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Unified Constitutive Model Considering Equivalence of Back Stress Due to Plastic and Viscoplastic Deformation

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Abstract. This paper treats a unified constitutive model for the viscoplastic and creep deformations. First, experimental observations on the correlation between back stress due to plasticity and creep are carried out using Type 304 stainless steel at room temperature. The experimental results show that there is the equivalence of back stress due to plastic and creep deformation. Then considering the equivalence of back stress, a unified constitutive model for plasticity and viscoplasticity is proposed. To verify the applicability of this model, the numerical simulations are conducted by the model, and compared with the experiments.

Introduction

The back stress plays an important role to explain the plastic deformation, in particular in cyclic plasticity, and almost all constitutive models for the cyclic plasticity utilize the back stress[1,2,3]. The constitutive models of the cyclic plasticity were originally developed for time independent deformation. However, to apply the constitutive models to estimate fatigue failure, including creep-fatigue interaction, the constitutive model must also describe viscoplastic deformation such as creep and stress relaxation. Then, the evolution equation of the back stress should be taken into accounts of the effects of both plasticity and creep. One question is whether the back stress developed by creep deformation is of the same value as that developed by the plastic deformation [4,5,6].

In this paper, a unified constitutive model for plastic and creep deformation is proposed. First, in order to observe the correlation between back stress affecting plasticity and creep, creep tests during pure tension are conducted. The experiments show that there is equivalence of back stress in plasticity and creep. Considering the equivalence of the back stress due to plasticity and creep, the unified constitutive model is constructed. The applicability of the model is also verified by the numerical simulations of several tests of (i) pure tension, (ii) pure creep, (iii) cyclic tension-compression loading with constant strain amplitudes under constant strain rates, and (iv) intermittent creep tests during cyclic loading. The experiments are conducted using thin tubular specimens of Type 304 stainless steel at room temperature.

Experimental Material and Method

The specimen used in this study is a drawing tube of Type 304 stainless steel subjected to a solution

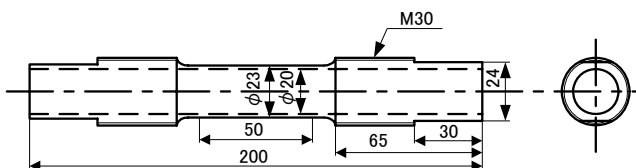


Fig.1 Geometry of specimen

Table1 Test conditions
of intermittent creep

Strain amplitude [%]	0.5		
Strain rate [%/sec]	0.1	0.01	0.001
Stress Level [MPa]	100, 200, 250, 275		

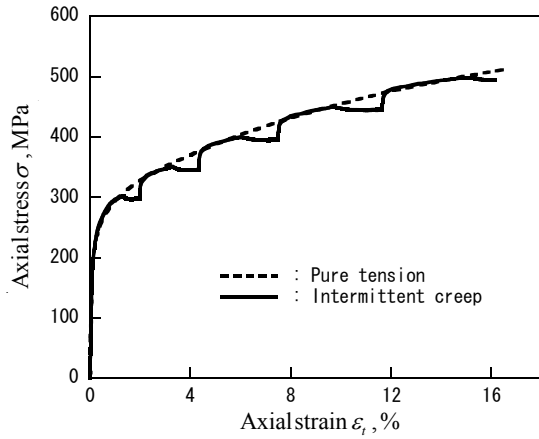


Fig.2 Stress-strain curves of pure tension and intermittent creep during pure tension

heat treatment. The drawing tube has a 32mm outer diameter and 6mm wall thickness and is annealed by the manufacturer at 1070 °C followed by water quenching for half an hour. Its chemical compositions are 0.02C, 0.45Si, 1.09Mn, 0.030P, 0.005S, 8.30Ni, 18.54Cr, and Fe balanced in weight percent. The tubular specimens are machined from this tube as shown in Fig.1. The specimens are fabricated by metal cutting and then annealed to get an initial isotropic stress state. In this annealing process, the specimens are heated and maintained at 380 °C for 120min followed by air cooling. Young's modulus of the specimen at room temperature is $E=200\text{GPa}$.

