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—— PODEJŚCIE LINIOWE ——

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FACULTY OF FINANCE AND STATISTICS

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MULTI-LEVEL STRUCTURE OF LONG-TERM ECONOMIC PLANNING
—— LINEAR APPROACH ——

Doctoral dissertation

under the supervision of
Prof. dr hab. Krzysztof Porwit

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INTRODUCTORY NOTES

1. Figures and tables are numbered continuously only within a chapter. Therefore in every chapter the number of figure/table begins from one.
2. Numerical formulas (i.e. equations and inequalities) are numbered continuously only within a subsection. Therefore in every subsection the number of numerical formula begins from one.
3. Figures/tables/numerical formulas are commonly numbered in the text and its mathematical appendix.

For example, the equation (5) does not appear in the text and is contained in the mathematical appendix of the subsection concerned. In this case after the equation (4) directly the equation (6) appears in the text.

4. The following notations are used in this Thesis.

$a =: b$ means $a \equiv b$.

p' denotes the transpose vector of p .

A' denotes the transpose matrix of A .

$// T //$ denotes the set of T .

The aim of this Thesis is twofold. The first is to examine the multi-level structure of economic planning in a socialist economy which is not fully centralized. The second is to investigate the investment criteria of the optimal long-term economic planning in a socialist economy which is open to the world economy.

These two aims are not irrelevant to each other. On the contrary they are closely connected to each other. The optimal investment problem can not be solved without paying attention to the decentralization of decision making in economic system with multi-level structure. Throughout this Thesis the Author investigates the optimal long-term economic planning and the optimal investment criteria, but always behind the discussion the Author simultaneously pays regard to the problem of multi-level structure of economic planning.

The meaning of the multi-level structure is also twofold. The first meaning concerns the time horizon of economic planning; i.e. short-term medium-term, long-term and infinite economic planning. The second meaning concerns the layer of decision making. The discussion about multi-level structure of decision making should be connected with the investigation of two approaches to economic planning, or in other words, two definitions of decentralization. There are two kinds of planning, i.e. directive(deterministic) planning and indicative planning. There are also two kinds of definition of decentralization, i.e. decentralization of information and decentralization of decision making.

The difference between two approaches to economic planning is related to the difference between two definitions of decentralization. To understand these differences a brief survey of "socialism controversy" in 1930's is useful.

Refusing Hayek and Robbins, Lange argued in his article "On the economic theory of socialism" that the rational resource allocation can be achieved by trial and error even in socialism.¹⁾ The Author of this Thesis regards this article as the beginning of the informationally decentralized approach to socialist economic planning. The Author, however, considers that Lange presented this article as a critique to Hayek and Robbins not as a desirable mechanism of socialist economy. Socialist economy, as well as other economic

systems, has more complicated and manifold elements in itself. The Author of this Thesis considers that the most important element in the socialist economy belongs to the problem of decision making, not to the problem of information.

Unfortunately thereafter the study of decentralized procedure has developed along the line of decentralized information. Typical examples of this approach are articles by Hurwicz(19), Malinvaud(37), Kornai and Liptak(29), Heal(18) and so on.

In these models the decision making is seemingly decentralized, but in essence it is fully centralized. The decision maker at a lower level executes his function as a representative of the decision maker at the central level, since these models assume that the manager at a lower level executes loyally his decision making according to the decision rules which is assigned by the central level to the lower level. Therefore these models are the centralized planning model in the sense of decision making. The Author calls this type of model the directive(deterministic) planning.

Of course, recently some economists try to explore a spontaneous decision rule (so-called incentive system) in the informationally decentralized planning.²⁾ The Author of this Thesis considers that this direction of study is valuable, but he also recognizes the fact that we can not construct any completely satisfactory spontaneous and autonomous decision rule in a socialist economy, because in a socialist economy the central level plays a leading role and we can not assume the fully spontaneous and autonomous decision making at a lower level. This fact arises from the nature of socialism. Therefore the Author will not be engaged in this direction of study throughout this Thesis. Needless to say, this statement does not deny the importance of the so-called incentive system in a socialist economy. On the contrary the Author admits the importance of the so-called incentive system and intends to investigate this problem from another viewpoint.

Modern socialist economy has a complicated and manifold character. Every decision maker at a lower level faces his own particular conditions of production. There can not exist any general incentive system. Therefore the Author abandons the so-called incentive approach in the socialist economic planning. On the contrary the Author intends to investigate the indirect method of planning mainly depending on the monetary-commodity relation. In this method the guiding parameters on prices, interest rate and foreign exchange rate play an important role. The Author calls

this system the I N D I C A T I V E P L A N N I N G.³⁾ In the indicative planning the Central Planning Board(CPB) determines only general guidelines and guiding indicators. Particular production plan of individual enterprises should be made up according to these guidelines and guiding indicators (guiding parameters). So the indicative planning in a socialist economy can contain a decentralized character in decision making. The character of guiding indicators in this Thesis is rather different from the character of control parameters in the study by Sulmicki(52). Sulmicki's approach belongs, according to the Author's opinion, to the category of directive and deterministic planning.⁴⁾

Usually the word of indicative planning concerns the economic planning in developed capitalist countries, one example of which is France. But in this Thesis the Author uses the word of indicative planning in a rather different meaning. In this Thesis the conception of indicative planning is directly connected with the so-called cost-benefit analysis. Recently an interesting approach has been explored in the economic planning in developing countries. This approach is called the cost-benefit analysis. The main contribution in this field is the study by Little and Mirrlees(34) and Dasgupta, Marglin and Sen(8). In their approach the Central Planning Board determines, as guiding indicators, only accounting prices, shadow wage rate, shadow exchange rate. As a guideline the CPB adopts the surplus of the project, i.e. the difference between the benefit and the cost of the project.

Socialist economy, as well as developing countries, is in principle a non-market economy. So the Author of this Thesis considers that the technique of the cost-benefit analysis can be utilized as a tool of the indicative planning in a socialist economy. The Reader should not misunderstand the indicative planning as a market mechanism. On the contrary it is a planning method which should be used in a non-market economy.

The result of the indicative planning depends on adequateness of the guiding indicators. Guiding indicators should be consistent to each other and therefore must be deduced from the central plan. The first role of the long-term central plan in the development planning consists in this procedure. So in principle the long-term central plan does not possess an executive character. Needless to say in some important industries or some important projects the central plan directly turns into the executive program. Moreover, even in the long-term planning, the central plan is indispensable

for the quantitative adjustment of production. Nevertheless the main role of the long-term central planning consists in the determination of the optimal direction of economic development and the deduction of the guiding indicators and not in making up the executive program.

For this purpose the Author constructs a multi-term turnpike model⁵⁾ with multi-sector economy which is open to the world economy. The Central Planning Board (CPB) maximizes the total sum of consumption. Therefore the accumulation is not an aim in itself but a tool for realization of consumption. We may call this type of optimal planning the consumption-turnpike model. The most troublesome problem of the consumption-turnpike model is the terminal condition of capital stock. If the CPB does not set the terminal condition of capital stock, then the optimal plan must consume all resources and leave no capital stock at the last period.

Determination of the terminal condition is also an important subject of economic planning. But the solution can not be derived from the model, since the terminal condition is a preliminary for the model. In order to avoid this dilemma the Author examines the infinite economic planning, time horizon of which is infinite and therefore the terminal condition is not necessary. The optimal growth path of the infinite economic planning determines the terminal condition of capital stock in the long-term economic planning. In the same manner the optimal growth path of the long-term economic planning determines the terminal condition of capital stock in the medium-term economic planning. Determination of the optimal growth path and determination of the terminal condition of capital stock are the most important task of the central plan.

Summing up, the Author presents the following two figures.

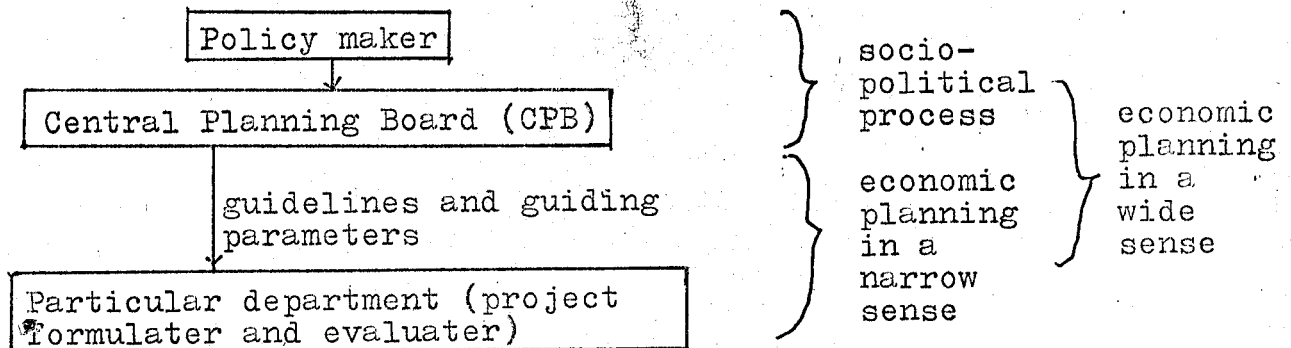


figure 1

The figure 1 explains the multi-level structure of economic planning from the viewpoint of decision making. The socio-political process and the economic planning in a wide and narrow sense

will be discussed in II-4. The following figure 2 explains the multi-level structure of economic planning from the viewpoint of time horizon.

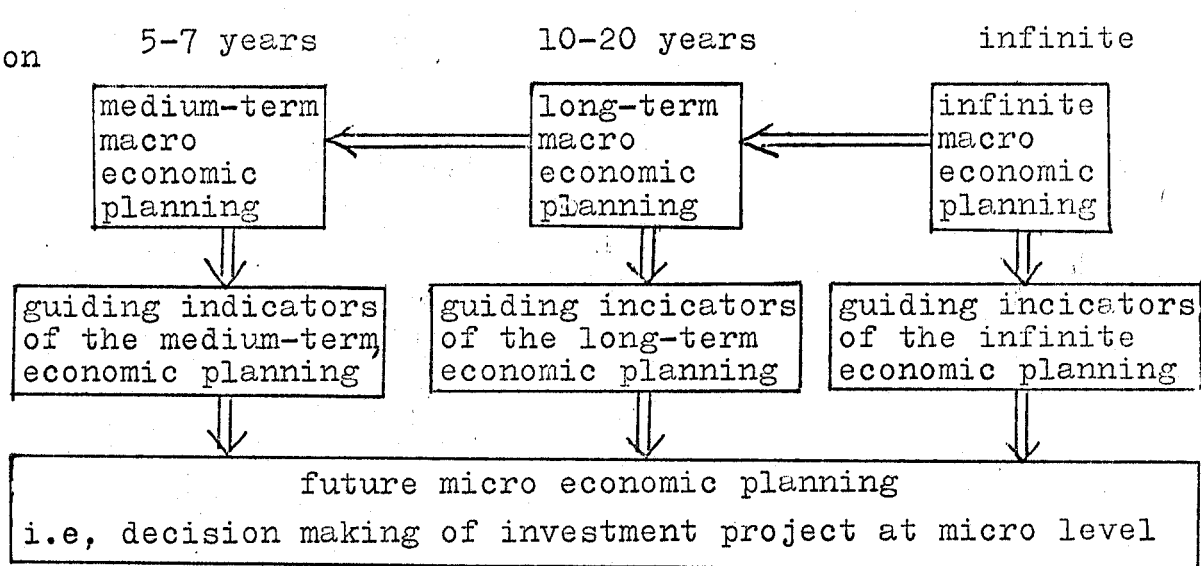


figure 2

The basis of the Author's model is given by the contributions by Porwit(46) and Gale(13). From a mathematical point of view the model of Gale is a startpoint of the Author's study. From an economic point of view the model of Porwit provides the theoretical foundation of this Thesis.

As a guideline the Author proposes the net surplus of the investment project. That is,

$$\text{MAX (net surplus)} = \left(\begin{array}{c} \text{accounting value} \\ \text{of outputs} \end{array} \right) - \left(\begin{array}{c} \text{accounting value} \\ \text{of inputs} \end{array} \right)$$

Here it is necessary to notice that all outputs and inputs are measured by the accounting prices and not by the existing prices. Therefore the determination of the level of the accounting prices becomes a crucial problem. In this Thesis the determination of the accounting prices is influenced by the induced labour input,⁶⁾ the existing price and the international price. In the case of the domestically produced goods the induced labour input exerts a strong influence. But here the Reader should not consider that this approach by the Author belongs to the category of the cost-price approach, since in this Thesis the determination of the accounting prices is strongly influenced also by the structure of the final demand. In chapter VII the Author will show that the greater final demand brings the higher accounting price.

In this Thesis the Author never uses the conception of capital, since the Author assumes the full mobility of goods and resources

in the long-run. Therefore we can not obtain the rent of capital from the model of this Thesis.

Needless to say, this Thesis belongs to the category of model analysis.⁷⁾ Model analysis inevitably has to contain some assumptions and some simplifications. They are more or less different from the reality of economy. We can not, therefore, directly apply the conclusion of this Thesis to the real economy. Taking the real situation into account we must always correct the conclusion of model analysis. As already mentioned, the Author's model has a multi-level structure. The extent of these corrections depends on the planning level. At the lower level the extent of correction should be larger, since the reality is more complicated and more affluent. At the central level these corrections should be limited to the minimal necessity. We should always take into account the limit of mathematical model analysis.

This Thesis has the following construction.

In chapter II the Author examines the general problem of economic planning and guides the Reader to the essence of the problem which the Author investigates in this Thesis.

In chapter III firstly the Author gives a brief survey of conception of multi-level structure mainly relying on the book by Porwit(48). Next he introduces the economic planning in Japan and examines the multi-level structure in Japanese economic planning.⁸⁾

Chapter IV is an introductory chapter to chapter V. There the Author presents a short and easy outline of the turnpike theorem and the dynamic Leontief model. The Reader, who is well familiar with these mathematical theories, can omit this chapter. The Reader who does not demand mathematical rigorousness in this Thesis, also can omit some mathematical parts of this chapter and the whole part of IV-2-3. IV-2-3 is written only for the Reader whose speciality is mathematical economics.

Chapter V is the main part of this Thesis. There the Author will construct an optimal consumption-turnpike model of the economic planning, in which the international trade is incorporated. In this chapter the Author examines the multi-level structure of economic planning in a sense of time horizon as well as in a sense of decision making. He deduces the optimal level of foreign currency debt. The shadow exchange rate and the shadow wage rate are also deduced in this chapter. The Author presents there some principles on determination of the accounting prices. He proves that in princip-

le the accounting price of the domestically produced good should be determined by the quantity of the induced labour input and that the accounting price of the imported good should be determined by the international price multiplied by the shadow exchange rate. Accounting price of the exportable good should be determined by the induced labour input if there is no production limit and should be determined by the international price if there exists a production limit. Finally the Author argues that the accounting surplus of the investment project can serve as a guideline of decision making.

In chapter VI the Author firstly gives a brief survey of the investment criteria in socialist countries. Next he presents an outline of the cost-benefit analysis. The main theme of this chapter is to modify the general principles of chapter V in order that they can be applicable in the real economy. There the Author presents some decomposition principles from the macro-accounting index at the industry level into the micro-accounting price at the enterprise level. Of course, the number of micro-accounting prices is enormous. But in this Thesis the enterprise level can calculate their own micro-accounting prices using a small number of the guiding indicators which are transmitted from the central level. Only the inferior level can handle with their own particular conditions of economic activity, e.g. expected future price of internationally tradable goods. In this Thesis the micro-accounting prices are calculated at the inferior level.

In chapter VII the Author examines the remained problems of decision making at micro level. They concern the difference of operation periods, the quantitative adjustment, the external effect, the increasing returns and the uncertainty.

In chapter VIII the Author presents a very short summary of the contributions of this Thesis.

FOOTNOTES TO CHAPTER I

- 1) See, Lange(30) p.56.
- 2) One example of this direction is the study by Groves(16) and (17).
- 3) The definitions of deterministic planning and indicative planning are a quite original one by the Author. The main point of

these definitions concerns the short-term production program and therefore the material balance. In the short-term planning the material balance is a crucial element. Therefore the short-term planning has to maintain a directive (deterministic) character. In the extreme case or in the ideal case the directive planning determines the all production programs. Similarly in the extreme case the indicative planning does not determine any production program. Needless to say, in the reality these two kinds of planning are mixed each other.

- 4) Of course, the Author does not regard Sulmicki's contribution as a study of centralized planning. Surely Sulmicki investigated the multi-level structure of planning. But the Author of this Thesis considers that his work has a deterministic character in decision making, since the study by Sumicki is engaged in mainly the short-term management planning.
- 5) Turnpike model will be explained in chapter IV.
- 6) Induced labour input will be explained in chapters V and VI.
- 7) The basis of the studies by Porwit and Gale is the Leontief model. Therefore this Thesis has a subtitle of "linear approach".
- 8) Economic planning in Japan has an indirect character. Moreover the long-term economic planning in Japan is the first attempt of the practical application of the consumption-turnpike model in the world.

CHAPTER II

INVESTMENT PROBLEM IN THE MACROECONOMIC PLANNING

This chapter is a problematic introduction to the main theme of this Thesis. General study of roles and functions of the macroeconomic planning requires one volume of comprehensive book and apparently such a detailed analysis is not intended by the Author. On the contrary the Author presents in II-1 and II-2 some concrete examples, problem of which concerns the theme of this Thesis. The Reader can understand through these examples the essence of the problems which the Author intends to examine in this Thesis and the way by which the problems are related to the macroeconomic planning. Therefore this chapter is called the problematic introduction to this Thesis. The Reader will find some related arguments about roles and functions of economic planning also in the following chapters of this Thesis.

The Reader, if he wants, can omit II-1 and II-2, then however, he dares dangers to lose himself in the abstract model of chapter V. In II-3 the Author examines the cost-benefit analysis, which is the base of the analysis of this Thesis. In II-4 the Author investigates functions of economic planning and multi-level structure of decision making.

II-1 Interdependence and impossibility of full-decentralization of investment decisions

Firstly the Author introduces the model by Chenery(7), since his model analyzes the interdependence and the multi-level structure of economic planning using empirical data. So his model is not factitious and is very helpful to understand the essence of the theme of this Thesis.

Chenery's model is also very interesting from the viewpoint of cost-benefit analysis. The Author of this Thesis regards Chenery as an inventor of the OECD approach of the cost-benefit analysis.

Investment decision of one industry exerts an influence on the cost of other industries and therefore on the investment decision of other industries. Chenery calls this phenomenon the "external effect" and shows an interesting example using empirical data of Latin America. His model is quite complicated, so the Author abbreviates Chenery's example to some extent. Chenery's model is as

follows. There are only six industries, i.e. (1) metal products (e.g. automobile or machines), (2) iron and steel, (3) electric power, (4) transport, (5) agriculture and (6) other products. Let us imagine an underdeveloped country, where the economy has already established four industries, i.e. (3) electric power, (4) transport, (5) agriculture and (6) other products. At this point of time the country imports (1) metal products and (2) iron and steel, since the economy has no production equipment of these two industries. On the other hand this country already has established the production equipment of electric power and transport service. We assume that only agricultural product is exportable to the world market.

When this country imports metal products of one million peso, she must export agricultural products of 850 thousand peso in order to finance the import cost of metal products. This supposition means that the domestic price of metal products is rather higher than the world price. On the contrary, when this country imports iron and steel of one million peso, she must export agricultural products of 1.2 million peso, since the domestic price of iron and steel is rather lower than the world price.

	import activity of metal products	import activity of iron and steel	export activity of agricultural products	production activity of other products	final demands	price
activity	M_1	M_2	E_7	X_8		
metal products	1.0				1000	
iron and steel		1.0			1000	
foreign exchange	-0.85	-1.2	1.0			
other products			-0.1	1.0		
labour			-1.0	-1.0		1.5
capital			-2.2	-1.5		1.0

table 1

In order to produce agricultural products of one million peso, it is necessary to outlay the other products of 10 thousand peso, one unit of labour force and 2.2 unit of capital. The domestic price of one unit of labour force is 1.5 and the domestic price of one unit of capital is 1.0. Unit of labour force is man-year and unit of capital is million peso. Therefore one worker earns one and half million peso in a year. In order to produce the "other

products" of one million peso, it is necessary to outlay one unit of labour force and 1.5 unit of capital. Now the government faces the final demand for metal products of one billion peso and the final demand for iron and steel of one billion peso. These conditions are summed up in table 1. For labour and capital no restriction is placed on supply, since we examine not a general development program but a partial investment project and this investment project is considered not to exert any influence on the scarceness of labour and capital. Therefore the price of these unproduced inputs is assumed to be fixed and determined historically by the economy.

In this situation the total cost for the government is 8200 million peso. Calculation is done as follows.
 Export of agricultural products; $(1000 \times 0.85) + (1000 \times 1.2) = 2050$.
 Output of other products; $2050 \times 0.1 = 205$.
 Labour cost; $(2050 \times 1.0 \times 1.5) + (205 \times 1.0 \times 1.5) = 3383$.
 Capital cost; $(2050 \times 2.2 \times 1.0) + (205 \times 1.5 \times 1.0) = 4817$.
 Total cost; $3383 + 4817 = 8200$.

We can calculate the total cost from another side. The second method of the calculation of the total cost is as follows.
 price of other products; $(1.0 \times 1.5) + (1.5 \times 1.0) = 3.0$
 price of agricultural products;
 $(3.0 \times 0.1) + (1.5 \times 1.0) + (1.0 \times 2.2) = 4.0$
 price of metal products; $4.0 \times 0.85 = 3.4$
 price of iron and steel; $4.0 \times 1.2 = 4.8$
 Total cost for society must be equal to the value of the final demand. The value of the final demand; $(3.4 \times 1000) + (4.8 \times 1000) = 8200$. So we have obtained the same result.

Let us call this plan the first program. The result of the first program is shown in table 2.

	M ₁	M ₂	M ₃	X ₄	M ₅	X ₆	E ₇	X ₈	
activity level	1000	1000	0	0	0	0	2050	205	
labour cost	0	0	0	0	0	0	3075	307.5	3383
capital cost	0	0	0	0	0	0	4510	307.5	4817
price	3.4	4.8	4.4	5.04	4.0	4.43	4.0	3.0	8200

table 2

Next we investigate the investment problem. The government now examines the possibility of domestic production of metal products and domestic production of iron and steel. Technology of each

industry is denoted by the activities X_1 and X_2 of table 3. For the domestic production of these products, electric power and transport service are necessary and import of iron ore and coal is also necessary. These activities are shown in table 3.

import activity of metal products	production activity of metal products	import activity of iron and steel	production activity of iron and steel	import activity of iron ore	production activity of electric power	import activity of coal	production activity of transport	export activity of agricultural products	production activity of other products		
	M_1	X_1	M_2	X_2	M_3	X_4	M_5	X_6	E_7	X_8	final demand
1) metal products	1.0	1.0									1000
2) iron and steel		-.22	1.0	1.0		-.01		-.02			1000
3) iron ore				-.08	1.0						
4) electric power		-.01		-.02		1.0					
5) coal				-.1		-1.25	1.0	-.07			
6) transport		-.01		-.02				1.0			
7) foreign exchange	-.85		-1.2		-1.1		-1.0		1.0		
8) other products		-.17		-.09				-.17	-.1	1.0	price
9) labour		-.7		-.2				-.7	-1.0	-1.0	1.5
10) capital		-.7		-2.7				-2.5	-2.2	-1.5	1.0

table 3

Firstly we assume that the government adopts only the domestic production of iron and steel and decides to import the metal products. Let us call this plan the second program. We can solve the second program in the following manner. The result of the second program is shown in table 4.

$$\begin{cases} X_2 - 1000 = 0.01 X_4 + 0.02 X_6 \\ X_4 = 0.02 X_2 \\ X_6 = 0.02 X_2 \end{cases}$$

The solution of this equations system is as follows.

$$X_2 = 1000.6, \quad X_4 = 20.01, \quad X_6 = 20.01$$

$$M_3 = 0.08 X_2 = 80.05, \quad M_5 = 0.1 X_2 + 1.25 X_4 + 0.07 X_6 = 126.47$$

$$M_1 = 1000, \quad E_7 = 0.85 M_1 + 1.1 M_3 + 1.0 M_5 = 1064.53$$

$$X_8 = 0.09 X_2 + 0.17 X_6 + 0.1 E_7 = 199.91$$

Activity level of each industry, which results from the

second program, is shown in table 4.

	M ₁	X ₁	M ₂	X ₂	M ₃	X ₄	M ₅	X ₆	E ₇	X ₈	
activity level	1000	0	0	1000.6	80.5	20.01	126.1	20.01	1065	199.9	
labour cost	0	0	0	300	0	0	0	21	1597	299.8	} 7611
capital cost	0	0	0	2701.6	0	0	0	50.03	2342	299.8	
price	3.4			4.21	4.4	5.04	4.0	4.42	4.0	3.0	

table 4

Total cost of the second program is 7611, which can be obtained as the sum of the labour cost and the capital cost. Now we calculate once more the total cost by the second method.

$$P_1 = 3.4, P_3 = 4.4, P_5 = 4.0, P_7 = 4.0, P_8 = 3.0.$$

$$\begin{cases} P_2 = 0.02 P_4 + 0.02 P_6 + 4.022 \\ P_4 = 0.01 P_2 + 5.0 \\ P_6 = 0.02 P_2 + 4.34 \end{cases}$$

From this equations system we can obtain the solution. $P_2 = 4.211$, $P_4 = 5.042$, $P_6 = 4.424$. These prices are shown in table 4. The total cost is equal to the value of the final demand. That is, $(1000 \times 3.4) + (1000 \times 4.211) = 7611$. So we have obtained the same result.

In the second program the total cost decreases from 8200 to 7611 comparing to the first program. So this investment program is profitable. Therefore the government should execute investment in the steel and iron industry, though there remains one question, i.e. should the government also execute investment in the metal products industry?

Firstly we assume that the investment decisions are taken independently. The director of ministry of metal products industry will judge that the investment is unprofitable. Reason is as follows. He can not be informed of investment plan of the steel industry, so he considers that the price of steel is 4.8, which is shown in table 2. He will not expect that the price of steel reduces to 4.21, which is realized by the investment of the steel industry, since the investment decision is fully decentralized. Therefore the director of ministry of metal products industry executes the following cost calculation. The cost of metal products is calculated through multiplication of the input coefficients of activity of X_1 by each price of inputs.

$$(4.8 \times 0.22) + (5.04 \times 0.01) + (4.43 \times 0.01) + (3.0 \times 0.17) + (1.5 \times 0.7) + (1.0 \times 0.7) = 3.41$$

The domestic production cost of 3.41 is higher than the import cost of 3.4. Therefore the director of ministry will judge that the investment in this industry is unprofitable. But this decision is inadequate from the viewpoint of the national economy, since he neglects the interdependence of industries.

So next we assume that the investment decision is fully centralized, in other words, the government executes the investment program in the steel industry as well as in the metal products industry. Let us call this full-centralized plan the third program. We can solve the third program in the same manner as in the case of the first and the second program. Therefore the procedure of calculation is omitted. The solution is presented in table 5.

	X ₁	X ₂	M ₃	X ₄	M ₅	X ₆	E ₇	X ₈
activity level	1000	1221	97.68	34.42	167.5	34.43	274.9	313
price	3.28	4.21	4.4	5.042	4.0	4.42	4.0	3.0

table 5

The total cost decreases from 7611 to 7490. Therefore the investment in the metal products industry is profitable for the national economy. Cost of metal products decreases from 3.4 to 3.28. Cost of steel does not change at the level of 4.21. Therefore the investment in the metal products industry is favourable for the metal products industry and also for the whole economy.

In this case of an underdeveloped country, decentralization of investment decision brings a worse result, so centralization or coordination of investment decisions becomes indispensable. Through the Chenery's example the Reader can understand the fact that the investment decision should be consistently related with the macroeconomic planning. If we utilize the guiding parameter (price) irrespectively of the macroeconomic planning, then the investment decision brings an unfavourable result.

II-2 Treatment of foreign deficit

For some countries it may be possible to borrow the foreign currency as much as she wants. But for underdeveloped countries and socialist countries the situation is completely different. In socialist countries the upper limit of the foreign currency debt is decided by the government from the viewpoint of international economic cooperation and also from a political point of view. For the central planning board this upper limit of foreign currency

debt is not a control variable but a restrictive condition. Therefore we can consider that the foreign currency, as well as labour force and land, is a scarce resource and we must estimate the index which reflects its scarceness. It is plausible that this upper limit of foreign deficit is fully utilized in the existing situation of the socialist economy. In this section the Author presents two alternative methods of determination of shadow exchange rate.

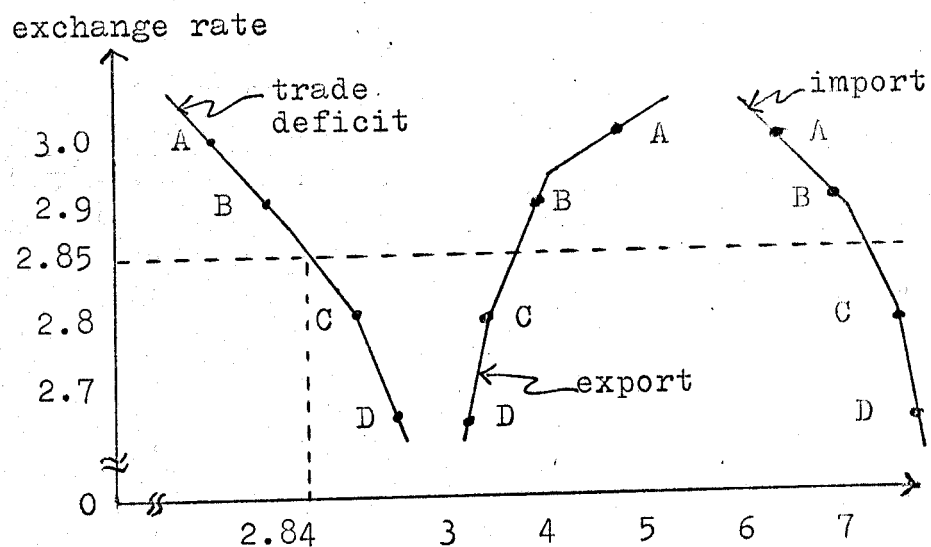
For simplicity we assume that there exists only one foreign market, i.e. the world economy. Therefore there exists only one foreign currency, i.e. the U.S.dollar. Of course, it is not so difficult to introduce a distinction between currency areas, for example, CMEA area, OECD area and underdeveloped area.¹⁾

About this theme Chenery also published an interesting paper.²⁾ In the Chenery's model the exchange rate is determined not endogeneously but exogeneously. He calculates many kinds of alternative plan,

each of which has different exchange rate. Each alternative plan has a different amount of trade deficit. The result is shown in figure 1. He calculates four different plans and each of them are shown in figure 1 by the points A, B, C and D.

If the upper limit of foreign deficit is 2.84, then the exchange rate should be set at the level of 2.85. This solution is obtained by weighted average of two alternative plans, i.e. C and B. The result is illustrated in figure 1.

The Chenery's method was explored for the underdeveloped countries. The most difficult barrier in the application of the Chenery's method into the socialist economy is the reliability of demand and supply curve of foreign currency. When an adequate evaluating criterion lacks, the enterprise does not calculate honestly its (ex ante) export supply or (ex ante) import demand. Moreover its export supply and import demand are quite difficult to expect in the strategic long-term planning. Therefore, although the Chenery's



method may be applicable in the tactical short-term planning of the socialist economy, we must explore another method for determination of the shadow exchange rate in the strategic long-term economic planning. Trzeciadowski(56) also examined a similar model for determination of the shadow exchange rate, but he analyzed his model in the framework of the short-term planning. The Author considers that the shadow wage rate should be determined endogeneously in a consistent way.

One alternative method is the comprehensive approach by Porwit(46). This method stipulates the shadow price of foreign currency by means of dual price of linear programming problem. Although the Author of this Thesis obtained many inspirations from this comprehensive approach, he has some objections to the application of the Porwit's method in the strategic long-term economic planning. It is quite plausible that 20 years hence the upper limit of the foreign currency debt will not be fully utilized. The level of foreign currency debt should be determined by the economic calculation taking into profitability of borrowing. Though in the developing phase the economy should borrow the foreign currency up to the upper limit, in the developed phase, i.e. in far future, the optimal level of foreign currency debt will drastically decrease. Of course, the foreign currency debt provides a possibility for economic development. But when the economy becomes sufficiently developed, the further borrowing will not be profitable comparing the international bank rate with the production ability of the economy. For example, in 1999 the government may borrow only 8 billion dollars, even when she can borrow 12 billion dollars. Needless to say, when the constraint condition is not fully maintained with equality, the dual price is zero by the dual theorem of the linear programming. Therefore in this case the shadow exchange rate in 1999 must be zero. Of course, this difficulty takes place only in the range of the long run. So in the medium-term planning of the socialist economy the upper limit of foreign currency debt is always fully utilized and we can obtain the shadow exchange rate by the comprehensive approach.

According to the Author's opinion, although the Porwit's method is powerful in the operational planning in the medium-term, it can not be adequately applicable in the strategic long-term planning. So we must explore another method. The Author will propose one alternative method for determination of the shadow exchange rate in chapter V. Theoretical background of this method

is the cost-benefit analysis. So next we examine the essence of the cost-benefit analysis.

II-3 Rationale for cost-benefit analysis

In the precedent sections we have investigated the necessity of consistent connection between the central level and the enterprise level. But we should examine this problem from the opposite side, i.e. from the viewpoint of enterprise. In the economic planning, especially in the long-term investment decisions, the initiative of enterprises plays an important role, since the enterprise can most effectively handle the technological informations. Economic calculation at the enterprise level should prevent the arbitrary decision making at the central level. Interaction between these two decision makings is very the main theme which the Author intends to tackle.

Therefore we have to bid farewell to such an idea that the decision making at the central level and that in the individual inferior organizations are completely separated. The UNIDO group argues that "the avoidance of a complete dichotomy between project choice and national planning is one of the main reasons for doing social benefit-cost analysis"³⁾ The Author of this Thesis really agrees with this view. Individual behavior of the inferior organizations should not be contrary to the intention of the central agency. Decision making of the inferior organizations should be checked by a criterion of the central agency from a nationwide viewpoint. "The main reason for doing social benefit-cost analysis in project choice is to subject project choice to a consistent set of general objectives of national policy." (UNIDO, p.11). Therefore the individual objective of the inferior organizations should be in harmony with the nationwide objective of the central agency. This is a rather difficult problem. Here the theory of pure competitive equilibrium gives us some hints to this problem.

According to the neoclassical theory of pure competitive equilibrium the Pareto optimum resource allocation is achieved if and only if all agents, i.e. all enterprises and all consumers, maximize their own objectives, i.e. profit in case of enterprise and utility in case of consumer.⁴⁾

This proposition is also valid in the socialist economy, however needless to say, the reality of the socialist economy contradicts the premise of the neoclassical theory of competitive equilibrium, mainly because the framework of the neoclassical theory of

competitive equilibrium does not contain various activities of the government and, of course, does not suppose the social ownership. In spite of these restrictions we may accept the following proposition as plausible one; the socialist economy can not find herself in an optimal situation if every organization does not utilize her own possibility. Gain of a national economy is composed of individual gains of every organization. Therefore every organization should maximize her own gain. But here it must be took into account that this gain should be measured not from an individual viewpoint but from a nationwide and social viewpoint. If the measurement of the individual gain does not accord with the social welfare, then the decision making at the enterprise level brings undesirable effects. The Author calls this nationwide gain the "social profit". This social profit should be distinguished from the commercial profit which is measured by real market prices. The OECD group argues that "the essence of a cost-benefit analysis is that it does not accept that actual receipts adequately measure social benefits, and actual expenditures social cost."⁵⁾ From this point of view they propose to use accounting prices, regarding the real market price as not adequate in order to measure the desirability of goods and services especially in developing countries.

Now we examine in detail the reasons why the OECD group considers that the real market price is not an adequate measure of goods and services. The main reason is that the OECD group places a small importance on the market mechanism. They present many cases, in which the market mechanism does not function and incompleteness of the market prevails. Most typical one in developing countries is the existence of unemployment. The OECD group and also the UNIDO group do not consider that real market wage rate reflects the "cost" of labour in the developing countries, since an increase of employment in the public sector by one labourer does not mean a decrease of employment in the private sector by one labourer in the developing countries where exists involuntary unemployment or underemployment. Additional one labourer can be pulled out from the pool of the jobless in the rural sector, so in an extreme case an increase of employment in the public sector does not bring any decrease of output in the private sector. In such a case the "social cost" of labour is zero, although the public sector must pay the real market wage rate for the labourer in order to pull him out from the pool of the disguised unemployment in the rural sector. Therefore there exists a divergence between the real market wage rate and the

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"social cost" of labour. In the socialist economy by another reason there exists a divergence between the real wage rate and "social cost" of labour.) This problem will be investigated in detail in chapter V.

The second reason, by which they propose the cost-benefit analysis, is the problem of marginality. Let us assume that consumers of a country buy one hundred thousand colour-television sets and the market price of a colour-television set is one thousand dollars. If the government increases the output from 100,000 to 100,001, then the additional one colour-television set can be sold to the consumer at the price of 1000 dollars. In this case we can consider that the "benefit" of the additional one colour-television set for the consumer is 1000 dollars, since the consumer bought the additional one colour-television set at the expense of 1000 dollars and he must have felt the colour-television set to be equally or more valuable than 1000 dollars. But actually in the developing countries the size of the industrial project is indeed large comparing the size of the market of the economy. Therefore if the government accepts the project of the colour-television plant, then the output of colour-television increases from 100,000 to e.g. 200,000. Clear-out market price of 200,000 colour-television sets may be, e.g. 600 dollars. So the benefit of the last colour-television set, i.e. the 200,000-th colour-television set, is 600 dollars and far less than the existing market price of 1000 dollars. In this case the benefit of the colour-television plant should not be measured by the existing market price, i.e. 1000 dollars, but should be measured by a lower "social accounting price". This marginal property is also important for the project evaluation in a small or middle-sized socialist country.

The third basis of the OECD approach is that the conditions of perfect competition are too rigid and non-realistic for the actual state of the developing countries. The assumption of price taker, i.e. the assumption that every economic agent considers that his own action does not influence the market price, is far from the reality of the developing countries and the assumption of profit maximization is controversial one even in the developed capitalist countries. This statement is valid for the socialist economy, because in the socialism the profit is not the sole and absolute objective of the enterprises.

The fourth reason of the OECD group concerns the income distribution. Let us compare two projects, both of which bring an

increase of wage income by one million dollars, however, project A increases the wage income of the poor and project B increases the wage income of the rich. If we use the market wage rate in order to judge the project selection, then there is no difference between two projects with regard to the increase of the wage income. But many governments consider, from a social point of view, that the one dollar's increase of the wage income of the poor is more favourable than the one dollar's increase of the wage income of the rich. Therefore the government must put a different weight on the increase of the wage income which results from different two projects.

The fifth reason by the OECD group concerns the interest rate and risk. Market interest rate is influenced by the preference of the consumer, however the OECD group considers that the time preference of the consumer is rather myopic. So the OECD group insists that the project should be discounted not by the market interest rate but by the social discount rate. The Author of this Thesis agrees with the use of the social discount rate, though he does not approve of their myopia theory. Of course in many developing countries the market interest rate is very high, but this phenomenon should be interpreted from the viewpoint of inflation and uncertainty. Theoretically saying, if there exists uncertainty, the theory of perfect competition faces a severe difficulty and the market interest rate becomes a non-suitable measure for the selection of the efficient project.

The sixth reason by the OECD group is that there exists externality of some goods in the developing countries. They indicate air pollution as an example of external diseconomy and training of labour as an example of external economy. This externality concerns also the developed capitalist economy and the socialist economy. Therefore when we evaluate a project in a socialist economy, we must take it into account that this project may have some external effects and therefore the social accounting price should reflect these external effects.

The seventh ground, on which the OECD group considers the cost-benefit analysis to be necessary, is the existence of public goods. Almost every project is benefited from public goods. But on the other hand products of some projects are destined to the collective use. Public goods or collective goods do not concern the individual use and therefore do not possess their own market prices. The government of the socialist economy, as well as of the developing countries, should evaluate their public service by the use of

the social accounting prices from the viewpoint of national policy.

The last basis of the OECD group is partial denial of consumers sovereignty in exceptional cases. These cases concern, for example, cigarette or alcohol. In case of these goods the government of every economic system should intervene the market mechanism and levy indirect taxes.

From the above-mentioned reasons the OECD group proposes the use of the cost-benefit analysis in the developing countries, however, arguing that in the developed economy the use of cost-benefit analysis is limited to the public sector, since in the developed economy every private project is executed and evaluated on the commercial basis. In the socialist economy, however, there do not exist the private sector in a true sense of the word, although some socialist countries have many private farms and some private enterprises with a minute size. The cost-benefit analysis was explored for the non-market economy. Needless to say, socialist economy is a non-market economy. Therefore according to the Author's view the cost-benefit analysis can be applied to many projects in the socialist economy.

II-4 Functions of economic planning

In this subsection the Author investigates the most essential problem; i.e. how this Thesis concerns the whole process of economic planning in a socialist economy?

In order to consider this problem firstly we depict the planning process from the viewpoint of decision maker. In chapter I the Author presented a three-tiered schemat of planning system. Although the actual planning system in a socialist economy is more complicated, the Author presents here the simplest model of planning system for conciseness of the argument. It should be also noticed that figure 1 of chapter I is drawn from a functional viewpoint and therefore that figure does not imitate any existing state of institutional framework. In case of a big project the function of particular department of that figure is executed by ministry, on the contrary in case of a small project the director of the enterprise executes this function of particular department.

The three-tiered subjective schemat of economic planning in chapter I corresponds to the following three-tiered functional schemat of economic planning. For the consideration of this problem the argument by Lange is very suggestive. Lange divided econo-

mic policy into two groups.⁶⁾ The first is SOCIO-STRUCTURAL POLICY, subject of which is to form the relations of production and other economic relations. The second is current ECONOMIC POLICY, subject of which is to control the economic process under the condition of given economic relations. The Author of this Thesis divides the current economic policy into two groups, i.e. the macro-economic policy and the micro-economic policy.

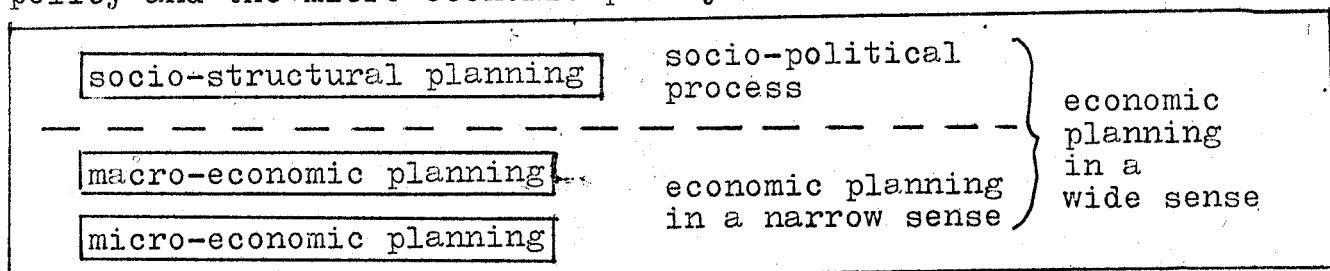


figure 2

The socio-structural planning has generally a qualitative character. But we should not confuse the socio-structural planning with the qualitative planning. Similarly the quantitative planning is a different conception from the economic planning in a narrow sense.⁷⁾ For example, international economic cooperation and foreign currency debt is eminently a subject of socio-political process, though the upper limit of foreign currency debt can be quantitatively articulated.

Needless to say, the economic planning in a narrow sense is subordinate to the socio-political process. But the relation between these two elements is not one-sided but interrelated. On the one hand the macro-economic planning is made out under the condition of given socio-structural planning, though on the other hand the macro-economic planning offers quantitative informations to the socio-political process in order to make out the socio-political policy.

The socio-political process should be performed by the nation. Concretely saying, political party, labour unions and other social groups concern this process. Without exact information about current economic situation and future economic prospect they can not make a rational decision on the socio-structural planning. The FIRST function of the macro-economic planning is to inform the socio-political process of future economic prospect. So the central planning organization should make out several different variants of macro-economic planning corresponding to a different socio-structural condition. In such a way the economic planning in a narrow sense can cooperate with the socio-political process. Without this cooperation of economic planning the participation of the nation in

the socio-political process becomes impossible and the socio-structural planning brings an irrational result.

The SECOND function of the macro-economic planning is to make out the final and consistent macro-economic planning corresponding to the decided socio-structural planning. Therefore the macro-economic plan in the second function should contain detailed informations, though the macro-economic plan in the first function contains only general informations. We may say that the macro-economic plan in the first function has an indicative character and the macro-economic plan in the second function has an executive character. Formerly the macro-economic planning was understood only in this sense of the second function. But according to the Author's opinion the superior value of the macro-economic planning consists in the variant approach of the first function.⁸⁾

The THIRD function of the macro-economic planning is to deduce guiding parameters for the micro-economic planning. In this point it must be noticed that the deduction of the guiding parameters does not require a detailed macro-economic plan but only requires a general macro-economic plan. On the contrary we can not deduce the guiding parameters from a detailed macro-economic plan. Simplification of the macro-economic model is indispensable for the deduction of the guiding parameters.

The MAIN SCOPE of this Thesis consists in the investigation of the first and the third function of the macro-economic planning. Therefore the model of this Thesis is rather simple and many assumptions are introduced. So the model of this Thesis can not possess the detailed consistency of the material balance and consequently becomes insufficient from the viewpoint of the second function of the macro-economic planning. But this Thesis is intended to concern only the framework of the long-term economic planning. We can say that the long-term planning does not demand such a detailed consistency of the material balance as in the short-term planning.

Nevertheless the Author admits that even in the long-term economic planning the coordination between the macro-economic plan and the micro-economic plan is very important. Since the micro-economic planning is also subordinate to the socio-structural planning, the coordination process must be performed as a socio-political process. In this situation the planning organization can only offer the information about performance and prospect of particular project at micro-level to the socio-political process.

