Breathing Scattors in Dissipative Systems

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Scattering process between traveling breathers (TBs) is studied for the complex Ginzburg-Landau equations (CGLE) with a parametric forcing term. The phase-dependency of the input-output relation can be explained from the scattor's viewpoint. In this note, we especially focus on the issue for the asymmetric head-on collisions, i.e., the phases of two colliding TBs are different.

§1. Introduction

There is a variety of collision process for particle-like patterns in dissipative systems even restricted to head-on ones. A key issue is to classify the input-output relation before and after collision and clarify its underlying mechanism for the scattering process. A new viewpoint was presented to explain the process of head-on collisions among traveling pulses and spots,[1),2)] especially a notion of "scattor" was introduced to understand the input-output relation. The scattor itself is just an unstable steady or time-periodic solution (i.e., saddle) and its profile is not close to that of the concerning traveling object, however once there occurs a collision, the solution deforms significantly and approaches a part of the unstable manifold of the scattor and is driven by it. The final output is therefore determined by the destination of the unstable manifold.

The aim of this note is to investigate the scattering process for TBs in the following CGLE with a parametric forcing term.

$$W_t = \quad (1+ic_0)W + (1+ic_1)W_{xx} - (1+ic_2)|W|^2W + c_3W^*,$$

where c_3 is taken as a bifurcation parameter in a bistable regime. Since such TBs vary time-periodically, the input-output relation in general depends on the phase at collision. It turns out that the scattor for the CGLE becomes an unstable time-periodic solution. Our goal is to show the existence of such scattors numerically and we especially focus on the complex dynamics like the synchronization between the phases of colliding TBs through an asymmetric head-on collision.

§2. For symmetric head-on collisions

We first consider the symmetric head-on collision, namely the initial two pulses are perfectly symmetric except their propagating directions, i.e., equivalent to hit-

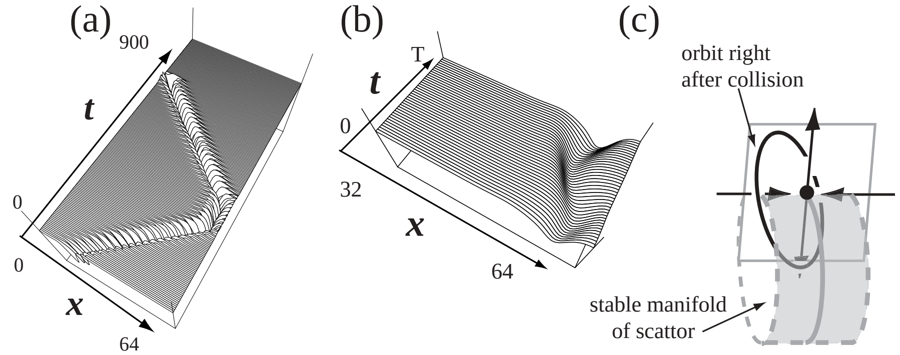

Fig. 1. (a) The TB is reflected at the first collision, while the annihilation occurs at the second collision at $c_3 \approx 0.14243$. Here we draw the space-time plot of $|W|^2$. (b) Spatio-temporal pattern of the in-phase breathing scattor obtained by using AUTO.[3] (c) Schematic picture of the relation for the orbits right after collisions and the stable manifold of the scattor when the annihilation and reflection coexist.

ting a wall under Neumann boundary condition. To investigate the transition point between the annihilation and reflection regime, the orbital behavior is traced carefully by changing c_3 with the initial condition being fixed. The other parameters are set to be $c_0 = 1.0$, $c_1 = -0.5$ and $c_2 = 1.1$. A remarkable thing happens as shown in Fig. 1(a), namely the TB reflected at the first collision, while the annihilation occurs at the second collision despite c_3 being fixed. This makes a sharp contrast with the steady traveling pulses.[1),2)] It should be noted that a phase-dependent output occurs over a range of c_3 in this transition regime and the quasi-time-periodic object is observed for certain time right after the first collision as shown in Fig. 1(a).