A servo-controlled axial-torsional testing machine are used for computerized testing data and data acquisition. Strain is measured using two strain gauges applied on diametrically opposite side of the specimen. The axial force is measured using the load cell incorporated in the machine.

There are four types of test in this work: (i) pure tension tests under stress rates of 5 and 0.5MPa/sec; (ii) creep tests during 300 seconds at stress levels of 250, 275 and 300MPa; (iii) cyclic tension-compression loading tests with a constant strain amplitude under a constant strain rate; and (iv) intermittent creep tests during cyclic tension-compression loading. Here, tests (iii) and (iv) are conducted successively as follow. First, the specimens are subjected to cyclic loading with a constant strain amplitude. After stabilization of the stress-strain curve, the intermittent creep tests are conducted at the four stress levels as shown in Table 1.

Equivalence of back stress due to plasticity and creep

In Fig.2, a solid line represents the stress-strain curve of pure tension and the broken lines represent intermittent creep tests during pure tension. The pure tension are conducted under the stress rate of 10MPa/sec, and the intermittent creep tests are conducted for 300 seconds at the stress levels of 300, 350, 400 and 450MPa during the pure tension loading under the stress rate of 10MPa/sec. When loading state changes to pure tension under 10MPa/sec after intermittent creep at each stresses, the stress-strain curves after the intermittent creep coincide with that of the pure tension without creep as shown in Fig.2. This result shows that the strain hardening occurs during creep although the stress keeps constant, and the amount of strain hardening during creep is as same as that during the pure tension without creep. Namely the experiments suggest that the development of the back stress during the pure tension equals to that during the intermittent creep. Considering the equivalence of the back stress a unified constitutive model is constructed as the below section.

Formulation of Unified Constitutive Model

The viscoplastic potential is defined as in chaboche [7].

$$F = \frac{H}{n+1} \left\langle \frac{\bar{\sigma} - R}{H} \right\rangle^{n+1} \quad (1)$$

where $\langle \rangle$ is MacAuley bracket and defined as $\langle x \rangle = \frac{1}{2}(x + |x|)$. H is the drag stress, R is the flow stress and n is the material constant. The equivalent stress $\bar{\sigma}$ in Eq.(1) is represented as

$$\bar{\sigma} = \sqrt{\frac{3}{2}(s_{ij} - \alpha'_{ij})(s_{ij} - \alpha'_{ij})} \quad (2)$$

where s_{ij} and α'_{ij} are the deviatoric components of the stress tensor σ_{ij} and the kinematic back stress tensor α_{ij} , respectively.

The normality hypothesis now becomes

$$\dot{\epsilon}_{ij}^v = \frac{\partial F}{\partial \sigma_{ij}} = \left\langle \frac{\bar{\sigma} - R}{H} \right\rangle^n \frac{\sigma_{ij} - \alpha_{ij}}{\bar{\sigma}} \quad (3)$$

The modified viscoplastic work rate is defined as the following equation.

$$\dot{W}^v = \bar{\sigma} \dot{\epsilon}^v = (\sigma_{ij} - \alpha_{ij}) \dot{\epsilon}_{ij}^v \quad (4)$$

Then, from Eqs.(3) and (4), the following equation is given.

$$\dot{\epsilon}^v = \left\langle \frac{\bar{\sigma} - R}{H} \right\rangle^n \quad (5)$$

From Eqs.(3) and (5), the flow rule can be explained by

$$\dot{\epsilon}_{ij}^v = \frac{\dot{\epsilon}^v}{\bar{\sigma}} (\sigma_{ij} - \alpha_{ij}) \quad (6)$$

From Eq.(6) the equivalent strain rate can be described as the follow;

$$\dot{\epsilon}^v = \sqrt{\dot{\epsilon}_{ij}^v \dot{\epsilon}_{ij}^v} \quad (7)$$