Apparently the project selection at the micro-level should be

done by the consideration of all aspects. Among all aspects, however, the most important one is the economic surplus of the project, i.e. the difference between the benefit and the cost of the project. In the following chapters the Author argues that the surplus of the project can be an influential criterion for the project evaluation. Needless to say, however, the surplus of the project is not an unique criterion. The project selection should be done from the socio-economic point of view. Summing up, we should regard every efficiency criterion as a tool for the decision making in the socio-political process.

FOOTNOTES TO CHAPTER II

- 1) One example of extension from one foreign market to many foreign markets is found in the following book; W.Trzeciakowski (56) and (57), especially chapter 6.
- 2) See. Chenery(6).
- 3) See. Sen, Dasgupta and Marglin(8), p.11.
- 4) See, for example, Debreu(10) or Arrow and Hahn(1).
- 5) See. Little and Mirrlees(34)p.19
- 6) O.Lange(31), Political Economy volume 1, chapter 4. Oskar Lange Dzieła tom 3, p.363.
- 7) The same idea was presented by a Japanese economist. See, H.Onoe(42).
- 8) Porwit presented this idea in the context of relation between central planning and sectoral planning. See, Porwit(45), chapter 5. Bocian(3) also presented a similar argument in the context of macro-economic planning.

CHAPTER III

STRUCTURE OF ECONOMIC PLANNING — OBJECTIVES AND CONSTRAINTS —

III-1 General problem

In this section the Author examines the methodological problem of economic planning. Although the main theme of this Thesis is the long-term economic planning, the study must be founded on general understanding of planning system. One-sided and isolated way of understanding becomes very dangerous, since every particular planning always contains interrelated connections with another planning. Therefore economic planning should be analysed by a synthetic and comprehensive approach.

In this field of general consideration of planning system we have an interesting and stimulative contribution by Porwit(48). The Author of this Thesis intends to examine the general problem of economic planning mainly depending on the argument by Porwit. He introduces the analysis by Porwit, however, in connection with the main theme of this Thesis. So the Author does not present in this section a general summation of the Porwit's book, to say nothing of a book review. Therefore in this section the Author examines only a part of the Porwit's analysis, though the Porwit's analysis has a multilateral and synthetic character. The Reader can refer a book review, e.g., by Lukaszewicz(36).

Porwit adequately defines the planning system as a subsystem of a socio-economic system.¹⁾ The Author of this Thesis also agrees with this view and regards the socio-economic aspect as a superior premise of economic planning. So in the last section of chapter II the Author of this Thesis already presented a schematic explanation of this relation. We must entrust many determinant factors to decision of the socio-political process. These factors concern the future direction and structure of economic development of the economy. Concretely saying, they concern income distribution within the nation, future structure of final consumption, international division of labour and so on. We must recognize that these factors have a priority to the economic planning process and therefore the domain of planning system is rather restricted. The role of planner is to offer the best program within the framework of the socio-economic system.²⁾

This way of thinking enables us to consider the relation between objectives and constraints of economic planning in such a manner that its understanding reflects the reality of the contemporary socialist economy. In the contemporary socialist economy the relation between objectives and constraints is quite complicated comparing to that in the first stage of socialism. In contemporary socialist economy the welfare of the nation seems to be a superior objective of economic planning and, needless to say, the welfare of the nation mainly depends on the final consumption by the nation. The intertemporal planning requires a decision on mixture of present consumption and future consumption, i.e. accumulation. In this point the minimal limit of the final consumption in the beginning periods of planning becomes an indispensable constraint in planning process. Therefore the final consumption becomes a constraint of economic planning as well as an objective of economic planning. These constraints and objectives should be determined in the sociopolitical process. So the Author of this Thesis is opposed to the dichotomy of objectives and constraints in economic planning.³⁾

As Porwit argues, we must examine these factors in a more synthetic framework of planning system. In planning system every planning unit has its own particular objective, its own particular means, its own particular mechanism of function and its own particular economic environment. Taking into consideration this diversity it is easy to understand that the dichotomy of centralization and decentralization and the dichotomy of objectives and constraints do not bring any satisfactory result.⁴⁾

Diversity of these elements is historically stipulated and their relation is complicated. As Porwit says, it is nonsense to construct a non-historical model of economic planning. Moreover the Author of this Thesis feels that the construction of a mathematical model of general economic planning is impossible.⁵⁾ The Author of this Thesis considers that we can build only a specific model of a particular economic planning, since only the specific model can embrace the particular interrelationship of each element and the particular economic conditions. In such a manner the mathematical model approach can be useful in economic analysis.⁶⁾

In this sense the Author presents in this Thesis a specific model of the long-term economic planning. The Reader should not confuse the model of this Thesis with a general model of economic planning. So the Author does not examine the problems concerning the short-term economic planning. Even within the framework of the

long-term economic planning the domain of the theme of this Thesis is rather restricted. Firstly the regional problem is omitted. Secondly the Author pays not a much attention to the coordinational function of the long-term economic planning. One of the aims of this Thesis is to explore the possibility of parametric-indicative planning in the long-term economic planning and therefore the consideration is concentrated in, if we use the terminology by Kornai, the control sphere.⁷⁾ Of course, consistency of material balance is important even in the long-term economic planning. But the coordinational function of the long-term economic planning is executed, in reality, in the socio-political process and therefore is quite difficult to describe by the model approach.

In this sense the model of this Thesis is a partial model of the long-term economic planning. According to the Author's opinion, however, the importance of the coordinational function in the long-term economic planning is rather limited comparing to that in the short-term planning. In the long-term economic planning the coordinational function can be reflected in the objective setting and in the constraint setting. Therefore the Author hopes that the insufficient consideration of the coordinational function is not a serious defect of this Thesis.

On the contrary the essence of the problem consists in the cooperation between the central plan and the individual development initiative of particular planning units. In this respect the parametric approach using guiding indicators becomes powerful. In the long-term economic planning the recognizability of economic environment at the central level is quite limited, since the degree of complexity and uncertainty is large. This lack of recognizability must be compensated by the role of selfregulation at the lower level. The selfregulation can function only with the help of adequate guiding parameters. In this way the selfregulation and the parametric control are inseparably related to the long-term economic planning.

The most difficult problem is found in the following questions; which variable is committed to the selfregulation?, which variable is controlled by the directive constraint at the central level? The concrete answer, needless to say, depends on the particular economy. But by means of the Porwit's analysis the Author of this Thesis can present general characteristics of this answer. Within the framework of the Porwit's analysis these characteristics are closely connected with the classification of planning. Porwit classifies planning into three groups; i.e. strategic planning,

operational planning and tactical planning.⁸⁾ Tactical planning is situated outside of the scope of this Thesis.

Strategic planning concerns the setting up of objectives and their evaluation criteria. Operational planning concerns the means and the manner for realization of the objectives. Apparently we should not identify the strategic planning with the long-term planning, although the strategic planning requires a longer time horizon.

In this sense the model of this Thesis should be called strategic planning, but the Author prefers conventional terminology and calls it "long-term" planning. Therefore the "long-term" model of this Thesis covers the time horizon from the medium-term to the infinite time horizon.

The Author of this Thesis considers that the principal aim of the strategic planning is to provide the foundation of the operational planning. In the line of the theme of this Thesis the aim of the strategic planning is to determine the direction of future development of the economy. Therefore in the long-term economic planning the resource allocation and its material balance become a secondary problem.

In the determination of the direction of future development the most essential role is played by the future structure of the final consumption, the international division of labour and the international agreement on foreign trade including the upper limit of foreign currency debt. These are superior elements in the long-term economic planning. Therefore these factors should be controlled at the central level by the constraint condition which is mathematically expressed by an inequality. On the other hand the resource allocation can be managed by the selfregulation. In other words, in the strategic ("long-term") planning the production program and the investment program should be made out by the inferior planning unit depending on the guiding parameters.

To conclude, the achievement of the "long-term" planning depends on two factors. The first is the setting up of objectives and the recognizability of economic environment. The second is the adequateness and the effectiveness of guiding parameters.

Till now we have considered the methodological problem. Next the Author introduces a concrete case study of Japanese economic planning.

III-2 Comparative analysis
— Economic planning in Japan —

It is well known that Japan has made a series of economic planning after the second world war. But the system and character of Japanese economic planning is not so widely known to the economists in the world because of linguistic obstacle. In this section the Author presents a compact survey of economic planning of Japan. The first subsection(III-2-1) introduces the institutional organization for economic planning in Japan. III-2-2 presents a classification of economic planning in Japan. The third subsection(III-2-3) investigates the medium-term economic planning in Japan. The last subsection(III-2-4) examines in detail the long-term economic planning in Japan, since the method of the long-term economic planning which will be proposed in this Thesis has something in common with the method of Japanese long-term economic planning.

III-2-1 Organization for economic planning in Japan

Firstly we must define the word of "economic planning". It is commonly accepted that the Japanese government does not possess a short-term economic planning, i.e. annual economic planning. Japan is a capitalist country and therefore all production programs and investment programs are determined by the managers of private enterprises. The government in principle does not intervene the enterprise's program. Especially in the short run we may say that the Japanese government has never intervened the production program of private enterprise with few exceptions. One exception is stagnation. When the industry suffers from a serious stagnation, the government recommends to the industry the formation of a cartel for reduction of output level of the industry.

The another exception is maintenance of fair trade. Japanese economy, as a modern capitalism, contains some industries with oligopoly. Sometimes big enterprises with conspiracy reduce the output level of industry and raise the price level, nevertheless cartel is prohibited by the law. In this case the Fair Trade Commission of the government decrees dissolution of illegal cartel.

The government does not intervene the enterprise's activity, but she exerts a strong influence onto the enterprise's activity through her economic policy. Japanese economic policy consists of two elements, i.e. monetary policy and fiscal policy. Main tool of

monetary policy is interest rate of Bank of Japan (governmental bank of Japan). The interest rate of Bank of Japan is frequently and elastically altered according to the situation of the economy. The interest rate is determined officially by Bank of Japan but substantially by the cabinet.

The fiscal policy is embodied in the annual budget of the government. National budget of Japan constitutes 15.3% of GNP in 1977. The most powerful tool of the fiscal policy is public investment by the government, which constitutes about 2.5% of GNP in 1977. Public investment is executed elastically and adjusted from season to season corresponding to the situation of the economy.

The Author considers that the annual budget can be interpreted as a short-term planning of the capitalist economy. The final plan of annual budget is made up by the Ministry of Finance regarding requests by other executive Ministries.⁹⁾ As well as the Ministry of Finance, the Economic Planning Agency(EPA) also exercises the adjusting function. The institutional organization for budget making is illustrated in the following figure.

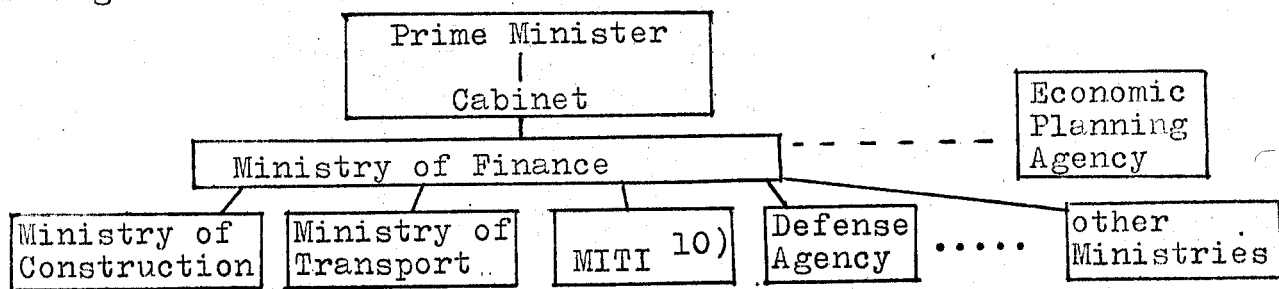


figure 1

Meanwhile in the case of the medium-term and the long-term planning the Economic Planning Agency plays a very important role. The Economic Planning Agency of Japan has the following organization.

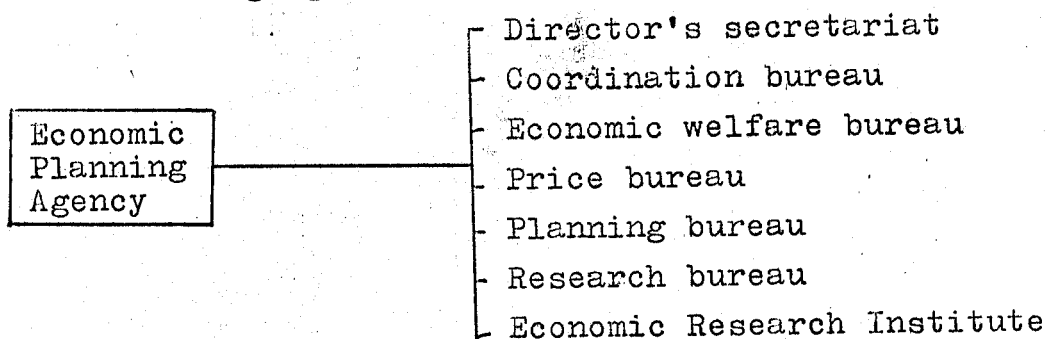


figure 2

In 1977 about 520 persons worked in Economic Planning Agency. In making up the medium-term and the long-term economic planning the planning bureau of the EPA exercises the leading function. Other bureaus manage their own subjects and prepare supplementary

materials for the planning bureau. In the planning bureau there work about 80 persons including typists and physical workers. In the planning bureau 11 chief planners (i.e. director of team) worked in 1977. Under the chief planner several assistant planners work. It is worth remembering that the Japanese Economic Planning Agency possesses relatively small number of staffs. The number of, as well as the quality of, staffs is a decisive preliminary condition of economic planning.

Officially the medium-term economic planning is made up by Economic Congress which is attached directly to the Prime Minister. Economic Congress possesses more than 100 members who are regarded as "wise men" in the society. Members are constituted of retired government officials, retired and active businessmen, chairmen of various trade unions, retired and active professors and representative of various social groups. It must be remembered that any politicians and active government officials are not included in the construction of Economic Congress. It is worth mentioning that most of the members are not economists. Therefore planning staffs from EPA and economists from various Ministries must cooperate with Economic Congress. Practically saying the members of Economic Congress determine only a general line of economic planning which is expressed in words. The substantial parts and numerical calculation are executed by the economists from EPA and other Ministries. The organization for economic planning in Japan is illustrated as follows.

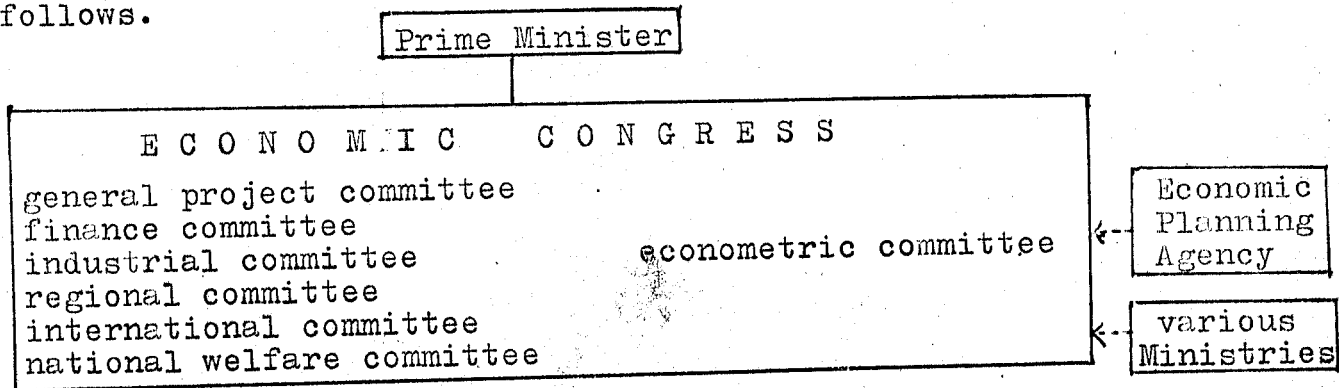


figure 3

Here it is necessary to point out the special character of the econometric committee of the Economic Congress. This committee is an exceptional committee, since in 1977 the econometric committee possessed 14 members, all of whom are professors in various universities. Econometric model is a backbone of Japanese economic planning. The Author can say that other committees exercise their function only in determination of exogeneous variables for the

econometric model, though, of course, the exogeneous variables play a crucial role in economic planning. All members of econometric committee are able scholars and they make up the substantial parts of economic planning with assistance by planning staffs from EPA.¹¹⁾

III-2-2 Kinds of economic planning in Japan

In Japan there are three kinds of economic planning.

- i) Annual budget. Usually annual budget is not regarded as economic planning. But as mentioned in III-2-1, the budget making possesses a character of short-term economic planning.
- ii) Medium-term economic planning. In Japan the word of "economic planning" denotes the medium-term economic planning. The medium-term economic planning covers the time horizon from 5 to 7 years. But historically the medium-term plan has been altered with a cycle of 3 or 4 years. Especially when a new prime minister takes his post, a new medium-term economic plan is made up. The final plan of the medium-term economic planning must be approved by the cabinet and must be published for the nation. Therefore in Japan also the economic planning in a narrow sense is submitted to the socio-political process, that is, to the economic planning in a wide sense.
- iii) Long-term economic planning. Recently the Economic Congress investigates the long-term economic planning. The long-term economic planning does not possess any executive character. Even now the long-term economic planning is at the experimental stage and is regarded as a general guideline for the medium-term economic planning. The long-term economic planning covers the time horizon of 15 years. The existing long-term plan contains a certain shortcoming for the practical use, so we may say that the long-term economic planning in Japan is under development. On the other hand the long-term economic planning has an outstanding characteristic from a theoretical point of view. The long-term economic planning in Japan is a turnpike model with multi-sector economy. The Economic Congress of Japan is proud of the fact that the Japanese long-term economic planning is the first attempt of the turnpike model in the world.¹²⁾
- iv) The Economic Congress intends to explore an extra-long-term economic planning, which covers the time horizon of more than 20 years. Till now no result has been obtained in this field and no report has been published.

III-2-3 Medium-term economic planning in Japan

The Japanese medium-term economic planning possesses an empirical character, since the Japanese medium-term economic planning model is a forecasting model. This character arises from the fundamental property of the capitalist economy. In the capitalist economy variables which the government can directly control are restricted to a rather small number. Almost every economic decision is made by private enterprises and consumers. Therefore the first task of the Central Planning Board is to forecast the future situation of the economy. The medium-term economic planning is made up for this purpose.

Through publishing this forecast the government can control indirectly the economic activity of private sector, since the decision making of enterprises is influenced by the government's forecast. In this point the Japanese medium-term "economic planning" can be an economic planning in a true sense. The most difficulty of this planning, therefore, exists in the forecasting of economic behaviors of private sectors.

i) Household sector. In the capitalist economy the consumer's behavior exerts a very strong influence onto conjuncture and forecasting of it is quite difficult. For example, propensity of saving in Japan increased from 5% in 1950's to about 20% in 1970's. Consumer is assumed to behave in order to maximize his utility. So the price level and the income level become decisive factors. Therefore the planning bureau must forecast the future price level and the future income in the household sector. Consumption function is estimated using econometric model. Housing investment by household sector is also an important element.

ii) Enterprise sector. The Japanese medium-term economic planning model is a multi-sector model. The first problem is estimation of the production function of each sector. This problem concerns the level of capital stock, technological progress, operation rate, labour input and so on. Capital investment and stock investment of each sector also should be forecasted.

iii) Labour demand is also forecasted.

iv) Export level and import level of each sector should be forecasted.

v) Price level should be forecasted.

vi) Wage income and profit level should be forecasted.

All these elements (endogeneous variables) are forecasted by the simultaneous estimation using the econometric model. Some equations are non-linear.

The government can control some variables, for example, interest rate of Bank of Japan, level of public investment, tax rate and so on. There exists another kind of variable, which is regarded as given data for the government. International price is one example of this type of exogeneous variable.

The model contains the following variables.

variables { 691 endogeneous variables
790 exogeneous variables { non-controlled variables
policy tools by the government

This model contains 10 sectors. Input-output coefficient of each sector is calculated from the input-output table of Japan with 60 industries. The model contains 20 periods and one period covers six months. Therefore the model forecasts the future economic situation of 10 years. The medium-term economic plan of 5 years is deduced from this 10 years plan. Way of determination of the optimal final plan is the variant method. The planning bureau makes up many variants with different values of exogeneous variables. The Economic Planning Agency of Japan calculated more than 100 variants in order to determine the optimal final plan.

The determination of the optimal plan is executed by all-round aspects. But the most important aspects in the Japanese economic planning are the inflation rate and the unemployment rate. The simplest choice is done in the following manner.

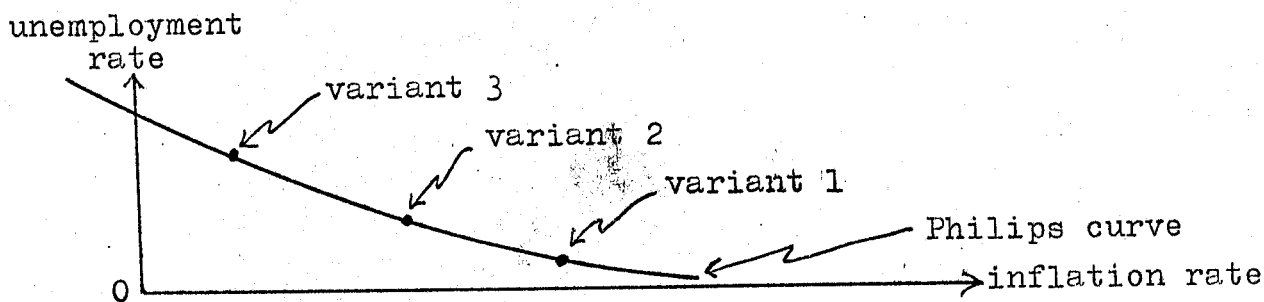


figure 4

Economic Congress determines the optimal plan by its "utility function". But practically saying, this final choice is done by the political decision.

Here we must pay attention to an important fact. The exogeneous variables exert a very strong influence onto the forecast of the future economic situation. Therefore the number of exogeneous

variables must be very huge (i.e 790). For the numerical computation, on the other hand, the number of industry had to be reduced to a small number (i.e. 10). Perhaps the planning staffs of EPA considered that the model with many exogeneous variables and with small number of sectors is more powerful in forecasting than the model with small number of exogeneous variables and with many sectors.

The long-term economic planning of Japan possesses a different character comparing to the medium-term economic planning.

III-2-4 Long-term economic planning in Japan

The Japanese long-term economic planning may seem very curious to the eye of economists in the socialist economy. The Japanese long-term economic planning has also a forecasting character as well as the medium-term planning. But the Japanese long-term economic planning explicitly possesses an objective function. The objective function is as follows.

$$\text{MAX} \quad \sum_{t=0}^T \frac{F^t}{(1+r)^t}$$

where F^t ; the value of the public expenditure by the government in t-th period.

r ; the discount rate.

The public expenditure by the government concerns the following items. 1) public investment, 2) public housing construction, 3) education, 4) health service and 5) other public service. Therefore the expenditure by the government for the general administration cost and the national defense does not enter into the objective function of the Japanese long-term economic planning.

It is an outstanding characteristic of the Japanese long-term economic planning that the private consumption does not explicitly appear in the objective function. The report of the Japanese long-term economic planning(21) says that the econometric committee of Economic Congress adopted this approach, since only the public expenditure is a control variable of the government and the private consumption is not a control variable of the government. This approach of the Japanese EPA is expressed in the following form of the material balance equation. Although the Japanese long-term economic planning is a dynamic Leontief model with 34 sectors, the Author explains the Approach of the Japanese EPA using an aggregate model. The material balance equation in t-th period is as follows.

$$x_t \geq a_t x_t + g_t (x_{t+1} - x_t) + c_t x_t + F^t$$

i.e.

$$(\text{gross output}) \geq (\text{internal input}) + (\text{investment}) + (\text{private consumption}) + (\text{public expenditure})$$

where x_t ; gross output in t-th period. . .

a_t ; coefficient of internal input in t-th period.

g_t ; coefficient of capital investment in t-th period.

c_t ; coefficient of private consumption in t-th period.

F^t ; public expenditure by the government in t-th period.

The Japanese EPA considers that the private consumption is AUTOMATICALLY determined by the gross output. Suppose that the consumption propensity is constant and that the household income is proportional to the gross output(x_t). Then we can express the private consumption as a function of gross output. That is, (private consumption) = $c_t x_t = cx_t$. If the economy produces the gross output of x_t , then the amount of $(c \cdot x_t)$ would always be devoted to the private consumption. The Japanese EPA considers that the government can not control the level of the private consumption and this level is automatically determined by the output level of the gross output level. If the government intends to increase the output level of the next year (x_{t+1}), then the investment of t-th period should be increased and consequently the government's public expenditure must be reduced. On the other hand, however, the increased output level of the next year brings a possibility of a larger amount of public expenditure in the next year. Therefore the government should choose the optimal combination of public expenditure in t-th period and in (t+1)-th period. This is the essence of the model of the Japanese long-term economic planning. Consequently the Japanese EPA concludes that the objective function is constituted only from the public expenditure by the government, since the private consumption is not a control variable of the government.

On the other hand the Japanese long-term economic planning possesses a very interesting innovation from a theoretical point of view. Firstly this model is a multi-sector long-term model. The number of sectors is 34 and the time horizon is 15 periods, where one period covers one year. The Japanese long-term economic planning proved the applicability of the multi-sector optimal growth theory in practice.

Secondly this model is a turnpike model. Since the terminal condition of economic planning is difficult to determine, the settlement of the terminal condition is always troublesome problem in

economic planning. The Japanese long-term economic planning solved this problem by the turnpike property. The terminal condition is determined by the turnpike path of the finite turnpike model. The turnpike property will be explained in chapters IV and V. The model of the Japanese EPA is very strongly influenced by the idea of J. Tsukui, who was the research leader of econometric committee of the Economic Congress.¹³⁾

Thirdly the objective function of the Japanese long-term economic planning has a quite similar form to that of Porwit(46) and that of this Thesis. The econometric committee assumed that the internal structure of the public expenditure is fixed. Then we can illustrate the indifference curve of the Japanese long-term economic planning, for example, in the following two-dimensional figure.

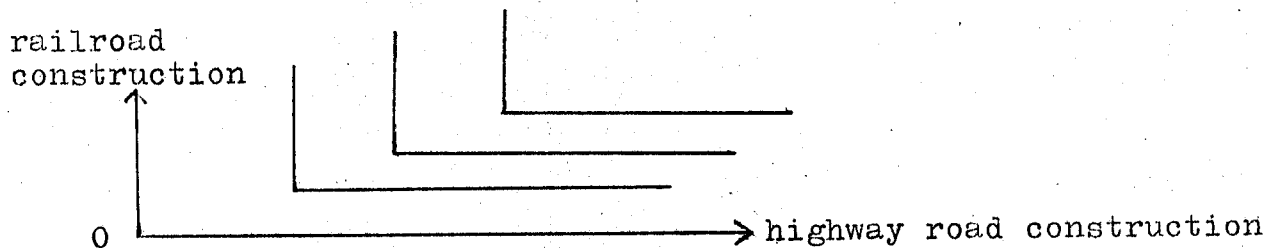


figure 5

Fourthly the report of the Japanese long-term economic planning (21) mentions that the change of the discount rate does not influence the optimal growth path after 5 years. Of course, the change of the discount rate drastically alters the optimal path in near future, i.e. within the span of 5 years. Determination of discount rate is also a troublesome problem in economic planning. Fortunately, according to the report of the Japanese EPA, in the determination of optimal growth path in far future we can be free from the discount rate problem. The econometric committee changed the discount rate from 0% to 10% in numerical calculation. Surprisingly then did not observe any outstanding changes in optimal path after 5 years. This fact is very suggestive also to the economic planning in the socialist economy.

Fifth, the convergency property of the optimal path is very well observed, even if the convergency can not be proved mathematically. For the convergency proof the assumption of strict convexity of preference and technology is usually indispensable. The econometric committee can not prove the convergency property (i.e. turnpike property) of their model mathematically. But numerical simulations showed the turnpike property of their model. This is also suggestive to the economic planning in the socialist economy.

In chapter V the Author will prove the turnpike property of the model of this Thesis under some assumptions. According to the report of the Japanese EPA, we can expect the convergency of the optimal path, even if we loose some assumptions in our model.

Finally, the model of the Japanese EPA is a turnpike model with a finite time horizon. Recently, however, Gale(13) has explored the infinite turnpike model. In this Thesis the Author also examines the infinite turnpike model.

III-2-5 Industrial planning at micro-level in Japan

Firstly we investigate the general character of industrial policy of Japan. Japanese industrial policy has been executed generally by indirect guidance of the MITI (Ministry of International Trade and Industry). So we can say that the Japanese industrial policy has an indirect character.

Frequently many economists in Western Europe point out the fact that the decision making of Japanese enterprises is strongly influenced by the MITI and they sometimes insist on the direct leadership of the MITI. Surely we can observe some direct intervention by the MITI in some fields of industry. For example, the Japanese steel industry always holds a close consultation with the MITI on the export problem. After the oil crisis the MITI directly intervened the investment and production program of the shipbuilding industry and the textile industry. But in the steel industry the investment program and the production program seem to be determined spontaneously by the enterprise itself. Also in shipbuilding industry and the textile industry we could observe in 1960's only indirect guidance by the MITI. Of course, sometimes high officers of the MITI are inaugurated as a director or a vice director of private enterprises. Even in this case, however, the influence by the MITI is only psychological.

Next we investigate the policy tools by which the MITI exerts its influence onto enterprise's decision making. The Author classifies the MITI's policy tools into the following three groups; i.e. (i) output policy, (ii) investment policy and (iii) technological policy.

After the second world war the Japanese government has protected the infant industry. The Japanese government guaranteed an appropriate level of profit for the infant industry by the output policy in order to protect the industry. On the one hand the MITI

restricted the number of enterprises and sometimes recommended a merger of enterprises. In such a manner the MITI limited the output of the infant industry to the level which brings sufficient profit. On the other hand the MITI has introduced import control and protective tariffs. Moreover the MITI has given a subsidy to the infant industry which competes with imported goods.

Investment in the infant industry demands an enormous amount of money. The MITI has promoted the investment in the infant industry by its investment policy. The MITI established a special system of capital amortization of the infant industry and released the infant industry from the tariff of imported oil and imported machines for investment. The Japanese government simultaneously financed the infant industry with a special low rate.

The MITI also has promoted the introduction of foreign technology. The introduction of foreign licence is sometimes connected with the capital import. In 1950's and 1960's the capital import required the approval by the MITI. In this way the MITI adequately selected foreign licences. The industry, which introduced foreign technology, was treated favorably by specially low tax system and tariff system.

Summing up, the MITI effectively utilized these indirect policy tools of tax, tariff and finance.

Surely the economic planning of Japan is not a deterministic planning. But the Japanese economic planning also is not an indicative planning in the true sense of the word, although the Japanese economic planning has some resemblances to the indicative planning.

The Japanese government and the MITI has exerted their influence power as a corrective tool, only when the market mechanism and the free competition can not operate satisfactorily. Moreover the Japanese government and the MITI usually make their decision depending on the existing market prices, the existing market interest rate and the existing foreign exchange rate.

In the indicative planning, which the Author proposes in this Thesis, the Central Planning Board consciously and explicitly utilizes the guiding parameters which are different from the existing prices.

Therefore the Author does not regard the Japanese economic planning as a typical example of the indicative planning.

FOOTNOTES TO CHAPTER III

- 1) See. Porwit(48), p.111.
- 2) The same view can be found in the arguments about welfare economics in 1950's. See, Graaff(15) and Little(32).
- 3) The dichotomy of objectives and constraints and the dichotomy of centralization and decentralization can be frequently found in the arguments by western specialists of socialist economy. See, Kaser(25) and Wiles(59).
- 4) See. Porwit(48), p.11.
- 5) See. Porwit(48), p.11 and p.83.
- 6) See. Porwit(48), p.178.
- 7) See. Kornai(28), p.41.
- 8) See. Porwit(48), p.142-145.
- 9) This process is discussed by Komiya(26).
- 10) MITI is a abbreviation of the Ministry of International Trade and Industry.
- 11) See. Economic Planning from 1976 to 1980 (Report of the Economic Congress) (11). Also see (21).
- 12) See. Economic Planning from 1976 to 1980 (11). Also see(21).
- 13) Professor J.Tsukui in Osaka University has published many articles about dynamic Leontief model, optimal growth theory and turnpike model. See, for example, Tsukui(58).

CHAPTER IV

FUNDAMENTAL THEORIES OF OPTIMAL ECONOMIC GROWTH

This chapter has an introductory character to chapter V, which is the main core of this Thesis. In chapter V the Author will examine indirect control parameters and decision rules of investment problem in a socialist economy with multi-level structure. For this purpose we should construct firstly an optimal long-term economic planning model. In this Thesis the foundation of the optimal long-term economic planning model is given by the dynamic Leontief model and by the consumption-turnpike model. These two models, however, are not so easy to understand. Therefore the Author considers that it is better to present an introductory chapter. So this chapter depicts only an outline of these two models.

The Reader who is well familiar with these two models can omit this chapter, since this chapter contains no originality by the Author. Mathematical properties were already examined in many textbooks of these models.¹⁾

Therefore mathematical proof is sometimes completely omitted in this chapter, though in chapter V the Author will, in general, present every proof of his original property.

The Reader who does not demand the mathematical rigorousness throughout this Thesis can also omit this chapter IV, since chapter IV concerns only mathematical property. Especially subsection IV-2-3 is written only for the Reader whose speciality is mathematical economics.

In IV-1 the Author will illustrate the consumption-turnpike property in case of one-sector model, since the one-sector model is simpler than the multi-sector model of chapter V.

In IV-2-1 and IV-2-2 we examine the accumulation-turnpike model and consumption-turnpike model with multi-sectors.

In IV-2-3 we transform the dynamic Leontief model into the von Neumann model, which offers the basis of the turnpike model. This subsection is indispensable only from a mathematical point of view, since the turnpike theorem is usually proved in the framework of the von Neumann model.

IV-1 One-sector model

The one-sector model has the following production function.

$$X = F(K, L) \quad \dots\dots\dots(1)$$

where X ; homogeneous lutput.

K ; homogeneous capital input.

L ; homogeneous labour input.

Assumption IV-1-A Constant returns to scale in production function prevails; i.e. $F(aK, aL) = a F(K, L)$ for every $a \geq 0$.

Assumption IV-1-B

(i) Labour and capital are indispensable in production; i.e. $F(K, 0) = 0$ and $F(0, L) = 0$.

(ii) $\frac{\partial F}{\partial K} =: F_K(K, L) > 0$ and $\frac{\partial F}{\partial L} =: F_L(K, L) > 0$.

(iii) $\frac{\partial^2 F}{\partial K^2} =: F_{KK}(K, L) < 0$ and $\frac{\partial^2 F}{\partial L^2} =: F_{LL}(K, L) < 0$.

(iv) $\lim_{K \rightarrow 0} F_K(K, L) = \infty$. $\lim_{K \rightarrow \infty} F_K(K, L) = 0$.

$\lim_{L \rightarrow 0} F_L(K, L) = \infty$. $\lim_{L \rightarrow \infty} F_L(K, L) = 0$.

Material balance of the model is as follows.

$$\begin{aligned} X_t &= C_t + I_t \\ &= C_t + (K_{t+1} - K_t) \end{aligned} \quad \dots\dots\dots(2)$$

where C_t ; consumption in t -th period.

I_t ; investment in t -th period.

Now we assume that the labour resource increases with a constant rate (q).

$$\text{Assumption IV-1-C} \quad L_t = (1+q)^t L_0$$

Hence we examine the model by the per capita form.

Firstly we define the notation.

$$x =: X/L, \quad k =: K/L, \quad c =: C/L$$

Under the assumption IV-1-A the production function turns into as follows.

$$x = X/L = \frac{1}{L} F(K, L) = F\left(\frac{K}{L}, 1\right) = F(k, 1) =: F(k)$$

From the equation (2) and the assumption IV-1-C we can obtain the following relation.

$$\frac{I_t}{L_t} = \frac{K_{t+1} L_{t+1}}{L_{t+1} L_t} - \frac{K_t}{L_t} = (1+q)k_{t+1} - k_t \quad \dots\dots\dots(3)$$

Using (1), (2) and (3) we obtain the following equation.

$$f(k_t) = c_t + (1+q)k_{t+1} - k_t \quad \dots\dots\dots(4)$$

In this model the planner maximizes the sum of discounted utility under the condition of (4), the initial condition of capital and the terminal condition of capital. We define new notations as follows.

u: utility function.

d; discount rate.

The model is formulated as follows.

$$\begin{aligned} \text{MAX} \quad & \sum_{t=0}^T \frac{u(c_t)}{(1+d)^t} \\ \text{subject to} \quad & f(k_t) = c_t + (1+q)k_{t+1} - k_t \\ & k_0 = \underline{k}_0 \\ & k_{T+1} = \underline{k}_{T+1} \end{aligned}$$

The initial condition of capital stock is illustrated by the point A of figure 1. The terminal condition of capital stock is illustrated by the point B of figure 1. The planning board should control the economy in a such way that the economy moves from the point A to the point B. There are, however, many attainable programs. The planning board determines the optimal program among all attainable programs.

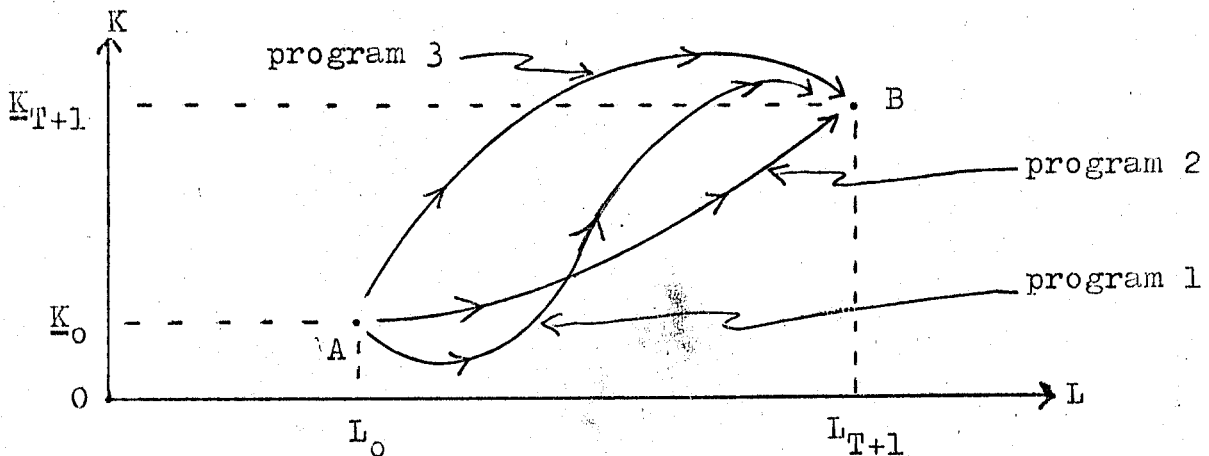


figure 1

The consumption-turnpike theorem asserts that the optimal program maintains the constant stationary construction of capital and labour except in the beginning periods and the ending periods. This stationary construction of capital and labour denoted by k^* .

The Author would like to present here only the result of turnpike theorem.²⁾

This k^* can be deduced by the following equation.

$$f'(k^*) = q + d + qd \quad \dots\dots\dots(5)$$

Since the term (qd) is negligible, we can say that

$$\left(\begin{array}{l} \text{marginal productivity of capi-} \\ \text{tal in the stationary constru-} \\ \text{ction of capital and labour} \end{array} \right) = \left(\begin{array}{l} \text{growth rate of} \\ \text{labour resource} \end{array} \right) + \left(\begin{array}{l} \text{discou-} \\ \text{nt rate} \end{array} \right)$$

The stationary construction of labour and capital (k^*) is illustrated by the line OC in figure 2.

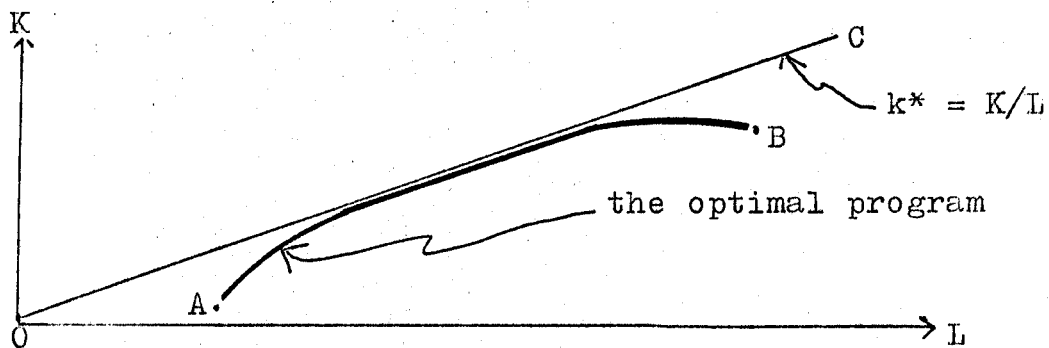


figure 2

So we have illustrate the consumption-turnpike theorem in figure 2 without mathematical proof. The Author, however, considers that this section(IV-1) is sufficient to present a general image of the turnpike theorem to the Reader who is not familiar with the turnpike theorem.

In chapter V the Author will present a rigorous proof of the turnpike theorem in the framework of his original model.

IV-2 Multi-sector model

IV-2-1 Dual formulation of the accumulation-turnpike model in the dynamic Leontief model

In this subsection the Author firstly formulates the dynamic Leontief model and next examines its dual system. The model of this subsection is the same as that of Solow(51), so the model is formulated as an accumulation-turnpike model. At the last part of this subsection the Author will try to prove some propositions which were presented in the book by Porwit(46). The accumulation-turnpike theorem, however, is not proved in this subsection, since it was proved in many textbooks of mathematical economics.

We construct a model with n -goods and T -period. $x_i(t)$ denotes the output of the i -th good by the i -th production sector in t -th

period. The i -th sector produces only one kind of homogeneous good, i.e. i -th good. x_t denotes the n -dimensional column vector of $x_i(t)$. In this economy two factors are necessary for production, i.e. current inputs and capital goods. a_{ij} denotes the quantity of current input of i -th good for the production of one unit of j -th good. $A =: (a_{ij})$ denotes the (n,n) matrix of a_{ij} . b_{ij} denotes the quantity of i -th good, which is necessary as capital for the production of one unit of j -th good. $B =: (b_{ij})$ denotes the (n,n) matrix of b_{ij} . We assume that the coefficients of matrices A and B are constant over time. We also assume that the gestation period (freezing period) of investment is one period. In other words the investment lag is one period. This assumption means that one period of this model must be measured by the gestation period. Therefore one period of this model may be one year or three years and must not be interpreted as one week or one month. So in our model the current input and the consumption goods should be produced in the precedent period. $c_i(t)$ denotes the final consumption of i -th good in t -th period. We do not distinguish private and collective consumption.

Material balance of our model is as follows.

$$x_i(t-1) \geq \sum_{j=1}^n a_{ij} x_j(t) + \sum_{j=1}^n b_{ij} (x_j(t) - x_j(t-1)) + c_i(t) \quad ; (i=1,2,\dots,n) \quad \dots\dots\dots(1)$$

$$-x_{t-1} + A x_t + B(x_t - x_{t-1}) + c_t \leq 0 \quad \dots\dots\dots(1')$$

$(x_j(t) - x_j(t-1))$ denotes the increase of production level of j -th good. In order to attain the increase of production of j -th good in t -th period it is necessary to accumulate $b_{ij}(x_j(t) - x_j(t-1))$ of i -th good in $(t-1)$ -th period. Therefore $B(x_t - x_{t-1})$ should be interpreted as investment flow and should not be interpreted as capital stock.

$K_{ij}(t)$ denotes the capital stock of i -th good in the j -th sector in t -th period. $K_i(t)$ denotes the capital stock of i -th good in the whole economy in t -th period. That is,

$$K_i(t) = \sum_{j=1}^n K_{ij}(t) \quad \dots\dots\dots(2)$$

The capital stock constraint is as follows.

$$\sum_{j=1}^n b_{ij} x_j(t) \leq K_i(t) \quad ; (i=1,2,\dots,n. \quad t=1,2,\dots,T) \quad \dots\dots(3)$$

This inequality (3) admits the possibility of unemployed capital. For example, in the first period the quantity of capital may be too much because of extremely intensive accumulation in the preceding periods.

On the contrary in the investment side, i.e. the flow side, of capital we assume full employment of capital. In mathematical words we formulate the following equation.

$$I_i(t-1) =: K_i(t) - K_i(t-1) = \Delta K_i(t) = \sum_{j=1}^n b_{ij}(x_j(t) - x_j(t-1)) \quad \dots\dots\dots(4)$$

From (1) and (4) we can obtain the following relation.

$$-x_{t-1} + A x_t + (K_t - K_{t-1}) + c_t \leq 0 \quad \dots\dots\dots(5)$$

Apparently from this relation (5) the dynamic Leontief model assumes the full mobility of capital and homogeneousness of products within a sector.

Now we can formulate the original problem of accumulation-turnpike. The objective function is the capital stock in the last period, i.e. the accumulation for the after-planning periods.

The whole model is as follows.

$$\begin{aligned} \text{MAX} \quad & \sum_{i=1}^n v_i K_i(T) \\ \text{subject to,} \quad & -x_{t-1} + A x_t + (K_t - K_{t-1}) \leq -c_t \quad ; (t=1, 2, \dots, T) \\ & B x_t - K_t \leq 0 \quad ; (t=1, 2, \dots, T) \\ & x_t \geq 0 \quad ; (t=1, 2, \dots, T) \\ & K_t \geq 0 \quad ; (t=1, 2, \dots, T) \end{aligned}$$

where v_i ; the evaluation coefficient of i -th capital stock.

K_0 and x_0 are historically given and c_t ; $(t=1, 2, \dots, T)$ are politically determined. So c_t is not a decision variable.