A hidden unstable time-periodic solution called the breathing scattor plays a crucial role to understand the phase-dependency of the input-output relation. The scattor can be obtained by continuation of a stable standing pulse based on a global bifurcation viewpoint with the AUTO software. Transition points of the input-output relation in a parameter space such as from reflection to annihilation correspond to when the orbits cross the stable manifold of the breathing scattor. If we plot the profiles of the orbit right after collision in the appropriate phase near scattor, then it becomes a closed loop as the phase at the collision varies from 0 to 2π. As shown in Fig. 1(c), this loop intersects transversally with the stable manifold of the scattor near the transition point of input-output relation. Correspondingly, the coexistence of the annihilation and reflection for the fixed c_3 value is caused by the difference of the phase at collision. The details are discussed in our previous paper.[3)]

§3. For asymmetric head-on collisions

For asymmetric collisions, namely, the phases of two incoming TBs are different, the outputs become more complicated and the response of scattor for the asymmetric perturbations must be considered. The initial asymmetric pair of TBs are tuned carefully to control the phase at collision. The phase-difference between two colliding TBs generally remains in the output, although their differences are not the same ($|\Delta\phi_{\rm in} - \Delta\phi_{\rm out}| \approx 0.2\pi$ in the example shown in Fig. 2, then depend on the initial

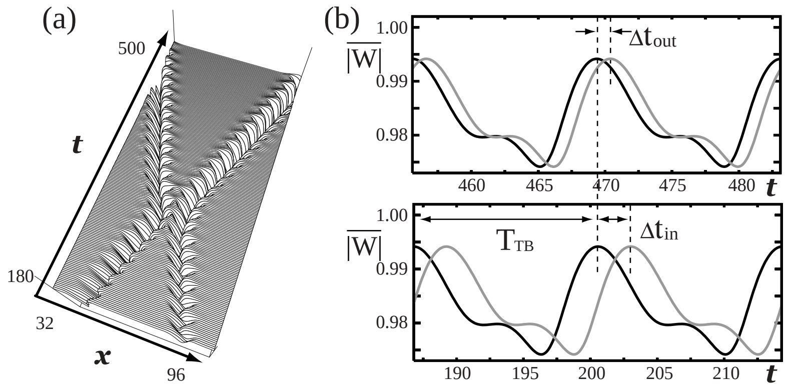

Fig. 2. (a) Typical spatio-temporal pattern of the asymmetric reflection at $c_3 = 0.143$. The system size is 128. (b) The phase ϕ of TB is defined as $\overline{d\phi/dt} = 2\pi/T_{\rm TB}$, where $T_{\rm TB}$ is the period of oscillations of the spatial average of modulus $\overline{|W|}$. The solid (resp. gray) line indicates the time evolution of $\overline{|W|}$ on the left-hand side domain of $x \in [0 : 64)$ (resp. right-hand side of $x \in [64 : 128)$). The phase-differences between the colliding TBs are then computed by $\Delta\phi_{\rm in,out} = 2\pi\Delta t_{\rm in,out}/T_{\rm TB}$.

condition. When c_3 is decreased and below the transition point ≈ 0.14208, the input-output relation changes from reflection to annihilation. It is noteworthy that a waveringly-wandering motion right after collision is observed as shown in Figs. 3(a) and (b). In addition, the phases of the two emitted TBs are synchronized each other, namely the initial phase-difference disappears during the collision process. For the rest of this note, we demonstrate the transition mechanism of the input-output relation like Figs. 3(a) and (b) in terms of scattors.

A symmetric collision process and the associated scattor shown in §1 correspond to the in-phase oscillations on a whole line. From the global bifurcation viewpoint, it is reasonable to take into account the out-of-phase oscillations as the counterpart of the in-phase breathing scattor. In fact, the unstable time-periodic orbit (UPO) illustrated in Fig. 3(c) is detected by using the delayed feedback control (DFC) method, in which the term of $W(t - T_{\rm UPO}) - W(t)$ is added to the CGLE. The details of the method will be shown in Ref. 4).