The following Ziegler's type of assumption for the evolution equation of the kinematic back stress is used here.

$$\dot{\alpha}_{ij} = (\sigma_{ij} - \alpha_{ij}) \dot{\mu} \quad (8)$$

where $\dot{\mu}$ is the scalar multiplying factor which is determined from Eqs.(2),(5) and (8). Eventually, the kinematic back stress rate is represented by

$$\dot{\alpha}_{ij} = \left[\frac{\sigma_{kl} - \alpha_{kl}}{\bar{\sigma}^2} - \frac{H \dot{\epsilon}^v}{n \bar{\sigma}} \left(\frac{\dot{\epsilon}^v}{\bar{\sigma}} \right)^{\frac{1-n}{n}} - \frac{\dot{R}}{\bar{\sigma}} \right] (\sigma_{ij} - \alpha_{ij}) \quad (9)$$

In this paper, the following power law is used to represent the nonlinear viscoplastic modulus simply,

$$\hat{\varepsilon}^v = \frac{K}{E} \left(\frac{\hat{\sigma}}{D} \right)^m \hat{\sigma} \quad (10)$$

where E is Young's modulus and K is a material constant. $\hat{\varepsilon}^v$ is the equivalent viscoplastic strain measured from ε_{ij}^R , which is the viscoplastic strain just before the loading direction change. And $\hat{\sigma}$ is in general represented as,

$$\hat{\sigma} = \left[\frac{3}{2} (s_{ij} - \hat{\alpha}'_{ij})(s_{ij} - \hat{\alpha}'_{ij}) \right]^{1/2} \quad (11)$$

where $\hat{\alpha}'_{ij}$ is the deviatoric components of the center of the current yield surface $\hat{\alpha}_{ij}$ before the loading direction change. The back stress α_{ij} is memorized as $\hat{\alpha}_{ij}$ when the loading direction changes.

Since, during the current loading the residual viscoplastic strain ε_{ij}^R is constant, $\dot{\varepsilon}_{ij}^v$ is equals to $\dot{\hat{\varepsilon}}_{ij}^v$ and $\dot{\varepsilon}^v = \dot{\hat{\varepsilon}}^v$ from $\varepsilon_{ij}^v = \hat{\varepsilon}_{ij}^v + \varepsilon_{ij}^R$, Eqs.(10) and (11) give

$$\dot{\varepsilon}^v = \dot{\hat{\varepsilon}}^v = \frac{K}{E} (m+1) \left(\frac{\hat{\sigma}}{D} \right)^m \frac{1}{\hat{\sigma}} \left[\frac{3}{2} (\sigma'_{ij} - \alpha'_{ij}) \dot{\sigma}'_{ij} \right] = \left\langle \frac{\bar{\sigma} - R}{H} \right\rangle^n \quad (12)$$

In Eqs.(10) and (12), D and m are the functions depending on the accumulated inelastic strain at the memorized point. They can be explained by

$$D = D_{(i)} \left\{ 1 - \alpha \exp \left(- \int_{i-1} \dot{\varepsilon}^v dt / a \right) \right\} \quad (13)$$

$$m = m_{(i)} \left\{ 1 - \beta \exp \left(- \int_{i-1} \dot{\varepsilon}^v dt / b \right) \right\} \quad (14)$$

where $D_{(i)}$ and $m_{(i)}$ become D_1 and m_1 during the initial loading before the loading direction changes, D_2 and m_2 after the first loading direction change, and D_0 and m_0 after the second loading direction change, respectively.