Here we define the following notations of vector.

$$c_t = \begin{pmatrix} c_1(t) \\ \vdots \\ c_n(t) \end{pmatrix}, \quad K_t = \begin{pmatrix} K_1(t) \\ \vdots \\ K_n(t) \end{pmatrix}, \quad x_t = \begin{pmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

Using matrix and vector we can formulate the model as follows.

$$\begin{aligned} \text{MAX} \quad & \sum_{i=1}^n v_i K_i(T) \\ \text{s.t.} \quad & \begin{bmatrix} A & 0 & \dots & 0 & I & 0 & \dots & 0 \\ -I & A & \dots & 0 & -I & I & \dots & 0 \\ 0 & -I & \dots & 0 & 0 & -I & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & A & 0 & 0 & \dots & I \\ B & 0 & \dots & 0 & -I & 0 & \dots & 0 \\ 0 & B & \dots & 0 & 0 & -I & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & B & 0 & 0 & \dots & -I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_T \\ K_1 \\ K_2 \\ \vdots \\ K_T \end{bmatrix} \leq \begin{bmatrix} x_0 + K_0 - c_1 \\ -c_2 \\ -c_3 \\ \vdots \\ -c_T \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{aligned}$$

where I ; (n, n) unit matrix.

Next we consider the dual problem. We introduce the dual variables $w_i(t)$;($t=1,2,\dots ,T$) and $q_i(t)$;($t=1,2,\dots ,T$).

$$\begin{aligned} \text{MIN} \quad & w_1(x_0 + K_0 - c_1) + \sum_{t=2}^T w_t(-c_t) \\ \text{s.t.} \quad & A'w_t - w_{t+1} + B'q_t \geq 0 \quad ;(t=1,2,\dots ,T-1) \dots\dots\dots(6) \\ & A'w_T + B'q_T \geq 0 \quad \dots\dots\dots(7) \\ & w_t - w_{t+1} - q_t \geq 0 \quad ;(t=1,2,\dots ,T-1) \dots\dots\dots(8) \\ & w_T - q_T \geq v \quad \dots\dots\dots(9) \\ & q_t \geq 0, \quad w_t \geq 0 \quad \dots\dots\dots(10) \end{aligned}$$

where $w_t' = (w_1(t), \dots, w_n(t))$, $q_t' = (q_1(t), \dots, q_n(t))$

We can now formulate the dual problem in the matrix form.

$$\begin{aligned} \text{MIN} \quad & w_1(x_0 + K_0 - c_1) + \sum_{t=2}^T w_t(-c_t) \\ \text{s.t.} \quad & \begin{bmatrix} A' & -I & 0 & \dots & 0 & B' & 0 & \dots & 0 \\ 0 & A' & -I & \dots & 0 & 0 & B' & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & A' & 0 & 0 & \dots & B' \\ I & -I & 0 & \dots & 0 & -I & 0 & \dots & 0 \\ 0 & I & -I & \dots & 0 & 0 & -I & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & I & 0 & 0 & \dots & -I \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_T \\ q_1 \\ q_2 \\ \vdots \\ q_T \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ v \end{bmatrix} \end{aligned}$$

Now we prove some propositions by Porwit(46). From (8), $w_t - w_{t+1} \geq q_t$. Using (10) we obtain $w_t \geq w_{t+1}$. So we obtain the following property.

Property IV-2-1-A Dual price of good does not increase.
That is, $w_i(t) \geq w_i(t+1)$;($t=1,2,\dots ,T-1$. $i=1,2,\dots ,n$).

Next we introduce the following assumption.

Assumption IV-2-1-A $x_i(t) > 0$ and $K_i(t) > 0$ for every i and t .

Under this assumption, using the well-known dual theorem of linear programming, we can obtain the following equations.

$$A'w_t - w_{t+1} + B'q_t = 0 \quad \dots\dots\dots(6')$$

$$w_t - w_{t+1} - q_t = 0 \quad \dots\dots\dots(8')$$

If $q_t = 0$, then from (8') $w_t = w_{t+1}$. By (6') we obtain

$(I - A')w_t = 0$. This equation contradicts to the Hawkins-Simon condition. Therefore we obtain $q_t \geq 0$, i.e. $q_i(t) \geq 0$ for all i and at least one i has a strict inequality; $q_i(t) > 0$. This

inequality and relation (8) together mean that at least one i has a strict inequality $w_i(t) > w_i(t+1)$. Therefore we have proved the following property.

Property IV-2-1-B Under the assumption IV-2-1-A and the Hawkins-Simon condition the average price of goods decreases.

It must be noted that in this property we did not use any discount rate or decreasing marginal utility.

Equation (6') can be transformed as follows.

$$w_{t+1} = A'w_t + B'q_t$$

or (shadow price) = (current input cost) + (capital rent cost)(6'')

Equation (6'') means that in the optimal solution every sector earns zero "profit".

Therefore in this model we can utilize the "profit" as a criterion for effectiveness. If profit is negative, then the sector should not produce its output. So this model contains a possibility of multi-level indirect management.

IV-2-2 Dual formulation of the consumption-turnpike model in the dynamic Leontief model

In this subsection the Author examines also a very simple model. Notations are the same as in the preceding subsection. In this subsection the value of consumption is maximized as an objective function. Since the model has n kinds of consumption good, however, it is necessary to aggregate the heterogeneous consumption into the value term. We use $P_i(t)$; ($i=1,2,\dots,n$, $t=1,2,\dots,T$) as this aggregate coefficient and we assume that $P_i(t) > 0$ for all i and all t . For simplicity, however, we must change the framework of the preceding subsection. In this subsection we neglect the time-lag of production. So the output of t -th period can be used as current input, consumption and investment outlay in t -th period. Then the original problem turns into as follows.

$$\begin{aligned} \text{MAX} \quad & \sum_{t=1}^T \sum_{i=1}^n P_i(t) c_i(t) \quad \text{or} \quad \text{MAX} \quad \sum_{t=1}^T p'_t c_t \\ \text{s.t.} \quad & x_t \geq A x_t + c_t + (K_{t+1} - K_t) \quad ; (t=1,2,\dots,T) \dots(1) \\ & B x_t \leq K_t \quad ; (t=1,2,\dots,T,T+1) \dots\dots\dots(2) \\ & x_t \geq 0, \quad c_t \geq 0, \quad K_t \geq 0 \quad ; (t=1,2,\dots,T,T+1) \dots\dots(3) \end{aligned}$$

Inequality of (1) means the following relation.

$$(\text{gross output}) \geq (\text{intermediate input}) + (\text{final consumption}) + (\text{accumulation})$$

Here we must recall that the objective function is $\sum_{t=1}^T p'_t c_t$ and $p_t > 0$. Therefore (1) must be maintained always with equality in the optimal situation. That is,

$$x_t = A x_t + c_t + (K_{t+1} - K_t) \quad \dots\dots\dots(4)$$

$$\text{So } - (I - A)x_t + c_t + (K_{t+1} - K_t) = 0$$

We assume here the Hawkins-Simon condition. Therefore $(I-A)^{-1}$ exists. From (4) we can obtain the following equation.

$$- x_t + (I-A)^{-1} c_t + (I-A)^{-1}(K_{t+1} - K_t) = 0$$

$$\text{Then } B(I-A)^{-1} c_t + B(I-A)^{-1}(K_{t+1} - K_t) = B x_t \leq K_t$$

The last inequality is obtained from (2). So we obtain

$$B(I-A)^{-1}c_t - (B(I-A)^{-1} + I)K_t + B(I-A)^{-1}K_{t+1} \leq 0 \quad \dots\dots\dots(5)$$

Therefore the original model turns into as follows.

$$\begin{aligned} \text{MAX } & \sum_{t=1}^T p'_t c_t \\ \text{s.t. } & B(I-A)^{-1}c_t - (B(I-A)^{-1} + I)K_t + B(I-A)^{-1}K_{t+1} \leq 0 \\ & c_t \geq 0 \quad ; (t=1,2,\dots,T) \\ & K_t \geq 0 \quad ; (t=2,3,\dots,T) \end{aligned} \quad \dots\dots\dots(6)$$

Here the level of K_1 and K_{T+1} is politically and historically determined. That is, the initial condition of capital stock and the final accumulation of capital for the after-planning periods are exogeneously determined. Using matrix we can formulate the model as follows.

$$\begin{aligned} \text{MAX } & \sum_{t=1}^T p'_t c_t \\ \text{s.t. } & \begin{bmatrix} B(I-A)^{-1} & 0 & \dots & 0 & B(I-A)^{-1} & 0 & \dots & 0 \\ 0 & B(I-A)^{-1} & \dots & 0 & -(B(I-A)^{-1} + I) & B(I-A)^{-1} & \dots & 0 \\ \vdots & \vdots & & \vdots & 0 & -(B(I-A)^{-1} + I) & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & B(I-A)^{-1} \\ 0 & 0 & \dots & B(I-A)^{-1} & 0 & 0 & \dots & -(B(I-A)^{-1} + I) \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_T \\ K_2 \\ \vdots \\ K_T \end{bmatrix} \\ & \leq \begin{bmatrix} (B(I-A)^{-1} + I)K_1 \\ 0 \\ \vdots \\ 0 \\ -B(I-A)^{-1}K_{T+1} \end{bmatrix} \quad \text{and } c_t \geq 0, K_t \geq 0. \end{aligned}$$

Next let us examine the dual problem.

$$\text{MIN } w_1' (B(I-A)^{-1} + I)K_1 - w_T' B(I-A)^{-1}K_{T+1}$$

$$\text{s.t. } (B(I-A)^{-1})' w_t \geq p_t \quad ; (t=1, 2, \dots, T)$$

$$(B(I-A)^{-1})' w_t - (B(I-A)^{-1} + I)' w_{t+1} \geq 0 \quad ; (t=1, 2, \dots, T-1)$$

$$w_t \geq 0 \quad ; (t=1, 2, \dots, T)$$

Now we introduce the following assumption.

Assumption IV-2-2-A $K_t > 0$ for all t .

Let us assume that $c_t > 0$ in t -th period. Then

$$(B(I-A)^{-1})' w_t = p_t \quad \dots\dots\dots(7)$$

$$\text{So, } (B(I-A)^{-1})' w_t - (B(I-A)^{-1} + I)' w_{t+1} = 0 \quad \dots\dots\dots(8)$$

$$\text{From (7) we obtain } w_t' B(I-A)^{-1} = p_t'. \text{ Then, } w_t' B = p_t' (I-A)$$

$$\text{Therefore, } p_t = B' w_t + A' p_t \quad \dots\dots\dots(9)$$

$$\text{That is, (profit) } =: \pi_t = p_t - (B' w_t + A' p_t) = 0 \quad \dots\dots\dots(9')$$

From (7) and (8) we obtain the following equation.

$$p_t - p_{t+1} = w_{t+1} \quad \dots\dots\dots(10)$$

From (10) and $w_{t+1} \geq 0$, we obtain the following relation.

$$p_t \geq p_{t+1} \quad \dots\dots\dots(11)$$

So we can present the following proposition.

Proposition IV-2-2-A Under the assumption IV-2-2-A, if $c_t > 0$, then p_t must not be smaller than p_{t+1} ; i.e. $p_t \geq p_{t+1}$.

Proposition IV-2-2-B If $p_t < p_{t+1}$, then $c_t = 0$.

This proposition means that in order to maintain a positive consumption the evaluation coefficient p (price of good) must not increase. Equations (9) and (9') mean that the "profit" must be zero. (9') means the following relation.

$$\left(\begin{array}{c} \text{profit of } i\text{-th} \\ \text{sector in } t\text{-th} \\ \text{period} \end{array} \right) = \left(\begin{array}{c} \text{price of } i\text{-th} \\ \text{good in } t\text{-th} \\ \text{period} \end{array} \right) - \left(\begin{array}{c} \text{capital cost} \\ \text{in } t\text{-th} \\ \text{period} \end{array} \right) - \left(\begin{array}{c} \text{current} \\ \text{cost in} \\ t\text{-period} \end{array} \right) = 0$$

So $w_1(t)$ can be interpreted as capital rent of i -th good in t -th period. Therefore we can utilize p_t and w_t as an indicative parameter of commodity-manetary relation in the indirect management system of a multi-level socialist economy. Control rule of this system is profit maximization. This rule means that if profit is negative, then the relevant sector should not produce its output.

IV-2-3

Transformation of the dynamic Leontief model into the von Neumann model

In the preceding subsection we have examined the dynamic Leontief model, where primary production factors are neglected. In the dynamic Leontief model we have implicitly assumed that the planner can determine an optimal program, aiming at maximization of the objective function, regardless to primary production factors. In reality, however, the production level is restricted by many factors, e.g. existing amount of labour force, land and other natural resources. The von Neumann model embraces the primary production factor. Needless to say, the most important primary production factor is labour force, therefore in this subsection we construct a von Neumann model which contains only one primary production factor, i.e. labour force, although in chapter V we will try to include other primary production factors into the model.

The von Neumann model contains three fundamental relations, i.e. 1) the material balance, 2) the labour force balance and 3) the initial condition.

The material balance is formulated as follows.

$$A X_t + B(X_t - X_{t-1}) + C_t \leq X_{t-1} ; (t=1,2,\dots,T) \dots(1)$$

where A ; (n,n) matrix of current input coefficients.

B ; (n,n) matrix of capital input coefficients.

X_t ; n -dimensional column vector of output levels.

C_t ; n -dimensional column vector of consumption levels.

Therefore in the von Neumann model the current input and the consumption in t -th period must be produced in the preceding $(t-1)$ -th period. The labour force balance is formulated as follows.

$$0 \leq v X_t \leq L_t \quad \text{or} \quad 0 \leq \sum_{i=1}^n v_i X_{it} \leq L_t ; (t=1,2,\dots,T) \dots(2)$$

where v ; n -dimensional row vector of labour input coefficients.

v_i ; necessary labour input for production of one unit output in i -th sector.

L_t ; labour resource in t -th period.

The initial condition is formulated as follows.

$$A_0 X_0 + B(X_0 - \underline{X}) + C_0 \leq \underline{X} \dots\dots\dots(3)$$

where $\underline{X}$; n -dimensional column vector of output levels in the preceding period before planning.

$\underline{X}$ is historically given. For example, the economic planning of 1981-1991 has to be restricted by the result in 1980.

Now we assume that the labour resource increases at a constant

$$\text{rate } (q). \quad L_t = (1+q)L_{t-1} \quad \dots\dots\dots(4)$$

Next we transform the relation (1) as follows.

$$\begin{aligned} (A+B)x_t + c_t &\leq (I+B)x_{t-1} \\ \frac{(A+B)x_t}{L_t} + \frac{c_t}{L_t} &\leq \frac{(I+B)x_{t-1}}{L_t} = \frac{(I+B)x_{t-1}}{(1+q)L_{t-1}} \\ (A+B)x_t + c_t &\leq \frac{(I+B)}{(1+q)} x_{t-1} \quad \dots\dots\dots(5) \end{aligned}$$

where x_t ; per capita version of X_t .
 c_t ; per capita version of C_t .

The relations (2) and (3) turn into as follows.

$$0 \leq v x_t \leq 1 \quad \dots\dots\dots(6)$$

$$(A+B)x_0 + c_0 \leq \frac{(I+B)}{(1+q)} x \quad \dots\dots\dots(7)$$

In the relation (5) we can interpret the first term $(A+B)x_t$ as an "input" (or expense) in t -th period and the last term $\frac{(I+B)}{(1+q)}x_{t-1}$ as an "output" (or donation) of $(t-1)$ -th period.

According to this interpretation we can rewrite the whole structure of the von Neumann model as follows.

$$\begin{cases} a_t + c_t \leq b_{t-1} & ; (t=1, 2, \dots, T) \\ a_0 + c_0 \leq b \\ a_t = (A+B)x_t & ; (t=1, 2, \dots, T) \\ b_t = \frac{(I+B)}{(1+q)}x_t & ; (t=1, 2, \dots, T) \\ 0 \leq v x_t \leq 1 & ; (t=1, 2, \dots, T) \end{cases}$$

where a_t ; "input" for production and accumulation.

b_t ; "output" as a result of production and accumulation.

Now we define a set $// T //$ as follows.

$$// T // =: \left\{ (a_t, b_t) \mid \begin{array}{l} a_t = (A+B)x_t \\ b_t = \frac{(I+B)}{(1+q)}x_t \\ 0 \leq v x_t \leq 1 \end{array} \right\}$$

Lemma IV-2-3-A $// T //$ is a nonempty, compact and convex set in R^{2n} .

Proof. See mathematical appendix (M-IV-2-3).

In every economic system the government should maximize its own objective. This objective may be national income, gross national products, per capita consumption and so on. In chapter V of this Thesis the Author adopts the per capita consumption with

minimal living level as an objective of the government. In any case the objective of the government can be articulated in the form of the objective function. In this Thesis the objective function is a function of per capita consumption and is denoted by $F(c_t)$.

Assumption IV-2-3-A The objective function $F(c_t)$ is continuous and concave on the preference field $\parallel C \parallel$. $\parallel C \parallel$ is a nonempty, compact and convex subset in R^n .

Now we can construct the von Neumann model.

von Neumann model

- (i) $(a_t, b_t) \in \parallel T \parallel$; $(t=1, 2, \dots, T)$
- (ii) $c_t \in \parallel C \parallel$; $(t=1, 2, \dots, T)$
- (iii) $a_0 + c_0 \leq \underline{b}$
- (iv) $a_t + c_t \leq b_{t-1}$; $(t=1, 2, \dots, T)$

In the von Neumann model there exist many attainable programs. The set of all attainable programs is denoted by $\parallel A(\underline{b}) \parallel$. The set of attainable programs $\parallel A(\underline{b}) \parallel$ depends on the initial condition. If the initial condition is too severe, then there may exist no attainable program. Therefore we need the following assumption.

Assumption IV-2-3-B $\parallel A(\underline{b}) \parallel$ is nonempty and compact.

Assumption IV-2-3-C There exist $(\bar{a}_t, \bar{b}_t) \in \parallel T \parallel$, $\bar{c}_t \in \parallel C \parallel$ such that $\bar{a}_t + \bar{c}_t < \bar{b}_{t-1}$; $(t=1, 2, \dots, T)$ and $\bar{a}_0 + \bar{c}_0 < \underline{b}$.

The above-mentioned assumptions are introduced only from a mathematical necessity, but they seem very plausible.

The optimal problem can be formulated as follows.

$$\begin{aligned} \text{MAX} \quad & \sum_{t=0}^T F(c_t) \\ \text{subject to} \quad & (a_t, b_t, c_t) \in \parallel A(\underline{b}) \parallel \end{aligned}$$

Next we prove that the optimal program has a competitive character. In other words a certain price vector corresponds to the optimal program. This competitiveness is a very important concept in the theory of optimal economic growth.

Definition of competitiveness A program (a_t^*, b_t^*, c_t^*) is called competitive, if there exists a sequence of n -dimensional price vectors $p_t \geq 0$; $(t=0, 1, 2, \dots, T, T+1)$ such that

- (i) $F(c_t^*) - F(c_t) \geq p_t(c_t^* - c_t)$; $(t=0, 1, 2, \dots, T)$
- (ii) $p_{t+1}b_t^* - p_t a_t^* \geq p_{t+1}b_t - p_t a_t$ for all $(a_t, b_t) \in \parallel T \parallel$; $(t=0, 1, 2, \dots, T)$

$$\begin{aligned}
 \text{(iii)} \quad & (a_t^*, b_t^*, c_t^*) \in // A(\underline{b})// \\
 & p_0(\underline{b} - c_0^* - a_0^*) = 0 \\
 & p_t(b_{t-1}^* - c_t^* - a_t^*) = 0 \quad ; (t=1, 2, \dots, T) \\
 & p_{T+1} = 0
 \end{aligned}$$

where p_t ; n -dimensional row vector.

The condition (i) can be transformed as follows.

If $F(c_t) > F(c_t^*)$, then $p_t c_t > p_t c_t^*$. That is, the condition (i) means the maximization of the objective function under the budget constraint.

The condition (ii) means the profit maximization. The condition (iii) means that if there exists an excess supply, then its price becomes zero.

Now the Author presents two very important theorems. The proof of these theorems is omitted, since the Reader can find these proofs in many books on optimal economic growth.³⁾

Theorem IV-2-3-A If a program $(a_t^*, b_t^*, c_t^*) \in // A(\underline{b})//$ is competitive, then it is optimal.

Theorem IV-2-3-B Under the assumptions IV-2-3-A and IV-2-3-C, if (a_t^*, b_t^*, c_t^*) is an optimal program, then it is competitive.

Till now we have examined an economic planning with a finite time horizon. In other words the optimal program has finite periods from 0 to T . As will be mentioned in chapter V, however, economic planning inevitably concerns the infinite time horizon. Hence we investigate the infinite planning problem.

$$\begin{aligned}
 \text{MAX} \quad & \sum_{t=0}^{\infty} F(c_t) \\
 \text{subject to} \quad & (a_t, b_t) \in // T// \\
 & c_t \in // C// \\
 & a_0 + c_0 \leq \underline{b} \\
 & a_t + c_t \leq b_{t-1}
 \end{aligned}$$

The solution of this problem is quite difficult to obtain. Therefore firstly we construct a stationary version of this problem.

$$\begin{aligned}
 \text{MAX} \quad & F(c) \\
 \text{s.t.} \quad & (a, b) \in // T// \\
 & c \in // C// \\
 & a + c \leq \underline{b}
 \end{aligned}$$

The solution of this problem is called optimal stationary program. Now we introduce the following assumption.

Assumption IV-2-3-D There exist $(a,b) \in //T//$ and $c \in //C//$ such that $a + c < b$.

It is evident that under this assumption there exists an optimal stationary program.

Theorem IV-2-3-C Suppose that the assumptions IV-2-3-A and IV-2-3-D are satisfied. If (a^*,b^*,c^*) is an optimal stationary program, then there exists a price vector p ; $p \geq 0$ and $p \in R^n$ such that $F(c) + p(b-a-c) \leq F(c^*) + p(b^*-a^*-c^*)$ for every $(a,b) \in //T//$ and $c \in //C//$ and also that $p(b^* - a^* - c^*) = 0$.

Proof is omitted, since we can prove this theorem directly by the saddle-point theorem of non-linear concave programming.⁴⁾

Theorem IV-2-3-D If $F(c)$ is strictly monotonious increasing function⁵⁾, then $p > 0$; i.e. every P_i is positive.

Proof. See, for example, Takayama(54), p.585.

The above-mentioned theorems provide the mathematical foundation of the analysis in chapter V.

FOOTNOTES TO CHAPTER IV

- 1) Typical textbooks are, for example, books by Morishima(38), Nikaido(40), Takayama(54) and Burmeister and Dobell(5).
- 2) The Reader can find the proof of this Theorem in the textbook by, e.g. Takayama(54),chapter 5.
- 3) The Author recommends, for example, the textbook by Takayama(54), especially chapter 7.
- 4) The Reader can find this theorem in every textbook on non-linear programming. One example is Takayama(54),p.69.
- 5) The definition of strictly monotonious increasing function is presented, for example, in Nikaido(39).

CHAPTER V

ADAPTATION OF THEORY TO PRACTICE IN MULTI-SECTOR MODEL

V-1 COMPREHENSIVE APPROACH

V-1-1 Introduction to the problem

The aim of this Thesis is to investigate the multi-level structure of economic planning within the framework of long-term optimal growth model of a socialist economy which is open to the world economy. As already mentioned in chapters I, II and III, the word of "multi-level" contains a twofold meaning; that is, the multi-level structure of time horizons and the multi-level structure of decision makers.

From the viewpoint of time horizon the economic planning is grasped by the Author as a complex of three different kinds of economic planning, i.e. the infinite optimal economic planning, the long-term optimal economic planning and the medium-term optimal economic planning. Although the short-term economic planning (annual planning) is an important element of economic planning, in this Thesis the short-term economic planning is not examined, since the short-term economic planning lies outside the scope of this Thesis.

The supreme principle of economic planning which is sometimes forgotten by mathematical economists in capitalist countries is applicability of planning model in practice. In analysing their model the western mathematical economists usually introduce the assumptions of strictly concave utility function and strictly convex technological set. If we introduce these assumptions into the model, we can deduce a very beautiful conclusion. In practice, however, we can not observe any concrete form of utility function and technology set. In almost every country the planner can handle only the linear production function which is calculated by the Central Statistiscal Office in the form of input-output table.

The Author bids farewell to the subjective approach in economic planning, since we can not settle any concrete form of the utility function of the nation.

As already mentioned in III-1, each economic planning has a different informational structure according to the length of time horizon of the economic planning. Needless to say, the

infinite economic planning and the long-term economic planning can not be applicable in practice, if they require excessively detailed informations about technology and institutional constraints. In this Thesis the Author always pays attention to the limit of planning ability of the Central Planning Board (CPB) and to the limit of capacity of computer.

Taking into these limitations the Author proposes three kinds of optimal program of economic planning. These are optimal stationary infinite program, optimal long-term program and optimal medium-term program. The Author considers that the CPB in many socialist countries can calculate these optimal programs within the range of the existing ability of the planning agency and its computer.

Now let us investigate the mutual relation of these three kinds of planning. Every planning model with a finite time horizon requires the terminal condition of the last period in order to solve an optimal program. This terminal condition (final target) exerts a very strong influence especially on the medium-term economic planning. We may not be off to the point in saying that the terminal condition substantially determines the optimal path of the economic growth. Therefore the determination of the final state of the economy in the last period of the medium-term economic planning (e.g. in 1986) becomes a crucial element. This terminal condition should be determined rationally and not arbitrarily, so the Author proposes the use of the long-term optimal planning in order to determine the final state of the medium-term planning.

Alas, the same thing applies to the long-term economic planning. The final state (final target) of the long-term optimal planning exerts a strong influence on the determination of the optimal growth path of the long-term planning. In order to determine rationally the terminal condition of the long-term optimal planning the Author proposes the use of the infinite optimal planning. Only the infinite optimal economic planning does not require the terminal condition, since its time horizon is infinite.

As will be mentioned in this chapter, we can not calculate the optimal infinite program. Fortunately we can prove that the optimal infinite program converges to the optimal stationary program. So we can substitute the infinite optimal stationary program for the infinite optimal program, when we examine the state of the economy in far future, e.g. 20 years hence.

For the understanding of the mutual relation between these three optimal programs the Author illustrates an example of the optimal economic development program in figure 1.

output of the
2-nd industry's
product

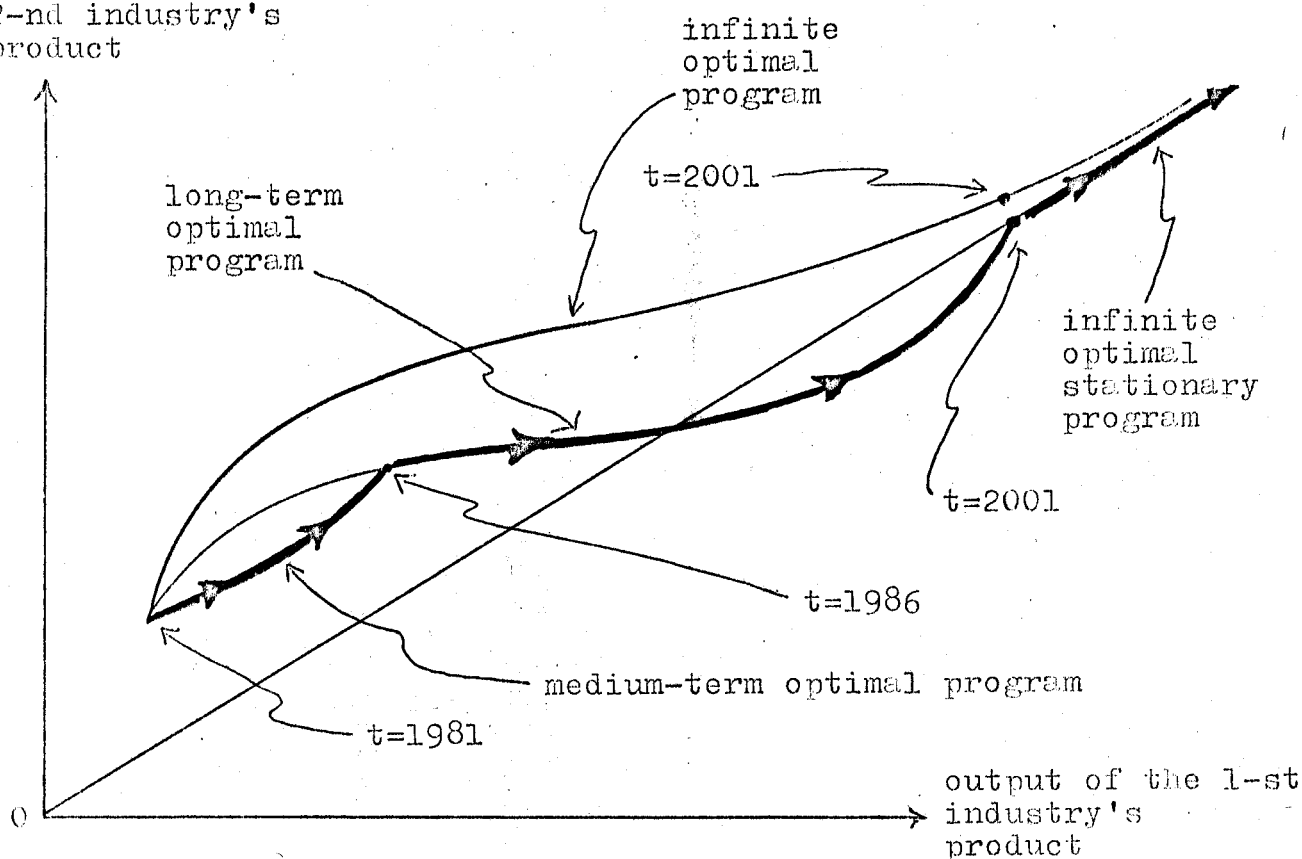


figure 1

As shown in figure 1, the infinite optimal path converges to the infinite optimal stationary program. As will be shown in this chapter, we can easily calculate the infinite optimal stationary program. So the terminal condition of the long-term economic planning (i.e. the final state in 2001) can be determined by the infinite optimal stationary program in 2001. Similarly the terminal condition of the medium-term economic planning (i.e. the final state in 1986) is given by the long-term optimal program in 1986. These relations are illustrated in figure 1. The first aim of the long-term economic planning is to determine the direction of economic growth and its growth rate taking into account the factors which concern the long-term economic situation. As a result of this determination the long-term economic planning settles the terminal condition of the medium-term economic planning.

Now let us examine the second aim of our multi-level structured planning model from the viewpoint of decision makers. The second aim of this model concerns the guiding parameters. That is,

the second aim of the long-term optimal planning is to offer the guiding parameters (accounting prices) for the after-planning periods of the medium-term economic planning. It is evident that the five-years plan can not provide any guiding parameters for the period six years hence. The second aim of the infinite optimal planning is also to provide the guiding parameters for the after-planning periods of the long-term optimal planning. This aim is very important, since the operation period of a certain kind of investment project exceeds 20 years.

In recent years so-called cost-benefit analysis has been proposed for the purpose of project evaluation. Little and Mirrlees(34) have proposed the use of international price as an accounting price in project evaluation. The Author of this Thesis considers that their method is in some cases useful, but in other cases is not applicable. In this Thesis the Author will make clear the situation where the Little-Mirrlees method is applicable in the socialist economy.

Theoretical foundation of this Thesis is given by the Forwit's achievement(46). One of the main subjects of this Thesis is to enlarge the Forwit's method to the range of the long-term and the infinite economic planning in such a manner that the CPB can easily apply the mathematical model in practice.

From a mathematical point of view a characteristic of this Thesis is the proof of convergency of the infinite optimal program to the optimal stationary program without the assumption of strictly concave objective function and also without the assumption of strictly convex technology set. Instead the model of this Thesis requires another additional assumptions. The Author considers, however, that these additional assumptions are very plausible and justifiable in practice. Therefore the model of this Thesis can be — the Author hopes — easily applied in practical economic planning in the socialist economy.

V-1-2 Framework of comprehensive approach at macroeconomic level

In this subsection V-1-2 the Author gives a short survey of the comprehensive approach.¹⁾ In this subsection the Author does not present any original conception. Therefore the Reader who is very well acquainted with the comprehensive approach by Forwit can omit subsections V-1-2 and V-1-3 and can read directly the following part of this Thesis, i.e. V-2 of this chapter. But the

concluding remarks of V-1-2 and V-1-3 will be quite helpful for the understanding of the general line of this Thesis. Especially the concluding remarks of V-1-3 contains very important suggestions.

The comprehensive approach by Porwit(46) consists of four parts, i.e. calculation at macroeconomic level, calculation at branch level, calculation at regional level and calculation at microeconomic level. In this subsection the Author will survey only the calculation at macroeconomic level, since the Author intends to utilize the comprehensive approach in order to determine the direction of economic development in the infinite and the long-term economic planning.

The objective of the planning is, according to Porwit, to maximize the degree of realization of various objectives of the whole society within the range of given means. These objectives of the whole society are manifold and some parts of them should be determined by socio-political decisions and not by economic calculations. Social needs consist of two parts, i.e. individual needs and collective needs (for example, education, national defense and so on). Porwit considers that the level of collective needs should not be determined by economic calculation but by the socio-political decisions. That is, for the economic calculation the level of collective consumption is a given condition. In other words we can make up a different model according to a different level of collective consumption. This treatment of collective consumption seems very proper.

There is another subject which can not be solved by the economic calculation of the model. That is accumulation for the after-planning periods. It must be emphasized that the accumulation within the range of the time horizon is a means of planning. But the level of accumulation for the periods which are out of the time horizon is a given condition for the model building.

The individual needs can be satisfied by domestic products or imported products. Therefore the sum of the domestic products which are destined to private consumption and the imported products which are destined to private consumption can be an adequate indicator for the degree of realization of various objectives of the whole economy, if the condition of collective consumption and the condition of accumulation for the after-planning periods are satisfied. Porwit considers — and the Author of this Thesis also agrees with his approach — that the sum of private consumption in all planning periods should be maximized under the

above-mentioned conditions. Therefore Porwit excludes the maximization of GNP or minimization of investment cost as a criterion of macroeconomic model.

Besides the above-mentioned conditions there are many constraints for economic planning.

- a) Supply of labour force is always limited. Especially in the medium-term economic planning (5-years planning), it is rather difficult to transform a truck driver to a trained mechanic in chemical industry. So we must deal with many kinds of labour force and consequently many kinds of labour force constraint.
- b) The quantity of resources, for example land or coal mine, is also restricted. There are many kinds of resource and consequently there are many kinds of resource constraint. We must know the resource input coefficients in each industry.
- c) The upper limit of foreign currency debt should not be exceeded.
- d) Sometimes there are upper limits of export and import.
- e) Desirable structure of final consumption should be maintained. Evidently the combination of 100 zloty's milk and 100 zloty's bread is more desirable than the combination of 180 zloty's milk and 30 zloty's bread, nevertheless the value of the latter combination is larger than that of the former. The Author of this Thesis considers that this supposition is a key point of the comprehensive approach. By introducing this supposition we can be freed from difficulty of the utility approach. In the range of the medium-term economic planning, to say nothing of the long-term and the infinite economic planning, the market mechanism can not provide any powerful information. As Ostrowski and Sadowski (44) argues, the market mechanism can not form the long-term development program.
- f) Total amount of private consumption must increase at least with a certain rate. This assumption excludes the harsh over-investment and consequently the decline of private consumption in the beginning periods.
- g) Private needs are satisfied also by the imported consumption goods. The ratio of the imported consumption goods to the whole private consumption should not be below than the lower limit.
- h) For the domestic production the import of material is indispensable. Porwit divides the import for the domestic production into two groups; the import which can not be substituted by the domestic products and the import which can be substituted

by the domestic products.

i) For investment the industry also must import some materials. The ratio of imported materials for investment is given.

The above-mentioned constraints bring the following balance conditions. 1)material balance. 2)capital stock balance. 3) foreign currency balance. 4)labour force balance. 5)resource balance. Moreover the mathematical model must include other balances besides these five balances.

Since the objective function of the comprehensive approach is linear and the technology set is also linear, we can construct the dual model of the original comprehensive model. Consequently we can obtain the dual price which corresponds to each constraint condition. This dual price is interpreted as an accounting price of the comprehensive approach.

Concluding Remarks. Porwit attaches a great importance to the property of dual prices. The Author of this Thesis also considers that the dual price is very important, since the aim of the long-term and the infinite economic planning is to determine the optimal direction of economic development and not to determine a concrete and detailed production program at microeconomic level. So the macroeconomic planning should provide adequate guiding indicators for the decision making at the microeconomic level. This is a fundamental principle of the medium-term, long-term and infinite economic planning.

In V-2-6 a quite simplified version of a concrete model of the comprehensive approach will be introduced. In this Thesis the Author will construct a model of the infinite and the long-term optimal planning in a manner which is rather different from the manner of Porwit. Especially the treatment of foreign currency debt is different from that of the original comprehensive approach. In the infinite and the long-term optimal planning the level of foreign currency debt is a means of the optimal planning. The CFB determines the optimal level of foreign currency debt in order to maximize the objective function within the range of the upper limit of foreign debt. In the model of this Thesis, if the international bank rate is higher than the sum of population growth rate and technological progress rate, the optimal level of foreign currency debt is lower than the upper limit of foreign debt in far future. This is a difference between the Porwit's model and our model.

In conclusion, however, the main idea of the model of this Thesis is same as that of the comprehensive approach by Porwit.

V-1-3 Introductory model of the comprehensive approach

In this subsection the simplest model of the comprehensive approach will be presented. Since the comprehensive approach was thoroughly examined by Porwit, the Reader can grasp the whole model of this approach in the Porwit's book(46).

Since the foundation of this Thesis is given by the comprehensive approach, the Author would like to present here the fundamental structure of the comprehensive approach.

In order that the Reader can easily understand the fundamental structure of the comprehensive approach, here some simplifications are introduced.

- a) International trade and consequently foreign currency debt are omitted.
- b) There is only one kind of capital stock. In other words various kinds of capital stock are integrated into a homogeneous unit of "capital".
- c) There is only one kind of labour.
- d) The terminal condition of accumulation for the after-planning periods is omitted.
- e) Input coefficient (a_{ij}), labour input coefficient (v_i) and resource input coefficient (b_i) are constant over time.

Now the Author presents an introductory model of the comprehensive approach.

Introductory model

$$\text{Max } F = \sum_{t=1}^T P_t \quad \dots\dots\dots(1)$$

subject to

$$- X_{it} + \sum_{j=1}^n a_{ij} X_{jt} + \sum_{t'=t}^T b_{i,tt'} J_{t'} + f_{it} P_t \leq - F_{it} \quad \dots\dots(2)$$

($i=1,2,\dots,n; \quad t=1,2,\dots,T$)

$$- P_t \leq - P_0 (1+w)^t \quad (t=1,2,\dots,T) \quad \dots\dots\dots(3)$$

$$\sum_{i=1}^n v_i X_{it} \leq L_t \quad (t=1,2,\dots,T) \quad \dots\dots\dots(4)$$

$$\sum_{i=1}^n b_i X_{it} - N_t \leq 0 \quad (t=1,2,\dots,T) \quad \dots\dots\dots(5)$$

$$\left. \begin{aligned} N_t - (1 - d_{t-1}) N_{t-1} - J_{t-1} &\leq 0 \quad (t=2,3,\dots,T) \\ N_1 &= \underline{N}_1 \end{aligned} \right\} \quad \dots\dots\dots(6)$$

where P_t ; total value of private consumption in t-period.

X_{it} ; gross output of i-th industry in t-period.

a_{ij} ; current input of i-th products in order to produce one unit of j-th industry's products.

f_{it} ; desirable fraction of consumption of i-th industry's product to the total value of consumption in t-period.

w ; lower limit of growth rate of private consumption.

v_i ; labour input coefficient in i-th industry.

L_t ; quantity of labour force in t-period.

J_t ; total value of capital stock increase in t-period.

$b_{i,tt'}$; investment input of i-th industry's products in t-period in order to increase one unit of capital stock in t'-period.

b_i ; capital stock coefficient of i-th industry in order to produce one unit of gross output.

N_t ; total value of capital stock in t-period.

N_1 ; initial condition of capital stock.

a_t ; coefficient of capital amortization in t-period.

P_{it} ; collective consumption of i-th industry's product in t-period.

We have defined the introductory model of the comprehensive approach. Similarly we can formulate the model by the use of vector and matrix. This manner of formulation is very useful when we examine the dual system of the model. This formulation is presented in the mathematical appendix to this chapter(M-V-1).

Concluding Remarks. The dual system of the comprehensive approach is shown in the mathematical appendix(M-V-1). We can calculate the accounting prices from the dual system and can deduce an interesting result about evaluation criterion of the program. We can obtain the following relation, mathematical proof of which is given in the mathematical appendix(M-V-1).

$$\left(\begin{array}{l} \text{net benefit of} \\ \text{the optimal} \\ \text{program} \end{array} \right) = \left(\begin{array}{l} \text{benefit of} \\ \text{the optimal} \\ \text{program} \end{array} \right) - \left(\begin{array}{l} \text{cost of the} \\ \text{optimal} \\ \text{program} \end{array} \right) = 0$$

$$\left(\begin{array}{l} \text{net benefit of} \\ \text{non-optimal} \\ \text{program} \end{array} \right) = \left(\begin{array}{l} \text{benefit of} \\ \text{non-optimal} \\ \text{program} \end{array} \right) - \left(\begin{array}{l} \text{cost of} \\ \text{non-optimal} \\ \text{program} \end{array} \right) \leq 0$$

Here the benefit of the program and the cost of the program are measured by the dual prices. Therefore the surplus (the net benefit) is also measured by the dual prices. From the above-mentioned relation we can say that the optimal program should not have a negative value of its surplus. In other words, the program with a negative value of the surplus can not be an optimal program. This statement suggests that the net benefit can be an

adequate criterion of project evaluation when the cost and the benefit are measured by the adequate accounting prices. In chapter VI the Author argues that the dual price should be the basis for the calculation of accounting prices, although the accounting prices are influenced by many other factors. Moreover if the optimal solution changes, then the dual prices also change. In this sense we must remember that the criterion of net benefit has not an absolute character but a relative character.

The above-mentioned formula provides a possibility of the multi-level structure of decision making, because the Central Planning Board can utilize the value of the surplus as an adequate evaluating criterion of decision making at microlevel. If a new investment project has a negative value of its surplus, then the introduction of this project surely reduces the surplus of whole program of the national economic plan and therefore this investment project should not be accepted. In such a manner the CPB can execute the decentralization of decision making and introduce the multi-level structure into the planning system of the national economy. Also we can find a possibility of integration of the comprehensive approach and the cost-benefit analysis. This is the reason why the comprehensive approach is the theoretical foundation of this Thesis.

The optimal program and the dual prices can be easily calculated by the simplex method. Therefore the comprehensive approach has a great possibility of practical applicability.

V-2 THE MODEL OF MULTI-LEVEL OPTIMAL ECONOMIC PLANNING

V-2-1 After-planning problem

V-2-1-1 Uncertainty

In the preceding section V-1 the Author introduced the comprehensive approach by Porwit, which was explored in order to cope with the problem of multi-period economic planning. But here we must put a question concerning the definition of the multi-period planning. Concretely saying, the question concerns the time horizon of the planning. Although Porwit made no reference to a concrete time horizon, from the opinion of the Author of this Thesis the original comprehensive approach can be most powerful in the medium-term economic planning and the time horizon of the comprehensive approach may be setted appropriately in the range from 5 years to 7 years. Now the Author explains the reasons why the original comprehensive approach is applicable only for the medium-term economic planning and not for the perspective economic planning.

In the socialist economy the long-term (perspective) economic planning is considered to cover the range of 10 or 20 years.²⁾ Technological conditions, domestic market conditions and international economic situation are expected to change drastically within the range of 20 years and are very difficult to forecast exactly. As Porwit pointed out, in the long-term (perspective) planning the degree of uncertainty is very large and possibility of recognition of quantitative relations is quite limited. As Ostrowski and Sadowski(44) argue, uncertainty is the core of the problem of the long-term economic planning.

The comprehensive approach requires detailed information about these uncertain factors. For example, the comprehensive approach requires the quantitative information about the current input coefficient, the investment input coefficient, the labour input coefficient, the foreign demand for exportable goods and the limit of foreign currency debt. We can not estimate exactly these detailed data in far future, e.g. 1999. Only population and labour supply can be forecasted relatively with precision.

Therefore in construction of the infinite and the long-term economic planning, we must firstly simplify the model with respect to the exogeneous data. Secondly we must forecast the most

plausible value of the technological and the institutional information. Since the aim of the infinite and the long-term economic planning is to determine the optimal direction of economic development, we have to be satisfied with these approximately forecasted data which contain a certain degree of uncertainty. Ex-post exact material balance is not demanded in the infinite and the long-term economic planning because these plannings do not possess an executive character. On the other hand in the range of 5 or 7 years these quantitative data can be meaningfully forecasted with precision. Therefore only in the medium-term economic planning the original comprehensive approach can be a powerful tool. In fact, Bocian(3) calculated a numerical program in the range of 5 years by the help of the original comprehensive approach.

V-2-1-2 Accumulation for the after-planning periods

Every kind of planning has a difficult problem, i.e. accumulation for the periods which are out of the scope of the planning. The Author explains this problem using the introductory model of the comprehensive approach which was already introduced in V-1. Let us investigate the optimal level of J_T . J_T denotes the investment of the final period. In the model, which is presented in V-1-3, J_T concerns only the material balance condition, i.e. the equation (2) of V-1-3. Therefore the optimal level of J_T should be zero. Verbally saying, in the last period no investment is done. This is an irrational consequence from the viewpoint of further development. One method to prevent this irrational consequence is to set a lower limit of accumulation in the last period. Porwit also setted a similar condition in his comprehensive approach. In the Porwit's model the capital stock should increase by $i\%$ every year after the last period of the planning.

Nearly all economists consider that this condition is settled by the socio-political decision and therefore the accumulation for the after-planning periods is determined exogenously. But we have to face a serious problem. Suppose that the CPB increases the level of investment in the last period. Then the consumption of the after-planning periods increases and contrarily the consumption within the time horizon of the planning must decline. This choice is also a subject of the optimal planning. The Author considers that the level of the accumulation for the after-planning periods should be determined by the solution of another optimal planning

and not by arbitrary political decisions. So we must investigate the relation between the welfare level within the planning periods (near future) and the welfare level of the after-planning periods (far future). The long-term planning also faces the same difficulty that the medium-term planning possesses with respect to the accumulation problem of the after-planning periods.

