_t+\dot{\phi}_t}{\hat{Q}_t+\phi_t}-\frac{\dot{\hat{Q}}_t}{\hat{Q}_t}=-\frac{\phi_t(\varepsilon\hat{Q}_t+\dot{\hat{Q}}_t)}{\hat{Q}_t(\hat{Q}_t+\phi_t)} \quad \text{(since } \dot{\phi}_t=-\varepsilon\phi_t\text{)}$$

$$=-\frac{\phi_t}{\hat{Q}_t+\phi_t}(\varepsilon+\frac{\dot{\hat{Q}}_t}{\hat{Q}_t})$$

$$=-\varepsilon\frac{\phi_t}{\hat{Q}_t+\phi_t}-\frac{\phi_t}{\hat{Q}_t+\phi_t}\frac{\dot{\hat{Q}}_t}{\hat{Q}_t}$$

$$\therefore \frac{\dot{\hat{Q}}_t+\dot{\phi}_t}{\hat{Q}_t+\phi_t}+\varepsilon\frac{\phi_t}{\hat{Q}_t+\phi_t}=\frac{\dot{\hat{Q}}_t}{\hat{Q}_t}(\frac{\hat{Q}_t}{\hat{Q}_t+\phi_t})$$

$$\therefore \frac{\dot{\hat{Q}}_t}{\hat{Q}_t} = \frac{\dot{\hat{Q}}_t + \dot{\phi}_t}{\hat{Q}_t + \phi_t} + \left(\frac{\dot{\hat{Q}}_t + \phi_t}{\hat{Q}_t + \phi_t} + \varepsilon\right)\frac{\phi_t}{Q_t}$$

APPENDIX 3.

$$\int_a^b f(x)\varphi(x)dx = f(\xi)\int_a^b \varphi(x)dx$$

Where, between a and b, f(x) is continuous, $\varphi(x)$ is differentiable and has constant sign.

(proof)

if $\varphi(x) \geqq 0$,

$m\ \varphi(x) \leqq f(x)\varphi(x) \leqq M\varphi(x)$.

where, m M are the below and above boundary of $f(x)$.

$$\therefore\ m\int_a^b \varphi(x)\,dx \leqq \int_a^b f(x)\varphi(x)dx \leqq M\int \varphi(x)dx.$$

Selecting a number μ as

$$\int_a^b f(x)\,\varphi(x)dx = \mu\int_a^b \varphi(x)dx,$$

$m \leqq \mu \leqq M$

As f(x) is continuous between a and b, there should be a number, which is $a \leqq \xi \leqq b$, to satisfy $f(\xi) = \mu$. Concludingly,

$$\int_a^b f(x)\varphi(x)dx = f(\xi)\int_a^b \varphi(x)dx$$

APPENDIX 4.

From $\eta(E^*) = \xi(E^*)$

$$\frac{d\eta}{dL_0} = \frac{\partial\eta}{\partial L_0} + \frac{\partial\eta}{\partial E^*}\ \frac{dE^*}{dL_0} = 0$$

where, $\frac{\partial\eta}{\partial L_0} = \frac{a(a-1)Q\tau}{rL^2} \gtreqless 0$ as $\alpha+\beta \gtreqless 1$

$\frac{\partial\eta}{\partial E^*} < 0$ (see appendix 1)

$$\therefore\ \frac{dE^*}{dL_0} \gtreqless 0 \quad \text{as } \alpha+\beta \lesseqgtr 1$$

and also

$$\frac{d^2\eta}{dL_0^2}=\frac{\partial^2\eta}{\partial L_0^2}\left(1+\frac{dE^*}{dL_0}\right)+\frac{\partial\eta}{\partial E^*}\frac{d^2E^*}{dL_0^2}=0$$

where, $\frac{\partial^2\eta}{\partial L_0^2}=a(a-1)(a-2)\frac{Q\tau}{rL^2}$

$$a-2=\frac{2\alpha+\beta-2}{1-\alpha}\gtrless 0 \text{ as } 2\alpha+\beta\gtrless 2$$

$\therefore \frac{\partial^2\eta}{\partial L_0^2}\geq 0$ if $\alpha+\beta\leq 1$ or $2\alpha+\beta\geq 2$

$\frac{\partial^2\eta}{\partial L_0^2}<0$ if $2\alpha+\beta>2$ (of course $\alpha+\beta>1$)

Meanwhile, $\frac{dE^*}{dL_0}=\frac{d\eta/dL_0}{d\eta/dE^*}=\frac{k}{\frac{\phi''}{\varepsilon+r}-k}>-1$

$(k=a(a-1)Q\tau/rL^2)$

Therefore,

$\frac{d^2E^*}{dL_0^2}\geq 0$ if $\alpha+\beta\leq 1$

$\frac{d^2E^*}{dL_0^2}<0$ if $\alpha+\beta>1$ but $2\alpha+\beta<2$

$\frac{d^2E^*}{dL_0^2}>0$ if $2\alpha+\beta>2$ (of course $2\alpha+\beta>1$)