When we add a small perturbation to the out-of-phase scattor, the resulting behavior, depending on the phase at perturbation, is either annihilation or emission of two TBs. As shown in Fig. 3(d), the orbit stays very close to out-of-phase oscillation at the beginning. After leaving the out-of-phase breathing scattor, the orbit starts to wander waveringly for certain time and approaches a quasi-steady state and then emits two TBs. There actually exists a real steady state of codim 1 as depicted in Fig. 3(e), numerically confirmed by the Newton method. Since it has only one unstable direction Ψ besides the translation zero eigenvalue, the stable manifold separates the phase space into two parts. The responses for the small perturbation along Ψ are homogeneous state (annihilation) and counter-propagating TBs (reflection) of the same phase. Consequently, through this codim 1-steady scattor, the orbital behavior is converted to the in-phase motion. The waveringly-wandering motion like Figs. 3(a) and (b) can be regarded as a transient process from the out-of-phase

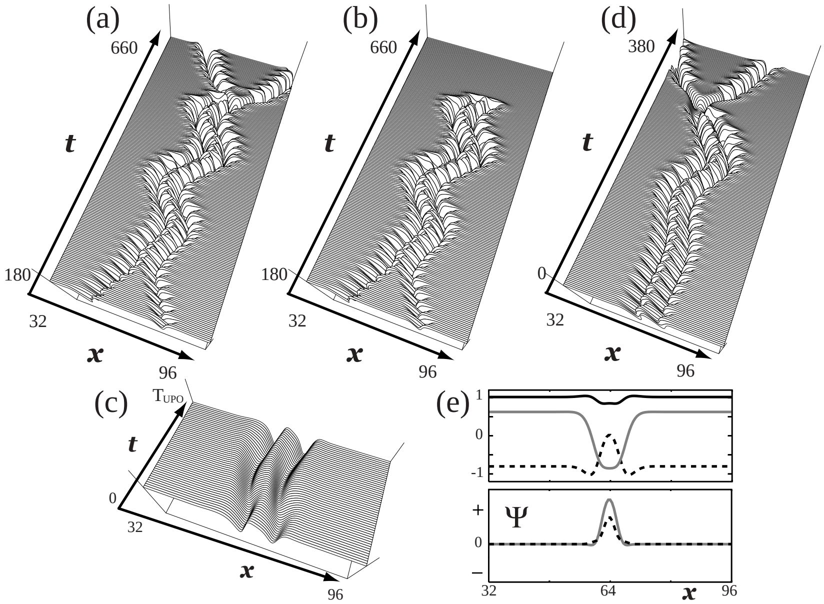

Fig. 3. Waveringly-wandering motions observed at the transition point from (a) reflection to (b) annihilation when $c_3 \approx 0.14208$ as c_3 is decreased. (c) Out-of-phase oscillating UPO with $T_{\mathrm{UPO}} \approx 12.9$ obtained by using the DFC method. We call it out-of-phase breathing scattor. (d) Response of the out-of-phase breathing scattor for the small perturbation. Note that the orbital behavior is switched from out-of-phase regime to another via a quasi-steady state. (e) Codim 1-steady scattor has only one (real) unstable eigenvalue (≈ 0.388) and the associated eigenfunction Ψ. The solid line indicates the profile of modulus of the scattor and the gray (resp. dotted) line shows the real (resp. imaginary) part of it.

breathing scattor depicted in Fig. 3(c) to the codim 1-steady scattor. This argument is not completely conclusive because it is still unclear whether such a wandering motion is transient or not. Whatever small the phase-difference between the incoming TBs is, the in-phase breathing scattor itself cannot stabilize it, namely the asymmetric perturbations at collision seems to grow up along the unstable manifolds. In other words, the asymmetric eigenfunctions on a whole system play important roles to determine the fate of the orbit. To detect the unstable directions of such breathing scattors, the monodromy operator associated with the linearized operator around them must be considered theoretically, which is a part of the ongoing project.

Overall the scattors play a pivotal role to understand the transient aspect of scattering dynamics in dissipative systems.

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