V-2-1-3 Difference between the operation period of project and the planning period

As Porwit proved, we can deduce the shadow price (accounting price) for all resources by the dual system of the comprehensive approach. In the case of von Neumann model, as the Author already pointed out in chapter IV, we can also deduce the shadow price for all resources. Apparently, however, we can not obtain the accounting price for resources in the after-planning periods. Needless to say, the 5-years optimal plan can not provide the accounting price for resources of the period 8 years hence.

In the case of a big investment project the plant will work usually more than 10 years. Still some kinds of plant will operate even more than 20 years. So for the project evaluation we must know the accounting price in far future.

In the calculation of investment efficiency the accounting price of near future exerts the most determinant influence. Therefore a detailed and precise estimation of the accounting price of near future is indispensable. In this point of view the comprehensive approach becomes very helpful in the medium-term planning. But, as already mentioned, the original comprehensive approach can not be directly applied to the perspective economic planning. This is a dilemma. One of the aims of this Thesis is to solve this dilemma.

V-2-2 Time-structure of multi-level economic planning

In the precedent subsection the Author argued the necessity of investigation into relation between economic plannings with different time horizon. In this subsection the Author would like to illustrate once more the relation between different plannings in the following figure, although in chapter I this problem was already discussed. The reason why the infinite economic planning is introduced, will be explained in the following subsection. We int-

-end to analyse the multi-level character of economic planning from the dual dimensional point of view. The first dimension is decision maker in economic planning and the second dimension is time horizon of economic planning. The first dimension is quite simplified in figure 2 though the second dimension is depicted in detail.

The arrow in figure 2 indicates the direction of influence. In this Thesis the Author will investigate the relations between different plannings, the range of which is designated by broken line in figure 2, i.e. the relations which are denoted by a double arrow line in figure 2. The relation which is denoted by a single arrow line, will not be investigated in this Thesis.

The Author will examine this problem especially placing the focus on the project evaluation problem. So the deduction of the guiding indicator (accounting price) from different plannings is a main theme of this Thesis.

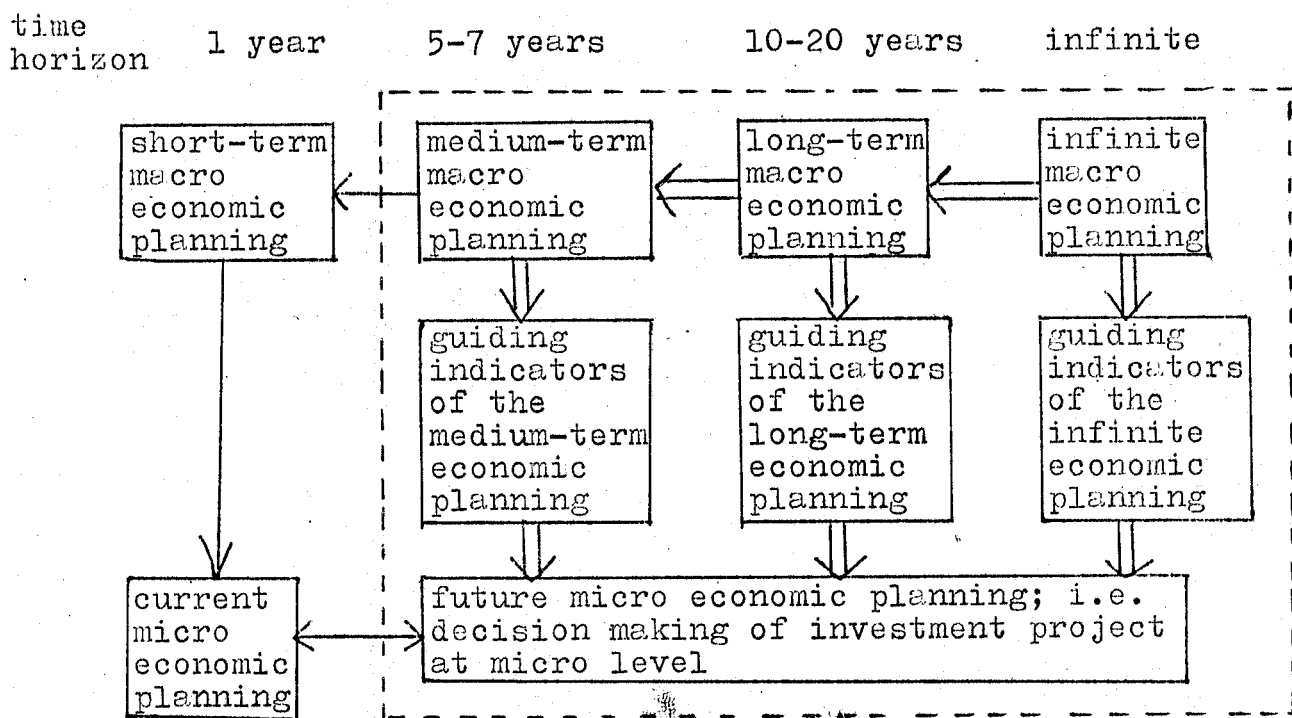


figure 2

V-2-3 Infinite optimal planning of the multi-sector model

V-2-3-1 Conception

The objective of the infinite optimal planning is twofold. The first and main objective is to determine the optimal direction of national economic development. It should be emphasized that the determination of growth path in extra-long-run is not a final

objective in itself. The determination of development direction should serve for the long-term economic planning and in turn for the medium-term economic planning. The infinite optimal planning exerts an especially strong influence on the after-planning problem of the long-term economic planning and in turn that of the medium-term economic planning. In this sense the infinite optimal planning is connected with the long-term and the medium-term optimal planning.

The second objective of the infinite optimal planning is to calculate the accounting price in far future, e.g. 30 years hence. For the evaluation of dam construction for electric generation the accounting price in far future is indispensable.

We must, however, remind ourselves of the fact that the technology and the exogeneous economic conditions in far future are uncertain to a large extent. The first alternative approach is to forecast these data approximately. But the degree of inaccuracy may be very large. The second alternative approach is to simplify the model and limit the number of the data. In this Thesis in general the second approach will be adopted.

V-2-3-2 Objective function of the infinite optimal planning

In the first section of this chapter V-1 we have examined the comprehensive approach by Porwit, since the Author of this Thesis intends to extend the Porwit's method to the sphere of the long-term economic planning. The comprehensive approach adopts the value of consumption by the nation as an objective of the planning. This principle applies to the infinite and the long-term economic planning. As well as Porwit, the Author of this Thesis does not regard economic growth as a final objective per se. Economic growth is desirable, only since the economic growth serves for the increase of consumption.

Here we must pay attention to one problem; i.e. structure of consumption. Let us compare the following two plans. Plan A is composed of 70 billion zloty of durable consumption goods and 30 billion zloty of foodstuffs. Plan B is composed of 100 billion zloty of durable consumption goods and 2 billion zloty of foodstuffs. If we adopt the maximization of consumption value as the government's objective, then the plan B is favourable, since $100 + 2 > 70 + 30$. The plan B, however, surely compels the nation to face starvation. Apparently the plan A should be adopted.

So the government has to decide the ideal structure of consumption goods 20 years hence in order to make up the infinite optimal economic plan. In other words the government must determine a priori how much portion of the consumption fund is devoted to clothes, furnitures, cars, foodstuffs and so on. Then the CPB should maximize the consumption fund under this structural condition of consumption. The determination of consumption structure belongs to the subject of the socio-political decision, therefore this problem is a preliminary condition to the economic planning. The determination of consumption structure is situated outside of the scope of this Thesis.

Now it is necessary to investigate the requirement of minimal increment of consumption. Porwit took into account the following condition in his model.

$$P_t \geq (1+r)^t P_0 \quad \dots\dots\dots(1)$$

where P_t ; the total value of final consumption in t-th period.

Ostrowski and Sadowski(44) also introduce the similar conception of minimal tempo of growth, however, with respect to the national income.³⁾ In the infinite and the long-term economic planning the Author of this Thesis takes into consideration this condition in a different manner from that of Porwit. The Author considers that the per capita version of this condition is more adequate.

Needless to say, investment plays a crucial role in economic growth. Increase of investment brings, on the one hand, an increase of consumption in future periods, but on the other hand brings a decrease of consumption of the periods in which the investment is executed. So excess investment inevitably reduces the consumption of the beginning periods and sometimes reduces it to an intolerable level. Consumption should not be less than the lower limit which is determined from the socio-political point of view. This minimal level of consumption is usually expressed by per capita term, e.g. 50 kg of meat per man-year. In a simple example of the economy with two industries (the durable consumption good industry and the foodstuff industry) we can illustrate the minimal consumption limit in figure 3.

The Porwit's constraint (1) concerns also the pattern of consumption over time. Now we present a paradoxical numerical example in table 1. There are two plans, result of which is same except in the year 1990 and the year 2000.

If the CPB adopts the total final consumption by the nation

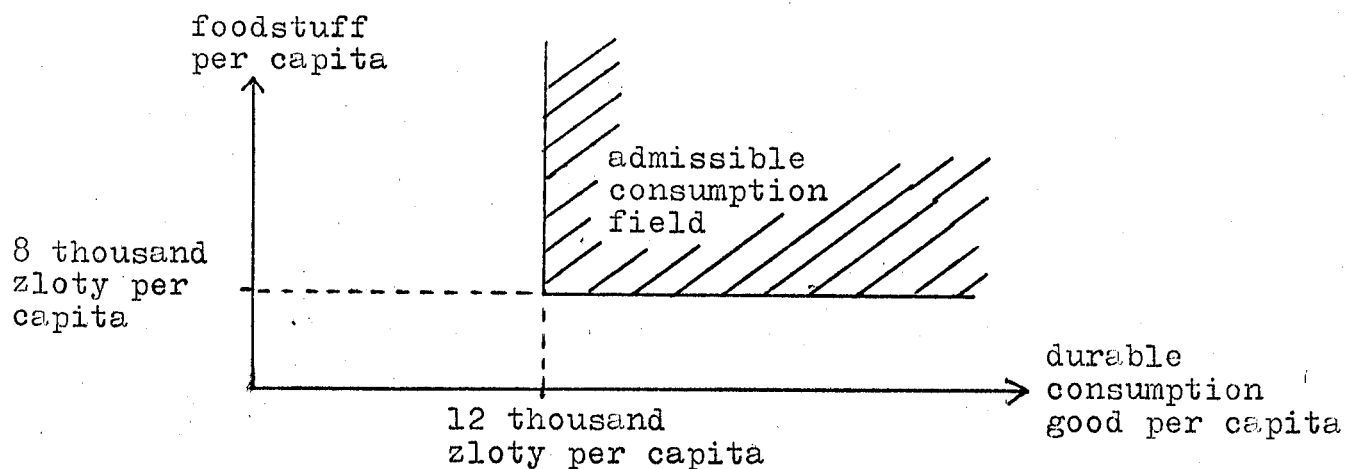


figure 3

year		1989	1990	1991	1992	... 1999	2000
total final consumption by the nation	plan A	3900	<u>4000</u>	4100	4200	... 4900	<u>5000</u>
	plan B	3900	<u>3520</u>	4100	4200	... 4900	<u>5500</u>
population		39	40	41	42	... 49	50
per capita consumption	plan A	100	<u>100</u>	100	100	... 100	<u>100</u>
	plan B	100	<u>88</u>	100	100	... 100	<u>110</u>

table 1

as her objective, then the plan B must be preferred. Since $4000 + 5000 < 3520 + 5500$. But here the population growth should be taken into account. From the viewpoint of per capita consumption apparently the plan A is preferred, since $100 + 100 > 88 + 110$.

In this Thesis the Author adopts per capita consumption as the objective of the CPB, since the standard of living of the nation or the welfare level of the nation is reflected not in the total consumption value of the nation but in the per capita consumption of the nation. If we regard the per capita consumption as the objective of the CPB and if we take into account the minimal limit of per capita consumption, then we can omit the Porwit's constraint(1) in the infinite and the long-term economic planning. In other words, in this Thesis we introduce a more strong and detailed condition of the lower limit of consumption than the total version of the condition by Porwit. The lower limit of consumption differs period to period. As Porwit(46) and Ostrowski and Sadowski(44) point out, the lower limit of welfare level should increase year by year.

Of course the difference between these two approaches has a rather formal character and not a substantial character. If the

lower limit of per capita consumption increases every year by $m\%$ and if the population grows every year by $n\%$, then the lower limit of total consumption should increase by $(m+n)\%$, i.e. $r = m + n$. But if the lower limit of per capita consumption increases unproportionally year by year as in the case of table 1, the per capita version of the lower limit of consumption seems to be more adequate.

Now the Author sums up the above-mentioned argument. The first phase of the infinite economic planning contains the following steps.

Step 1 The government determines the minimal consumption limit of each period.

Step 2 The government determines a desirable structure of future consumption.

Objective of planning The Central Planning Board maximizes the sum of the per capita consumption flow in the future.

Now we examine the treatment of collective consumption in the infinite and the long-term economic planning. Porwit regards the collective consumption not as an objective of planning but as a preliminary condition which is determined by the government from a socio-political viewpoint. This view is valid also in the infinite and the long-term economic planning. The treatment of the collective consumption, however, is rather different in these two plans. We assume that the level of per capita collective consumption is constant over time in the infinite economic planning. Since the infinite economic planning covers infinite time horizon, we can imagine only a plausible state of the economy. Actually the per capita collective consumption may increase over time. But the role of the infinite economic planning is to determine the optimal direction of economic development from 1981 to 2001. Therefore we may expect a plausible level of the per capita collective consumption in 2001 and assume that this level does not change over time. We regard this level as the ideal level of per capita collective consumption in far future.

From the same reason we can imagine only the plausible lower limit of per capita private consumption in far future. Almost every Reader may agree with such a opinion that it is nonsense to forecast the lower limit of private consumption in the years 2030, 2080, 2130. Practically we can imagine the plausible lower limit in 2001 (i.e. 20 years hence) and assume that this level is constant over time in the infinite economic planning. Needless to say, in the framework of the long-term and the medium-term planning

the lower limit can change from period to period. (See V-2-4-6 and V-2-6). Now the Author introduces the following assumptions. Of course in the long-term economic planning we can loosen these strict assumptions.

Assumption V-2-3-2-A In the infinite economic planning the level of the per capita collective consumption does not change.

Assumption V-2-3-2-B The lower limit of the per capita private consumption does not change over time in the infinite economic planning.

If we accept these assumptions, the collective consumption can be included in the lower limit of consumption. We can illustrate this statement in figure 4. Needless to say, these assumptions are introduced from a mathematical necessity. But these assumptions have a certain rationale from a practical point of view. As already mentioned, in far future we can not expect exactly the economic situation. We can only imagine the plausible state of the economy. Under these assumptions we can obtain the following property.

Property V-2-3-2-A Under assumptions V-2-3-2-A and V-2-3-2-B the admissible consumption field does not change over time.

Now we can illustrate the indifference map of the objective function on the admissible consumption field.

Suppose that the government determines such a desirable structure of future consumption that 40% of the total consumption fund is devoted to the foodstuffs and 60% to the durable consumption goods. In this case the indifference map can be drawn as in figure 4.

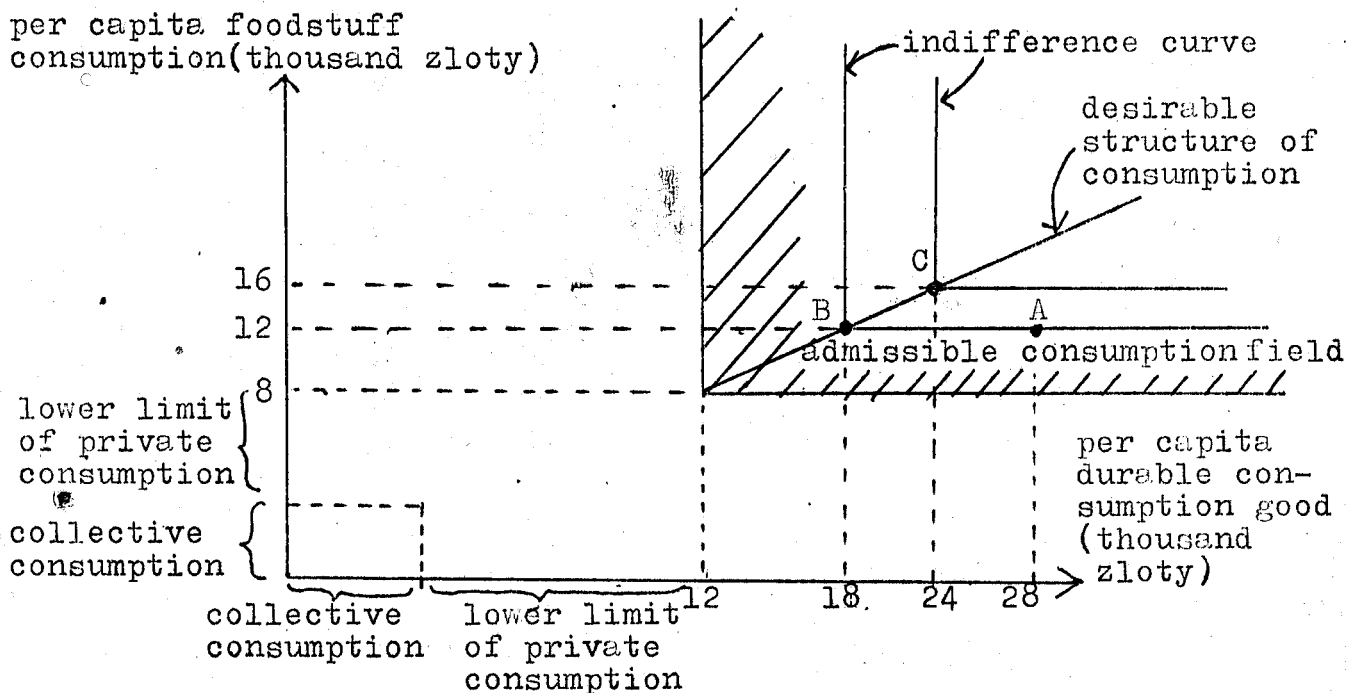


figure 4

Suppose now that the economy supplies 28 thousand zloty of the durable goods and 12 thousand zloty of the foodstuffs for one population. This situation is illustrated by the point A of figure 4. This is an unproportional consumption. The desirable composition of consumption is marked by the point C in figure 4, where the durable consumption goods is 24 thousand zloty and the foodstuffs is 16 thousand zloty, since $(28 + 12) \times 0.6 = 24$ and $(28 + 12) \times 0.4 = 16$. In the case of the point C of figure 4 each of the consumption goods is fully satisfactorily consumed, so 40 thousand zloty can be an index of welfare standard. In the case of the point A of figure 4, however, the durable consumption good is excessively produced and part of them is consumed wastefully. In the case of the point A the supply of the foodstuffs is 12 thousand zloty and if we want to bring the desirable structure of consumption, then the supply of the durable consumption goods should be reduced to 18, since $12 \frac{0.6}{0.4} = 18$ (the point B of figure 4). Therefore in the case of the point A 10 thousand zloty ($28 - 18 = 10$) of the durable consumption goods is wastefully consumed. So in the case of the point A the index of welfare standard should be 30; ($12 + 18 = 30$). The point B of figure 4, however, also brings the same index of welfare standard ($12 + 18 = 30$), because the point B corresponds to the desirable structure of consumption goods. Therefore the point A and the point B are indifferent.

Mathematically we can formulate the index of welfare standard as follows.

$$\text{MAX } U = (F + D) \times \min\left(\frac{F}{(F+D) \times 0.4}, \frac{D}{(F+D) \times 0.6}\right) \dots\dots\dots(2)$$

where U; the index of welfare standard.

F; the per capita consumption of the foodstuffs.

D; the per capita consumption of the durable consumption goods.

This objective function may seem quite strange for some Readers. So we can formulate the above-mentioned objective function in a different manner.

$$\text{MAX } U = \max_{\lambda} \{ \lambda \mid \lambda c \leq c \}$$

where λ ; scalar.

c; the vector of consumption.

$\underline{c}$; the vector of the desirable composition of consumption.

In this case $c = (F, D)$ and $\underline{c} = (0.4, 0.6)$. This manner of formulation was proposed by Kantorovitch and Makarov (24). The

Japanese Agency of Economic Planning(21) also proposed the same function.⁴⁾ But in this Thesis we adopt the first form of the objective function(2), since these two forms have the same meaning.

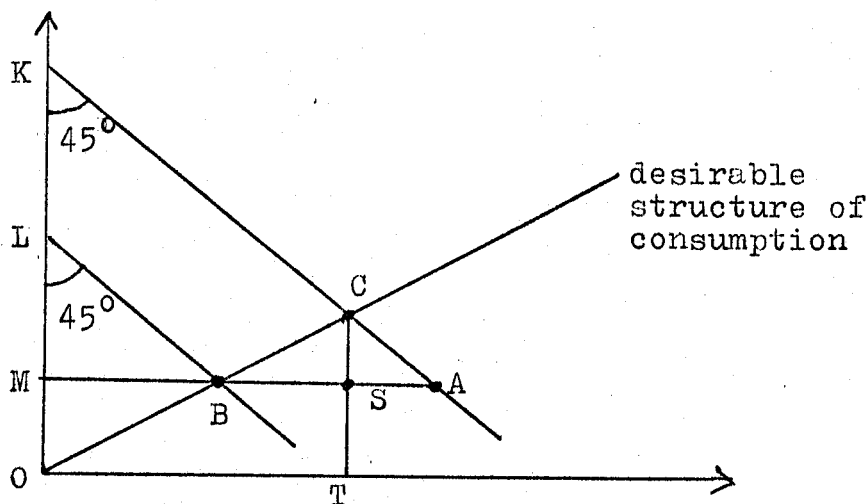


figure 5

The value of $(F+D)$ of the point A is equal to the length of the segment OK , i.e. $F+D = \underline{OK}$. Hereafter underline indicates the length of segment. $0.4(F+D)$ of the point A is equal to $\underline{TC}$.

$$\text{So, } F / 0.4(F+D) = \underline{TS} / \underline{TC} < 1$$

$$\text{Similarly } D / 0.6(F+D) = \underline{MA} / \underline{\frac{MS}{F}} > 1$$

$$\text{Therefore } U(A) = (F+D) \times \min\left(\frac{\underline{F}}{0.4(F+D)}, \frac{\underline{D}}{0.6(F+D)}\right)$$

$$= \underline{OK} \times \underline{TS/TC}$$

$$= \underline{OK} \times \underline{OB/OC} = \underline{OL} = U(B)$$

where $U(A)$; the index of the welfare standard of the point A.

So we have proved that the objective function (2) brings the same index for the point A and the point B. More generally we can formulate the objective function as follows.

$$\begin{aligned} \text{MAX } U &= P \times \min(c_1/Pf_1 , c_2/Pf_2 , \dots , c_n/Pf_n) \\ &= \min(c_1/f_1 , c_2/f_2 , \dots , c_n/f_n) \end{aligned} \quad \dots\dots\dots(3)$$

where c_i ; the per capita consumption of the i-th industry's output.

P ; the total value of the per capita private consumption;

$$\text{i.e. } P =: \sum_{i=1}^n c_i .$$

f_i ; the desirable proportion of the i-th consumption good to the total value of private consumption.

We can prove the following proposition. The proof is given in the mathematical appendix(M-V-2-3-2-A) and the definition of concave function is presented in every textbook of mathematical economics. See, for example, Takayama(54) and Nikaido(39) and (40).
Proposition V-2-3-2-A The index of the welfare standard (3) is a

continuous concave function.

Now we reconsider the step 2. Till now we have assumed that the desirable structure of future consumption does not change over time. But this assumption is rather strict and unrealistic. Porwit analyzed his model under the condition that the desirable structure of consumption can vary over time. So we also examine this condition and relax the step 2. But here we must examine the changeability of the desirable structure from a different viewpoint. Porwit assumed that the desirable structure of consumption varies from period to period, so he used a notation of $(f_{i,t})$ and did not use a notation (f_i) . In this Thesis, however, the desirable structure of consumption must be assumed to vary according to the level of the total value of consumption. It is useful here to remember the famous Engel's law. Engel asserted that the proportion of food expense decreases when the income of household increases. When the level of per capita consumption rises, usually the percentage of consumption of car and electric commodities increases and that of foodstuffs decreases. So we may suppose the following figure 6.

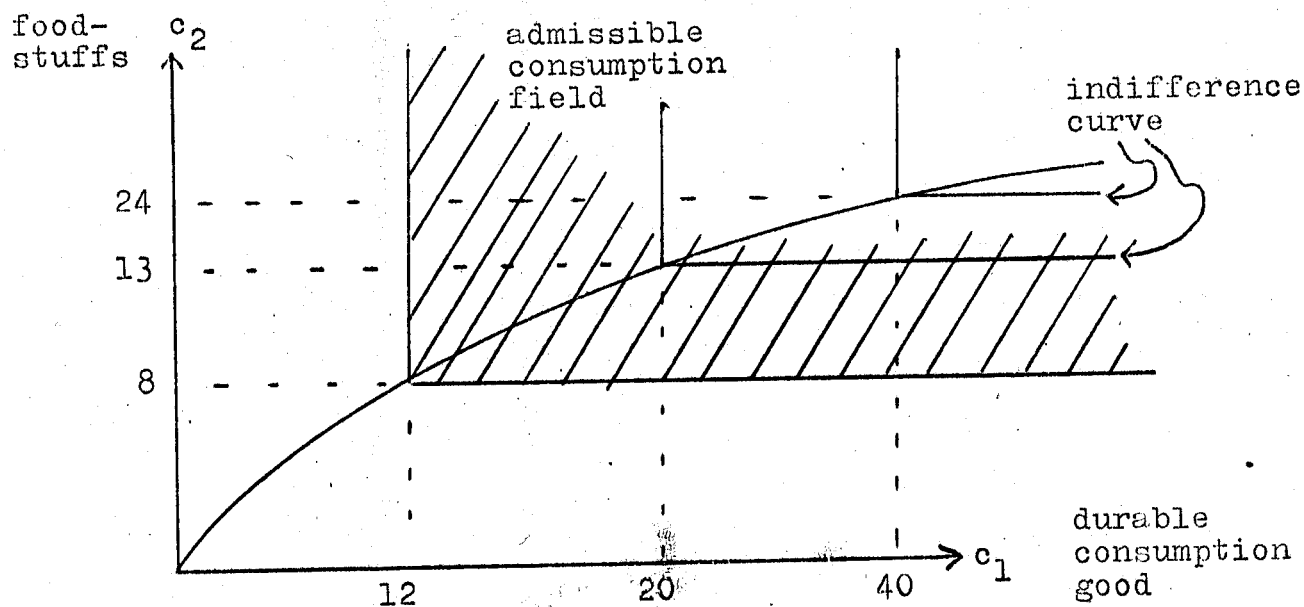


figure 6

In figure 6 the desirable structure of consumption has a nonlinear form. Theoretically saying, this nonlinearity has no difficulty in analyzing the model. But this nonlinearity will bring a difficulty, in practice, especially in numerical calculation. So the Author utilizes the linear approximation. The ground of the linear approximation is given as follows. Under the assumption of constant technology the technologically possible production set is apparently limited from above. Moreover the admissible consumption

field is limited from below. So the relevant consumption field is rather limited in the infinite economic planning.

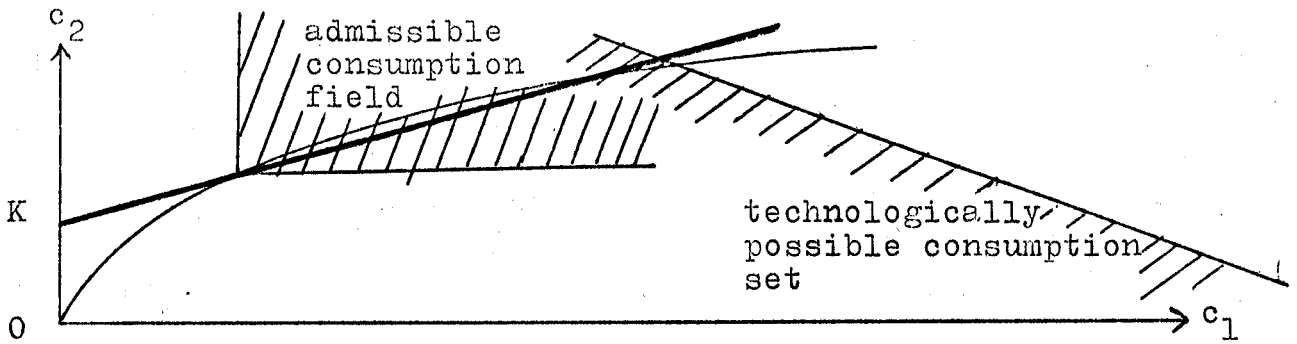


figure 7

The linear approximation brings the following index of the welfare standard.

$$\text{MAX } U = \min\left(\frac{c_1 - k_1}{f_1}, \frac{c_2 - k_2}{f_2}, \dots, \frac{c_n - k_n}{f_n}\right) + \sum_{i=1}^n k_i \quad \dots (4)$$

where k_i ; the constant term ($i=1,2,\dots,n$).

f_i ; the linearly approximated (marginal) desirable proportion of the i -th consumption good.

Proposition V-2-3-2-B Linearly approximated index of the welfare standard (4) is a continuous concave function.

Proof of this proposition is given in the mathematical appendix(M-V-2-3-2-B).

Finally the Author would like to examine a theoretical generalization. As already mentioned in chapter IV, it is sufficient that the objective function is a concave function. Therefore even if we suppose a more general indifference map such as in figure 8, there is no difficulty in order to build up our argument in the following parts of this Thesis.

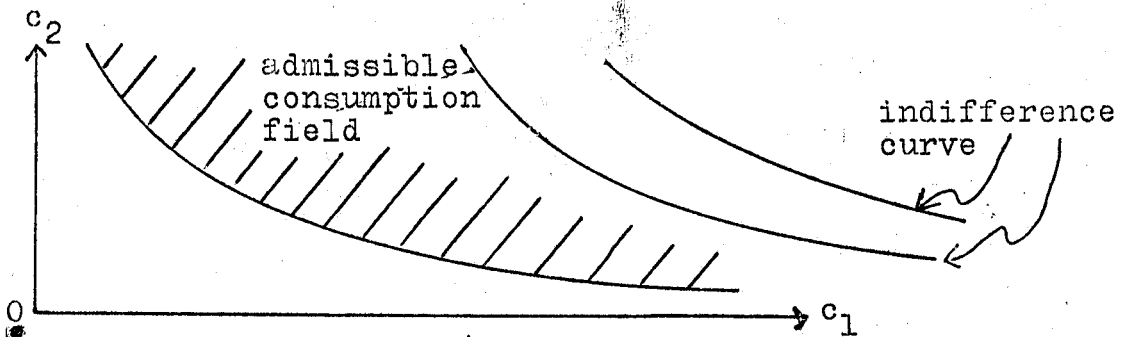


figure 8

The Author, however, does not adopt this approach. The first reason is that in economic planning the objective function should be set up objectively and not subjectively, as the Author already mentioned. It is very difficult or even impossible to settle the

numerical form of the generalized objective function such as in figure 8.

The second reason is a technological one. Though recently the capacity of computer has been developed, numerical calculation of the optimal program with the generalized objective function must be limited by the computational capacity. These are the reasons why the Author adopts the total value of consumption with the desirable structure as an objective function of the infinite economic planning.

V-2-3-3 Technological progress

The most troublesome problem in the long-term and the infinite economic planning is technological progress. Technological progress has occurred continuously in the history of humankind especially after the industrial revolution in 18-th century and perhaps will continue forever. Therefore we must answer such a question that with how much speed the technological progress of each industry will occur in future. This is a really difficult question. The first approximation is to neglect the technological progress. According to this approach of model building of the infinite economic planning the CPB forecasts the technology at a certain point of the future (e.g. 2001) and assumes that this technology does not change.

Step 3 The Central Planning Board forecasts the technology at a certain point of the future and assumes that this technology does not change from now to the infinite time horizon.

Needless to say, though this procedure contains a very strict assumption, this procedure brings a quite satisfactory result, since the role of the infinite economic planning is only to determine the optimal direction of economic development from now to 20 years hence. Therefore for the infinite economic planning it may be sufficient to suppose the technology 20 years hence (e.g. the year 2001).

Unfortunately it is plausible that some Readers do not agree with this approach, so the Author intends to explore a modification of the step 3.

Inclusion of the technological progress into the model brings a further difficulty to the discussion. The difficulty is as follows. Under the technological progress the output which is produced by one hour's labour, continuously increases every year. For the simplicity let us suppose that the rate of technological progress is 5% per year and the amount of labourer and population do not change. Also suppose that in 1981 one worker produces 100 thousand

zloty of product per year (i.e. 50 zloty per one hour). Then in 1982 one worker will produce 105 thousand zloty of product and in 2001 he will produce 265 thousand zloty of product (i.e. 132 zloty per one hour, if in 2001 the worker works 38 hours per week). Can the sum of flow of per capita future consumption be a rational criterion of economic welfare in such a case? Many Readers will agree with such a view that the additional 100 zloty's consumption for one population in 1981 is more valuable than the additional 100 zloty's consumption for one population in 2001, because the standard of living in 2001 is higher than in 1981.

This view is similar to the utility approach, i.e. the law of decreasing marginal utility. But even if we take leave of the utility approach, we can obtain the same conclusion. In order to produce the additional product of 100 zloty per capita in 1981 the nation must sacrifice 2 hours of leisure per capita. On the other hand in order to produce the additional product of 100 zloty per capita in 2001 the nation has to sacrifice only 45 minutes (since $100/132 = 0.75$ hour). Therefore 100 zloty's product in 1981 is more scarce and more valuable than 100 zloty's product in 2001.

Therefore we must conclude that if we include the technological progress into the model, then we must discount the flow of future consumption. The above-mentioned argument suggests that the CPB should discount the future consumption flow at the same rate of the technological progress. Now we can modify the step 3 into the following step.

Step 3' The Central Planning Board assumes that the technological progress occurs at a constant rate every year and also assumes that the rate of the technological progress is same in all industries. The CPB discounts the future consumption flow at the rate of the technological progress.

It seems better to repeat explicitly the assumptions which are contained in the step 3'.

Assumption V-2-3-3-A Technological progress occurs at the same rate in all industries.

Assumption V-2-3-3-B Technological progress occurs at a constant rate every year.

Assumption V-2-3-3-C The Central Planning Board discounts the future consumption flow at the same rate of the technological progress.

Though these assumptions may seem very strong, from theoretical reasons the Author has to introduce these assumptions into the

analysis. From the practical viewpoint the assumptions V-2-3-3-A and V-2-3-3-B can be justified, since the capacity of computer is limited. If we relax the assumptions V-2-3-3-A or V-2-3-3-B, then we can not execute any numerical calculation even in the long-term economic planning, to say nothing of the infinite economic planning.

Here the Reader naturally raises the question whether the rate of the technological progress is given exogeneously or not. Of course the rate of the technological progress depends on the capital accumulation and the amount of the capital accumulation is also a subject of economic planning. In order to answer to this question the Author will propose the variant method in the following parts of this Thesis. The CPB must calculate a series of optimal programs with a different rate of the technological progress and with a different amount of the capital accumulation.

For the time being, however, we will analyze the model supposing that the rate of the technological progress is exogeneously given.

V-2-3-4 International trade

Recently the international trade has been closely connected with the socialist economy. After the second world war many socialist countries were born in Eastern Europe and consequently the international cooperation between socialist countries became an important subject of economic planning. Furthermore the international trade between socialist countries and capitalist countries and also between socialist countries and developing countries has greatly increased after 1960's. This tendency will continue for the time being. Although the capitalist economy may not exist in far future, however, in order to determine at the present point the optimal direction of economic development the international trade with all countries should be investigated in the infinite economic planning. Therefore we must acknowledge the necessity of introducing the international trade into the infinite economic planning as well as in the long-term and the medium-term economic planning.

It is necessary here to take into account two restrictions; i.e. the limit of computer's capacity and the uncertainty of future economic environment. The limit of computer's capacity compels us simplification of the model. The second restriction endows a non-executive character to the model. Setting up the infinite economic planning the CPB forecasts in advance a most plausible future

international economic situation and assumes that this situation does not change over time. For example, the CPB predicts an international price in the year 2001 and assumes that this price prevails over time; i.e. from 1981 to the infinite time horizon.

Since the model of the infinite economic planning is a macro model, this future price should be a price index of each industry. The CPB also should predict in advance the future interest rate of international finance. In the infinite economic planning we neglect the difference among socialist, capitalist and developing economies, since the infinite economic planning should be simple.

Let us sum up the above-mentioned arguments.

Assumption V-2-3-4-A In the infinite economic planning there is only one foreign market; i.e. the CPB aggregates many foreign markets into one foreign market. Therefore there is no distinction among socialist, capitalist and developing economies.

Step 4 The CPB predicts a future price index of foreign trade of each industry. The CPB also predicts a future interest rate of international finance.

Assumption V-2-3-4-B In the infinite economic planning the CPB assumes that the future price index and the future interest rate of international finance do not change over time.

Now we examine the scale of the international trade. The scale of the domestic production is restricted by the technology and the quantity of labour power. Similarly the scale of export and import is also limited by various elements which are determined historically or politically. For example, the quantity of oil import is determined mainly by international negotiation. Export of meat or coal from Poland can not exceed e.g. 30% of the domestic production and so on. These limits are determined by the internal and the external situation. They are not a subject of economic planning in a narrow sense but a subject of the socio-political process. We can consider that the upper limit of export and import is an exogeneous prerequisite in the infinite economic planning.

$$\begin{aligned} 0 \leq E_{it} &\leq h_{it}^e & (i=1,2,3,\dots,n) \\ 0 \leq M_{it} &\leq h_{it}^m & (t=0,1,2,\dots,\infty) \end{aligned}$$

where E_{it} ; value of export of i -th industry's product in t -th period (measured by the domestic prices).

M_{it} ; value of import of products of i -th industry in t -th period (measured by domestic prices).

h_{it}^e ; the upper limit of export of i -th industry's product in t -th period.

h_{it}^m ; the upper limit of import of products of i -th industry in t -th period.

These h_{it}^e and h_{it}^m are determined internally (for example, in the case of meat export) or externally (for example, in the case of oil import). Now we assume that these upper limit of international trade increases in the same rate of growth of the production ability. The ability of production is determined by the number of workers and the labour productivity. Also we assume that the number of workers and the labour productivity increases in a constant rate. Under these suppositions we can obtain the following assumption.

Assumption V-2-3-4-C

$$0 \leq E_{it} \leq (1+q)^t (1+s)^t h_{i0}^e$$

$$0 \leq M_{it} \leq (1+q)^t (1+s)^t h_{i0}^m$$

where q ; the annual growth rate of labour force.

s ; the annual growth rate of labour productivity.

h_{i0}^e and h_{i0}^m ; the upper limit of export and import of products of i -th industry in the beginning period.

This assumption may cause objections by some Readers. Needless to say, the assumption V-2-3-4-C does not reckon with the structural change of the limit of the international trade. Empirically we have observed a drastic structural change of the international trade. Although in the infinite time horizon the structure of the international trade will change, it is quite difficult to forecast exactly the future change of the trade structure. In the range of the infinite economic planning the CPB can only forecast a plausible limit of the international trade of far future. So we introduce the following step.

Step 5 The CPB forecasts the future limit of the international trade 20 years hence. That is, the CPB forecasts $h_{i,2001}^e$. Then the h_{i0}^e is defined as follows.

$$h_{i,1981}^e = \frac{1}{(1+q)^{20} (1+s)^{20}} h_{i,2001}^e$$

Therefore the infinite economic planning can not manipulate the structural change of the limit of the international trade within the range of 20 years. In the framework of the infinite economic planning apparently the $h_{i,1981}^e$ can not coincide with the existing value of the export limit in 1981. But this is not a serious difficulty, since the main role of the infinite economic planning is to determine the optimal direction of economic development towards the far future, i.e. towards the year 2001. So the future

structure of the limit of international trade becomes the most substantial element. The Author considers that the assumption V-2-3-4-C and the step 5 are justifiable as a framework of the infinite economic planning. Treatment of the structural change of the international trade belongs to the subject of the long-term and the medium-term economic planning.

Usually the export activity and the import activity require a certain labour input and a certain resource input, yet these resources are taken into account in the determination of price index. That is,

$$\begin{aligned} (\text{export price}) &= (\text{international price}) - (\text{transportation cost}) \\ (\text{import price}) &= (\text{international price}) + (\text{transportation cost}) \end{aligned}$$

Step 6 Price index of export is calculated in accordance with the f.o.b. prices. Price index of import is calculated in accordance with the c.i.f. prices.

The above step 6 justifies the following assumption.

Assumption V-2-3-4-D The export activity and the import activity do not require any input; i.e. these activities are executed by foreign enterprises and the transportation cost is reflected in the price index.

The assumption V-2-3-4-B can be transformed mathematically into the following form.

Assumption V-2-3-4-B'

$$d_{it}^e = d_{i0}^e \equiv d_i^e = \text{constant} \quad (t=0,1,2,\dots,\infty)$$

$$d_{it}^m = d_{i0}^m \equiv d_i^m = \text{constant} \quad (t=0,1,2,\dots,\infty)$$

$$r_t = r_0 \equiv r = \text{constant} \quad (t=0,1,2,\dots,\infty)$$

$$d_i^e \leq d_i^m \quad (i=1,2,3,\dots,n)$$

where d_i^e ; the export price index of i-th industry's products;
(e.g. 2.1 U.S.cent for one zloty).

d_i^m ; the import price index of products of i-th industry;
(e.g. 2.3 U.S.cent for one zloty).

r ; the interest rate of foreign currency debt.

Finally we must investigate a problem of interest payment of foreign currency debt. Apparently the following relation should be maintained every year. In t-th period the government must make the payment of the last year's (i.e. (t-1)-th period's) import by export activity or by new foreign currency debt. Of course the government must pay the interest of the last year's foreign currency debt.

$$\sum_{i=1}^n d_i^e E_{it} + Z_t \geq (1+r) \sum_{i=1}^n d_i^m M_{i,t-1} + (1+r) Z_{t-1}$$

where Z_t ; the level of foreign currency debt in t -th period (measured by dollar).

Usually the socialist economy has the upper limit of foreign currency debt. We already introduced the assumption V-2-3-4-A, so every foreign currency debt is measured by a representative foreign currency, e.g. dollar.

$$0 \leq Z_t \leq H_t \quad (t=0,1,2,\dots, \infty)$$

where H_t ; the upper limit of foreign currency debt in t -th period (measured by dollar).

Here it seems necessary to explain the definition of foreign currency debt in the infinite economic planning. In this model of the infinite economic planning the foreign currency debt is assumed to be "one year debt". Therefore the government must pay back her foreign currency debt (Z_{t-1}) with interest in 31-th of December every year and the government can borrow simultaneously new debt (Z_t) for the next year in 31-th of December. Of course usually the foreign currency debt is contracted with 6-8 years' borrowing period. So this model can not manipulate the servicing problem. In reality the foreign currency payment is prescribed by the past economic policy of the government and its amount changes unproportionally year to year. Therefore in the medium-term planning the servicing problem exerts a very strong influence to the international trade. So the Author considers that in the medium-term planning the original comprehensive approach is more powerful, where the net trade deficit is restricted by the upper limit which is determined by the program of foreign currency servicing. On the other hand in the infinite economic planning the above-mentioned definition seems to be admissible as the first approximation, since in the long run it is quite difficult and even is nonsense to set up the servicing program of far future (e.g. 2001). We can interpret the "one year debt" as the average value of the future foreign currency debt. Now we introduce the following additional assumption.

Assumption V-2-3-4-E The upper limit of foreign currency debt increases in the same rate of growth of production ability.

$$H_t = (1+s)(1+q)H_{t-1} \quad \text{So,} \quad 0 \leq Z_t \leq (1+s)^t(1+q)^t H_0$$

Here the Reader should not confuse the H_0 in the infinite planning with the H_0 in the long-term and the medium-term planning.

In the infinite economic planning the H_0 is determined as follows. Firstly the CPB predicts the plausible upper limit of foreign debt in the last period of the long-term economic planning; e.g. H_{2001} . Here the H_{2001} should be selected taking into account the existing foreign currency debt and the long-run foreign currency policy of the government. Therefore the choice of H_{2001} also belongs to the socio-political process. Next we obtain H_0 as in the following way.

$H_{1981} = (1+s)^{-20} (1+q)^{-20} H_{2001}$. Therefore the level of H_{1981} is different from the existing limit of foreign currency debt in the year 1981. So we can say that the H_t in the infinite economic planning is a quite artificial conception.

This assumption V-2-3-4-E seems rather strange. Needless to say, in the short and the medium-term economic planning the limit of foreign debt is strictly prescribed. On the other hand in the long run it is very difficult to predict accurately the limit of foreign debt in the far future (e.g. H_{2001}). The CPB has to be satisfied to predict a plausible tendency of this limit. It is quite acceptable that the larger scale of economy brings the greater limit of foreign currency debt. The scale of economy is determined by the labour productivity and the quantity of labour force. Therefore in the framework of the infinite planning the assumption V-2-3-4-E can be justified. The Author will prove in the following parts of this Thesis that the optimal infinite plan has a very small amount of foreign debt (mathematically saying, converges to zero) in far future, if $(1+r) > (1+s)(1+q)$.

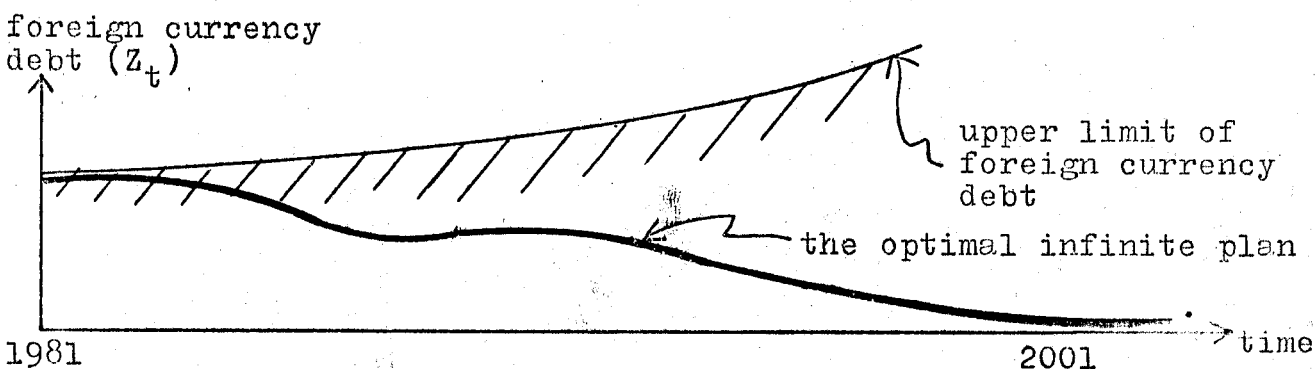


figure 9

It is better to remind us of the fact that the infinite economic plan does not possess any executive character. So the above-mentioned assumption can be regarded as acceptable, even though the optimal infinite economic plan brings an unrealistic result of foreign debt in the near future. One of aims of introducing the foreign currency debt into the infinite economic planning is to obtain the final condition of the long-term economic planning, i.e.

to obtain the level of Z_{2001} .

