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Resonance and the Formation of Death-Spot in One-Dimensional FitzHugh-Nagumo Equations

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We analyze the effect of periodic forcing on a chain of FitzHugh-Nagumo elements in an excitable regime. The element at the end of chain is coupled unidirectional to a single pacemaker, limit cycle oscillator. With a suitable pacemaker's frequency, we find that the suppression of the pulse propagation occurs in some parameter regime, while the isolated pulse propagates stably in the chain. This propagation failure comes from the formation of the "death-spot", where successive pulses annihilate.

§1. Introduction

Excitable systems play a fundamental role in neural information processing and many other biological systems. For a simple excitable element, a small but finite perturbation to a rest state leads to a large excursion (an excitation). And the inclusion of repetitive stimuli shows variety of response patterns.¹⁾⁻³⁾ It is known that a localized stimulus of finite amplitude forms a traveling pulse in spatially extended excitable media. Often, one measures only large amplitude suprathreshold activity at a particular location in the system, for instance, the action potential in nerve or ventricular contraction in heart. In the case of oscillatory recovery, however, subthreshold oscillations exist behind a traveling pulse and cause an exotic spatiotemporal behavior.⁴⁾ We report spatiotemporal behavior, which are caused by pulse-pulse interaction via the subthreshold oscillation, on a one-dimensional chain of excitable elements, whose end is coupled unidirectional to a single pacemaker.

§2. Model

Here, we explore the one-dimensional chain of the excitable FitzHugh-Nagumo (FHN) elements,^{5),6)}

$$\dot{u}_i = u_i(u_i - \alpha)(1 - u_i) - v_i + \epsilon^2(u_{i+1} + u_{i-1} - 2u_i) + I, \quad (2.1)$$

$$\dot{v}_i = \tau(u - \gamma v), \quad (i = 1, \dots, N) \quad (2.2)$$

where α, ϵ, γ and τ are parameters, I is input stimulus, and N is a number of elements. $u_i = u_i(t)$ is the activator and $v_i = v_i(t)$ is the inhibitor, and the dot represents time derivative. The input I is added to the first element $i = 1$ from a pacemaker in the following manner. The pacemaker we have used is a limit cycle

oscillator:

$$\dot{x} = x + \omega y - x(x^2 + y^2), \quad (2.3)$$

$$\dot{y} = y - \omega x - y(x^2 + y^2), \quad (2.4)$$

where ω represents the frequency of the oscillator. Hereafter, we use the following impulsive inputs so that the intensity of each stimulus is constant for smaller ω : $I = I_0 \delta_{1,i} \exp(-x^2/(\omega\sigma))$, where $\delta_{i,j}$ is Kronecker's delta. The I_0 and σ are parameters in determining the intensity of the periodic stimuli. Without the stimuli, i.e., $I_0 = 0$, the homogeneous state $u_i = v_i = 0$ is a globally stable solution with the following boundary condition: $u_{-1} = u_1$ and $u_{N+1} = u_N$.

§3. Simulation

Employing the following parameters: $\alpha = 0.001$, $\epsilon = 0.5$, $\tau = 0.0172$, $\gamma = 0.5$, $I_0 = 0.5$, $\sigma = 1.0$ and $N = 100$, every pulse generated by each stimulus propagates in the chain for smaller values of ω , say $\omega < 0.01$. In this case, the time interval between the impulsive stimuli is long and the successive "isolated" traveling pulses are formed. Here, the term "isolated" simply indicates that the interaction between the successive pulses is very weak because the distance between them is very long. Every stimulus generates a stable traveling pulse, and the excitation is transmitted to the end of the chain, i.e., the element N is excited every $(2\pi)/\omega$ time interval as depicted in Fig. 1(a). The spatiotemporal behavior also depends on the parameter τ . Beyond a critical value $\tau = \tau_c \sim 0.0173$, "isolated" pulse fails to propagate in the chain. At τ_c , the stable solitary pulse solution disappears via the saddle-node bifurcation. Indeed, the isolated pulse propagates transiently for some finite distance and disappears for slightly larger values than τ_c .

Slightly below the critical parameter, $\tau \sim [0.0171, \tau_c]$, we find propagation failure: each pulse generated by the pacemaker vanishes successively at a certain position, "death-spot", after traveling a finite distance (see Fig. 1(b-d)). Note that the isolated pulse propagates stably, i.e., the death-spot does not form for smaller values of ω , and the propagation failure occurs at some particular pacemaker's frequencies. The position of death-spot depends on the parameter ω . In Fig. 2(a), the stationary position of death-spot is plotted as a function of ω (the position $i = 100$ in the vertical axis indicates that the excitation is transmitted to the opposite elements). Depending on the external frequency, the position is fixed in space, periodically oscillates and irregularly moves. For a continuous excitable media, i.e., a partial differential equations, the formation of the death-spot has been reported.⁴⁾ It is, however, that the irregular motion of the death-spot is only observed in the discrete system.