Discussions

Pure Tensile Tests Fig.3 shows the stress-strain curves of the pure tensile tests under the stress rates of 0.5MPa/sec (\circ) and 5MPa/sec (\square), respectively. The solid lines represent the stress-strain curves simulated by the proposed model. It can be seen that the simulations have a good agreement with the experiments. The material constants used in the simulation are shown in Table 2. D_1 has the strain rate dependence and the value of D_1 are calculated from the following equation.

$$D_1 = 185 - 18.3 \exp \left(- \frac{\dot{\varepsilon}}{1.45 \times 10^{-4}} \right) \quad (15)$$

Table2 Material constants

E [Gpa]	K [MPa]	H [MPa]	R ₀ [MPa]	n	λ	c	α	a	m ₁	m ₂	m ₀	β	b
200	0.05	130	110	10	0.26	0.002	0.3	0.04	8	5	5	0.1	0.02

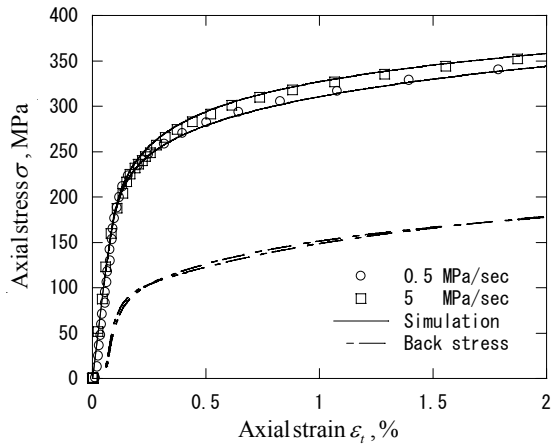


Fig.3 Stress-strain curves of pure tension

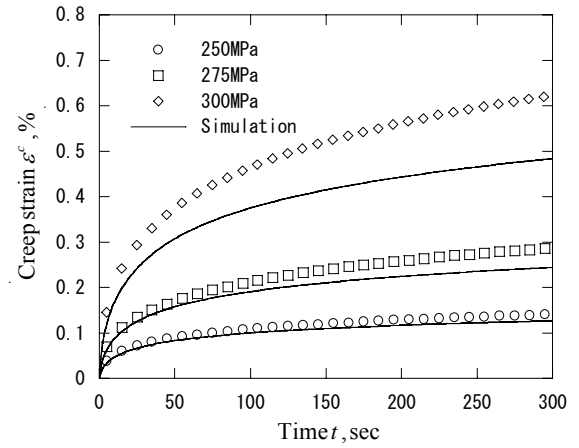


Fig.4 Pure creep curves
(Stress levels=250, 275, 300MPa)

Creep Tests Fig.4 shows the creep curves during 300 seconds at the stress levels of 250, 275 and 300MPa after the tensile loading under the stress rate of 5MPa/sec with three nontested original specimens. The solid lines represent the creep curves simulated by the proposed model. Because of the equivalence of the back stress in plastic deformation and creep as mentioned above, the back stress calculated by the pure tension is used in this simulation. The simulations have good agreements with the experiments.

Cyclic loading and Intermittent Creep Test Figure 5 shows the stress-strain curve of the intermittent creep tests at the stress levels of a(100MPa), b(200MPa), c(250MPa), and d(275MPa) after the stabilization of the cyclic tension-compression preloading with the strain amplitude of 0.5% under the strain rate of 0.01%/sec.

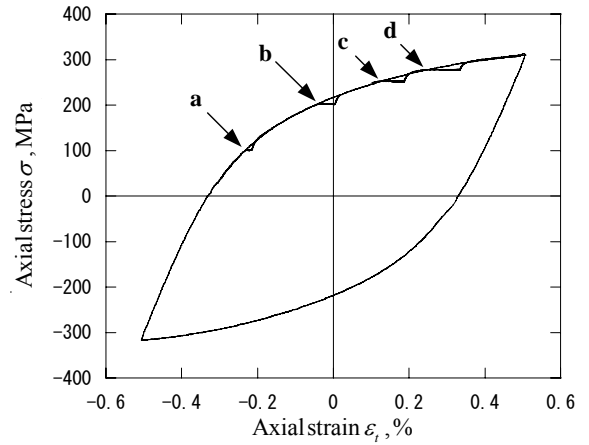


Fig.5 Intermittent creep during cyclic loading
(Strain amplitude = 0.5%,
Strain rate=0.01%/sec)