In our model the government regards the limit of foreign debt as only a constraint condition. The government does not regard the foreign debt as a negative (undesirable) element in itself. Foreign currency debt is desirable, since it brings a wider possibility of investment. On the other hand the foreign currency debt is undesirable, since the economy must suffer from the interest payment. So if these two viewpoints are taken into consideration in the economic planning, then the CPB should not regard the foreign currency debt as undesirable, because the upper limit of foreign currency debt is already incorporated in the model as an exogeneous constraint. Therefore we introduce the following assumption.

Assumption V-2-3-4-F The level of foreign currency debt (Z_t) does not enter the objective function of the economic planning.

V-2-3-5 The model of the infinite economic planning

The basis of the infinite economic planning is the dynamic Leontief model, which was already surveyed in IV-2 of this Thesis.

$$A X_t + E_t + C_t + B(X_t - X_{t-1}) \leq X_{t-1} + M_{t-1} \dots\dots(1)$$

or

$$\begin{pmatrix} \text{current} \\ \text{input of} \\ t\text{-th} \\ \text{period} \end{pmatrix} + \begin{pmatrix} \text{export} \\ \text{of} \\ t\text{-th} \\ \text{period} \end{pmatrix} + \begin{pmatrix} \text{consump-} \\ \text{tion of} \\ t\text{-th} \\ \text{period} \end{pmatrix} + \begin{pmatrix} \text{invest-} \\ \text{ment of} \\ t\text{-th} \\ \text{period} \end{pmatrix} \leq \begin{pmatrix} \text{output} \\ \text{of} \\ (t-1)\text{-th} \\ \text{period} \end{pmatrix} + \begin{pmatrix} \text{import} \\ \text{of} \\ (t-1)\text{-th} \\ \text{period} \end{pmatrix}$$

where

X_t ; n-dimensional column vector of gross output in t-th period.

$$X'_t = (X_{1t}, X_{2t}, X_{3t}, \dots, X_{nt})$$

X_{it} ; gross output of i-th industry in t-th period which is measured by the domestic value term, i.e. zloty.

A; (n,n) matrix of the current input coefficient. $A = (a_{ij})$.

a_{ij} ; necessary input of i-th industry's products in order to produce one unit of j-th industry's products.

E_t ; n-dimensional column vector of export in t-th period.

$$E'_t = (E_{1t}, E_{2t}, E_{3t}, \dots, E_{nt})$$

E_{it} ; export of i-th industry's products in t-th period, which is measured by the domestic value term, i.e. zloty.

C_t ; n-dimensional column vector of the final consumption in t-th period. $C'_t = (C_{1t}, C_{2t}, C_{3t}, \dots, C_{nt})$

C_{it} ; final consumption of i-th industry's products in t-th period, which is measured by the domestic value term, i.e. zloty.

B; (n,n) matrix of the fixed asset coefficient. $B = (b_{ij})$.

b_{ij} ; necessary capital investment of i-th industry's products in

order to increase the gross output of j -th industry's products by one unit.

M_t ; n -dimensional column vector of import in t -th period.

$$M_t' = (M_{1t}, M_{2t}, M_{3t}, \dots, M_{nt})$$

M_{it} ; imprt of i -th industry's products in t -th period, which is measured by the domestic value term, i.e. zloty.

Here it is necessary to pay attention to the following points.

(i) The model assumes that the capital investment is frozen by one period. So $(X_t - X_{t-1}) = (\text{increment of gross output})$ and

$$B(X_t - X_{t-1}) = \left(\begin{array}{l} \text{capital investment in order to produce the} \\ \text{additional increment of output } (X_t - X_{t-1}) \end{array} \right)$$

The capital investment $B(X_t - X_{t-1})$ is executed in the former period and therefore is supplied by the output of the former period (X_{t-1}) or the import of the former period (M_{t-1}) .

It must be noted that one period of this model should not necessarily one year. Apparently from the relation(1), the one period should be the average period of the gestation period of investment and also the average circulation time of commodities. This is a rather strong assumption, since the average circulation time of commodities may be three or six months and the average gestation period is quite longer than six months. A certain investment must be frozen through several years, though some small investments can be completed in several months. We may assume that the average gestation period is one year. So there is a divergence between these two periods. But in the framework of the long-term and the infinite economic planning we can not incorporate such a detailed information into the model. Some simplifications are inevitable, so the Author considers that the above-mentioned formulation is acceptable. Determination of the length of one period depends on the characteristics of the economy. As already mentioned in chapter III, the Japanese Agency of Economic Planning defined one period to be six months.

(ii) The current inputs (AX_t) , the export (E_t) and the final consumption (C_t) are supplied by the gross output of the former period (X_{t-1}) or by the import of the former period (M_{t-1}) .

(iii) The capital amortization is not explicitly taken into account. But we can suppose that the current input coefficient (A) contains the capital amortization.

(iv) The current input coefficient (A) and the capital investment coefficient (B) do not change over time. But this statement does not deny the technological progress. The technological progress is reflected in the following labour supply condition.

$$\sum_{i=1}^n \frac{v_{io}}{(1+s)^t} X_{it} \leq L_t = (1+q)^t L_0 \quad \dots\dots\dots(2)$$

where v_{io} ; the amount of labour input in order to produce one unit of i -th industry's products in o -th period.

s ; the annual rate of the technological progress. (s) is same in all industries and all planning periods.

L_t ; the amount of labour supply in t -th period.

q ; the annual growth rate of labour supply.

(v) The foreign currency balance is as follows.

$$\sum_{i=1}^n d_i^e E_{i,t} + Z_t \geq (1+r) \sum_{i=1}^n d_i^m M_{i,t-1} + (1+r) Z_{t-1} \quad \dots\dots\dots(3)$$

where $E_{i,t}$, $M_{i,t}$, d_i^e , d_i^m , Z_t and r have been already defined in the precedent subsection V-2-3-4.

The limit of the international trade is as follows

$$\left. \begin{aligned} 0 \leq E_{i,t} &\leq (1+s)^t (1+q)^t h_{io}^e \\ 0 \leq M_{i,t} &\leq (1+s)^t (1+q)^t h_{io}^m \end{aligned} \right\} \dots\dots\dots(4)$$

Also we have the following limit of the foreign currency debt.

$$0 \leq Z_t \leq (1+s)^t (1+q)^t H_0 \quad \dots\dots\dots(5)$$

Now we have presented the whole framework of the infinite economic planning. For deducing important conclusions the mathematical analysis is inevitable, though the mathematical analysis requires abstract notations and its calculation is quite complicated. Therefore the Author presents the whole part of mathematical calculation in the mathematical appendix(M-V-2-3-5). The Reader who demands mathematical rigorousness should read through the mathematical appendix.

As already mentioned in the definition of the objective of planning in V-2-3-2 and in the assumption V-2-3-3-C, the CPB maximizes the sum of the per capita consumption, which is discounted at the same rate as the technological progress under the condition of the desirable structure of consumption. Now we introduce the following assumption.

Assumption V-2-3-5-A Population increases at the same rate as the labour force.

This assumption means that the ratio between population and labour force is fixed. Under this assumption there is no difference between the conclusion from the maximization of the per capita consumption and the conclusion from the maximization of the per worker consumption, which is discounted at the same rate as the

technological progress.

Finally we present the whole system of the infinite economic planning.

The infinite economic planning

$$\text{MAX} \quad \sum_{t=0}^{\infty} \left(\min_i \left(\frac{c_{it} - k_i}{f_i} \right) + \sum_{i=1}^n k_i \right) \quad \dots\dots\dots(7)$$

subject to the relations (1),(2),(3),(4) and (5).

where $c_{it} = C_{it} / (1+s)^t(1+q)^t$ and $k_i = K_i / (1+s)^t(1+q)^t$.
 K_i ; the total amount of the collective consumption.

Interpretation of the objective function(7) was already given in the explanation of the equation(2) of V-2-3-2. Also see the alternative formulation of the objective function(2) of V-2-3-2.

A rigorous formulation of the infinite economic planning is presented in the mathematical appendix to this subsection. See the relations (7'),(8),(9),(10) and (11) of M-V-2-3-5.

V-2-3-6 Stationary optimal program and its uniqueness.

The definition of the optimal program of the infinite planning is rather complicated from a mathematical viewpoint. So we analyze the model without the definition of the optimal program. Reader, who demand the rigorous definition of the optimal program in the infinite planning, can refer, for example, the textbook by Takayama(54).⁵⁾

In the following parts of this Thesis the Author will prove that this model has an optimal program and this optimal program converges to a stationary state in far future. We call this stationary state the optimal stationary program.

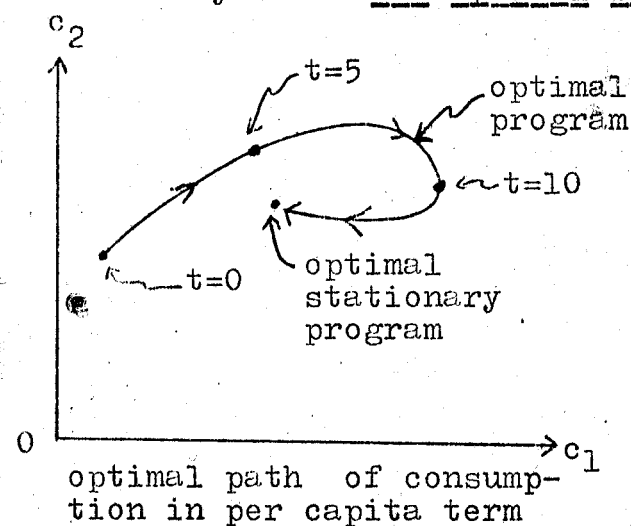


figure 10

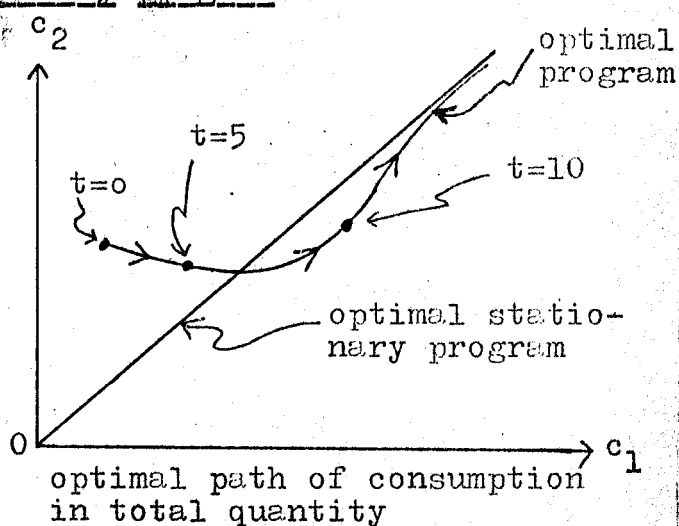


figure 11

This convergency is illustrated in figure 10 and figure 11 as a two-dimensional case. Here it is necessary to notice that $C_{it} = (1+s)^t(1+q)^t c_{it}$.

Now we define the optimal stationary program. In the optimal stationary program the value of each variable does not change over time, when we measure the variables by per capita term. Precisely speaking, if we discount the total quantity of each variable by $(1+s)^t(1+q)^t$, then the value of these discounted variables of the optimal stationary program does not change over time. Therefore we can calculate the optimal stationary program using the discounted variables which do not possess no index of period.

So we can calculate the stationary program, whose variables do not depend on period in the same framework of the problem (1'), (2'), (3'), (4') and (5') of the mathematical appendix (M-V-2-3-5).

Optimal stationary program

$$\begin{aligned} \text{MAX } F(c) &= \min_i \left(\frac{c_i - k_i}{f_i} \right) + \sum_{i=1}^n k_i \\ \text{s.t. } A x + e + c + B \left(x - \frac{x}{(1+s)(1+q)} \right) &\leq \frac{x}{(1+s)(1+q)} + \frac{m}{(1+s)(1+q)} \quad \dots\dots\dots(12) \\ \sum_{i=1}^n v_{io} x_i &\leq L_0 \quad \dots\dots\dots(13) \\ \sum_{i=1}^n d_i^e e_i + z &\geq (1+r) \sum_{i=1}^n \frac{d_i^m m_i}{(1+s)(1+q)} + \frac{(1+r)z}{(1+s)(1+q)} \quad \dots\dots\dots(14) \\ 0 \leq e_i &\leq h_{io}^e \\ 0 \leq m_i &\leq h_{io}^m \\ 0 \leq z &\leq H_0 \quad \dots\dots\dots(15) \end{aligned}$$

where; c, x, z, e, m are the discounted version of C, X, Z, E, M , respectively.

In the mathematical appendix to this subsection(M-V-2-3-6) the Author introduces some mathematical assumptions and proves some important properties. So he presents here only the essence of the argument. Firstly the existence of the optimal stationary program is proved. See the property V-2-3-6-A of the mathematical appendix(M-V-2-3-6). Next we introduce the following assumption.

Assumption V-2-3-6-B $(1+r) > (1+s)(1+q)$. That is,
 (international bank rate) > (rate of population growth)(rate of technological progress).

This assumption is very plausible in many socialist countries

in 1960's and 1970's. Rate of population growth was from 0% to 2%. Growth rate of productivity was more or less 5%, needless to say, depending on the economy of each country. On the other hand the international bank rate is usually from 8% to more than 10%. Under this assumption we can prove that $z^*=0$, that is, the foreign currency debt of the stationary optimal program should be zero.

Next the Author proves the uniqueness of the optimal stationary program. See the property V-2-3-6-C of the mathematical appendix to this subsection(M-V-2-3-6).

In the mathematical appendix the Author introduces the following assumption; relative scarceness of the domestic products is different from the ratio of the international trade term. In other words, the relative advantage of the domestic production is always different from the relative advantage of the international market. If the relative advantage of the domestic production is same as the relative advantage of the international market, then there exists no incentive for foreign trade. Therefore this assumption seems plausible. Rigorous form of this assumption is presented in the mathematical appendix. See the assumption V-2-3-6-C of M-V-2-3-6.

In the optimal stationary program the limit of export and import plays a very important role. That is, the upper limit or the lower limit (i.e.zero) is always satisfied with strict equality.

Property V-2-3-6-E In the optimal stationary program the level of export and import is determined by the upper limit of international trade. Needless to say, if $e_1^* = h_{10}^e$, then $m_1^* = 0$. If $m_1^* = h_{10}^m$, then $e_1^* = 0$. Here (*) denotes the solution of the optimal stationary program.

This property does not mean that export and import in the optimal stationary program is determined in advance. Only after solving the optimal stationary problem we can decide which product should be exported or imported. It must be also noticed that the above-mentioned property concerns not the optimal program but the optimal stationary program. So the property V-2-3-6-E depicts the ideal state of the economy in far future. The real optimal program converges to this ideal state. Therefore the foreign trade of the optimal program is not necessarily determined by the upper limit. The optimal program apparently admits the possibility of structural change of foreign trade. Summing up, every optimal program converges to the optimal stationary program where the foreign trade is determined by the upper limit. In this sense the expected future limit of the international trade exerts a substantial influence.

V-2-3-7 Convergency property of the optimal infinite program

In the precedent subsection we have argued the uniqueness of the optimal stationary program. Of course, the optimal stationary program is not the optimal program of the infinite economic planning. Here the Author presents once more the objective function of the infinite planning, though it was already presented in V-2-3-5.

Here the new notation (k) is defined as follows; $k = \sum_{i=1}^n k_i$.

$$\text{MAX} \sum_{t=0}^{\infty} F(c_t) = \sum_{t=0}^{\infty} \left(\min \left(\frac{c_{it} - k_i}{f_i} \right) + k \right) \dots\dots\dots(1)$$

In V-2-3-6 the Author did not present the definition of the optimal planning of the infinite economic planning. So now the Author presents the definition. The optimal program of the infinite economic planning is defined as follows.

Definition of optimal program of the infinite economic planning

An attainable program $(c_t^{**}, s_t^{**}, x_t^{**}, e_t^{**}, m_t^{**}, z_t^{**})$ is called to be the optimal program of the infinite economic planning, if there exists $\underline{T}$ such that, for any attainable program $(c_t, s_t, x_t, e_t, m_t, z_t)$

$$\sum_{t=0}^T F(c_t^{**}) \geq \sum_{t=0}^T F(c_t) \quad \text{for all } T \geq \underline{T}.$$

where (s) denotes the amount of foreign currency which is endowed to the developing countries or the International Red Cross. (s) can be interpreted as a slack variable in the constraint of foreign currency balance.

That is, the optimal program dominates every other program in sufficiently far future. Needless to say,

$$\sum_{t=0}^{\infty} F(c_t^{**}) \geq \sum_{t=0}^{\infty} F(c_t) \quad , \text{ if } \sum_{t=0}^{\infty} F(c_t^{**}) \text{ exists. Remark; Since}$$

the time horizon is infinite, $\sum_{t=0}^{\infty} F(c_t^{**})$ may be infinite.

Now we prove the following theorem.

Theorem V-2-3-7

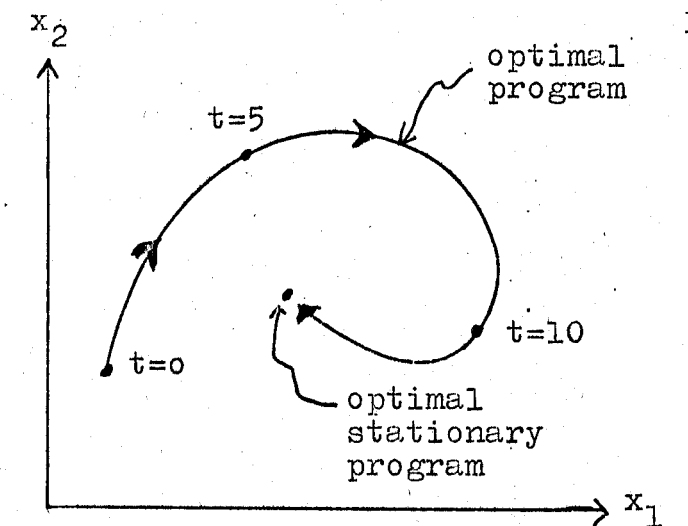
The optimal program $(c_t^{**}, s_t^{**}, x_t^{**}, e_t^{**}, m_t^{**}, z_t^{**})$ converges to the optimal stationary program $(c^*, s^*, x^*, e^*, m^*, z^*)$, when $t \rightarrow \infty$.

Proof. See the mathematical appendix(M-V-2-3-7).

This theorem assures that $c_t^{**} \rightarrow c^*$, $x_t^{**} \rightarrow x^*$, $e_t^{**} \rightarrow e^*$, $m_t^{**} \rightarrow m^*$ and $z_t^{**} \rightarrow z^*$. As already mentioned in the

mathematical appendix(M-V-2-3-6), $s^* = 0$ and $s_t^{**} = 0$; ($t=0,1,2,\dots,\infty$).

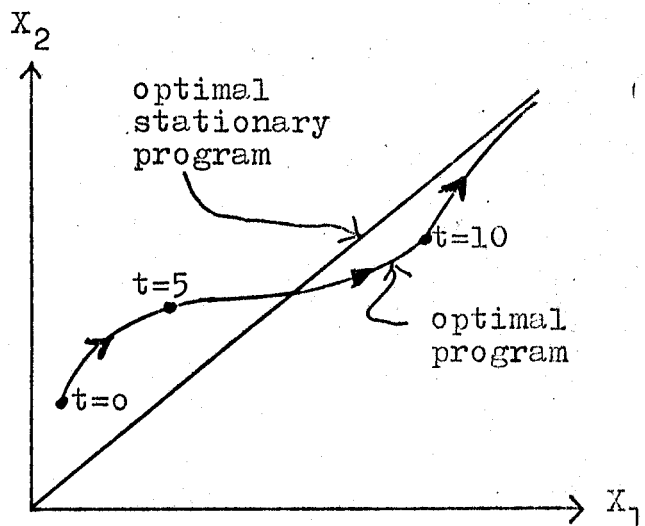
In figure 10 and 11 the Author illustrated this theorem in the case of consumption. Now the Author illustrates once more this theorem in the case of production. In the case of export, import and foreign currency debt the situation is same and therefore the illustration of these cases is omitted.



optimal program path of production which is measured by the "per capita" term. That is,

$$x_{it} = X_{it} / (1+s)^t (1+q)^t$$

figure 15



optimal program path of production. X_{it} is measured by the total amount of production.

figure 16

V-2-3-8 Determination of the final state of the long-term economic planning

Although the final aim of the infinite economic planning is to determine the optimal direction of the economic development, the direct aim of the infinite economic planning exists in the following two determinations; i.e. the determination of the final state of the long-term economic planning and the determination of the accounting prices in the after-planning periods of the long-term economic planning. This subsection concerns the first direct aim of the infinite economic planning.

When we examine the infinite economic planning, always we must keep in our mind the fact that the capacity of computer is limited. The Author has to say that practically it is impossible to calculate the optimal program of the infinite economic planning, since its time horizon is infinite and therefore the number of variables is also infinite. So reasonably a doubt arises in the meaning of the infinite economic planning; i.e. why we must investigate the infinite economic planning?

In this point the Author considers that the infinite economic planning is necessary, because the capacity of computer is limited. This statement is quite paradoxical, so the reason of this statement should be explained.

As already proved, the optimal program converges to the optimal stationary program in far future. Although the speed of convergence depends on the model, we may consider that 20 years hence the optimal program approaches very near to the optimal stationary program. Therefore the Author urges that we may substitute the optimal stationary program for the optimal program. This proposal is justified from the practical point of view. Firstly the infinite economic planning is investigated in order to determine the optimal direction of economic development in far future, so the infinite economic planning does not possess any executive character. The optimal stationary program offers the optimal direction of economic development approximately and we may be satisfied with this approximated direction of economic development. Secondly the optimal stationary program has a very simple structure, since its variables have not distinction among periods. The Reader should recall once more the constraints of the optimal stationary program, which is expressed by (12),(13),(14),(15) and (16) of V-2-3-6. Therefore we can easily calculate the optimal stationary program by using a medium or an even small-sized computer.

So the CPB determines the final state of the long-term economic planning by the optimal stationary program of the infinite economic planning. That is, the level of capital stock and the level of foreign currency debt in the year 2001 are determined by the optimal stationary program.

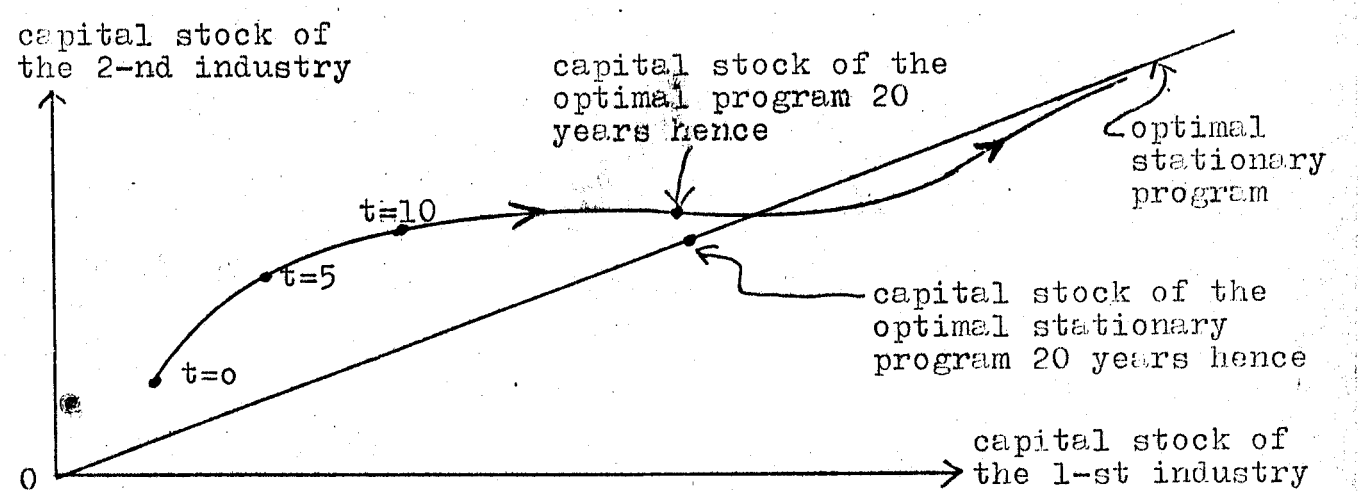


figure 17

currency debt as the final state of the long-term economic planning is determined by the optimal stationary program of the infinite economic planning.

So in the end of the year 2001 the society must leave Bx^* for the next year. Here Bx^* is measured by "per capita" term, so we must transform it into the total amount term.

$BX_{20}^* = (1+s)^{20}(1+q)^{20} Bx^*$; where BX_{20}^* denotes the total amount of the capital stock in the end of the year 2001. In other words the society should execute the investment in the year 2001 by the amount of $(1+s)^{20}(1+q)^{20} B (1 - \frac{1}{(1+s)(1+q)}) x^*$. This calculation can be easily understood by the formulation of the mathematical appendix (M-V-2-3-5).

Now we examine the level of foreign currency debt in the final state of the long-term economic planning. As already proved, we can present the following criterion.

Criterion for the final level of foreign currency debt

- (i) If $(1+r) > (1+s)(1+q)$; i.e.
if $\left(\begin{smallmatrix} \text{international} \\ \text{bank rate} \end{smallmatrix} \right) > \left(\begin{smallmatrix} \text{rate of technolo-} \\ \text{gical progress} \end{smallmatrix} \right) + \left(\begin{smallmatrix} \text{rate of popula-} \\ \text{tion growth} \end{smallmatrix} \right)$
 , then $Z_{20} = 0$; i.e. the foreign currency debt should be zero in the last year of the long-term economic planning.
- (ii) If $(1+r) < (1+s)(1+q)$, then $Z_{20} = H_{20}$; that is, the foreign currency debt in the last year should be at the level of the upper limit of debt, which is determined by the political decision in advance of the planning.

V-2-3-9 Accounting prices in the after-planning periods

The second direct aim of the infinite economic planning is to determine the accounting prices in the after-planning periods of the long-term economic planning. Needless to say, the long-term economic planning can not articulate the accounting prices in the after-planning periods, e.g. in the year 2002 or 2003.

Operation of some kinds of big project lasts more than 20 years. For example, water electric generation plant can operate more than 50 years. Therefore for the project evaluation of some industries the accounting prices in the after-planning periods are necessary.

The optimal stationary program can provide the accounting prices in the after-planning periods. The accounting prices of the infinite economic planning can be determined by the dual price of

the optimal stationary program. Mathematical calculation of them is presented in the mathematical appendix.

Now the Author presents the following two important properties; (V-2-3-9-C) and (V-2-3-9-D), mathematical proof of which is given in the mathematical appendix(M-V-2-3-9).

Property V-2-3-9-C If $F(c) > F(c^*)$, then

$$p^* \left(\frac{1}{w} (I_n + B)x + \frac{1}{w} m - (A+B)x - e - c \right) < 0 \dots\dots\dots(1)$$

Property V-2-3-9-D

$$p^* \left(\frac{1}{w} (I_n + B)x^* + \frac{1}{w} m^* - (A+B)x^* - e^* - c^* \right) = 0 \dots\dots(2)$$

where p^* ; the accounting price vector which corresponds to the stationary optimal program.

w ; $w = (1+s)(1+q)$.

I_n ; n -dimensional unit matrix.

x^*, m^*, e^*, c^* ; the solutions of the optimal stationary program.

x, m, e, c ; non-optimal program.

Now the Author explains the properties V-2-3-9-C and D verbally. The optimal stationary program brings a stationary situation where all "per capita" variables do not change. As already mentioned, the output can not be utilized till the next period. Therefore (x) should be interpreted as the output of the precedent period. The output (x) is measured by per capita term. Since the total amount of consumption is discounted by the rate (w) , for the period concerned the output (x) of the precedent period is interpreted as the bequest of the amount $\left(\frac{x}{w}\right)$ from the precedent period. (Bx) denotes the capital stock. (Bx) is also discounted by (w) . (m) denotes the import of the precedent year and also should be discounted by (w) .

Now we define the following new notations; a, b and d .

$$a =: \frac{1}{w} (I_n + B)x + \frac{1}{w} m$$

(a) can be interpreted as the output(bequest) of the precedent year.

$$b =: (A+B)x + e$$

(Ax) denotes the current input. (Bx) is the capital stock which must be bequeathed to the next year. Therefore (b) can be interpreted as the input(lutlay) of the year concerned. In other words the vector (a) is the stock of products which the government can freely dispose in the beginning of the year concerned. The vector (b) is the expense which the government must pay in the year concerned. Therefore $(a-b)$ is the surplus which the nation can consume.

$$d =: a - b$$

(d) can be interpreted as the net output or as the surplus.

Now the relations (1) and (2) can be transformed as follows.

(i) If $F(c) > F(c^*)$, then $p^*(d - c) < 0$ (3)

(ii) $p^*(d^* - c^*) = 0$ and $d^* - c^* \geq 0$ (4)

We can illustrate (3) and (4) in figure 18. Here we can easily prove that the attainable set of the net output is a convex set. The figure 18 suggests that there exists a hyper-plane ($p^*c=K$) which separates the attainable set of the net output from the set $\{c \mid F(c) > F(c^*)\}$.

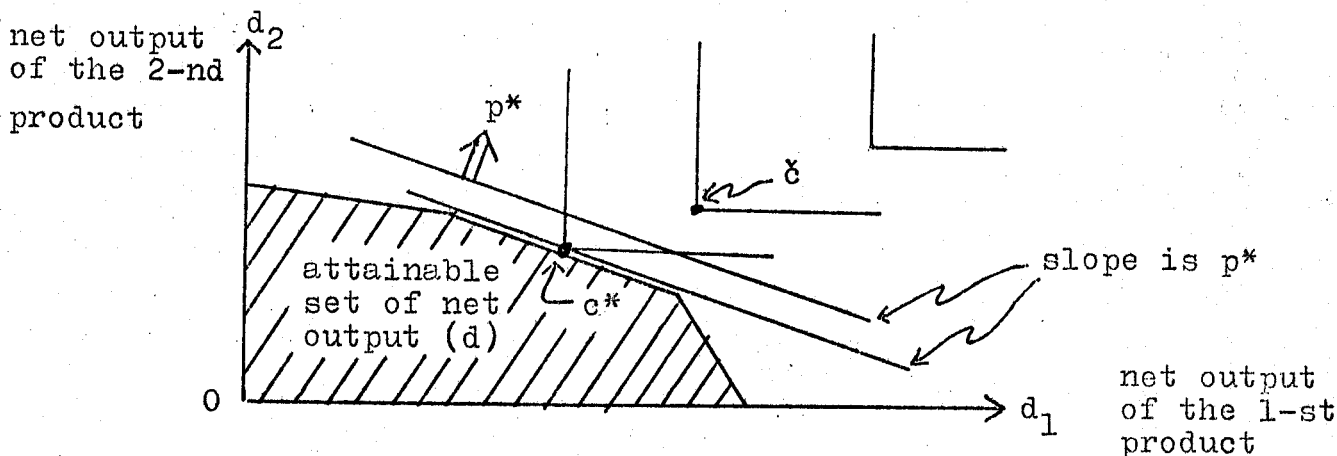


figure 18

So the price vector (p^*) associated with the optimal stationary program has the same property as the competitive price vector.

Step V-2-3-9-A The CPB adopts the price vector (p^*) as the accounting price vector in the after-planning periods of the long-term economic planning.

The figure 18 gives us a very important suggestion about the criterion of project evaluation. (c^*) maximizes the accounting value (p^*c) among (c)'s which belong to the attainable set of the net output. Therefore maximization of accounting value may be considered as a criterion of decision making of project evaluation. We examine this problem thoroughly in the following chapter VI.

Here we must pay attention to the definition of (c). (c) is measured by per capita term, so (p^*) is a price vector of one unit of per capita consumption of (c). One unit of (c) is the unit of per capita consumption. One unit of (c) in the year 2001 is equal to $(w^{20}c_{1981})$. Now the Author gives a numerical example. Suppose that $w=1.05$ and the population in 1981 is 40 million. In order to increase the consumption of bread (c_b) from 100 kg to 101 kg in 2001 the additional production of 106 million kg is necessary, since $(1.05)^{20} \times 40 = 2.65 \times 40 = 106$. On the

other hand in 1981 the additional production is only 40 million kg. Therefore the price (p_b^*) in 2001 is the price of 106 million kg of bread in 2001 and the price (p_b^*) in 1981 is the price of 40 million kg of bread in 1981. In other words the price of one million kg's bread in 1981 is $p_b^*/40$ and the price of one million kg's bread in 2001 is $p_b^*/106$. Now we choose arbitrarily such an unit of bread that $p_b^*(1981) = p_b^*$, where $p_b^*(1981)$ is the price of one million kg of bread in 1981, then

$$p_b^*(2001) = \frac{p_b^*/106}{p_b^*/40} p_b^*(1981) = \frac{p_b^*(1981)}{2.65} = \frac{p_b^*(1981)}{w^{20}}$$

Step V-2-3-9-B The price vector of one unit of product in t -period is $p^*/(1+s)^t(1+q)^t$. That is, $p^*(t) = p^*/w^t$.

The step V-2-3-9-B indicates the discount principle of the infinite economic planning.

CONCLUSION 1 The future value of the after-planning periods should be discounted by the sum of the rate of population growth and the rate of technological progress; i.e. $w = (1+s)(1+q)$.

Here the Author would like to add some remarks.

Remark 1. International bank rate does not influence the discount rate.

Remark 2. Conclusion 1 is absolutely different from the utility approach, since in the conclusion 1 the discount rate is determined objectively, though the utility approach requires subjective determination of discount rate.

V-2-3-10 Calculation of accounting prices in the after-planning periods of the long-term economic planning

In this subsection we examine the problem of calculation of the accounting prices in the after-planning periods.

We have already defined the problem mathematically in a consistent way. So if we have an adequate algorithm for numerical computation, we can obtain the concrete value of the price vector (p^*). In this Thesis the Author does not present any algorithm, since this task belongs to the sphere of mathematicians. Instead the Author presents a method of economic estimation of the accounting prices. The stationary optimal program necessarily satisfies the constraint(12) of V-2-3-6 with equality. So we have

$$\left(\frac{I}{w} - A + B + \frac{B}{w} \right) x = e + c - \frac{m}{w} \quad \dots\dots\dots(1)$$

$$\text{or } J x = g \quad \dots\dots\dots(1')$$

where $J =: \left(\frac{I}{W} - A + B + \frac{B}{W} \right)$ and $g =: e + c - \frac{m}{W}$

Assumption V-2-3-10-A Matrix(J) is non-negative matrix and satisfies the Hawkins-Simon condition.

In the equation (1') the vector(g) is interpreted as the net production vector. Now we examine the set of the attainable net production(g). As already proved in the mathematical appendix(M-V-2-3-6), a hyper-plane draws the borderline of the attainable set of net production(g). Slope (gradient vector) of the hyper-plane is

$$\gamma' = (\gamma_1, \gamma_2, \dots, \gamma_n) = (v_0' J^{-1} h_1, v_0' J^{-1} h_2, \dots, v_0' J^{-1} h_n)$$

where $h_i' =: (0, 0, \dots, 0, 0, \quad 1, \quad 0, 0, \dots, 0, 0)$ (i-th element)

This equation was proved in the mathematical appendix(M-V-2-3-6).

$J^{-1} h_i$ indicates the vector of additional product of each industry which is necessary to produce one more unit of net production of i-th industry. Therefore $(v_0' J^{-1} h_i =: \gamma_i)$ indicates the amount of labour input which is necessary to produce one additional unit of net product of i-th industry. Therefore γ_i can be interpreted as the amount of labour which is embodied in one additional unit of product of i-th industry.

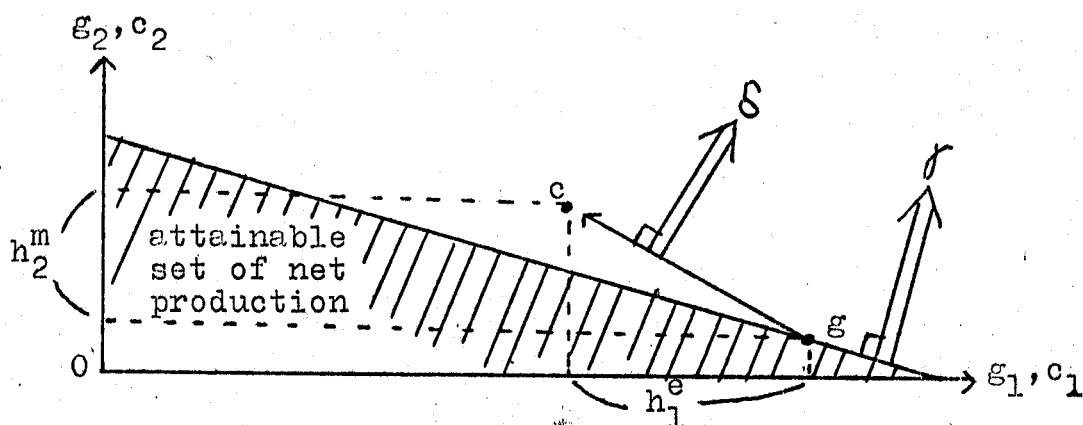


figure 19

From (1) we obtain the following equation.

$$c = g + \frac{m}{W} - e \quad \dots \dots \dots (2)$$

So the government can change the result of net production by utilizing the international trade. In figure 19 the consumption(c) can be attained from net output(g) by exporting the 1-st good and importing the 2-nd good. That is, $g_1 - c_1 = e_1$ and $c_2 - g_2 = \frac{m_2}{W}$.

Of course, here the constraints of the international trade(14) and (15) of V-2-3-6 should be maintained. In the optimal stationary program the foreign currency debt is zero under the assumption that $(1+r) > (1+s)(1+q)$. So (14) and (15) of V-2-3-6 turn into the following form.

$$\sum_{i=1}^n d_i^e e_i \geq (1+r) \sum_{i=1}^n \frac{d_i^m m_i}{(1+s)(1+q)} \quad \dots\dots\dots(14')$$

$$0 \leq e_i \leq h_{io}^e$$

$$0 \leq m_i \leq h_{io}^m$$

$$\left. \begin{array}{l} 0 \leq e_i \leq h_{io}^e \\ 0 \leq m_i \leq h_{io}^m \end{array} \right\} \dots\dots\dots(15')$$

In the case of figure 19 the condition (14') turns into as follows.

$$d_1^e e_1 \geq \frac{(1+r)}{(1+s)(1+q)} d_2^m m_2 \quad \text{That is, } d_1^e e_1 \geq (1+r) d_2^m m_2 \quad \dots\dots\dots(\beta)$$

Therefore the gradient vector δ of figure 19 should be as follows.

$$\delta = (d_1^e, (1+r)d_2^m)$$

Then we investigate the attainable set of consumption. In the two-dimensional case of figure 19 the attainable set of consumption can be drawn as follows.

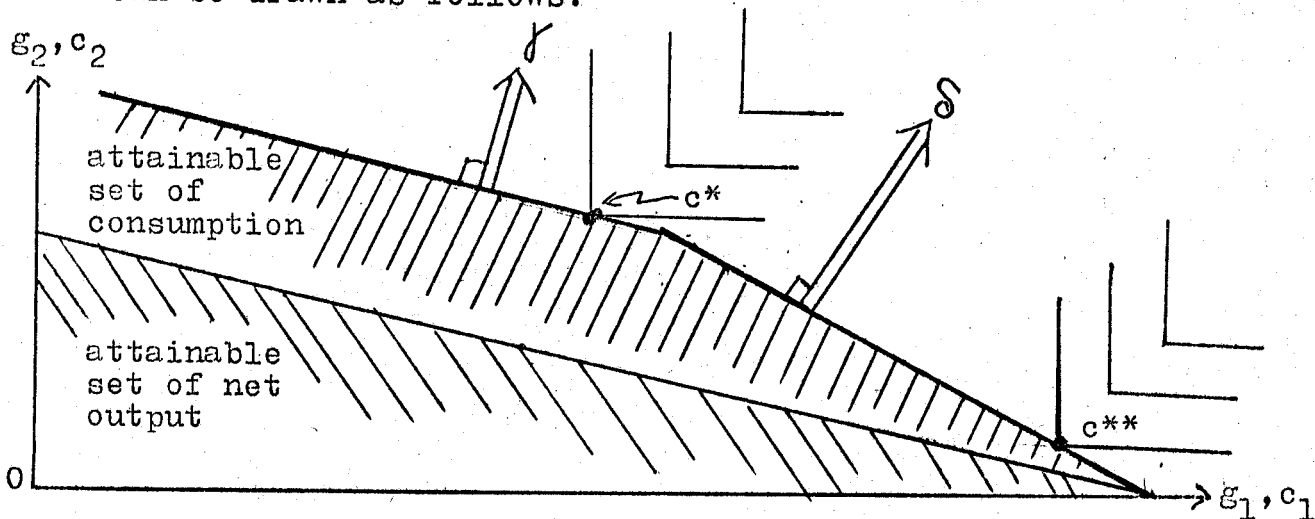


figure 20

There are two possibilities. The first is (c*) of figure 20. In this case both industries produce positive net output. The accounting price should be γ .

The second case is (c**) of figure 20. In this case the second industry does not produce net output. The final demand of the 2-nd good is satisfied entirely by the import. In this case the accounting price should be δ .
$$\delta = \left(\gamma_1, \gamma_1 \frac{(1+r) d_2^m}{d_1^e} \right)$$

Generally we can infer the accounting prices in the following procedure.

Procedure 1

- (i) If i-th industry produces positive net output, then the accounting price is γ_i .
- (ii) If i-th industry does not produce net output and therefore the final demand is satisfied by import, then the accounting

price should be δ_i ; $\delta_i = \gamma_j \frac{(1+r)d_i^m}{d_j^e}$

where (j) denotes the marginal exporting industry, the notation of which will be explained later.

Procedure 1 (verbal version)

- (i) If i-th industry produces positive net output, then the accounting price of i-th industry is determined by the marginal labour input which is necessary to produce one additional product of i-th industry.
- (ii) If i-th industry does not produce net output and therefore the final demand is satisfied by import, then the accounting price of i-th industry is determined by the marginal labour input of the marginal exporting industry (j) which is necessary to produce the exportable product for the exchange of one unit of imported product of i-th industry.

$$\left(\begin{array}{c} \text{accounting} \\ \text{price of} \\ \text{i-th indu-} \\ \text{stry} \end{array} \right) = \left(\begin{array}{c} \text{marginal} \\ \text{labour input} \\ \text{of exporting} \\ \text{j-th industry} \end{array} \right) \frac{\left(\begin{array}{c} \text{import price of} \\ \text{i-th industry's} \\ \text{product} \end{array} \right) + \left(\begin{array}{c} \text{financial} \\ \text{cost of} \\ \text{import} \end{array} \right)}{\left(\begin{array}{c} \text{export price of exporting} \\ \text{j-th industry's product} \end{array} \right)}$$

Here the Author presents some remarks.

Remark 1. Unit of the macro-accounting price of industry is labour hour. For example, the accounting price of one zloty's product of the machinery industry may be 18 seconds or 20 seconds. It must be noticed that the macro-accounting price is only an index for the calculation of particular accounting price of each product. The micro-accounting price of one tractor should be measured by manetary unit, i.e. zloty. This procedure will be investigated in chapter VI.

Remark 2. In the macro-economic planning the accounting price is determined by the scarceness of labour or the international price. Some Readers may feel that this procedure is too simple, since every particular price is influenced by many elements other than these two factors. It must be once more noticed that this accounting price is a macro-accounting price at the industry's level. At the industry's level we can not handle these manifold elements. Instead we can handle these elements only at the micro-level. As will be shown in chapter VI, the basis of determination of micro-accounting price of particular product is the existing price and the macro-accounting price of the industry, since these manifold elements are reflected in the existing price.

Remark 3. The marginal exporting industry (j) is the exporting

industry which possesses the most unprofitable trade term among all exporting industries. In other words the industry (j) is the last industry which exports its product. In chapter VI we will thoroughly investigate the nature of the marginal exporting industry.

Procedure 2 Accounting prices λ_i and $\lambda_j(1+r)d_i^m/d_j^e$ should be discounted by (w).

Procedure 1 is very plausible. If the economy does not produce the oil product and imports it, then the accounting price of oil product should be determined by the international price of oil.

Suppose that the economy imports the chemical fibre to the upper limit of import. Also suppose that the amount of import is not sufficient to satisfy the final demand for the chemical fibre. Then the economy must produce the chemical fibre domestically. In this case the accounting price should be determined by the domestic condition of production. It should be kept in mind that the international bank rate exerts influence on the determination of accounting prices through the financial cost of import.

Finally the Author presents an important conclusion, which becomes the foundation of the criterion of project evaluation.

CONCLUSION 2 Accounting price of domestically produced goods is determined by the domestic condition of production. Accounting price of entirely imported goods is determined by the international trade term and also by the international bank rate.

Till now we assumed that net production (g) is positive. But here arises a reasonable question. That is, can the net production be negative? It is true that net production can be negative when the international trade is possible. Fortunately this problem brings no difficulty to the model. In the mathematical appendix (M-V-2-3-10) we examine this problem.

V-2-4 Long-term optimal planning of the multi-sector model

V-2-4-1 Aim of the long-term optimal planning

Aim of the long-term optimal planning is twofold as well as the aim of the infinite economic planning. The first aim of the long-term optimal planning is to determine the optimal direction of economic development and to determine the final state of the medium-term economic planning. The determination of the final state of

the medium-term economic planning is identical with the determination of the optimal direction of economic development. Therefore, in other words, the first aim of the long-term economic planning is to determine the direction of economic development within the range of the long-term, e.g. 10 years or 20 years. It must be emphasized that the long-term optimal plan does not possess an executive character. In the socialist economy the short-term(one year) planning dominantly and the medium-term(five years) planning partly have an executive character. Therefore the determination of output of each industry's product in every period is not a main aim of the long-term economic planning.