To characterize these spatiotemporal behavior, we use the stimulus-response ratio (SRR).¹⁻³⁾ For the spatially extended system, we introduce SRR as the ratio of the number of the pulses that reach the opposite element N to the number of the stimuli as depicted in Fig. 2(b). Because the stable isolated pulse exists, every pulse generated by the periodic stimuli would reach the opposite elements for smaller ω . Thus, the SRR is equal to 1, that is, each stimulus would follow the excitation of the element N , as it is clearly shown in Fig. 2(b). On the other hand, the excitation of

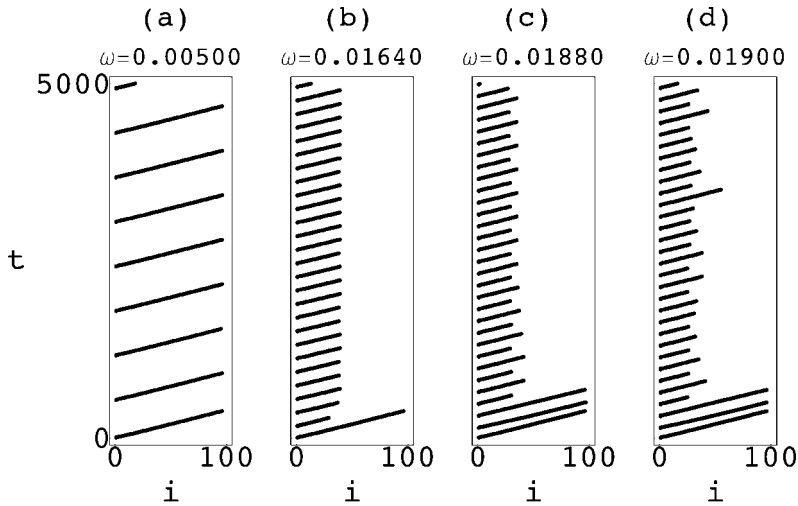


Fig. 1. The spatiotemporal patterns of the chain of FHN elements, whose one of the end is coupled with the pacemaker. The excited elements that satisfy the condition: $(u_i > u_{i-1}) \wedge (u_i > u_{i+1}) \wedge (u_i > u_c = 0.5)$ are plotted by changing the pacemaker's frequency ω . A stable traveling pulse exists in the following parameters $\epsilon = 0.5$, $\alpha = 0.001$, $\gamma = 0.5$ and $\tau = 0.0172$. (a) An isolated pulse is generated at every stimulus. (b) Propagation failure: the successive pulses suddenly vanish at a certain location, death-spot. (c) Periodic oscillation of the position of death-spot. (d) An irregular oscillation of the position of death-spot.

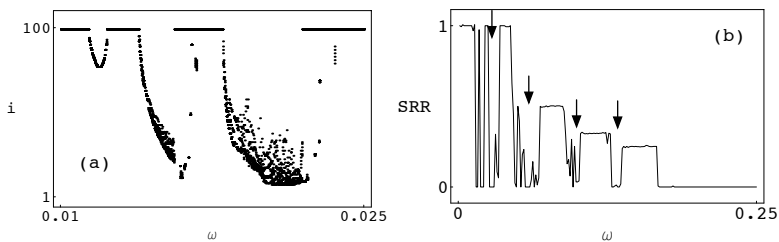


Fig. 2. (a) The stationary position of death-spot as a function of ω is shown. The stationary position is fixed and oscillate in a place, depending on the external frequency. (b) The stimulus response ratio is calculated by changing the forcing frequency ω . The zero firing rate regions indicated by the arrows correspond to propagation failure.

the first element does not occur with larger values of ω since there are time intervals during which a perturbation does not induce a new excitation. This interval is called the refractory period.

For an intermediate external frequency in which the SRR is 0, indicated by the arrows in Fig. 2(b), an excitation propagates for some finite distance and cannot reach the opposite element N despite the existence of stable isolated pulse. Even if the first element, to which the pacemaker is connected is excited, the excitation cannot be transmitted to the opposite element of the chain. In this case, the formation of the “death-spot” is observed.

The parameter values used here admit the “oscillating wake” of traveling pulse because the eigenvalues of the uniform stationary solution $(u, v) = (0, 0)$, which

are given by $\lambda_{\pm} = \left(-\alpha - \gamma\tau \pm \sqrt{-4(1 + \alpha\gamma)\tau + (\alpha + \gamma\tau)^2} \right) / (2\tau)$, have imaginary part.⁴⁾ Due to the subthreshold oscillations behind a traveling pulse, whether the pulse activates or inhibits the posterior pulse crucially depends on ω . Indeed, the state values of the first element, u_i and v_i , oscillate in time due to the subthreshold oscillation behind the traveling pulse. Thus the refractory and rest periods of the first element occur alternately. When the timing of stimulus coincides with the refractory period of the first element, the excitation is suppressed. And if the timing of stimulus coincides with the intermediate state between the refractory and the rest periods, the excitation occurs. But the excitation disappears at a certain position after propagating a finite distance. The formation of the death-spot depends in a subtle way on the ratio between external frequency ω and the intrinsic frequency of the system, $|\text{Im}(\lambda_{\pm})|/2\pi$. The term “resonance” stems from these two time scales.

§4. Summary

We have investigated the propagation failure of excitation in the chain of excitable FHN elements. The element at the end of chain is coupled unidirectional to a pacemaker, which is a limit cycle oscillator with the frequency ω . Periodical stimuli to the first element of the chain generate propagating pulses. The spatiotemporal behavior of these pulses depends on the pacemaker’s frequency ω . Even if the isolated pulse propagates in space stably, the pulses generated by certain frequencies fail to propagate. This propagation failure comes from the formation of death-spot, where the successive pulses annihilate. In addition, the stationary position of death-spot is depends on ω , and bifurcates from the immovable state to the oscillatory state.

An essential dynamical ingredient of the behavior is that the isolated traveling pulse has oscillating wake, which comes from the existence of imaginary part of eigenvalues for the stationary homogeneous solution. These phenomena can be observed in any excitable system near the critical point, where the stable isolated traveling pulse with oscillating wake disappears through saddle-node bifurcation. The study of these dynamics is of general application to the understanding of disordered phenomena in spatially extended excitable system, and may provide new insight about excitable systems such as the cardiac dynamics.

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