Notes

1. The farm manager, in this model, posseses his own capital and is responsible for all risk accompanied by his behaviors, and he makes all final decisions himself. In this sense, he may be rather called entrepreneur.
2. It is far from denying the fact that a lot of brilliant studies have been made for the matter of technological progress in Economics. Most of them, however, stand on the macroeconomic aspect and their central aims are at the problem of distribution of national income and growth of national economy. And sometimes they pay their attention to this matter for the problem of aggregation of the capital.
3. I am apart from the conceptual definition of the technique, and therefore from the philosophic controversy over the substancial definition, since I only intend to characterize a technique in a practical sense.
4. I called it worker effect, immitating Welch, at the first draft. I preferred this new name because the activity of the farm manager in the farm field is something more than that of a simple worker. He is given the right of tactical decision concerned with his farm field work and rather should be be called operator.
5. Time does not mean lapse of time, but rather number of experience for the production.
6. Distribution effect of purchased input among production lines is not included in this model. It is considered to be more plausible to assume that the farm manager decides the required quantity of purchased input for each production line respectively and purchases the total quantity than decides the total quantity to be purchased and distributes them among production lines.
7. The amount of investment for innovation is not necessarily the best argument because some innovation, even though require same amount of investmet with others, give bigger impact on the organization and cause bigger co-ordination cost when their characteristics are more different from the current one.
8. Time effect on M is neglected in this paper for only convenience. M can be solved as a function of time, using Maclaurin series, but it makes the further development of this model too complicate to be handled without new fruits.
9. In this development, the decision criterion with respect to the given year's

production of the farm manager is assumed to be value-added of the self-supplied input while the criterion for expansion and innovation concerned with long-run behavior is profit.

10. I think we can find a clue for this problem in the respects of fund accumulation and land supply, and also in the theory of the behavioral science.
11. I don't submit a formula to determine the precise time of innovation. The optimum time can be calculated under the assumption of long-run profit maximization. But I thought the behavioral theory of the firm may be useful for this matter and pended this problem untouched.
12. This implication is very similar to Shen (24).
13. I have excluded the case of price change and over-expansion. If we take account for such cases, there happens the possibility of over-size, i.e., the size of the farm is rightward beyond the $\hat{L}$. Therefore, in the practical world of fluctuating price and over-expanding due to mistake etc., some farms may be under over-scale.

References

(1) ALLEN, R.O., Mathematical Economics, Macmillan Oxford Basil Blackwell, 1959.

(2) ARROW, K.J., "The economic implications of learning by doing," Rev. Econ. Stud. 29: 155-173, 1962.

(3) BAILEY, W.R., ",Necessary conditions for growth of the farm", Agr. Econ. Rev., XIX: 1-6, 1967.

(4) BAUMOL, W. J., "On the theory of expansion of the firm", Ame. Econ. Rev., 52: 1078-1087, 1962.

(5) BOEHLJE, M.D., T.K. WHITE, "A production-investment decision model of farm firm growth", J. Farm Econ, 49, 1967.

(6) BULLER, ORLAN, "Analysis of growth of agricultural firm", Cana. J. Agri. Econ., 20: 25-35, 1971.

(7) EISGRUBER, L.M., G.E. LEE, "A system approach to studying growth of the farm firm", in System Analysis in Agricultural Management ed. by Dent J.B., T.R. Anderson, 1971.

(8) FERGUSON, C.E., The Neoclassical Theory of Production and Distribution, Cambridge Uni., 1971.

(9) GOULD, J.P., "Adjustment cost in the theory of investment of the firm", Rev. Econ. Stud., 35: 47-55, 1968.

(10) HALTER, A.N., "Model of firm growth", J. Farm Econ., 48: 1503–1509, 1966.

(11) HEADY, E. O., Economics of Agricultural Production and Resource Use, Prentice-Hall, 1952.

(12) HIRSCH, W.Z., "Technological progress and microeconomic theory", Ame. Econ. Rev., 59: 36–43, 1969.

(13) HUFFMAN, W. E., "Decision making; the role of education", Ame. J. Agri. Econ., 66: 85–97, 1974.

(14) IJIRI, YUJI, HERBERT A. SIMON, "A model of business firm growth", Economerica 35: 348–355, 1969.

(15) ______________________, "Interpretation of departures from the Pareto curve firm-size distribution", J. Pol. Econ., 81; 315–331, 1973.

(16) LEE, J.H., "An essay on the theory of the firm", Nogyo Keiei Kenkyu (Farm Management Study) 2, Hokkaido Uni., 1975.

(17) LUCAS, R.E., "Adjustment cost and the theory of supply", J. Pol. Econ., 75: 321–334, 1967.

(18) MANSFIELD, EDWIN, "Entry, Gibrat's law, innovation, and the growth of firms", Ame. Econ. Rev., 52: 1023–1051, 1962.

(19) MARSHALL, A., Principles of Economics, 8th ed., Macmillan 1962.

(20) NELSON R.R., E.S. PHELPS, "Investment in humans, technological diffusion and economic growth", Ame. Econ. Rev., 56: 69–75, 1966.

(21) PATRIK, G.., L M. EISGRUBER, "The impact of managerial ability and capital structure on growth of the farm firm", J. Farm Econ., 50: 491–506, 1968.

(22) PENROSF, E. T., The Theory of the Growth of the Firm, Oxford Basil Blackwell, 1952.

(23) RUBIN, P. H., "Expansion of firms", J. Pol. Econ., 81: 936–949, 1973.

(24) SHEN, T. Y., "Economies of scale, penrose effect, growth of plants and their size distribution", J. Pol. Econ., 78: 701–716, 1970.

(25) TAKAKI, Kaiseki Gairon (Introduction to Analysis), 97–100, 59–66, 1965.

(26) TREADWAY, A. B., "On rational entrepreurial behavior and the demand for investment", Rev. Econ. Stud., 36: 227–239, 1969.

(27) UZAWA, H., "The Penrose-effect and growth optimum", Theoretical Economics XIX: 1–14, Tokyo, 1969.

(28) WELCH, F , "Education in production", J. Pol. Econ., 78: 35–50, 1970.

(29) YOAV, KISLEV, NIRA SCHORI BACH RACH, "Process of an innovation cycle", Ame J Agri. Econ., 55: 28–37, 1973.