The second aim of the long-term economic planning is to settle the accounting prices in the after-planning periods of the medium-term economic planning. Since almost every industrial project has an operation period longer than 5 years, the accounting price in the after-planning periods possesses an crucial meaning in the project evaluation.

V-2-4-2 Character of the long-term optimal planning

The greatest difference between the infinite planning and the long-term planning is the possibility of numerical calculation. As already mentioned in V-2-3, we can not calculate numerically the optimal program of the infinite economic planning, since the time horizon is infinite and consequently the number of variables is also infinite. Therefore the Author proposed to substitute the optimal stationary program for the optimal program. Instead in the case of the long-term economic planning we can calculate the optimal program, if we utilize a computer with sufficient capacity.

In this place, as usual, we must pay attention to the limit of computer's capacity. For the purpose of calculation of the optimal five years plan Bocian(3) made out a multi-sector dynamic model, foundation of which is the comprehensive approach by Porvit. Although he constructed a rather simplified model than the original comprehensive model by Porvit, he had to observe the following constraint by the use of the computer EMC Minsk 22.⁶⁾ $(T+1)(n+1) \leq 301$. That is, $(\text{time horizon} + 1)(\text{number of industry} + 1) \leq 301$.

His calculation was executed in the beginning of 1970's, so now in 1980's we have a greater possibility than the Bocian's case. Nevertheless we must be subjected to the limit of computer's capacity, since the long-term economic planning has a large number

of time horizon, that is, $T=20$.

In this point the technological and the institutional information of the long-term economic planning has to be restricted to a smaller range than that of the medium-term economic planning.

There are two variants of the long-term economic planning. The first variant is accompanied with a longer time horizon (e.g. $T=20$) and, however, with a coarser information of the technology and the institutional constraints. The second variant is accompanied with a more detailed information of technology and the institutional constraints and, however, with a shorter time horizon (e.g. $T=10$). If unfortunately the CPB possesses a computer with a rather small capacity, this supposition being very plausible in developing countries, the CPB must restrict both the time horizon and the information of technology and the institutional constraints.

V-2-4-3 Simplified long-term optimal planning

In this subsection we investigate the first variant of the long-term economic planning, that is, economic planning with a longer time horizon but with a coarser information of technology and institutional constraints. The model of the simplified long-term optimal planning is similar to the model of the infinite economic planning except that the time horizon is finite.

The model of the simplified long-term economic planning has the following characters.

- a) The CPB maximizes the value of per capita consumption with restriction of the desirable structure of consumption. Per capita consumption is the value of total consumption which is discounted by the sum of the population growth rate and the technological progress rate. That is, $c_t = C_t / ((1+s)^t (1+q)^t)$. Justification of this formulation is already mentioned in V-2-3.
- b) Amount of labour force increases with the same rate as the population.
- c) The rate of population growth(q) is same in all periods.
- d) Technological progress proceeds with the same rate(s) in all industries. Also the rate of technological progress is same in all periods.
- e) The desirable structure of consumption never changes over time. Then the CPB maximizes the following function.

$$\text{MAX} \sum_{t=0}^T F(c_t) = \sum_{t=0}^T \left(\min \left(\frac{c_{i,t} - k_i}{F_i} \right) + \sum_{i=1}^n k_i \right) \dots\dots\dots (1)$$

where T ; the time horizon.

f_i ; the desirable fraction of consumption of the i -th industry's products.

k_i ; the admissible lower limit of consumption of the i -th industry's products.

f) Current input coefficient (a_{ij}) never changes over time, i.e. matrix (A) is constant.

g) Capital stock coefficient (b_{ij}) never changes over time, i.e. matrix (B) is constant.

h) Labour input coefficient decreases with a constant rate (s). That is, in this model the technological progress means a substantial increase of "labour force". Therefore we can assume, on the one hand, that the labour input vector (v_0) is constant over time and, on the other hand, that the labour force increases with a rate $w=(1+s)(1+q)$.

i) Production and import activity have a time lag of one period. That is, output of t -th period can be utilized in the next period (i.e. $(t+1)$ -th period). Products and resources which are imported in the t -th period can be also utilized only in the next period. As well as in the case of the infinite economic planning, the length of one period may be 6 months or one year.

j) Construction period of capital stock is assumed to be one period. This assumption seems very strong. Although this assumption is, of course, unrealistic in the sphere of particular enterprises, the Author considers that in the scale of the industry this assumption is not so unrealistic. The reason is as follows. There is a capital investment with a longer construction period as well as a capital investment with a shorter construction period. For example construction of steel plant needs 5 years, instead installation of machine in an established factory needs only few weeks. Therefore the average construction period of the whole economy is more or less one year. This is the common assumption of the dynamic Leontief model.

k) There is no distinction of labour force. That is, there is only one kind of labour force.

l) There is no distinction of foreign market. That is, there is only one foreign market, i.e. the world economy.

m) By the above-mentioned assumption there is only one kind of foreign currency. That is, different kinds of foreign currency (dollar, ruble and so on) are integrated into an unique foreign currency with an adequate conversion coefficient.

n) Resource constraints except labour force are not examined in

the model. Therefore land is assumed to be sufficiently abundant.

o) Upper limits of export, import and foreign currency debt increase with the same rate of the production ability of the economy. That is, these limits increase with a rate of the sum of the rate of population growth and the rate of technological progress; i.e. $w = (1+s)(1+q)$.

p) International price is assumed to be constant over time. This is an unrealistic assumption, but we can easily loosen this assumption by supposing that the international prices grow with a constant rate(i), that is, with a world inflation rate. In this case we also assume that the upper limit of foreign currency debt increases with a rate of the sum of the growth rate of production ability and the world inflation rate; i.e. $(1+i)w$. Then we can take into account the world inflation in our model without any modification. Although in this Thesis the Author does not examine this modification, the Reader can easily execute this procedure. In this case the substantial international bank rate is equal to $(1+r)/(1+i)$.

q) Expected international bank rate (r) never changes over time. The expected international bank rate may be different from the existing rate. The CPB expects a plausible bank rate in future, e.g. 10 years hence.

r) Collective consumption by per capita term is assumed to be constant over time. Needless to say, total quantity of the collective consumption increases with a rate of the sum of the population growth and the technological growth. So the collective consumption can be included in the admissible lower limit of consumption (k_i). We can loosen this assumption and can suppose that the per capita collective consumption can change from period to period. This modification will be investigated in the enlarged long-term optimal planning in V-2-4-6.

s) Final state of capital stock and final level of foreign currency debt (Z_T) are determined by the optimal stationary program of the infinite economic planning.

As apparent from the above-mentioned assumptions, the simplified long-term economic planning is very similar to the infinite economic planning. Instead the model of the simplified long-term economic planning has a big difference from the original comprehensive model of the medium-term economic planning. That is, the optimal level of the foreign currency debt is treated in a different manner. In the comprehensive approach the full utilization of the upper limit of foreign debt is implicitly assumed, because if not, then

the dual price of foreign currency becomes zero. This is an unrealistic conclusion. Of course, in the scope of the medium-term the limit of foreign currency debt is determined by the political decision or by the international negotiation and its limit can be supposed to be fully utilized. Therefore the setting of foreign currency debt of the comprehensive approach can be justified. On the contrary in the long-term economic planning the most crucial task is to answer such a question that at what level the foreign currency debt should be. Needless to say, we can not a priori conclude that the foreign currency debt should be fully utilized to the upper limit in the long-run. Therefore we suppose that the long-term economic planning determines an optimal level of foreign currency debt within the range of the upper limit.

Now the Author presents the whole system of the simplified long-term economic planning.

The model

$$\text{MAX} \quad \sum_{t=0}^T F(c_t) = \sum_{t=0}^T \left(\min_i \left(\frac{c_{i,t} - k_i}{f_i} \right) + \sum_{i=1}^n k_i \right) \dots\dots\dots(1)$$

subject to

$$A X_t + E_t + C_t + B(X_t - X_{t-1}) \leq X_{t-1} + M_{t-1} \quad ;(t=0,1,2,\dots, T) \dots\dots\dots(2)$$

$$\sum_{i=1}^n d_i^e E_{i,t} + Z_t \geq (1+r) \sum_{i=1}^n d_i^m M_{i,t-1} + (1+r) Z_{t-1} \quad ;(t=0,1,2,\dots, T) \dots\dots\dots(3)$$

$$\sum_{i=1}^n \frac{v_{i0}}{(1+s)^t} X_{i,t} \leq (1+q)^t L_0 \quad ;(t=0,1,2,\dots, T) \dots\dots\dots(4)$$

$$\left. \begin{aligned} 0 \leq E_{i,t} &\leq (1+s)^t (1+q)^t h_{i0}^e \quad ;(t=0,1,2,\dots, T) \\ 0 \leq M_{i,t} &\leq (1+s)^t (1+q)^t h_{i0}^m \quad ;(t=0,1,2,\dots, T) \end{aligned} \right\} \dots\dots\dots(5)$$

$$0 \leq Z_t \leq (1+s)^t (1+q)^t H_0 \quad ;(t=0,1,2,\dots, T) \dots\dots\dots(6)$$

$$X_{-1} = \underline{X}, \quad M_{-1} = \underline{M}, \quad Z_{-1} = \underline{Z} \quad \dots\dots\dots(7)$$

where X_{-1}, M_{-1} and Z_{-1} denote the initial condition. $\underline{X}, \underline{M}$ and $\underline{Z}$ are historically given.

$$X_T + B X_T + M_T \geq \underline{X}_T^* + B \underline{X}_T^* + \underline{M}_T^* =: \underline{K}_T^* \dots\dots\dots(8)$$

$$(1+r) \sum_{i=1}^n d_i^m M_{i,T} + (1+r) Z_T \leq (1+r) \sum_{i=1}^n d_i^m \underline{M}_T^* + (1+r) \underline{Z}_T^* =: \underline{D}_T^* \dots\dots\dots(9)$$

where $\underline{X}_T^*, \underline{M}_T^*$ and $\underline{Z}_T^*$ indicate the value of the solution concerned which is determined by the optimal stationary program of the

infinite economic planning. So $\underline{K}_T^*$ is the final state of stock vector of the long-term economic planning. $\underline{D}_T^*$ denotes the final state of the foreign currency debt. Condition(8) means that the final state of the last period should not be less than the final condition ($\underline{K}_T^*$) which is determined by the infinite optimal stationary program. $(1+r)\sum_{i=1}^n d_i^m M_T$ denotes the import payment of the last period. $(1+r)Z_T$ denotes the foreign currency debt of the last period. So the condition(9) means that the total amount of foreign currency debt in the last period should not exceed the final condition($\underline{D}_T^*$) which is determined by the infinite optimal stationary program. As already mentioned, $Z_T^* = 0$, if $(1+r) > (1+s)(1+q)$.

It must be noticed that in the long-term economic planning every foreign currency debt is treated as an "one year debt". Justification of this simplification was already mentioned in V-2-3.

In this model we can also obtain the accounting prices as the dual price of the model. Mathematical calculation is presented in the mathematical appendix(M-V-2-4). So in the text the Author gives only the essence of the arguments.

Firstly the Author introduces the following assumption.

Assumption V-2-4-3-A(Slater's condition)

There exists $(\underline{X}_t, \underline{E}_t, \underline{M}_t, \underline{Z}_t ; t=0,1,2,\dots,T)$ which satisfies all constraint conditions (2),(3),(4),(5),(6),(7),(8),(9) and also satisfies (2),(8),(9) with strict inequality.

Theorem V-2-4-3 Under the assumption V-2-4-3-A there exists an optimal program of the long-term economic planning.

Proof. See mathematical appendix(M-V-2-4).

Remark. Although we can prove the existence of the optimal program, we can not prove the uniqueness of the optimal program. So there is a possibility of multiple optimal solution. If the CPB obtains two optimal programs, the CPB decides which program should be adopted from the socio-political viewpoint.

V-2-4-4 Accounting prices of the optimal program of the simplified long-term optimal planning

In this subsection the Author presents only the essence of the arguments, mathematical calculation of which is presented in the mathematical appendix(M-V-2-4).

Theorem V-2-4-4-A If an attainable program is competitive, then

it is an optimal program of the simplified long-term economic planning.

Proof. See mathematical appendix(M-V-2-4).

Theorem V-2-4-4-B Under the assumption V-2-4-3-A if an attainable program is an optimal program, then it is a competitive program, that is, there exists a competitive price vector (the accounting price vector) which associates with the optimal program.

Proof. See mathematical appendix(M-V-2-4).

The relation(24) of the mathematical appendix(M-V-2-4) indicates that the optimal program maximizes the total value of consumption among all attainable programs, if the consumption is measured by the accounting price (the competitive price associating with the optimal program). The relation(21) of the mathematical appendix(M-V-2-4) indicates that the optimal program maximizes the "profit" (i.e. net surplus) among all attainable programs, if the output and the input are measured by the accounting prices. This result provides a foundation of the cost-benefit analysis in the decision making at the micro-economic level. The relation(25) of the mathematical appendix(M-V-2-4) means that if excess supply exists, then the accounting price of the product is zero.

Now we examine the procedure of determination of the accounting prices from an economic point of view.

Procedure 1 The CPB adopts the competitive price vector (dual price vector) associating with the optimal program as an accounting price vector of the long-term economic planning.

Remark. Competitive price vector is $(n+1)$ -dimensional vector. So p_t contains also the accounting price of the foreign currency.

Now let us illustrate the theorem V-2-4-4-B in the same manner of the precedent subsection. By the theorem V-2-4-4-B the optimal program is a competitive program. The condition(24) of the mathematical appendix(M-V-2-4) means that the optimal program maximizes the total value of consumption. Therefore we can draw the following figure, although the figure 21 is symbolic, since we can not draw a n -dimensional diagram.

As well as in the infinite economic planning, the Author does not present any concrete algorithm for calculation of the accounting prices. The algorithm may have the following form.

Discover $(p_t); (t=0,1,2,\dots,T)$ such that $(c_t^*); (t=0,1,2,\dots,T)$ maximizes $\sum_{t=0}^T p_t' c_t^*$ for all attainable (c_t) .

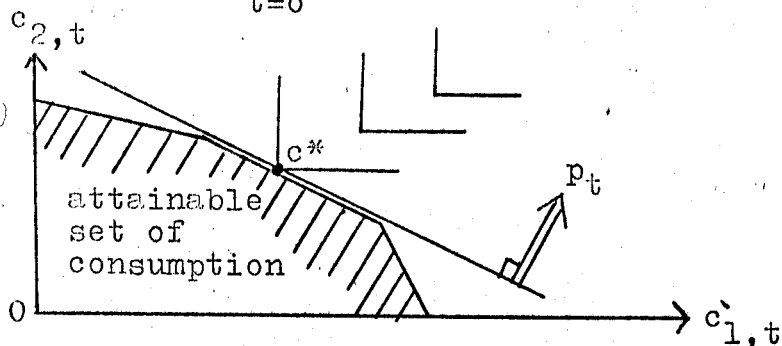


figure 21

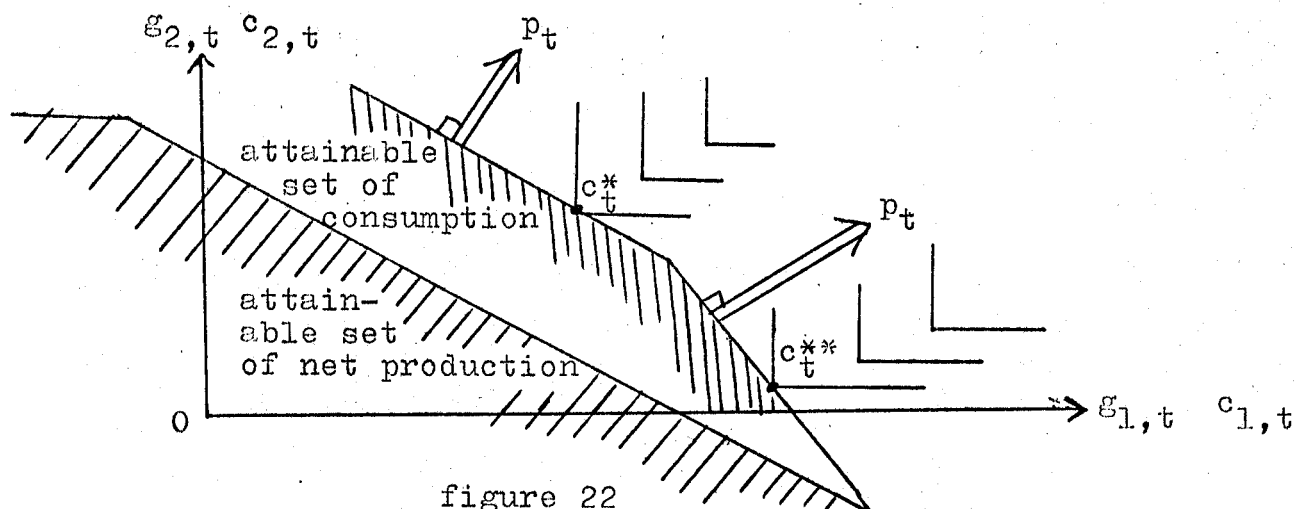


figure 22

We can, however, determine the accounting prices by economic inference. Firstly we should pay attention to the following property.

Property V-2-4-4-A In the simplified long-term economic planning the accounting price usually differs from period to period.

As well as in the infinite economic planning, the Author proposes the following procedure.

Procedure 2 If the optimal program of the long-term planning brings positive gross output of i -th industry in t -th period, then $p_{i,t} = \gamma_{i,t}$; where $p_{i,t}$ denotes "per capita" accounting price of i -th industry's products in t -th period. $\gamma_{i,t}$ denotes the marginally required labour input for the additional one output of i -th industry's product in t -th period.

Procedure 3 If the optimal program brings zero gross output of i -th industry's products and therefore the final demand has to be satisfied by import, then

$$p_{i,t} = \gamma_{j,t} \frac{(1+r) d_{i,t}^m}{d_j^e}$$

where j denotes the marginal exporting industry whose products are exported.

The procedure 2 and the procedure 3 are the fundamental rules of the project evaluation. In reality, from the socio-political reason, some industries produce a positive gross output even when the import is more profitable than the domestic production. In such a case the procedure 2 can not be applicable. One plausible case is the protection for the infant industry. So the accounting price of protected infant industry should be determined by the procedure 3.

More serious problem arises in the agricultural industry. The infinite and the long-term economic planning does not take into account the resource constraint except labour force. Sometimes the agricultural products are produced up to the upper limit of the land's constraint and any additional production is impossible or very expensive. In this case the additional demand must be satisfied by import. This situation is very plausible in feed grains (i.e. corn for animals). In this case also the accounting price of feed grain industry should be determined by the procedure 3.

Procedure 4 In case of the protected infant industry and agricultural industry, production of which is restricted by the resource condition or the socio-political aspect, the accounting price should be determined by the procedure 3.

There is another problem, e.g. in the coal industry. Production of coal is also restricted by the resource condition, so the additional demand must be satisfied by the reduction of the export. In this case the accounting price should be determined by the international price.

Procedure 5 In case of the coal industry or other industries, where the additional final demand is satisfied by reduction of export, the "per capita" accounting price is determined as follows.

$$p_{i,t} = p_{j,t} \frac{d_{i,t}^e}{(1+r)d_{j,t}^m}$$

where j denotes the marginal exporting industry. $p_{j,t}$ is the "per capita" accounting price of the j -th marginal exporting industry.

As well as in the infinite economic planning, the competitive price is a price of "per capita" consumption. Therefore we need discount the competitive price in order to obtain the accounting price of one physical unit in future periods.

Procedure 6 The CPB discounts the competitive price (the dual price) by the rate of $w = (1+s)(1+q)$. That is, discount rate is equal to the sum of the population growth rate and the technological growth rate. $P_{i,t} = p_{i,t}/w^t$. where $P_{i,t}$ denotes the accounting price of one physical unit of i -th industry's products in t -th period. $p_{i,t}$ denotes the "per capita" accounting price of i -th industry's products in t -th period.

Procedure 7 The CPB discounts the accounting price of foreign currency also by the rate of w .

V-2-4-5 Variant analysis

The long-term economic planning concerns future occurrences. Economic situation of future is changeable from two reasons.

The first reason is uncertainty. Value of exogeneous constraints may be changeable in future, so it is very useful to build up several different models according to the different expected value of exogeneous constraints, for example, the upper limit of export.

The second reason is investment. Expected values of technological coefficients $(a_{i,j})$, $(b_{i,j})$ and (v_{i0}) exert strong influence onto the solution of the optimal program. But here we must keep in our mind the fact that investment usually increases the capital stock coefficient $(b_{i,j})$ and on the other hand decreases the labour input coefficient (v_{i0}) and the current input coefficient $(a_{i,j})$. Till now we have assumed that these coefficients are constant over time in the scale of the whole economy. In many industries this assumption seems to be plausible, though in some industries this assumption does not hold good. For example, in the steel industry, especially in the developing phase, these coefficients change drastically. Therefore we had better investigate several different models according to the intensity of investment.

We call this approach the variant analysis. For example, the first variant contains higher $(b_{i,j})$ and lower $(a_{i,j})$ and lower (v_{i0}) for the steel industry. These coefficients can be changed from period to period by investment. The first variant corresponds to a highly intensive investment plan. The second variant, corresponding to a lowly intensive investment plan, contains lower $(b_{i,j})$ and higher $(a_{i,j})$ and (v_{i0}) . The CPB compares the values of the objective function of these two variants. The variant with a higher value of the objective function should be adopted.

Theoretically saying, the number of variants is infinite.

Since the number of technological processes is infinite. But practically saying, we have to be satisfied with several variants in the scale of the whole economy, e.g. variant with highly intensive investments, variant with moderate investments and variant with lowly intensive investments.

In the model of the simplified long-term economic planning all coefficients are assumed to be constant over time. So we can not execute the variant analysis of investment by the simplified long-term economic planning, since the effect of investment appears with a certain time lag. For the variant analysis we must enlarge the simplified long-term economic planning. The extent of enlargement and the number of variants depend on the capacity of computer and the amount and the ability of economists who work in the CPB.

In the following subsection the Author will present an enlarged long-term economic planning. We should always take care of the capacity of computer. The CPB decides which part of the simplified model should be enlarged and at what extent it should be enlarged taking into account the planning ability.

V-2-4-6 Framework of the enlarged long-term optimal planning

Practical applicability is the most important condition of economic planning models. In the precedent parts of this section (V-2-4) we have investigated the simplified long-term optimal planning, since it has a very simple form and therefore can be solved by a computer with a small capacity. In the developing countries, as well as in some developing socialist countries, the CPB can dispose only a small-sized computer, so the CPB of these countries is obliged to use the simplified optimal planning. In recent years, however, the capacity of computer is increasing with a high tempo. Therefore the CPB of many countries has a chance to investigate a more complicated model, i.e. the enlarged optimal planning model. The enlarged long-term optimal planning offers a possibility of executing the variant analysis. The enlarged long-term optimal planning has the following framework.

a) In the simplified long-term economic planning the total amount of consumption ($C_{i,t}$) was discounted into the "per capita" term ($c_{i,t}$) by the sum of the population growth rate and the technological progress rate. On the other hand in the enlarged long-term economic planning model the rate of technological progress differs from industry to industry, and consequently we must modify the

above-mentioned procedure. In the enlarged model the CPB calculates the average rate (u) of technological progress in all industries and substitutes the average rate (u) for the rate (s) of the simplified model. The average rate (u) can be interpreted as the growth rate of scarceness of leisure or "value" of leisure.

Therefore in the enlarged long-term optimal planning model, the "per capita" term of consumption is calculated as follows.

$$c_{i,t} = \frac{C_{i,t}}{(1+u)^t Q_t}$$

where Q_t ; the amount of population in t-th period.

u; the average rate of technological progress in all industries.

b) The admissible lower limit of "per capita" private consumption ($k'_{i,t}$) can change from period to period.

c) In the enlarged model we can take into account a change of the level of the collective consumption. In the simplified long-term optimal planning we assumed that the total amount of the collective consumption increases at the rate of $(1+s)(1+q)$. In the enlarged long-term optimal planning the collective consumption is determined by the government in advance, and then it can be changeable from period to period. The collective consumption can even decrease. Together with the above-mentioned formulation (b) we can illustrate the admissible consumption field as follows. Mathematically saying,

or

$$k_{i,t} = \frac{F_{i,t}}{(1+u)^t Q_t} + k'_{i,t}$$

$$\left(\begin{array}{l} \text{"per capita" lower} \\ \text{limit of consump-} \\ \text{tion} \end{array} \right) = \left(\begin{array}{l} \text{"per capita"} \\ \text{collective} \\ \text{consumption} \end{array} \right) + \left(\begin{array}{l} \text{"per capita" lower} \\ \text{limit of private} \\ \text{consumption} \end{array} \right)$$

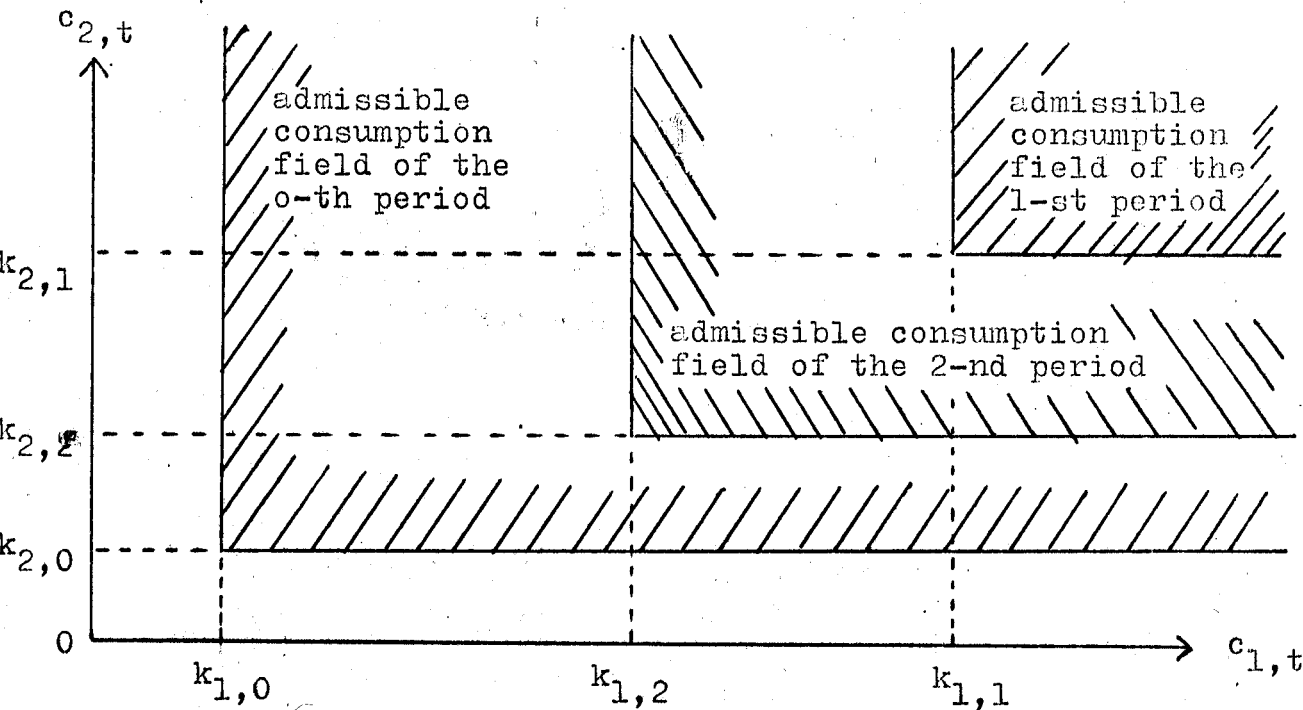


figure 23

A more abstract and formal definition of the admissible consumption field is presented in the mathematical appendix of this subsection (M-V-2-6-A).

d) Desirable structure of private consumption ($f_{i,t}$) can change from period to period. So the CPB maximizes the following objective function.

$$\text{MAX} \sum_{t=0}^T F_t(c_{i,t}) = \sum_{t=0}^T \left(\min_i \left(\frac{c_{i,t} - k_{i,t}}{f_{i,t}} \right) + \sum_{i=1}^n k_{i,t} \right)$$

Here $\sum_{t=0}^T F_t(c_{i,t})$ is a concave function, since $F_t(c_{i,t})$ is a concave function.

e) Amount of labour force can increase with a different rate from the population growth rate. L_t denotes the amount of labour force in t-th period. Instead we assume only one kind of labour force, since in the long run the one occupation can be transformed to the other occupation.

f) Current input coefficient can change from period to period.

$A_t = (a_{i,j,t})$ denotes the current input coefficient in t-th period.

g) Capital stock coefficient can change from period to period.

$B_t = (b_{i,j,t})$ denotes the capital stock coefficient in t-th period.

h) Labour input coefficient can change from period to period.

$V'_t = (v_{1,t}, v_{2,t}, v_{3,t}, \dots, v_{n,t})$.

i) International bank rate, export price and import price can change from period to period.

j) There are several foreign markets, i.e. ruble district (socialist countries) and dollar district (capitalist countries).

International price and international bank rate are different in these two districts. For example, $r_t^R \neq r_t^D$; where (R) denotes the ruble district and (D) denotes the dollar district.

k) Upper limit of import, export and foreign currency debt is different from period to period and also different in these two foreign markets.

l) Production and import activity have a time lag of one period. That is, output of t-th period can not be utilized till the next period (i.e. (t+1)-th period). Products and resources which are imported in t-th period also can not be utilized till the next period.

m) Construction period of capital stock is assumed to be one period. Justification of this assumption was mentioned in j) of V-2-4-3. We can easily introduce the more complicated time lag of construction into our model without any theoretical difficulty. If

We do this enlargement, then, however, the required capacity of computer may be huge. So the Author does not introduce the more complicated time lag of construction into the model. But in the mathematical appendix the Author presents the model with complicated time lag. See the mathematical appendix(M-V-2-4-6-B).

n) In the enlarged long-term model the resource constraints are taken into account. Some products, e.g. agricultural products or coal, are restricted by the upper limit, which is determined by the level of available resources. These upper limits can change from period to period.

o) Final state of capital stock and final level of foreign currency debt are determined by the solution of the optimal stationary program of the infinite economic planning. Here we must decompose z_T^* into z_T^{R*} and z_T^{D*} , since the optimal stationary program of the infinite planning determines only z_T^* . Fortunately as already mentioned, $z_T^* = 0$, if $(1+r) > (1+s)(1+q)$. Therefore we can set $z_T^{R*} = 0$ and $z_T^{D*} = 0$. That is, dollar debt in the final period (e.g. in the year 2001) is zero and ruble debt in the final period is also zero.

If we introduce all these above-mentioned enlargement into the model, the model requires a huge capacity of computer. The Author considers that even the most advanced computer in the world can not solve such a model. Therefore the CPB should decide, taking into account the limit of computational ability, which part of the simplified model should be enlarged.

V-2-4-7 The model of the enlarged long-term
optimal planning and its property

The whole system of the enlarged long-term optimal planning model is quite complicated. Therefore the Author presents the whole system of the model in the mathematical appendix(M-V-2-4-7). In this subsection the Author presents only two important theorems which are mathematically deduced from the whole system of the model. In the following two theorems we can confirm the existence of the optimal program and the existence of the accounting prices.

In the mathematical appendix(M-V-2-4-7) the Author rewrites the model in an abstract form. There he introduces the assumption (V-2-4-7-A) and proves the following theorems.

Theorem V-2-4-7-A

Under the assumption (V-2-4-7-A) of the mathe-

mathematical appendix, there exists an optimal program of the enlarged long-term economic planning.

Proof. See mathematical appendix(M-V-2-4-7).

Theorem V-2-4-7-B Under the assumption (V-2-4-7-A) of the mathematical appendix, if an attainable program $(x_t^*, e_t^{R*}, e_t^{D*}, m_t^{R*}, m_t^{D*}, z_t^{R*}, z_t^{D*})$ is an optimal program, then it is a competitive program. That is, there exists a competitive price vector which associates with the optimal program.

Proof. See mathematical appendix(M-V-2-4-7).

Remark. In the enlarged long-term optimal planning there exists $(n+2)$ -dimensional price vector. That is, prices of industry's product, price of ruble and price of dollar. As well as in the case of the simplified long-term economic planning, the price vector is the competitive price vector (the dual price vector) of "per capita" term. The accounting price of physical unit can be obtained by discounting the "per capita" accounting price.

V-2-5 Principles of determination of accounting prices at the macroeconomic level in the infinite and the long-term economic planning

V-2-5-1 Kinds of accounting prices

In the infinite and the long-term economic planning we obtain the accounting prices of each industry's product and the accounting price of foreign currency. If we incorporate two different foreign currencies into the model, then we can obtain the accounting price of dollar and the accounting price of ruble. In the infinite and the long-term economic planning even if the level of foreign currency debt is smaller than the upper limit of foreign currency debt, the accounting price of foreign currency is positive.

Accounting price of i -th industry is an average price of i -th industry's products. In the following chapter (VI-2) the Author will present a method of determination of accounting price of particular products.

In the infinite and the long-term optimal planning we can not obtain the accounting price of labour force, i.e. the shadow wage rate. The shadow wage rate should be determined in another way and

in the following part of this Thesis (VI-2) the Author will propose a method of calculation of the shadow wage rate.

V-2-5-2 Induced labour input

Accounting prices can be obtained mathematically in the following manner.

Find a price vector (p) such that maximizes pc^* among all attainable consumption vector (c) ; i.e. $pc^* \geq pc$.

That is, the accounting price is a price vector which maximizes the accounting value of the optimal consumption program among any other attainable consumption programs.

But there is another method for calculation of the accounting price, i.e. the economic inference. For the economic inference of accounting prices the conception of induced labour input is very important. For the production of 1 Mwh's electricity the input of 500 kg's coal and the direct labour input of one hour are necessary. For the production of 500 kg's coal the input of 100kg's steel and the direct labour input of 5 hours are necessary. For the production of 100 kg's steel the input of 100 kg's coal and the direct labour input of 1 hour are necessary, and so on. Then the induced labour input of 1 Mwh's electricity is defined as the sum of these all labour inputs. As already mentioned, in the framework of the infinite and the long-term economic planning various kinds of labour force are aggregated into the homogeneous labour force. This is a rather strong assumption, but seems justifiable in the long run, since in the long run one kind of labour force can be transformed into the other kind of labour force. Diverse labour forces should be examined in the framework of the medium-term economic planning and the microeconomic planning. The shadow wage rate of particular labour force will be investigated in the following chapter (VI-2). Therefore in this subsection we assume the homogeneous labour force.

Using the Leontief model we can easily calculate the induced labour input.⁷⁾ In the case of the enlarged long-term optimal planning model we can obtain the induced labour input of one unit's product of i -th industry as follows.

$$\delta_{it} = v_t' J_t^{-1} h_i$$

where δ_{it} ; the induced labour input of one unit's product of i -th industry in t -th period.

v_t ; n -dimensional column vector.

$$v_t' = (v_{1,t}, v_{2,t}, \dots, v_{n,t})$$

$v_{i,t}$; the direct labour input of one unit's product of i-th industry in t-th period.

J_t ; (n,n) matrix.

$$J_t = \left(\frac{Q_t (1+u)}{Q_{t-1}} I_n - A_t - B_t + \frac{Q_t (1+u)}{Q_{t-1}} B_t \right)$$

h_i ; n-dimensional column vector.

$h'_i = (0, 0, \dots, 0, 0, \underset{\text{(i-th element)}}{1}, 0, 0, \dots, 0)$

V-2-5-3 Principles of determination of accounting prices at macroeconomic level

In V-2-4(4) the Author presented the procedures for calculation of accounting prices. Here he sums up these procedures in the following principles. It must be noticed, however, that in this subsection δ denotes the induced labour input and not the dual price. Needless to say, the induced labour input can be obtained by the use of the Leontief model. But even when the CPB can not construct the Leontief model, the CPB can obtain the accounting prices, if the CPB possesses the data about the induced labour input of each industry's product.

Principle 1 If i-th industry produces an output, then $p_{i,t} = \delta_{i,t}$; that is, the accounting price is equal to the induced labour input.

Principle 2 If i-th industry does not produce any output and all final demands are satisfied by import from the dollar district, then
 or
$$p_{i,t} = \delta_{j,t} \frac{(1+r_t^D) d_{i,t}^{mD}}{d_{j,t}^{eD}}$$

$$\left(\begin{array}{l} \text{accounting} \\ \text{price of} \\ \text{i-th indu-} \\ \text{stry's} \\ \text{product} \end{array} \right) = \left(\begin{array}{l} \text{induced la-} \\ \text{bour input} \\ \text{of j-th ma-} \\ \text{rginal exp-} \\ \text{orting ind-} \\ \text{ustry} \end{array} \right) \frac{\left(1 + \frac{\text{interest}}{\text{rate of}} \right) \left(\begin{array}{l} \text{import price of dollar} \\ \text{of i-th industry's} \\ \text{products} \end{array} \right)}{\left(\begin{array}{l} \text{export price of dollar of j-th} \\ \text{marginal exporting industry's} \\ \text{product} \end{array} \right)}$$

Principle 2' If i-th industry does not produce any output and all final demands are satisfied by import from the ruble district, then we can apply the principle 2 by substituting the notation R for the notation D.

Principle 3 If i-th industry exports its products, then $p_{i,t} = \delta_{i,t}$; that is, accounting price is the induced labour input.

Principle 4 If i-th industry produces an output whose production level is below the upper limit of production and also if products of i-th industry are simultaneously imported, then $p_{i,t} = \delta_{i,t}$.

This principle is also important, since many industries apply to this case. The i -th industry produces output in spite of positive level of import, since 1) the domestic production and the import are equally profitable or 2) the import is restricted by the upper limit of import and then the additional demand must be satisfied by the domestic production. In the first case we can apparently measure the value of products by the induced labour input. In the second case the import is more profitable. But the value of the additional one unit of product should be measured by the induced labour input of the domestic production, since the additional demand must be satisfied by the domestic production.

Now the Author adds some important principles. In some "infant industries" production is executed under the protection of the government even when the import is more profitable than the domestic production. In this case the principle 5 should be applied.

Principle 5 If i -th industry produces an output under the protection of the government in spite of profitability of the import, then the accounting price of i -th industry is determined by the principle 2 and 2', that is, by the international trade term.

Principle 6 Suppose that i -th industry produces its output to the upper limit of production. Then the additional final demand must be satisfied by the import. In this case the accounting price of i -th industry is determined by the principle 2 and 2'.

Principle 7 Suppose that i -th industry produces its output to the upper limit of production. Also suppose that i -th industry exports its products. In this case the additional final demand must be satisfied by the reduction of export to the dollar (or ruble) district. Then

$$p_{i,t} = p_{j,t} \frac{d_{i,t}^{eD}}{(1+r_t^D) d_{j,t}^{mD}}$$

where j denotes the marginal importing industry whose trade term is most unprofitable among all importing industries.

If i -th industry exports its products to the ruble district, we substitute the symbol R for D in the principle 7.

Principle 8 Accounting price of foreign currency is determined by the induced labour input which is necessary to produce outputs being exported for one unit of foreign currency. That is,

$p_{z,t}^D = p_j / d_{j,t}^{eD}$; where j denotes the marginal exporting industry whose production level is under the upper limit of production. $p_{z,t}^R$ is determined similarly.

General principle

If the additional one unit of the final demand is satisfied

by $\begin{cases} (i) & \text{increase of the domestic production} \\ (ii) & \text{increase of the import} \\ (iii) & \text{reduction of the export} \end{cases}$

then the accounting price is determined by $\begin{cases} (i) & \text{principle 1} \\ (ii) & \text{principle 2 and 2'} \\ (iii) & \text{principle 7} \end{cases}$

As already mentioned, the accounting price of one physical unit of product is obtained by discounting the accounting price of per capita term.

Finally, from the relation (24) of M-V-2-4, we can obtain a suggestion about the criterion of project evaluation. (24) means the following relation.

$$(\text{accounting value of optimal consumption program}) \geq (\text{accounting value of other consumption program}) \dots (1)$$

Needless to say, (consumption) = (output) - (input) for every efficient program. Therefore (1) means the following relation.

$$(\text{accounting surplus value of optimal production program}) \geq (\text{accounting surplus value of other production program})$$

This relation suggests that the maximization of surplus (net benefit) is an adequate criterion of project evaluation. Therefore the optimal infinite and the long-term optimal economic planning provide a theoretical foundation of the cost-benefit analysis in project evaluation. In the following part of this Thesis (VI-2) we investigate the problem of project evaluation at the industry's and the enterprise's level using the net benefit (surplus) as a criterion of project evaluation.

V-2-6 Medium-term optimal planning of the multi-sector model

The main subject of this Thesis is examination of the infinite and the long-term optimal planning. But the Author would like to present a short survey of the medium-term economic planning, since the accounting prices in near future exert a very strong influence on project evaluation. The comprehensive approach of the medium-term planning was thoroughly developed by Porwit(46) and next Bocian(3) calculated numerically an optimal plan of the comprehensive approach in the periods 1971-1975. Although the Reader can utilize these results in order to construct a medium-term optimal planning, the Author here presents an alternative model which is

little different from the original comprehensive approach. The Author's model has a more direct connection with the dynamic Leontief model. This subsection should be interpreted as an alternative formulation of the comprehensive approach. Main characteristics of the medium-term optimal economic planning are as follows.

- a) Accumulation for the after-planning periods is determined by the long-term optimal program.
- b) Complicated time-lag of investment is introduced into the model. $b_{i,j,t}^{d,t+u}$ denotes the investment input of i -th industry's domestic products in t -th period which is necessary to invest in t -th period in order to produce the additional one unit of j -th industry's products in $(t+u)$ -th period.
- c) For the investment the imported products which can not be substituted by the domestic products are also necessary. This case applies to the so-called licence production, where main equipments are imported. $b_{i,j,t}^{m,t+u}$ denotes the investment coefficient of i -th industry's imported products in t -th period in order to produce the additional one unit of j -th industry's products in $(t+u)$ -th period.
- d) In the beginning periods of the medium-term planning the output level of some industries is restricted by the upper limit, which is determined by the investment level of the before-planning periods. So we introduce the following condition. $X_{j,t} \leq \bar{X}_{j,t}$.
- e) For the production the imported current input which can not be substituted by the domestic products is also indispensable. This case applies to, for example, the oil import. $a_{i,j,t}^m$ denotes the current input coefficient of the imported products.
- f) On the other hand, in consumption we do not distinguish the domestic products from the imported products.
- g) There are many kinds of labour force. $v_{i,j,t}$ denotes the labour input coefficient of i -th labour force in t -th period for one unit's production of j -th industry.
- h) We do not introduce the resource constraint into the model. This is a difference from the original comprehensive approach, though some industries have the resource constraint. On the other hand we introduce the upper limit of output, which is determined by the resource condition. For example, the output of coal in 1983 can not exceed the upper limit ($\bar{X}_{j,t}$) and so on. $X_{j,t} \leq \bar{X}_{j,t}$.
- i) Production and import activity do not have time-lag. This is a great difference between the long-term optimal planning and the medium-term optimal planning. In the long-term optimal planning we have assumed the time-lag, since in the long-term planning we do

not introduce the imported investment input which has time-lag. In the long-term planning we also did not introduce the time-lag of domestic input for investment. On the other hand in the medium-term optimal planning we introduced $b_{i,j,t}^{d,t+u}$ and $b_{i,j,t}^{m,t+u}$. They replace the time-lag assumption of the long-term planning model.

j) The objective function has the same form as the enlarged long-term economic planning. That is,

$$\text{MAX } F = \sum_{t=0}^T F_t(c_t) = \sum_{t=0}^T \left(\min_i \left(\frac{c_{i,t} - k_{i,t}}{f_{i,t}} \right) + \sum_{i=1}^n k_{i,t} \right)$$

We can easily understand that the following form of maximization problem brings the same result as the above-mentioned form.

$$\text{MAX } G = \sum_{t=0}^T \sum_{i=1}^n c_{i,t}$$

$$\begin{aligned} \text{s.t. } c_{i,t} &\geq f_{i,t} \sum_{i=1}^n (c_{i,t} - k_{i,t}) && \begin{matrix} (i=1,2,\dots,n) \\ (t=0,1,\dots,T) \end{matrix} \\ c_{i,t} &\geq k_{i,t} && \begin{matrix} (i=1,2,\dots,n) \\ (t=0,1,\dots,T) \end{matrix} \end{aligned}$$

Function G has a linear form, therefore we can utilize the linear programming method.

The whole system of the medium-term optimal planning is presented in the mathematical appendix(M-V-2-6). In this model we can also obtain the accounting price by the dual price of the model.

FOOTNOTES TO CHAPTER V

- 1) The comprehensive approach was explored by Porwit(46). A simplified version of the comprehensive approach was published in the subsequent book by Porwit(47). A quite short summary of the comprehensive approach can be found in appendix of the textbook by Baka et al.(2).
- 2) See, Baka, Góra, Knyziak and Porwit(2), pp.226-229.
- 3) See, Ostrowski and Sadowski(44), especially chapter V.
- 4) See, Kantorovitch and Makarov(24), p.54. Also see, Japanese Agency of Economic Planning(21), p.296.
- 5) See, Takayama(54), p.594.
- 6) See, Bocian(3), p.73.
- 7) Almost every textbook contains the explanation of this procedure. In the recent literatures the most interesting study on the Leontief model can be found in the Brody's book().

GUIDING PARAMETERS FOR MULTI-LEVEL DECISION MAKING

In the section 1 of this chapter the Author makes a survey of investment criteria in the socialist economy and gives some comments to them. Since in this Thesis the Author intends to propose the cost-benefit approach as a criterion of the investment evaluation in the socialist economy, critical retrospect of the theory of investment efficiency criterion in the socialist economy is an indispensable premise for a further investigation of this subject. They are examined in VI-1-1.