At first the simulation of the cyclic tension-compression loading is conducted. In Fig.6, the symbol (○) shows the stabilized stress-strain curve of the cyclic tension-compression loading, and the solid line shows the simulated stress-strain curve. The simulation has a good agreement with the experiment. The material constants D_2 and D_0 have the strain rate dependence and the value of D_2 and D_0 are calculated by the following equations.

$$D_2 = 191.5 - 14 \exp\left(-\frac{\dot{\epsilon}}{1.3 \times 10^{-4}}\right) \quad (16)$$

$$D_0 = 243 - 17 \exp\left(-\frac{\dot{\epsilon}}{1.3 \times 10^{-4}}\right) \quad (17)$$

Figure 7 shows the creep curves of the intermittent creep tests during 300 seconds at the stress levels of 100MPa (○), 200MPa (□), 250MPa (◇), and 275MPa (×). Experimental results show that

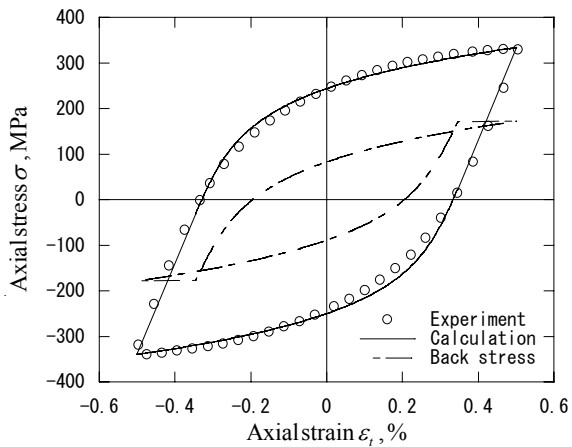


Fig.6 Tension-compression cyclic loading
(Strain amplitude=0.5%,
Strain rate=0.01%/sec)

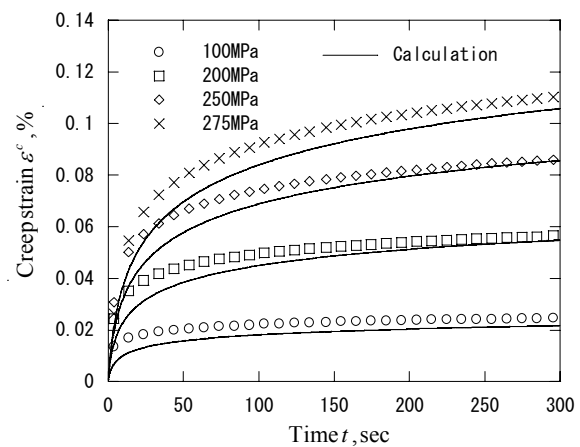


Fig.7 Creep curves of intermittent creep
(Strain amplitude=0.5%,
Strain rate=0.01%/sec)

the intermittent creep curves are almost saturated in 300 seconds, and the creep strain of the intermittent creep is much less than that of the creep with the nontested original specimen as shown in Fig.4. This result suggests the back stress during the intermittent creep is not same value as that during pure creep. Considering the equivalence of the back stress, the simulations of the intermittent creep curve are carried out. The simulations shown by solid lines in Fig.7 have good agreements with the experiments.

Conclusions

This paper discussed the construction of a unified constitutive model to describe the interaction between plastic and creep deformations. As a results, the following conclusions were obtained:

- (1) The creep test during pure tension suggests the equivalence of the back stress due to plasticity and creep.
- (2) Considering the equivalence of the back stress in plasticity and creep, a unified viscoplastic constitutive model is constructed from the flow rule obtained by the viscoplastic potential, the evolution equation by Ziegler rule, and the viscoplastic modulus by the power law.
- (3) The material constants can be simply obtained from the pure tension and the cyclic tension-compression loading tests.
- (4) The consideration of the equivalence of the back stress gives good description of the creep in the nontested original specimens and the intermittent creep during the cyclic tension-compression loading.

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