From the Author's point of view the criteria of investment efficiency can be divided into two groups. The first is ratio approach and the second is difference approach. The ratio approach adopts the ratio of cost and benefit of the project as an index of investment efficiency. On the other hand the difference approach esteems the difference between benefit and cost of the project to be an appropriate index of the investment efficiency. In the history of the socialist economy, in general, the ratio approach has been advocated by many economists and also accepted by the socialist government. The Author of this Thesis considers that the difference approach is better than the ratio approach. The reason of the Author's view and consequently some critical comments to the ratio approach will be presented in VI-1-2. After surveying the theory of the investment criterion it becomes evident that these two approaches do not contradict each other in essence and that they can be integrated in the cost-benefit approach.

Next the Author will explain the essence of the difference approach and give a survey of this approach in VI-1-3.

In the difference approach the most serious difficulty exists in the evaluation of benefit of the project. Because of this difficulty the socialist government has used the ratio approach. This problem of evaluation of products and resources will be examined in the section 2 of this chapter. Firstly a survey of evaluation techniques of scarceness of products and resources will be presented in VI-2-1. Merits and demerits of various techniques will be examined there.

Secondly the Author will propose a method of scarceness evaluation of products and resources in the socialist economy in VI-2-2. The Author's proposition is closely related with the result of chapter V of this Thesis. The long-term economic plann-

ing should be investigated from the viewpoint of the national economy before everything else. Therefore the project evaluation at the micro-level of individual enterprises should be investigated not only in the side of enterprises but also in the side of the whole economy. Guiding parameters, which is proposed in VI-2-2, will systematically connect the both sides of the consideration.

In the chapter V we have examined the macro-accounting price which evaluates the products of each industry. The macro-accounting price corresponds to the "average" product of the industry. In the project evaluation we must know the accounting price of the particular product. Though the macro-accounting price is an index which is measured by the unit of labour hour, the micro-accounting price should be measured by the monetary unit. The Author presents in VI-2-4 and VI-2-5 the method for deduction of the micro-accounting prices from the macro-accounting price. The Author considers that the micro-accounting prices, which are proposed in this chapter, are practically applicable in the existing condition of the socialist economy.

VI-1 EVALUATION AND CRITERION OF INVESTMENT

VI-1-1 Ratio approach

VI-1-1-1 Traditional criterion by Novozhilov

In this subsection the Author firstly examines the Novozhilov's approach() of the investment efficiency criterion. Novozhilov begins his analysis in a case where the measurement of output of the project is unnecessary. Therefore he compares alternative variants which have the same outcome. This approach has a certain rationality when we keep in our mind the history of the socialist economy, because the measurement of the outcome is sensitively effected by the price of the product and also because the price of the product sometimes did not reflect the scarceness of the product in the history of the socialist economy. If the price is not reliable, then the measurement of the outcome becomes impossible, because a different variant brings a different outcome and products of a different variant have different characteristics. Therefore Novozhilov begins his analysis by measureing only the cost of the project without measureing the benefit of the project.

Premise of this approach is identity of the effect of alternative variants. The following conditions should be satisfied for

the identity of the effect of different variants. 1)Quantity of output should be same. 2)Qualitative structure of output should be same. 3)Location of the project should be same. 4)Date of production should be same. Besides these factors every project has some unmeasurable effects of socio-political character. Therefore the identity condition is very rigorous and seems to be satisfied only in limited cases.

If the identity condition is satisfied, then we can neglect the benefit-side of the project and concentrate our concern in the cost-side of the project. Though the identity condition is a bold simplification, even under this assumption there remains a difficult problem. For example, let us compare the following four variants, each of which brings the same outcome.

variant	annual exploitation cost (C) (million zloty)	investment outlay (K) (million zloty)
1	100	500
2	80	600
3	60	720
4	40	880

table 1

In order to compare these variants a simple formula has been used in the history of the socialist economy. That is the efficiency coefficient (E) and the recoupment period (T). When we compare the variant 1 to the variant 2 of table 1, the definition of the efficiency coefficient and the recoupment period is given as follows. Apparently we assumed here that $C_1 > C_2 > C_3 > C_4$ and $K_1 < K_2 < K_3 < K_4$.

$$T_{2/1} = \frac{K_2 - K_1}{C_1 - C_2} = 100/20 = 5$$

$$E_{2/1} = \frac{1}{T_{2/1}} = \frac{C_1 - C_2}{K_2 - K_1} = 20/100 = 0.2$$

$$\text{Similarly, } T_{3/2} = 120/20 = 6, \quad E_{3/2} = 20/120 = 0.1666$$

$$T_{4/3} = 160/20 = 8, \quad E_{4/3} = 20/160 = 0.125$$

Next the economic meaning of the recoupment period should be examined. If we adopt the variant 2 instead of the variant 1, then we must pay 100 million zloty more for the investment outlay, though on the other hand we can save 20 million zloty per year from the exploitation cost. Therefore after five years we can pay back all the burden of the additional investment cost by the sum

of the annual savings from the exploitation cost. A variant with longer recoupment period should be esteemed to be less effective. Needless to say, a variant with 50 years recoupment period should not be adopted. So the government or the Central Planning Board indicates the upper limit of the recoupment period to the industry concerned. In other words the government indicates the lower limit of the efficiency coefficient. Now let us assume that the government indicates the upper limit of the recoupment period to be seven years. That is, $T^*=7$ or $E^* = 1/7 = 0.1428$. In this case the enterprise should adopt the third variant of table 1, since the recoupment period of the variant 4 is longer than seven years. We call this criterion the standard criterion.

Standard criterion. Enterprise should adopt such a variant that possesses the longest recoupment period (the lowest efficiency coefficient) not exceeding the upper limit of the recoupment period (the lower limit of the efficiency coefficient).

We can illustrate the problem as follows.

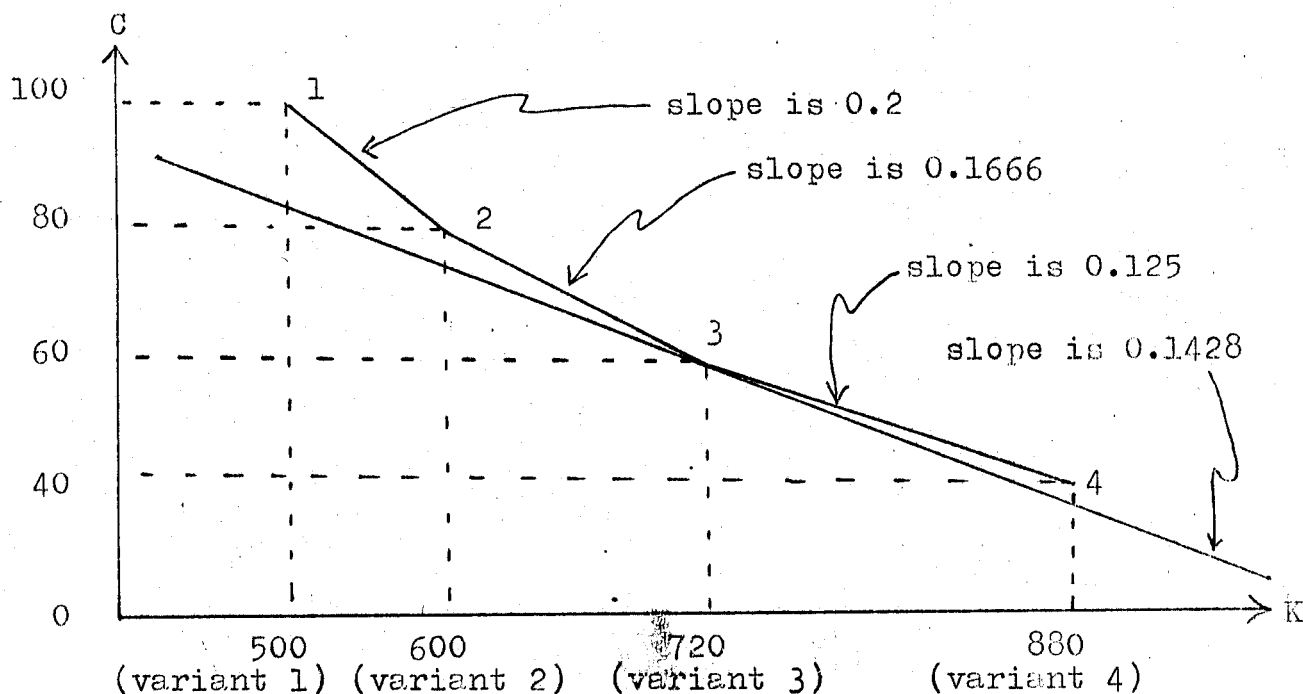


figure 1

We call the upper limit of the recoupment period the norm of the recoupment period. Similarly in the above-mentioned case the norm of the efficiency coefficient is 0.1428. Let us suppose that the government increases the norm of the efficiency coefficient from 0.1428 to 0.1818. Then the enterprise adopts the first variant instead of the third variant and the investment outlay by the enterprise decreases from 730 to 500. Investment outlay by the other enterprises will also decrease. Consequently the investment outlay of the industry decreases and may fall short of the

investment fund which the government assigned for the industry in advance. On the other hand if the government decreases the norm of the efficiency coefficient from 0.1428 to 0.11, then the enterprise will adopt the fourth variant. Consequently the investment outlay of the industry will exceed the investment fund which the government assigned for the industry in advance. Therefore the norm of the efficiency coefficient and, to the same thing, the norm of the recoupment period should be setted at such a level that the (ex ante) investment outlay by the industry equals the investment fund which the government assigned for the industry in advance.

We can interpret the Novozhilov's method in another way. The following alternative interpretation is more interesting from the viewpoint of theoretical economics. Here we introduce the following assumptions.

Assumption 1. $K_1 < K_2 < \dots < K_{m-1} < K_m$
 $C_1 > C_2 > \dots > C_{m-1} > C_m$

Assumption 2. $\frac{C_1 - C_2}{K_2 - K_1} > \frac{C_2 - C_3}{K_3 - K_2} > \dots > \frac{C_{m-1} - C_m}{K_m - K_{m-1}}$

So we have m kinds of variant. The goernment indicates the norm of the efficiency coefficient to be E^* . The enterprise should adopt the f-th variant as an optimal variant, when the following relation is maintained.

$$\frac{C_1 - C_2}{K_2 - K_1} > \frac{C_2 - C_3}{K_3 - K_2} > \dots > \frac{C_{f-1} - C_f}{K_f - K_{f-1}} \geq E^* > \frac{C_f - C_{f+1}}{K_{f+1} - K_f} > \dots > \frac{C_{m-1} - C_m}{K_m - K_{m-1}}$$

Since $K_{i+1} - K_i > 0$; for all i (i=1,2,...,m-1), we obtain the following relations.

$$\begin{array}{rcl} C_1 + K_1 E^* & > & C_2 + K_2 E^* \\ C_2 + K_2 E^* & > & C_3 + K_3 E^* \\ & \vdots & \\ C_{f-2} + K_{f-2} E^* & > & C_{f-1} + K_{f-1} E^* \\ C_{f-1} + K_{f-1} E^* & \geq & C_f + K_f E^* \\ C_f + K_f E^* & < & C_{f+1} + K_{f+1} E^* \\ & \vdots & \\ C_{m-1} + K_{m-1} E^* & < & C_m + K_m E^* \end{array}$$

Therefore we obtain the following equation.

$$C_f + K_f E^* = \min_{1 \leq i \leq m} (C_i + K_i E^*) \dots \dots \dots (1)$$

We can interpret E^* as the rent of capital. So the investment problem for the enterprise turns into the problem of cost minimization, because $(C_i + K_i E^*)$ can be interpreted as the sum of the annual exploitation cost and the annual rental cost. We can illustrate the problem as follows. Here we assume that variants can be changed continuously.

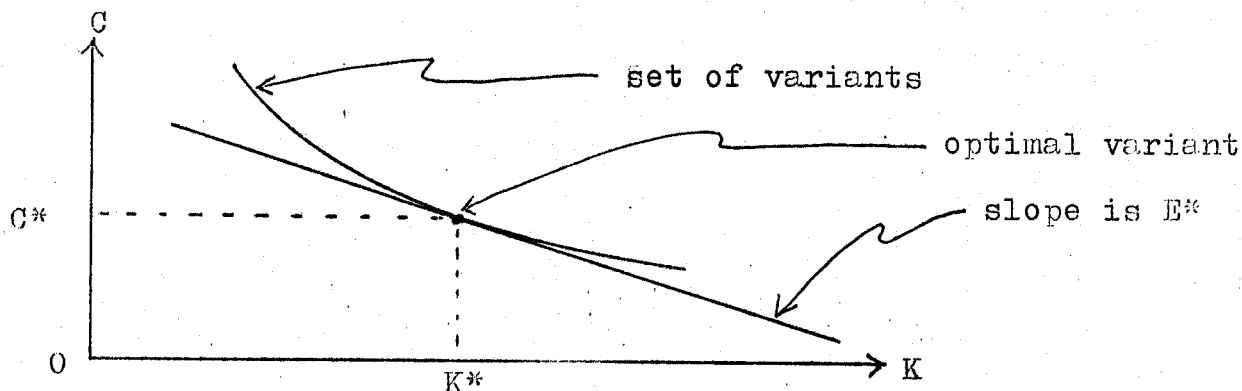


figure 2

The Author of this Thesis interprets the Novozhilov's method as a kind of cost minimization approach.¹⁾ So the project evaluation turns into the following problem.

$$\min_{1 \leq i \leq m} (C_i + E^* K_i) \quad \dots\dots\dots(2)$$

Finally the Author of this Thesis would like to add some comments to the Novozhilov's approach, i.e. to the standard criterion of the investment efficiency.

Firstly, the measurement of the outcome of the project is indispensable for the project evaluation, since a different project(variant) usually brings a different outcome. The Novozhilov's approach without measurement of benefit can be applied to very limited cases.

Secondly the most critical defect of the Novozhilov's approach is, from the Author's opinion, the neglect of exploitation period. Let us consider a case of power generation project. Suppose that there are three variants. The first is a water-power generation plant of one million KWh with 100 years of exploitation period. The second is an atomic-power generation plant of one million KWh with 40 years of exploitation period. The third is a thermal-power generation plant of one million KWh with 20 years of exploitation period. They are shown in table 2.

Apparently we can not judge which variant is the best project by the use of the Novozhilov's approach.

Thirdly the Novozhilov's approach can not take into account the time factor. Now let us examine the choice of power generation

variant	annual exploitation cost (C)	investment outlay (K)	exploitation period
water-power plant	10	1000	100
atomic-power plant	50	300	40
thermal-power plant	100	130	20

table 2

plant. Suppose that there are two variants, i.e. a thermal-power station with coal and a thermal-power station with oil fuel. Both plants have the same output of one million KWh and the same exploitation period of five years.

variant	annual exploitation cost (C)	investment outlay (K)	exploitation period
1. coal-type power station	10	50	5
2. oil-type power station	20	10	5

table 3

Suppose that the government decides to construct a power station by foreign currency debt and to finance the annual exploitation cost also by foreign currency debt. The government should pay back these foreign debt at the final year with interest of the international bank rate. Suppose also that the government indicates the norm of the efficiency coefficient to be 0.2, i.e. $ER = 0.2$. According to the Novozhilov's approach the enterprise should adopt the coal type power station. The reason is as follows.

$$10 + (0.2 \times 50) = 20 < 20 + (0.2 \times 10) = 22$$

But the sum of the payment in the final year depends on the international bank rate(i). The payment of the first variant is calculated as follows.

$$P_1 = (10 + 50) (1+i)^5 + 10 (1+i)^4 + 10 (1+i)^3 + 10 (1+i)^2 + 10 (1+i)$$

Similarly the payment of the second variant is calculated as follows.

$$P_2 = (20 + 10) (1+i)^5 + 20 \sum_{t=1}^4 (1+i)^t$$

Some numerical examples are shown in table 4. As is shown in table 4, if the international bank rate is less than 12%, then the coal-type generation plant is more efficient than the oil-type generation plant. But if the international bank rate is higher

than 13%, then the oil-type generation plant is more efficient than the coal-type generation plant.

variant	interest rate			
	11%	12%	13%	15%
coal-type power station	153.38	159.26	165.34	178.10
oil-type power station	155.10	159.92	164.87	175.18

table 4

The fourth comment concerns the total sum of the investment fund of particular industries. Apparently the Novozhilov's approach can not compare one project of A-industry with another project of B-industry. Consequently the Novozhilov's approach accepts the investment fund of the industry as a given condition. Needless to say, the comparison and the choice of projects among industries are also important.

The final critical comment concerns the level of the norm of the efficiency coefficient (E^*). In the socialist economy a different value of the norm is used for a different industry. The concrete values of the recoupment period(T^*) and the efficiency coefficient(E^*) of Poland and of the Soviet Union are presented in the appendix(M-VI-1-1-1). According to Rakowski(49) the value of T^* in 1960's differs from 1.5 to 8 years in Poland. Also according to the Gosplan(14) the value of T^* in 1961 differs from 3 to 10 years in the Soviet Union.

Why the efficiency coefficient should be discriminated among industries? Japanese economist Ishizu(20) investigated this problem and presented the following four reasons, though all of them seem to be not convincing.

The first reason is that the importance of industry differs from industry to industry and therefore the efficiency coefficient of the fundamental industry should be setted at a low level. Of course there is a difference in the importance of industry. But this problem should be investigated — from the Author's opinion — in the national economic planning at macro-level and not in the criterion of investment efficiency at micro-level.

The second reason concerns the scarceness of resources. According to the second reason the efficiency coefficient of the industry, which consumes scarce resources, should be setted at a high level and on the other hand the efficiency coefficient of the industry, which does not need scarce resources, should be setted

at a low level. From the Author's opinion the scarceness of resources should be taken into account in the measurement of inputs and outputs. So the scarceness of resources should be reflected in their accounting prices.

The third reason concerns the difference of the exploitation period. Usually the exploitation period of the light industry's project is shorter than that of the heavy industry project. Therefore according to the third reason the efficiency coefficient of the former industry should be set at a high level and that of the latter industry at a low level. But, as the Author already mentioned, the exploitation period differs from project to project even within the same industry. For example, the exploitation period of the water-power generation plant is longer than that of the thermal-power generation plant.

The fourth reason concerns the promotion of technological innovation. According to the fourth reason the efficiency coefficient of the industry, which needs a fast technological innovation, should be set at a low level. But usually the exploitation period of the industry, which has a faster technological progress, is short, so the efficiency coefficient of that industry should be set at a high level. This is a contradiction. From the Author's opinion the problem of technological innovation should be taken into account in the future price of inputs and outputs and also in the difference of exploitation period of the project.

Therefore the multiple system of the efficiency coefficient seems for the Author to be irrational. Of course the introduction of a single efficiency coefficient to the traditional criterion of the Novozhilov's approach brings a serious disorder. So a new approach of the efficiency evaluation of investment project should be explored. One proposition will be presented in VI-2 of this Thesis.

VI-1-1-2 Rakowski's approach

In the field of investment efficiency Polish economists made a contribution to the subject to a great extent. Firstly the Author would like to examine the Rakowski's approach. Rakowski(49) added some prominent contributions to the problem of investment efficiency criterion in 1963 as well as Fiszal(12) did.

Rakowski's most eminent departure from the traditional method is the consideration of benefit of the project. He defines the

investment efficiency as a ratio of cost and benefit of the project. That is, $E = \frac{\text{cost}}{\text{benefit}}$. Here emerges a rather difficult problem, i.e. the measurement of cost and benefit.

The Polish government in 1956 recommended the use of the following formula as an index of the effect of the project.²⁾

$$\frac{\sum_{t=1}^n P_t}{n} \quad \text{where } P_t ; \text{ the volume of the anticipated production in the } t\text{-th period.}$$

$n ; \text{ the operation (exploitation) period.}$

As Rakowski writes, in this formula is not taken into account the different degrees of importance of effects in different periods. Furthermore here we must be confronted with a difficult problem, i.e. by what P_t is measured? In general Rakowski tries to express the benefit of the project by physical units. Since we can not compare one unit of tractor with one unit of wheat, Rakowski's approach is also submitted to the identity condition. The identity condition of outcomes is a very strict condition, so Rakowski admits that in many cases in practice it is impossible to measure the benefit of the project by physical units.³⁾

Rakowski considers two cases in which it is better to measure the benefit not by physical units. These cases are the international trade and the multi-product. In the case of the products which are exportable or are able to replace imported goods, Rakowski argues that the value of the foreign exchange can be an indicator of the benefit. In the case of the multi-product also we can not measure the benefit of the project by using physical units. In the case of the multi-product Rakowski recommends the use of the sales price as an unit of measurement of the benefit. Here it should be kept in our mind that in the manufacture of consumer's commodities, sometimes, the sales price differs from the production costs owing to a large amount of the turnover tax. In this case Rakowski considers that "the most appropriate measure of the value will be given by the sales price less the turnover tax."⁴⁾

The second contribution by Rakowski is the explicit consideration of "indirect investment". Let us examine a case of the steel production plant. The new steel plant needs the additional input of coal. If the production capacity of coal is fully utilized and if the reduction of coal consumption in other branches is impossible, then the government should invest in the coal industry in order to supply the additional input of coal. Therefore the cost of the new steel plant should contain not only the direct investment cost but also the indirect investment cost in the coal

industry. Furthermore we can imagine another feedback. If this new steel plant decreases the cost of steel, then the production cost of coal also can be decreased and in turn the input cost of the steel industry will be reduced. So the inter-industry feedback is very complicated and therefore makes it very difficult to measure the net cost of inputs of the project.

According to Rakowski, there are three possibilities to solve this problem. The first is the use of the input-output table. But usually the input-output table is calculated depending on the data of industries, not depending on the data of particular goods. So we can not obtain the net input of individual products. Therefore practically the use of the input-output table has to be very restricted. In spite of this difficulty the input-output table can be — the Author of this Thesis considers — a powerful supplementary means if we add some devices. The Author will present these devices in VI-2 of this chapter.

The second possibility is to combine some related projects into one "complex" of projects. In some cases, e.g. kombinat, this method becomes very useful.

The third possibility is the pricing of raw materials on the basis of the foreign exchange conversion factor. This method also can be helpful in the evaluation of the cost of the project, though the application of this method is restricted to only tradable goods in the world market.

Which method is best? The answer depends on the situation of each project. The Author will examine once more this problem thoroughly in VI-2 of this chapter.

The fundamental form of the formula for investment efficiency index by Rakowski is as follows.

$$\text{MIN } E = \frac{\frac{1}{T} K + C}{P}$$

where T; the recoupment period. K; the investment outlay.
C; the annual exploitation cost. P; the annual output.

Needless to say, if we esteem P as constant, then there is no difference between the Rakowski's criterion and the Novozhilov's criterion. But as already mentioned, the benefit(F) of the project may be measured by value-term. Therefore the annual output(P) of one variant can be different from that of the other variant.

VI-1-1-3 Fiszal's approach

The most eminent characteristics of the Fiszal's approach

is the explicit consideration of temporal distribution of cost and benefit and therefore the introduction of the discount factor. The fundamental formula of the Fiszal's approach(12) for investment efficiency is as follows.

$$E = \frac{K + \sum_{t=1}^n \frac{C_t}{(1+r)^t}}{\sum_{t=1}^n \frac{P_t}{(1+r)^t}} \dots\dots\dots(1)$$

where E ; the index of investment efficiency.
 K ; the investment outlay.
 C_t ; the exploitation cost of t-th period.
 P_t ; the size of output of t-th period.
 r ; the discount rate.

Fiszal considers that the output(P_t) may be measured by the physical unit as well as by the value-term. Fiszal seems to investigate the problem of variant comparison of the same project and therefore he pays little attention to the problem of measurement of outputs treating the subject within a purely mathematical framework.

We can deduce the formula (1) in another way. Firstly we calculate the annual allocation of capital(A). In other words we suppose that the enterprise pays back the investment outlay every year by the same amount(A).

$$\sum_{t=1}^n \frac{A}{(1+r)^t} = K. \quad \text{Therefore,} \quad A = K \frac{r(1+r)^n}{(1+r)^n - 1}$$

Annual version of the index(E_a) of the investment efficiency is as follows. $E_a = \frac{A + C}{P} \dots\dots\dots(2)$

By the definition of A, we can calculate the following transformation.

$$\begin{aligned} E_a &= \frac{K \frac{r(1+r)^n}{(1+r)^n - 1} + C}{P} = \frac{K + C \frac{(1+r)^n - 1}{r(1+r)^n}}{P \frac{(1+r)^n - 1}{r(1+r)^n}} \\ &= \frac{K + \sum_{t=1}^n \frac{C}{(1+r)^t}}{\sum_{t=1}^n \frac{P}{(1+r)^t}} \end{aligned}$$

So if C_t and P_t are constant over time, i.e. $C_t = C$ and $P_t = P$ for all periods, then we obtain the following relation.

$$E_a = E \dots\dots\dots(3)$$

If we suppose that the benefit(P) is constant, then the

criterion by Fiszal turns into the minimization problem of the discounted present value of the total cost of the project. Fiszal considers that cost and benefit of the project should be discounted to the present value at a certain discount rate. This is the most prominent contribution to the subject.

The Author of this Thesis has some critical comments on the Fiszal's approach and these comments will be presented in VI-1-2 within the framework of general consideration. The second contribution by Fiszal is that he systematically connects the individual investment evaluation to the whole economy, especially when he investigates the determination of the discount rate. This subject will be once more examined in VI-2-1.

VI-1-1-4 Criteria for investment efficiency in the Polish economy in 1970's

In the beginning of 1970's the economic reform was introduced into Poland. This economic reform concerned many fields such as economic organization, wage and premium fund, price formation and so on. In the field of investment also new criteria for the project evaluation have been put into practice. The following criterion has been adopted for the evaluation of new construction of industrial projects which do not need the foreign currency credit.⁵⁾

$$E = \frac{\sum_{t=0}^m a_t (P_t - C_t)}{\sum_{t=0}^m a_t K_t} \dots\dots\dots(1)$$

where E; coefficient of investment efficiency.

P_t ; value of outputs in t-period.

C_t ; exploitation cost in t-period.

K_t ; investment outlays in t-period.

a_t ; discount factor in t-period: $a_t = \frac{1}{(1+r)^t}$.

r; discount rate.

m; calculation period: $m = b + n$.

b; construction period of the project.

n; exploitation period of the project.

The definition of the exploitation period(n) is given as follows taking into account the discount factor. The exploitation period(n) is defined as a period in which the sum of the discounted values of amortization of each period is equal to the investment outlays.

$$\frac{sK}{1+r} + \frac{sK}{(1+r)^2} + \dots + \frac{sK}{(1+r)^n} = K$$

where s ; amortization rate.

$$\text{Then, } \frac{s}{r} \left(1 - \left(\frac{1}{1+r} \right)^n \right) = 1$$

$$\text{Then, } (1+r)^n = \frac{s}{s-r} = \frac{s(s+r)}{(s-r)(s+r)} = \frac{s(s+r)}{s^2 - r^2} = \frac{s+r}{s}$$

, since r^2 is very small.

Therefore we can obtain the following equation.

$$n = \frac{\log \frac{s+r}{s}}{\log (1+r)} \dots \dots \dots (2)$$

In the Polish economy of 1970's three kinds of the discount rate are applied in practice to various industries. That is, 3% for the most primary industries, 5% for the projects concerned with modernization of old plant and 8% for almost manufacturing industries.⁶⁾

The criterion for the investment efficiency(1) is applied to the evaluation of new construction of manufacturing projects which do not heed the foreign credit. In 1970's the following simplified form of criterion for the investment efficiency is also applied in practice.

$$E = \frac{P - C}{J(r+s) + Br} \dots \dots \dots (3)$$

where P ; anticipated value of annual outputs.

C ; anticipated annual current cost, not including amortization and interest for credit but including the 20% excess burden of wage fund.

r ; discount rate.

s ; average rate of amortization.

B ; average value of circulation means.

J ; value of investment outlays, which is multiplied by the freezing coefficient(Z), taking into account the supplementary investment and the cost of licence and schooling: $Z = 1 + \frac{br}{2}$.

b ; construction period of the project.

In the mid of 1970's in the Polish economy we can observe a change in the efficiency criterion especially in the field of projects with international cooperation. Let us suppose that it is possible to choose a solution between one variant of domestic production without foreign licence and the other variant of domestic production with foreign licence. The following criterion is applied in practice.

$$E = \sum_{t=0}^m a_t (P_t^L - C_t^L - K_t^L) - \sum_{t=0}^m a_t (P_t - C_t - K_t) \dots (4)$$

Here the symbols with (L) denote the characteristics of the variant with the purchase of the foreign licence. Symbols without (L) denote that of the variant without the foreign licence.

If $E > 0$, then the purchase of the foreign licence is more effective. On the other hand if $E < 0$, then the domestic production without the foreign licence is more effective.

We call this type of criterion the difference criterion. In the field of international projects the difference criterion was to be generally applied in practice in the Polish economy.

So we can say that in 1970's some attempts to improve the investment evaluation have been made in Poland.

VI-1-2 Some comments on the ratio approach of the investment efficiency criterion

VI-1-2-1 Incomparability of benefit

The most remarkable difference between the ratio approach and the difference approach exists in the evaluation of benefit of the project. As the Author already mentioned in VI-1-1-1, in the traditional criterion by Novozhilov the evaluation of benefit is unnecessary, because this approach supposes the identity of outcomes of various variants. But it is apparent that the identity condition is not satisfied usually and if the identity condition is not satisfied, then we must compare different outcomes of various variants.

Therefore the applicability of the ratio approach is limited to the very small sphere of the project evaluation. Of course, as already mentioned in VI-1-1-4, some recent criteria of the ratio approach in Poland possess the property of comparability of benefit. This recent tendency indicates the fact that the comparison of outcomes of various variants is inevitable in the evaluation of investment projects.

VI-1-2-2 Divisibility of projects

As the second defect of the ratio approach, the Author points out the implicit assumption of divisibility of project. In other words the ratio approach assumes the linear production function.

This problem concerns especially the increasing returns to

scale. The Author presents here the following simple example. Suppose that the government decides to supply 100 thousand ton of ethylene in 1980's. Also suppose that there are three variants, characteristics of which are shown in table 5.

variant	production ability (thousand ton);(P)	investment outlay (million zloty);(K)	exploitation cost (million zloty);(C)
1	40	104	48
2	50	150	50
3	60	210	54

table 5

Investment is done and completed in 1980 and the operation of the plant will last from 1981 to 1990. Firstly we compare each variant using the Fiszal's approach. Numerical calculation is presented in the mathematical appendix(M-VI-1-2-2). If we adopt the Fiszal's criterion, then from the numerical result of the mathematical appendix we can apparently conclude that the third variant is best with every discount rate from 5% to 14%. That is, the third variant has the highest efficiency (the lowest value of E) of the Fiszal's criterion. So should the government adopt the third variant? The answer is "no". The reason is as follows.

If the government adopts the third variant, then the government also adopts the first variant, because it is impossible to construct two third of the variant 3. We call this property indivisibility. So we must choose the optimal solution from the following combinations.

combination	production ability (P)	investment outlay (K)	exploitation cost (C)
A (variant 2 + variant 2)	100	300	100
B (variant 1 + variant 3)	100	314	102

table 6

The combination A is superior to the combination B. Therefore the government should construct two plants of the variant 2.

Needless to say, if we assume the divisibility of the production facilities and therefore the linearity of the production function, then the best variant is the variant 3. But unfortunately under the condition of the modern technology it is difficult to assume the divisibility of the production facilities. In this point the ratio approach has a serious disadvantage.

VI-1-2-3

Additivity of index and the systems approach

In the ratio approach it has no sense to add each index. Now let us examine the numerical example, which the Author presented in the previous subsection VI-1-2-2. Here $E(0.08)_2$ denotes the Fiszal's index of the variant 2 under the condition of 8% discount rate. We can not imagine any economic meaning in such a calculation as $E(0.08)_1 + E(0.08)_3$.

On the contrary in the difference approach the addition of each index has a real economic meaning. The Author presents one example of the index of the difference approach. Here q denotes the price of the output and we assume that $q=2$ (for example, the price of one ton of the product is 2,000 zloty).

$$Q = -K + \sum_{t=1}^{10} \frac{(qP - C)}{(1+r)^t}$$

$Q(0.08)_2$ denotes the value of the above-mentioned index(Q) of the variant 2 under the condition of 8% discount rate. The value of each index is shown in table 7.

variant	output	discount rate				
		5%	8%	10%	12%	14%
1	40	143	110	92	76.8	63
2	50	236	185	157	132.5	111
3	60	299	232	195	162.9	134

table 7

We can interpret $Q(r)_2$ as the surplus(net benefit) of the variant 2 under the condition of $r\%$ discount rate. Addition of each surplus has an apparent economic meaning. From table 7 we can easily confirm the following relation.

$$Q(r)_2 + Q(r)_2 > Q(r)_1 + Q(r)_3 \quad \text{for } r = 5, 8, 10, 12 \text{ and } 14\%$$

This relation indicates that the surplus from two plants of the variant 2 is greater than the sum of the surpluses from the variant 1 and the variant 3. Therefore we can conclude that the construction of two plants of the variant 2 is more advantageous than the joint construction of the variant 1 and the variant 3.

Now let us suppose that the government decides to supply not 100 thousand ton but 600 thousand ton of the product. In this case we can imagine the following three combinations. From table 8 we can understand that in every case of different discount rates from 5% to 14% the combination C has the largest surplus. Therefore the government should construct 10 plants of the variant 3.

value of Q of the difference approach

combination	output	discount rate				
		5%	8%	10%	12%	14%
A; 15 plants of variant 1	600	2145	1650	1380	1152	945
B; 12 plants of variant 2	600	2832	2220	1884	1590	1332
C; 10 plants of variant 3	600	2990	2320	1950	1629	1340

table 8

The difference approach has an absolute advantage over the ratio approach, when we investigate a complex of projects. Let us examine a complex of projects which consists of the electric power generation and the aluminium production. The production of aluminium needs an enormous input of electricity, so these two projects are closely connected each other. In this case it is quite difficult to use the ratio approach for the project evaluation. On the contrary, using the difference approach, we can interpret the sum of the surpluses of two projects as an index of investment efficiency of the complex of these two projects.

In the Polish literature this idea is called as systems approach.⁷⁾

In modern industries the index of the difference approach can be a powerful measure especially when we use it in the systems approach. Now the Author would like to mention a defense for the difference approach with respect to the so-called over-investment tendency. As Ostrowski and Sadowski (43) mentioned, in the history of the Polish economy there has been a tendency of over-investment and therefore a rational index for investment efficiency has been searched for. When we use the index of the difference approach, a certain fear of over-investment will come out. Apparently the bigger variant has the greater surplus. So the fear of over-investment is not groundless. But if we use the index of the difference approach in the systems approach, then we can remove the fear of over-investment. Let us suppose that there are two variants; the first is the smaller project with the higher coefficient of the ratio approach and the second is the bigger project with the lower coefficient of the ratio approach. The absolute value of the surplus of the variant 2 is greater than that of the variant 1, although the ratio index of the variant 1 is greater than that of the variant 2. Which variant the government should adopt? The answer depends on the scale of the final demand and therefore on

the scale of production. Corresponding to the scale of the final demand the government should choose the best combination of these two variants by the help of the systems approach. If the final demand is sufficiently large comparing to the production capacity of each variant, then the government should construct several plants of the first variant in spite of the lower absolute value of the surplus of one unit of the first variant. In this way we can remove the fear of over-investment.

Finally the Author presents important concluding remarks. We should not confront these two approaches each other. If we pay sufficient attentions to the indivisibility and the nonlinearity of the project, the both approaches bring the same result. As mentioned above, we can synthesize the ratio approach with the difference approach. Nevertheless the Author feels that the index of the difference approach is more convenient in the project evaluation.

VI-1-3 Difference approach

VI-1-3-1 Contribution by Kantorovich

In 1959 Kantorovich published his famous book(23). His main contribution to economic science concerns — as widely known — the problem of evaluation of resources in the socialist economy. In his book Kantorovich strictly distinguishes the objectively determined valuation of resources from the existing price of resources or the cost price of resources. Kantorovich argues that the objective valuation of resources should be deduced from the optimal solution of economic planning. His propositions have exerted a great influence upon many economists in socialist countries.

Besides this contribution Kantorovich presented an innovation in the sphere of investment efficiency criterion. The Author of this Thesis regards him as the first proponent of the difference approach of the project evaluation in the socialist economy. Kantorovich argues that in order to evaluate an investment project the following procedures are necessary; 1) investment outlays of each period and outcomes of each period should be calculated according to the objective valuation of resources, 2) these values should be discounted to the present value by the rate of the standard efficiency (the normal efficiency), 3) the present value of outcomes and the present value of the cost should be compared each other.⁸⁾

As Kantorovich mentions in his book, in the simplest case the standard formula by Novozhilov and the Kantorovich's criterion of

the discounted present value bring the same conclusion.⁹⁾

We explain this proposition in the following way.

Firstly notations are defined as follows.

$K_t(i)$; value of investment outlay of i -th variant in t -period.

$P_t(i)$; value of output of i -th variant in t -period.

$C_t(i)$; value of exploitation cost of i -th variant in t -period.

$B_t(i)$; value of net benefit of i -th variant in t -period: i.e.

$$B_t(i) = P_t(i) - C_t(i) - K_t(i) .$$

E ; standard efficiency (normal efficiency).

r ; discount rate.

Maximization of the discounted present value (DPV) of i -th variant is defined as follows.

$$\text{MAX}_i \text{ DPV}(i) = \sum_{t=0}^n \frac{1}{(1+r)^t} (P_t(i) - C_t(i) - K_t(i)) \dots\dots(1)$$

In the mathematical appendix(M-VI-1-3-1-A) the Author of this Thesis proves that the above-mentioned criterion of maximization of the discounted present value is equal to the following minimization criterion by Novozhilov under some assumptions.

$$\text{MIN}_i (C(i) + E K_0(i)) \dots\dots\dots(1')$$

In the mathematical appendix the following assumptions are introduced.

Assumption 1 Investment is executed only in the 0-th period.

Assumption 2 Benefit of i -th variant is constant over time.

Assumption 3 Operation lasts forever.

Assumption 4 The value of outcome of each variant is same and does not change over time.

Assumption 5 $E = r$.

The criterion (1') is the very criterion by Novozhilov. Therefore as Kantorovich says, in the very simple case (i.e. mathematically saying under the assumptions from 1 to 5) the maximization criterion of the discounted present value brings the same result as the traditional criterion by Novozhilov brings.

The Author would like to call once more the Reader's attention to the fact that in the criterion by Kantorovich the outcome (P) is measured by the objective valuation, not by physical units. This is a great difference between the Novozhilov's approach and the Kantorovich's approach.

Assumptions from 1 to 5 are very strict conditions. In a more complicated and therefore realistic case the result of these two approaches becomes different each other.

Kantorovich argues that if the discounted present value is positive, then the project should be adopted. So the Author regards Kantorovich as a pioneer of the cost-benefit approach in socialist countries.

In order to explain the Kantorovich's approach at length, the Author of this Thesis cites one numerical example from the Kantorovich's book. This example is presented in the mathematical appendix(M-VI-1-3-1-B).¹⁰⁾

VI-1-3-2 Cost minimization approach by Kornai

In this subsection the Author will examine the Kornai's approach for investment decision problem which was developed in his book(27) in 1967.

Kornai intends to introduce the mathematical method into the economic planning in a consistent way. The first contribution by Kornai should be attributed to this approach. The second contribution by Kornai can be found in the calculating methods of the accounting prices. Distinguishing strictly the computational prices (the accounting prices) from the existing prices, Kornai argues that not the existing prices but the accounting prices should be used in the calculation of the investment problem. The Author of this Thesis fully agrees with this view and is greatly enlightened by the Kornai's approach.

Since the problem of the accounting prices will be discussed in VI-2, in this subsection the Author will concentrate the consideration only into the criteria for investment efficiency. Mathematical structure of the Kornai's criterion is very simple. We can sum up his criterion in the following form.¹¹⁾

$$\begin{array}{ll} \text{MIN} & c'x \quad \dots\dots\dots(1) \\ \text{s.t.} & Ax \geq b \quad \dots\dots\dots(2) \end{array}$$

where c' ; n-dimensional row vector of the social costs.
 x ; n-dimensional column vector of the activity level.
 A ; (m,n)matrix of the constraint coefficients.
 b ; m-dimensional column vector of the constraints.

Now the Author explains verbally the Kornai's criterion.
 x_i denotes the level of i-th activity (i-th variant) and is measured by the number of machines of i-th activity. Let us imagine an example of the textile industry. There are two kinds of spinning machine. A modern but expensive spinning machine(x_1) can produce 200 ton of cotton per year and an old-type but cheap spinning

machine(x_2) can produce 100 ton of cotton. Output target is set by the central national economic plan at the level of 100,000 ton per year. In this case the output constraint is as follows.

$$200 x_1 + 100 x_2 \geq 100,000$$

Investment decisions are, needless to say, constrained by the amount of the investment fund. The central national economic plan assigns the investment fund to each industry and this amount of the investment fund is regarded as given by the decision maker of investment. Purchase of one modern spinning machine requires 3 million zloty and that of one old-type machine requires 2 million zloty. The amount of the investment fund for the textile industry is 150 million zloty. Mathematically saying we obtain the following relation.

$$3 x_1 + 2 x_2 \leq 150$$

For each machine some imported installations may be necessary and for these installations the industry must pay dollars. A modern machine requires 700 dollars and an old-type machine requires 400 dollars. The foreign currency quota for the textile industry is also assigned by the central national economic plan.

Suppose that the quota for the textile industry is 39,000 dollars.

$$700 x_1 + 400 x_2 \leq 39,000$$

The industry should choose an optimal solution with the minimum "social input" under these constraints. Kornai defines the social cost (c_i) as follows.

$$c_i = r K_i + C_i \quad \text{.....(3)}$$

where K_i ; investment outlay for one unit of i -th machine.

C_i ; operation cost of one unit of i -th machine.

r ; rental on capital.

K_i contains the supply price of one unit of machine and the construction cost of factory per machine. C_i consists mainly of wage and material cost. In the case of Kornai the value of r is set at the level of 20%. So the criterion(1) turns into the following form(1').

$$\text{MIN} \sum_{i=1}^m (r K_i + C_i) x_i \quad \text{.....(1')}$$

The equation (3) is very similar to the criterion by Novozhilov. But the Author considers that the essence of the Kornai's approach is different from the Novozhilov's approach. The reason is as follows. The minimization criterion(1') can be interpreted as a maximization criterion of the surplus of the industry's

activity. Since the output target is fixed by the central national economic plan, the value of benefit is also given for the industry. So the minimization of cost turns into the maximization of surplus. Apparently the criterion(1') belongs to the difference approach. We can say that the Kornai's criterion (1') is very similar to the criterion of the cost-benefit approach. So the Author regards Kornai as a pioneer of the cost-benefit approach in the socialist economy as well as Kantorovich.

The Kornai's approach has a difficulty as well as the cost-benefit approach. That is the settlement of the value of the parameters, especially the rent of capital, the accounting wage rate and the accounting foreign exchange rate. These problems will be discussed in VI-2 of this chapter.

VI-1-4 Cost-benefit analysis

In recent years so-called cost-benefit analysis has been developed for the project evaluation especially in developing countries. Firstly Little and Mirrlees(33) published a manual of project evaluation from OECD in 1968. In 1972 Dasgupta, Sen and Marglin(8) published a guideline for project evaluation from UNIDO; where UNIDO is an abbreviation of United Nations Industrial Development Organization. Then a series of controversy about project evaluation in developing countries arose.¹²⁾

In Poland also this problem has been discussed,¹³⁾ since a similar method to the Little-Mirrlees approach has been introduced into the foreign trade policy of Poland in these 10 years.

Thereafter the Little-Mirrlees approach has accumulated many experiences in case studies of developing countries, e.g. Trinidad, Kenya, Ghana, Mauritius and so on.¹⁴⁾

It is not the main theme of this Thesis to scrutinize the controversy between the OECD approach and the UNIDO approach. The controversy concerns wide range of arguments. In this subsection the Author compares these two approaches placing the focus on the measurement of the accounting prices and the treatment of the foreign trade prices.

The Author considers that both approaches contain some difficulties when they are applied in the socialist economy. These difficulties should be conquered. Therefore the Author now intends to propose an alternative approach of project evaluation in the socialist economy.

VI-1-4-1 The UNIDO approach

The UNIDO group proposes as an objective of planning the maximization of the value of the final aggregate consumption; i.e. $\text{MAX} \sum_{i=1}^n P_i C_i$, where P_i denotes the accounting price of i-th consumption good and C_i denotes the amount of i-th consumption good. The UNIDO group considers that P_i should be measured by the "willingness to pay" of consumer. In this point they belong to the orthodox line of the cost-benefit approach such as Dasgupta-Pearce (9). If the condition of perfect competition is satisfied, the willingness to pay is reflected in the market price. This is the first difficulty in the socialist economy. Market mechanism can not be a ruling tool for operation of a planned economy. Apparently we can not observe any competitive price in the socialist economy with few exceptions. In many cases the retail price does not reflect the willingness to pay of consumer.

The willingness to pay may change when the project supplies a large amount of the new product. Therefore the UNIDO approach requires the information about the demand curve. Generally saying the measurement of demand curve is impossible. This is the second difficulty.

According to the UNIDO approach the accounting price of producer's good is determined as follows.

$$q_i = P \frac{\partial f(x_1, x_2, \dots, x_n)}{\partial x_i}$$

where P; accounting price of output(consumer's good).

f; production function.

x_i ; amount of i-th input.

q_i ; accounting price of i-th input.

Under the assumption of perfect competition the value(q_i) is same in all enterprises and q_i is given by the market price. The socialist economy does not possess any market for the producer's good. In this point the UNIDO approach faces the third difficulty. Furthermore the assumption of neoclassical production function is rather strong.

Under the assumption of perfect competition the shadow wage rate is given by the market wage rate. Of course the UNIDO group investigates the shadow wage rate with existence of unemployment. In the socialist economy the wage rate is determined by the government from a socio-political viewpoint taking into account

the collective consumption. In this point we must look for another method of measurement of the shadow wage rate.

On the criterion of project evaluation there is no difference between two groups. That is, the criterion of both groups is the maximization of surplus (net benefit) at the enterprise level.

$$\text{MAX (net benefit)} = (\text{benefit}) - (\text{cost})$$

Therefore these two approaches belong to the difference approach of project evaluation. The difference consists in the measurement of the accounting prices.

VI-1-4-2 The OECD approach

The OECD group also proposes as a policy objective the maximization of the standard of living, i.e. the final consumption. They consider that other elements such as full-employment are also important policy objectives.

The OECD group argues that every tradable good should be measured by its international price. This principle is adequate in developing countries, where only small number of domestic industry exists and many consumer's goods are imported from abroad. On the contrary in the developed socialist economy there exist almost all kinds of industry of the consumer's goods and there the consumer's goods are mainly domestically produced. In this situation the condition of the domestic production is a more important element than the condition of the international trade.

According to the OECD approach the non-tradable good should be measured by its marginal contribution to the production of the tradable good. Therefore the non-tradable good is also expressed by the unit of foreign currency. But the measurement of the marginal contribution is a rather difficult task.

The largest difficulty of the OECD approach in the socialist economy is the assumption of free trade in the international market. Foreign trade of the socialist economy is usually restricted by the mutual agreement between two countries. In this point the upper limit of export and import plays a deterministic role.

VI-1-4-3 General line for new approach

Generally speaking the difficulty of the UNIDO approach consists in the assumption of perfect competition of market mechanism and the difficulty of the OECD approach consists in the assumption of free trade in the international market.

In this Thesis the Author distinguishes three kinds of industry(or product). That is, the domestically producing industry(or product), the importing industry(or product) and the exporting industry(or product). But this naming is rather misleading. As will be mentioned later, the industry(or product) which exports its output, may belong to the category of the domestically producing industry(or product).

For the distinction of these three categories the conception of additional final demand is important. If the government intends to increase the final consumption of i -th product by additional one unit, then by what means this increment can be realized? If this increment can be supplied by the domestic production, then this product is called domestically producing product. Suppose that the main part of i -th industry's products is imported from abroad and this import is restricted by the upper limit. Then the additional one unit of the final demand should be domestically produced. In this case this industry belongs to the category of the domestically producing industry.

Suppose now that the main part of i -th industry's products is domestically produced and that the domestic production is restricted by the upper limit. Then the additional one unit of the final demand must be supplied by import. In this case this industry belongs to the category of the importing industry.

Now suppose that i -th industry's products are domestically produced and that the domestic production is restricted by the upper limit. Also suppose that this industry exports its output. Then the additional one unit of the final demand must be supplied by the reduction of its export. In this case this industry belongs to the category of the exporting industry.

General line. In our approach the accounting price of the domestically producing industry's products is measured by the cost of domestic production. The accounting price of the importing industry's products is measured by the import price. That of the exporting industry is measured by the export price.

This principle also applies to the determination of the accounting price of particular goods.

Generally saying if i -th good is domestically produced and if the domestic production is not restricted by the upper limit of production, then the product should be measured by the cost of domestic production. Therefore the Author does not adopt the Little-Mirrlees approach in the case of the domestically producing

industry(or product). Since the domestic products in the socialist economy possess a quite different character comparing to the same products in the capitalist economy, the measurement of the domestic products by the international price faces a serious difficulty. Also in this point the domestic products should be evaluated by the cost of domestic production.

Our approach is, therefore, quite different from the OECD approach, though the Author admits that our approach is strongly influenced by the OECD approach. International trade is a quite difficult problem in the economic planning of the socialist economy. This Thesis is intended to answer this problem and to offer a theoretical foundation of project evaluation in a socialist economy which is open to the world economy.

VI-2 GUIDING PARAMETERS OF MULTI-SECTOR MODEL

In this section the Author investigates a method of deducing the micro-accounting prices from the macro-accounting indices, whose deduction was examined in chapter V.

Firstly in subsection VI-2-1 the Author describes the general character of the macro-accounting index, since it is necessary to make clear the difference between the macro-accounting index and the accounting price at micro-level.

In subsection VI-2-2 the Author presents a scheme of the deduction of the accounting prices. There the Author proposes two alternative methods. The first method is simple but rather rough. The second method is not so simple but more accurate.

In subsection VI-2-3 the Author examines the problem of average macro-accounting index and average macro-accounting wage rate (average shadow wage rate).

In subsection VI-2-4 the Author investigates the simplified method of deducing accounting prices. This first method has a two-stage structure.

In subsection VI-2-5 the Author investigates the extended

method of deducing accounting prices. This second method has a three-stage structure.

In the last subsection VI-2-6 the Author considers the character of guiding parameters at micro-level.

VI-2-1 Character of macro-accounting index

Although in chapter V we have investigated the method of deducing the macro-accounting index, yet we have not examined the accounting price of particular goods. For the determination of the particular accounting price the consideration of character of the macro-accounting index is indispensable. The macro-accounting index has the following characters.

1) aggregativeness

The infinite and the long-term model of economic planning contains a limited number of industry, since the scale of economic model is restricted by the capacity of computer, the ability of planning staffs and the availability of statistiscal data. In case of Polish literature Bocian(3) constructed a model with 23 sectors of 5 years planning. Planning Agency of Japan calculated a model with 34 sectors of 15 years planning. Anyway the number of sectors should be limited to small. Let us examine a suppositional case of the transportation mechninery sector.

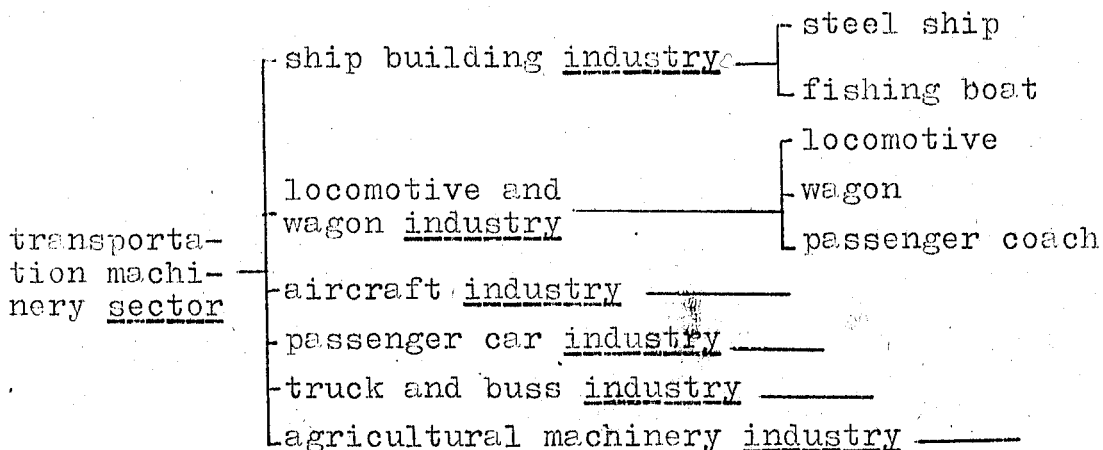


figure 3

We must remember that the infinite and the long-term economic planning can deduce only the macro-accounting index for the transportation machinery sector. From the macroeconomic planning we can not obtain the accounting index for the passenger car industry, to say nothing of particular type of automobile.

2) unit of measurement of macro-accounting index

As already mentioned in V-2-5-3 the macro-accounting index is measured by the unit of labour hour. Even the exclusively imported good (e.g. products of the crude oil industry) is measured by the labour hour. The Author now presents a suppositional example of the macro-accounting index. The macro-accounting index of one zloty's product in the textile sector is, to say, 60 seconds, i.e. one minute. This means that in order to produce domestically one zloty's product in the textile sector the induced labour input of 60 seconds is necessary. Suppose also that the domestic price of 100cc of crude oil is one zloty and the international price of 100cc of crude oil is 2 U.S.cents. Now suppose that the economy earns 2 cents by exporting the textile products of 1.2 zloty. Then the accounting index of the crude oil sector is calculated as follows. $(60 \text{ seconds}) \times 1.2 = (72 \text{ seconds})$. That is, the accounting index of one zloty's product (i.e. 100cc) of the crude oil sector is 72 seconds.

This type of accounting index is inconvenient in practical calculation, since the economic calculation in practice is done by zloty (i.e. by the monetary term) and not by the labour hour term. Therefore we must transform this macro-accounting index into the macro-accounting index in value term. This transformation will be investigated in the following parts of this section.

3) accounting index of labour power

In the macroeconomic model the accounting index of products is measured by labour hour. In other words the labour hour is numéraire. Therefore the accounting index of labour power is unity. This is also inconvenient in economic calculation in practice. The transformation of the accounting index of labour power into the shadow wage rate in value term will be examined in the following parts of this section.

VI-2-2 Structure of determination of micro-accounting price

Structure of determination of micro-accounting price depends on the institutional structure of the economy. In case of the developing country the economy has a quite simple structure and therefore the Central Planning Board can directly deduce the micro-accounting prices from the macro-accounting index. We call this system the T W O - S T A G E S T R U C T U R E of determination of the micro-accounting price. This system is illustrated in the following figure.

two-stage structure of determination of the micro-accounting price

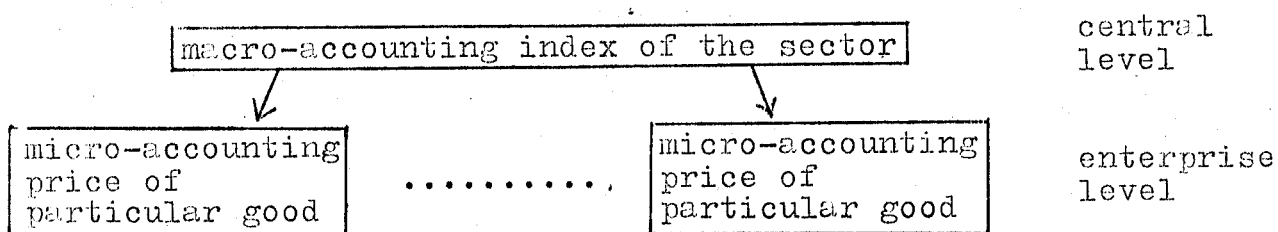


figure 4

On the other hand in the developed economy the above-mentioned two-stage method can not be applied. The reasons are as follows.

- 1) Firstly the developed economy has a very diversified structure of products. The number of products may exceed one million. Such a large amount of information can not be accumulated at the central level and then a certain decentralization of information becomes inevitable.
- 2) Secondly every sector of macro-economic planning contains several industries with different character. For example, the transportation machinery sector contains the ship building industry as well as motorcycle industry. These two industries have different structure of input-output coefficient. Accounting price should be determined by all-round aspects, i.e. by technological aspect, resource aspect, scarceness of labour power, foreign demand and so on, although the infinite and the long-term macro-economic planning take into account only the labour power constraint and the terms of international trade. If we deduce the micro-accounting price directly from the macro-accounting index, then we can not reckon with other important factors. In this point we should distinguish different industries within a sector.
- 3) Thirdly the terms of international trade play a very important role in determination of the macro-accounting index of some industries, e.g. the coal industry or the oil industry. On the other hand in many industries the macro-accounting index is determined by the induced labour input. For example, the macro-accounting index of the transportation machinery sector may be determined by the induced labour input, although the economy imports jet airplanes from abroad. Apparently in this case the jet airplane should be evaluated by the import price. Let us examine another example. The accounting index of coal sector is influenced by the export price, though some kinds of coal are not exported and consumed only domestically. In this case such kind of coal should be measured by the induced labour input. Therefore for the developed socialist economy we must explore the more accurate calculating method of

the micro-accounting price. Now the Author presents the following THREE - STAGE STRUCTURE of determination of the micro-accounting price.

three-stage determination of the micro-accounting price

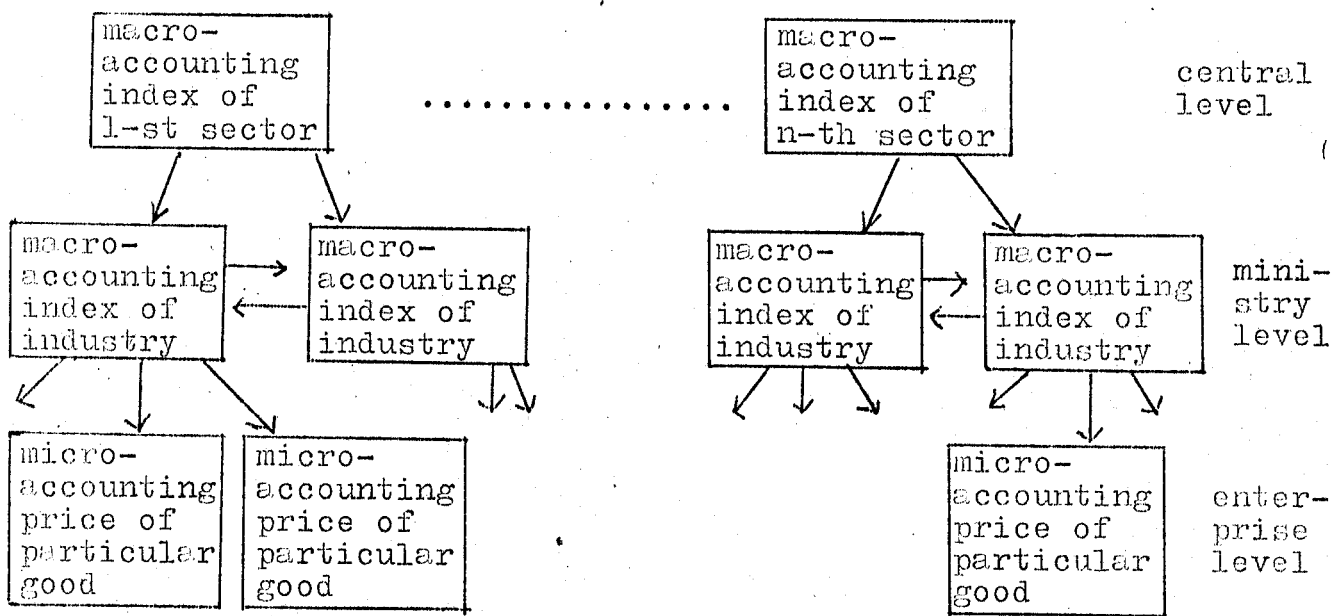


figure 5

The Author proposes this three-stage determination of the micro-accounting price as an adequate method of determination of accounting prices in the modern developed socialist economy. But firstly the Author would like to investigate the two-stage method of determination of the accounting price, because the two-stage method is simple and useful to understand the essence of the argument.

VI-2-3 Average macro-accounting index

The starting point of determination of micro-accounting prices is prescription of average macro-accounting index. Average macro-accounting index concerns three categories, i.e. average value of macro-accounting index of all sectors, average shadow wage rate and accounting foreign exchange rate.

The Author presents here a schema which explains the procedure of calculation of the micro-accounting price in the case of two-stage determination of the micro-accounting price.

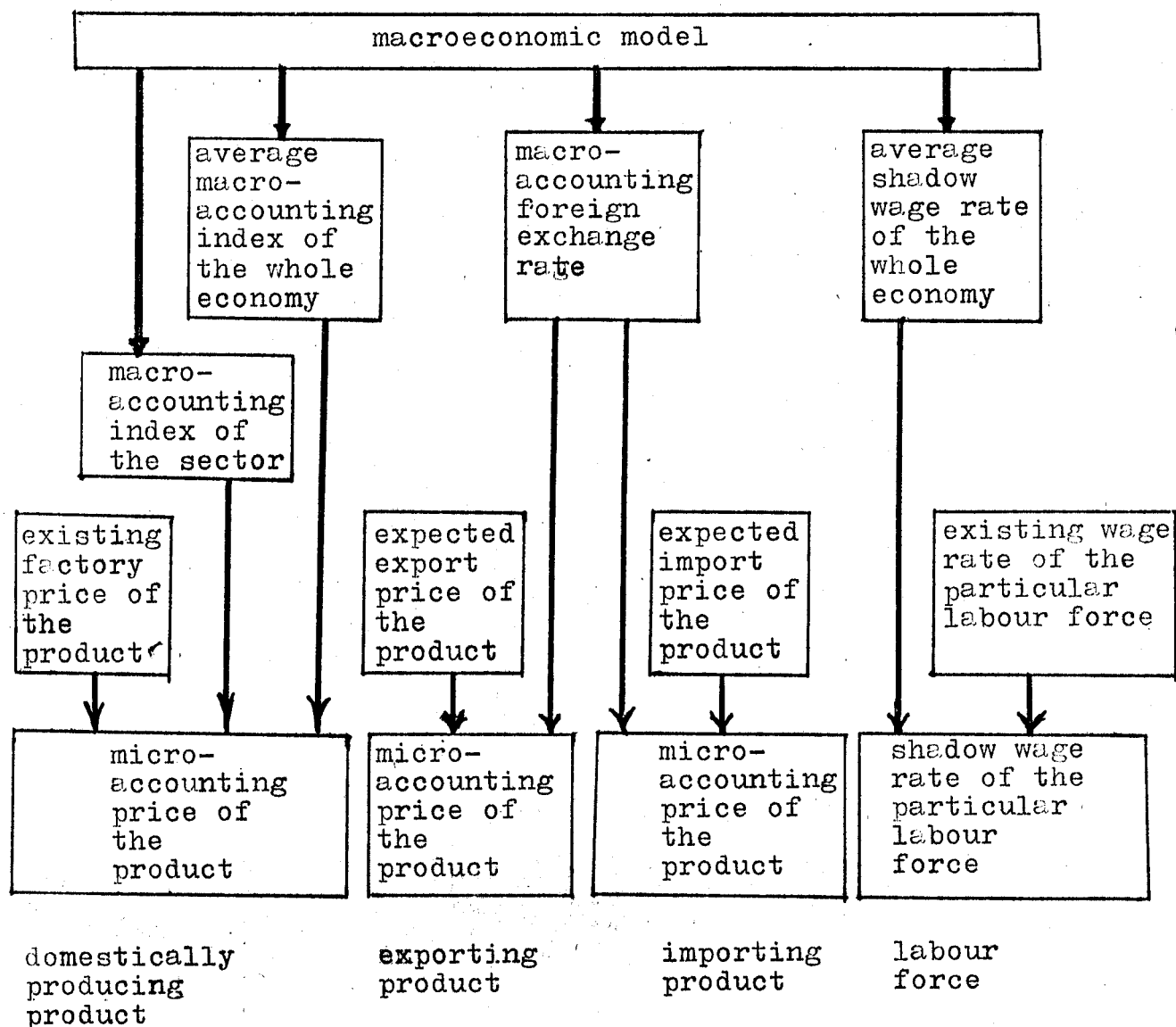


figure 6

1) Average value of macro-accounting index of the whole economy

In chapter V we have deduced the macro-accounting index of each sector. The macro-accounting index is considered to reflect the scarceness of products of the sector. Now we consider the scarceness of national products in the whole economy. In a closed economy without foreign trade the average value of macro-accounting index of the whole economy indicates the induced labour input which is necessary to produce one zloty of national product. In an open economy with foreign trade we can interpret the average value of macro-accounting index as an index of the scarceness of one zloty of national products. In both cases the average value of macro-accounting index is measured by unit of labour hour. The formula of the average value of macro-accounting index of the whole economy is as follows.

$$V = \frac{\sum_{i=1}^n P_i \varepsilon_i}{\sum_{i=1}^n \varepsilon_i} = \sum_{i=1}^n P_i \frac{\varepsilon_i}{G} \dots\dots\dots(1)$$

where V; average value of macro-accounting index of the whole economy.

P_i ; macro-accounting index of i-th sector.

ε_i ; final net output of i-th sector's products which is measured by zloty.

G; national products, i.e. $G = \sum_{i=1}^n \varepsilon_i$.

n; number of sectors.

Therefore the average value of macro-accounting index of the whole economy is obtained as a weighted average of macro-accounting index of each sector. P_i is expressed by unit of labour hour, for example, 55 seconds or 70 seconds. So V is also expressed by unit of labour hour. Hereafter we suppose that V is 60 seconds.

Here some Readers may imagine another formula. That is,

$$V' = \frac{L}{G} =: \frac{(\text{total amount of labour input})}{(\text{national products})} \dots\dots\dots(2)$$

where L; total amount of labour input in the whole economy which is measured by unit of labour hour.

In a closed economy without foreign trade the formula (1) and the formula (2) bring the same result.

Property VI-2-3-A In a closed economy, which does not contain the international trade, the formula (1) and the formula (2) bring the same value, i.e. $V = V'$.

Proof. See mathematical appendix(M-VI-2-3-A).

In an open economy, however, V is not equal to V' . V' reflects only the scarceness of labour resource. But in an open economy the scarceness of national products is also influenced by the international trade. Therefore in an open economy the scarceness of national product (V) does not coincide with the scarceness of labour resource (V'). In an open economy we adopt the formula (1) in order to determine the scarceness of national products.

2) average shadow wage rate of the whole economy

Shadow wage rate is a very important conception in project evaluation. Many disputants of the cost-benefit analysis argue that the existing wage rate is not an adequate index for project evaluation and that the CPB should use the shadow wage rate, which

reflects the scarceness of labour resource. This argument has been developed from the experience in the developing countries. The Author considers that this statement is also valid in the socialist economy.

The scarceness of labour power in the long-run is determined by the reproduction cost of labour force, although in the short term and the medium term the scarceness of labour force is influenced by the existing amount of the particular labour force. So in the short and the medium term the scarceness of labour is influenced by the mobility of labour force. Therefore in the medium-term economic planning the shadow wage rate is deduced directly from the dual price of the particular labour force. But in the long-term economic planning the model does not possess the dual price of labour power. Therefore from the nature of the long-term economic planning the scarceness of labour force must be determined by the reproduction cost of the labour force. So in the model of the long-term economic planning the scarceness of labour is reflected in the macro-accounting index of consumption goods.

In the socialist economy, as well known, the reproduction cost of labour force consists of two parts, i.e. the private consumption and the collective consumption. Every Reader certainly agrees with the opinion that the educational cost and the health service cost should be included in the shadow wage rate. Moreover in the project evaluation in the long-run the category of the collective consumption should be extended. National defense and general administration of the state are also indispensable for the reproduction of labour force, since without them the society can not operate. At last in the present socialist economy there is a quite difficult problem in definition of value of the private consumption. Many socialist governments grant a large amount of subsidy for fundamental consumption goods, especially for foodstuffs. Therefore the existing cost of the private consumption is undervalued. On the other hand the existing cost of the private consumption is overvalued in the case of industrial consumption goods. In many socialist countries so-called turn-over tax is levied in industrial consumption goods, for example, television set or automobile. The production cost of automobile is quite smaller than its retail price. Therefore we must reduce the turn-over tax from the private consumption fund in order to obtain the real reproduction cost of labour force.

Summing up the average shadow wage rate of the whole economy

is determined by the following formula. Here the average shadow wage rate of the whole economy is measured by the value-term, i.e. by zloty.

$$\left(\begin{array}{l} \text{average shadow} \\ \text{wage rate of the} \\ \text{whole economy} \end{array} \right) = \left(\begin{array}{l} \text{per capita} \\ \text{administration} \\ \text{cost of the state} \end{array} \right) + \left(\begin{array}{l} \text{per capita} \\ \text{collective} \\ \text{consumption} \end{array} \right) + \left(\begin{array}{l} \text{per capita subsidy} \\ \text{for fundamental} \\ \text{consumption goods} \end{array} \right) - \left(\begin{array}{l} \text{per capita} \\ \text{turn-over} \\ \text{tax} \end{array} \right) + \left(\begin{array}{l} \text{per capita} \\ \text{private} \\ \text{consumption} \end{array} \right)$$

Or

$$W = S_1 + S_2 + S_3 - T + C \quad \dots\dots\dots(3)$$

If we neglect the saving by households, the per capita private consumption is equal to the existing average wage rate. Therefore,

$$W = S_1 + S_2 + S_3 - T + w \quad \dots\dots\dots(3')$$

where w; the existing average wage rate.

Ostrowski and Sadowski(43) argue that in the project evaluation the existing wage rate should be multiplied by 20% of additional wage cost. The Author of this Thesis considers that the average shadow wage rate may be greater than the existing average wage rate by 30% or more, although the Author does not present any numerical consideration.

Numerical calculation of the formula (3) may be rather complicated. Fortunately we have another method of calculation of the average shadow wage rate of the whole economy. So now we examine the second method.

As will be mentioned later, the input-output coefficient is measured not by the retail price but by the factory price. Therefore the national products (G) does not contain the turn-over tax (T) and the state's subsidy (S₃). So we have the following identity.

$$\left(\begin{array}{l} \text{national} \\ \text{products} \end{array} \right) = (\text{investment}) + (\text{administration outlay}) + (\text{collective consumption outlay}) + (\text{wage fund})$$

Or

$$G = I + L S_1 + L S_2 + L w$$

where L; the total number of labourer.

The total reproduction cost of labour force is equal to the amount (L S₁ + L S₂ + L w). Therefore the average shadow wage rate of the whole economy can be obtained as follows.

$$W = \frac{L S_1 + L S_2 + L w}{L} = \frac{G - I}{L} \quad \dots\dots\dots(4)$$

Or

$$(\text{average shadow wage rate}) = \frac{(\text{national products}) - (\text{investment})}{(\text{number of labourer})}$$

3) Macro-accounting foreign exchange rate

In the long-term planning the macro-accounting foreign exchange rate is equal to the exchange rate of the exporting sector whose export is not restricted by the upper limit of export. In the long-term economic planning the CPB should intend to specialize its export in the sector which has the most favourable terms of foreign trade. This sector has the first priority in export. This sector may be the coal sector. Suppose that the domestic factory price of one ton of coal is 500 zloty and the international price of that is 50 dollars. Then the exchange rate of the coal sector is 10 zloty per dollar. Notwithstanding the coal sector in the most profitable sector in the international trade, the long-term economic plan can not completely specialize its export in the coal sector, since the coal industry has the upper limit of production which is determined by the resource constraint. Then the long-term economic plan has to search the second exporting sector whose terms of international trade is most favourable except the coal sector. The second exporting sector may be the transportation machinery sector, which has the exchange rate of 50 zloty per dollar. This sector has the second priority in export. As same as in the case of the coal sector, the long-term economic plan can not specialize its export in the transportation machinery sector, since the automobile export has the upper limit which is determined by the situation in the foreign market. Now the coal sector and the transportation machinery sector export its products up to the upper limit. Next the long-term economic plan must look for the third exporting sector. Finally the long-term economic plan determines the last exporting sector in order to earn the sufficient amount of foreign currency for the payment for import. We call this last exporting sector the marginal exporting sector. The M A R G I N A L E X P O R T I N G S E C T O R is the exporting sector whose terms of trade is worst among all exporting sectors. The marginal exporting sector may be the textile industry or the light industry, whose exchange rate is 70 zloty per dollar. Then the macro-accounting foreign exchange rate is 70 zloty per dollar.

In the Polish literature the similar argument was already presented by Trzeciakowski(56)(57). According to his model,¹⁵⁾ the foreign exchange rate should be setted at the level where the exchange rate of the marginal exporting product is equal to the exchange rate of the marginal importing product. At this level of the foreign exchange rate the export earning is equal to the import

payment. Therefore we can say that in the Trzeciakowski's model the foreign exchange rate is determined by the exchange rate of the last product which is exported in order to earn the payment for import. So the Author of this Thesis considers that in essence there is no difference between the Trzeciakowski's model and the model of this Thesis.

VI-2-4 Simplified two-stage determination of micro-accounting prices

This two-stage method is very simple and is not adequately applicable to the modern developed socialist economy. On the other hand through this two-stage method the Reader can easily understand the essence of the argument. So this subsection has an introductory character to the following subsection VI-2-5.

Principle of determination of micro-accounting prices is very similar to that of chapter V. The simplified two-stage method determines micro-accounting prices by the following four principles.

Principle 1 If j -th product of i -th sector will be domestically produced in t -th period, then

$$Q_j^t = \frac{P_i^t Q_j^0}{V^t (1+s)^t (1+q)^t}$$

where Q_j^t ; the micro-accounting price of j -th product of i -th sector in t -th period.

P_i^t ; the macro-accounting index of i -th sector in t -th period.

V^t ; the average value of macro-accounting index of the whole economy in t -th period.

Q_j^0 ; the existing factory price of j -th product of i -th sector in the first period. That is, the existing price in 1980.

s ; the discount factor.

q ; the growth rate of population.

Here P_i^t and V^t are measured by the unit of labour hour and Q_j^0 is measured by the value-term, i.e. zloty. Therefore Q_j^t is expressed by the value-term. Economic meaning of the discount factor(s) was already discussed in chapter V.

From this principle apparently the micro-accounting price in future is strongly influenced by the existing factory price. Now the Author presents a numerical example. Suppose that P_i^t of the automobile sector is 50 seconds and V^t is 60 seconds. Also suppose that the existing factory price of a truck is 300 thousand zloty,

$s = 0.04$ and $q = 0.01$. The the accounting price of this truck in 1990 is $\frac{50}{60} \times \frac{300,000}{(1.04)^{10} \times (1.01)^{10}} = 152,895$.

That is, the accounting price 10 years hence is about half of the existing price.

Principle 2 If j -th product is completely imported, or if the domestic production of j -th product is restricted by the resource constraint and then the additional demand must be satisfied only by import, then

$$Q_j^t = \frac{M^t d_{j,t}^m}{(1+s)^t (1+q)^t}$$

where M^t ; the macro-accounting foreign exchange rate in t -th period.

$d_{j,t}^m$; the expected import price of j -th product in t -th period.

Principle 3 If the domestic production of j -th product is constrained by the upper limit and then the additional final demand of j -th product can be satisfied only

by reducing its export, then

$$Q_j^t = \frac{M^t d_{j,t}^e}{(1+s)^t (1+q)^t}$$

where $d_{j,t}^e$; the expected export price of j -th product in t -th period.

Principle 4 The shadow wage rate of i -th labour force in t -th period is determined by the following formula.

$$w_i^t = \frac{w^t w_{i,0}}{\frac{\sum_{i=1}^L L_{i,t} w_{i,0}}{\sum_{i=1}^L L_{i,t}}}$$

where w_i^t ; the shadow wage rate of i -th labour force in t -th period.

w^t ; the average shadow wage rate of the whole economy in t -th period,

$w_{i,0}$; the existing wage rate of i -th labour force.

$L_{i,t}$; the amount of i -th labour force in t -th period.

L ; the number of labour force.

If the employment structure will not change in future, then we obtain the following simpler formula.

Principle 4' If the employment structure does not change in future, then the shadow wage rate of i -th labour force in t -th period is determined by the following

formula.

$$w_i^t = \frac{w^t}{w^0} w_{i,0}$$

where w^0 ; the existing average wage rate of the whole economy.

The Author presents here a numerical example. Suppose that w^t is 8000 zloty and the existing average wage rate (w^0) is 5000 zloty. If the existing wage rate ($w_{i,0}$) of typist is 4000 zloty, then the shadow wage rate of typist in t-period is 7200 zloty.

VI-2-5 Extended three-stage determination of micro-accounting prices

The infinite and the long-term economic planning can include only 20 or 30 sectors, though in many countries a more detailed input-output table is constructed. In the case of Japan the Planning Agency calculates an input-output table with 160 industries.

The three-stage method utilizes this detailed input-output table. The two-stage method can not take into account the change of scarceness within a sector. Therefore the three-stage method is proposed in order to handle this intra-sectoral change of scarceness. The Author presents here one simple example. In future the energy will be more expensive and as a result of this tendency the aluminium product will be relatively more expensive comparing to the copper product, although these two industries belong to the same sector, i.e. the metal sector.

In the following figure 7 the Author presents a schema which explains the procedure of determination of accounting price in the three-stage method, however, only in the case of the domestically producing product. Since the Reader can easily imagine the same schema in the case of exporting product and importing product, the Author does not present the schema of procedure in the case of exporting and importing product.

First step

The first step is the calculation of the cost using the macro-accounting index.

i) If j-th industry produces domestically its output, then the accounting index of this industry in t-th period in the first step ($q'_{j,t}$) is calculated as follows.

$$q'_{j,t} = \sum_{i=1}^{160} a_{i,j}^t P_{i,t} + V_{j,t}$$

where $a_{i,j}^t$; the input coefficient of a more detailed input-output table in t-th period with e.g. 160 industries.

three-stage determination of micro-accounting prices in the case of domestically producing product

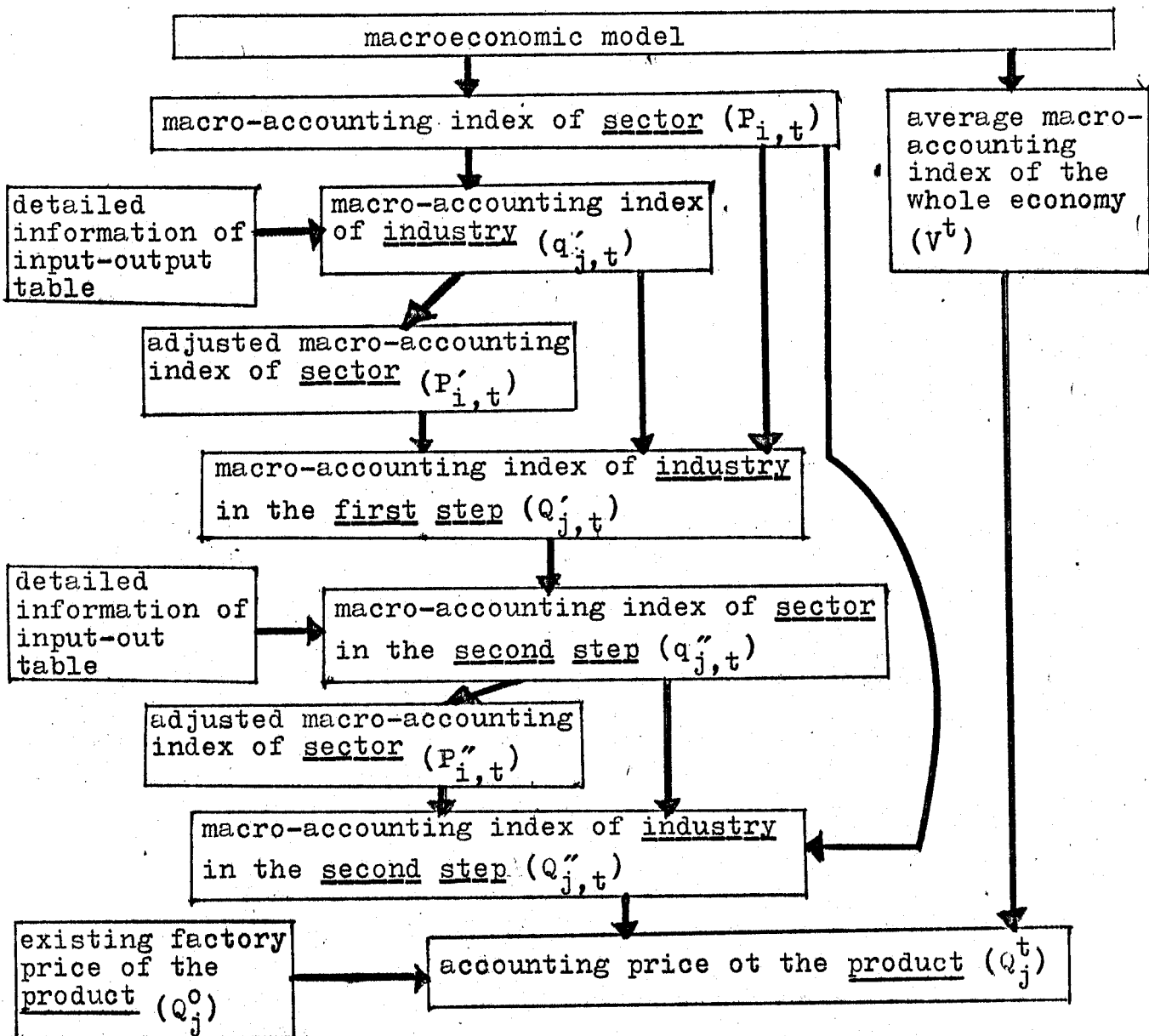


figure 7

$V_{j,t}$; the direct labour input coefficient in j -th industry in t -th period.

$P_{i,t}$; the macro-accounting index of i -th industry. If i -th industry belongs to k -th sector, then $P_{i,t} = P_{k,t}$.

Therefore in the first step we substitute the macro-accounting index of sector for the macro-accounting index of industry. Apparently $q'_{j,t}$ is measured by the unit of labour hour.

Here we must remember that $P_{i,t}$ is the macro-accounting index with a classification of 20 or 30 sectors, so we have

$$\sum_{i=1}^{160} a_{i,j}^t P_{i,t} = (a_{1,j}^t + a_{2,j}^t + \dots + a_{h,j}^t) P_{1,t}$$

$$+ (a_{h+1,j}^t + \dots + a_{k,j}^t) P_{2,t} \\ + \dots + (a_{m,t}^t + \dots + a_{160,t}^t) P_{n,t}$$

where n ; number of sectors

ii) If j -th industry's product is completely imported, or if the domestic production of j -th industry is restricted by the upper limit of the resource constraint, then the macro-accounting index of j -th industry in the first step is calculated as follows.

$$q'_{j,t} = q'_{M,t} M^t d_{j,t}^m$$

where $d_{j,t}^m$; the expected import (dollar) price of one zloty's products of j -th industry in t -th period.

M^t ; the macro-accounting foreign exchange rate in t -th period.

$q'_{M,t}$; the macro-accounting index of marginal exporting sector in t -th period.

iii) If the domestic production of j -th industry is restricted by the upper limit of the resource constraint and then the additional final demand can be satisfied only by reducing its export (e.g. in the case of the coal industry), then the macro-accounting index of j -th industry in the first step is calculated as follows.

$$q'_{j,t} = q'_{M,t} M^t d_{j,t}^e$$

where $d_{j,t}^e$; the expected export (dollar) price of one zloty's products of j -th industry in t -th period.

In every case from (i) to (iii), $q'_{j,t}$ is measured by the unit of labour hour. Now we must adjust $q'_{j,t}$ in order to maintain the consistency with the sector's macro-accounting index. Firstly we calculate the macro-accounting index of i -th sector in the first step.

$$P'_i = \frac{\sum_{j=1}^m q'_{j,t} X_j}{\sum_{j=1}^m X_j}$$

where m ; number of industry which belongs to i -th sector.

X_j ; { i) gross output of j -th industry which is measured by zloty, if j -th industry domestically produces its output.
ii) amount of import of j -th industry's products which is measured by zloty, if j -th industry's products are completely imported.

Then the macro-accounting index of j -th industry in the first step is calculated as follows.

$$Q'_{j,t} = \frac{P_i}{P'_i} q'_{j,t}$$

second step

i) If j-th industry domestically produces its output, then the macro-accounting index of the industry in the second step is as follows.

$$q_{j,t}'' = \sum_{i=1}^{160} a_{i,j}^t Q_{i,t}' + V_{j,t}$$

Differing from the first step, in the second step $Q_{i,t}'$ has different values in every 160 industries.

ii) If j-th industry's products are completely imported, or if the domestic production of j-th industry is restricted by the upper limit of the resource constraint, then the macro-accounting index of j-th industry in the second step is calculated as follows.

$$q_{j,t}'' = Q_{M,t}' M^t d_{j,t}^m$$

iii) If the additional final demand is satisfied only by reducing the export of j-th industry's products, then

$$q_{j,t}'' = Q_{M,t}' M^t d_{j,t}^e$$

In the same manner as in the first step we obtain the macro-accounting index of i-th sector in the second step ($P_{i,t}''$). The macro-accounting index of j-th industry in the second step is calculated as follows.

$$Q_{j,t}'' = \frac{P_i}{P_i''} q_{j,t}''$$

third and further steps

In the same manner as in the first and the second step we can obtain the accounting index of each industry in the third and further steps. Practically saying we may consider that the second step brings a quite satisfactory approximation of scarceness of every industry. Of course, the CPB can execute this iterative process many times, if the CPB has a large number of planning stuffs and a large computer. It seems, however, that two or three times of iteration is the limit of planning capacity.

Now we have obtained the macro-accounting index of each industry ($Q_{i,t}''$). From this index we can obtain the micro-accounting prices in the same manner as in the simplified two-stage method.

$$Q_j^t = \frac{Q_{i,t}'' Q_j^0}{V^t (1+s)^t (1+q)^t}$$

Principle 1 If j-th product of i-th industry will be domestically produced in t-th period, then the micro-accounting price of j-th product is calculated by the above-mentioned formula.

Principle 2 and principle 3 are very similar to that of the

simplified two-stage method. But in the three-stage method the macro-accounting foreign exchange rate is determined by the exchange rate of the marginal exporting industry, on the other hand in the simplified two-stage method the macro-accounting foreign exchange rate was determined by the exchange rate of the marginal exporting sector. This is a difference between two methods.

Principle 2 If j-th product is completely imported, or if the domestic production is restricted and then the additional final demand is satisfied only by import, then the micro-accounting price of j-th product is calculated as follows.

$$Q_j^t = \frac{\hat{M}^t d_{j,t}^m}{(1+s)^t (1+q)^t}$$

where $\hat{M}^t$; the accounting foreign exchange rate of the marginal exporting industry.

Principle 3 If the additional final demand of j-th product can be satisfied only by reducing its export, then the micro-accounting price is

$$Q_j^t = \frac{\hat{M}^t d_{j,t}^e}{(1+s)^t (1+q)^t}$$

Principle 4 This principle is same that of principle 4 of the simplified two-stage method. That is, in the three-stage method the shadow wage rate of particular labour force is determined by the principle 4 of the simplified two-stage method.

VI-2-6 Some comments on the micro-accounting price

- 1) Basis of accounting prices in future is the existing prices in the beginning period, i.e. the first year. Therefore we must confirm that the existing prices adequately reflect the scarceness of product, i.e. that the existing prices are not extremely distorted.
- 2) In the measurement of the output of the industry the state's subsidy and the turn-over tax should be excluded.
- 3) The input coefficient also exerts a decisive influence.
- 4) The accounting foreign exchange rate also plays an important role. Therefore the choice of the marginal exporting sector/industry is a crucial task.
- 5) The marginal exporting sector/industry may change from period to period and then the accounting foreign exchange rate also changes from period to period.
- 6) The shadow wage rate is greater than the existing wage rate.

FOOTNOTES TO CHAPTER VI

1) Mathematically we can deduce the another minimization problem. From the equation(1) we obtain

$$C_f + K_f \frac{1}{T^*} = \min_{1 \leq i \leq m} (C_i + K_i \frac{1}{T^*})$$

$$\text{Then, } T^* C_f + K_f = \min_{1 \leq i \leq m} (T^* C_i + K_i)$$

So the cost minimization problem(2) turns into the following minimization problem.

$$\min_{1 \leq i \leq m} (T^* C_i + K_i) \quad \dots\dots\dots(2')$$

From the Author's opinion it is rather difficult to explain the economic meaning of the minimization problem(2').

- 2) Rakowski(49),p.17. Also see, Ramowe wytyczne badań ekonomicznej efektywności inwestycji, Komisja Planowania, Warszawa, 1956.
- 3) Rakowski(49), p.20.
- 4) Rakowski(49), p.21.
- 5) See, Informator inwestora(61),pp.317-318 and Wit(60),p.94. Also see, Ostrowski and Sadowski(43), p.22.
- 6) See, Wit(60),p.94. Also see, Ostrowski and Sadowski(43), p.37.
- 7) See, Wit(60),pp.101-107. Also see, Ostrowski and Sadowski(43), pp.55-59.
- 8) See, Kantorovich(23), chapter 3, section 1, conclusion 24, p.166(English edition).
- 9) See, Kantorovich(23), chapter 3, section 4,p.237(English edition).
- 10) Kantorovich(23), chapter 3, section 2, table 44, p.194(English edition). Figures are slightly changed for the simplicity.
- 11) Kornai(27),pp 67-68.
- 12) Many articles were contributed in the symposium on the Little-Mirrlees manual of industrial project analysis in developing countries. See, Bulletin of the Oxford University Institute of Economics and Statistics, vol. 34, No.1, 1972.
- 13) See, J.K.Thieme(55). Also see,Kamiński(22), especially chapter V.
- 14) See, for example, Little and Scott(35). After this controversy a revised version of the OECD approach(34) was published in 1974.
- 15) See, Trzeciakowski(56) and (57), chapter 6. These two books have similar contents.

CHAPTER VII DECISION MAKING AT THE MICROECONOMIC LEVEL

In this chapter the Author investigates the problem of decision making at the microeconomic level. In section 1 the Author proposes a criterion for project evaluation at the microeconomic level. In section 2 the Author examines the difference of the operation period. In section 3 the Author investigates the relation between the final demand and the accounting price. In section 4 the Author examines the problem of coordination of plannings between the central level and the microeconomic level especially placing the focus on the quantitative adjustment. In the rest part of this chapter the Author discusses some difficult problems in decision making at the microeconomic level. That is, in section 5 the external effects, in section 6 the non-linearity of production function and in the last section the uncertainty.

VII-1 Criterion for project evaluation at the microeconomic level

Project evaluation at the microeconomic level concerns many aspects, i.e. pure economic effect, income distribution effect, regional development effect, scientific and technological development effect and so on. As well as in the economic planning at the macroeconomic level, the project evaluation at the microeconomic level should be investigated from all-round viewpoints of socio-economic aspects. This is a very important point in the project evaluation and unfortunately this point has been foregotten by planners in the developing countries. Consequently the project evaluation without socio-economic aspects brought serious disturbances in the society in Asian and African countries. We can say that the recent development of the theory of project evaluation was born from the reflection of these mistakes in the developing countries.

Here we should bear in our mind that quantitative measurement of these socio-economic effects is usually very difficult and that the pure economic effect is, needless to say, a fundamental element in the project evaluation. The pure economic effect is quantitatively measured by the following formula.

$$(\text{accounting net sur-}) = (\text{accounting value of}) - (\text{accounting value of}) \\ (\text{plus of the project}) \quad \text{the benefit of the} \quad \text{of the cost of} \\ \text{project} \quad \text{the project}$$

$$\text{Or } V = B - C$$

The Author would like to repeat that this formula is not an absolute criterion for the final decision making but a guiding principle for the final decision. Final decision should be made through consultation between the enterprise level and the ministry level or the central planning level taking into account the socio-economic aspects.

We can express the above-mentioned formula precisely as follows

$$V = \sum_{t=0}^T \sum_{i=1}^n Q_i^t Y_i^t - \sum_{t=0}^T \sum_{i=1}^n Q_i^t X_i^t - \sum_{t=0}^T \sum_{i=1}^L W_i^t L_i^t$$

where Q_i^t ; micro-accounting price of i -th good in t -th period.

Y_i^t ; output level of i -th good in t -th period.

X_i^t ; input level of i -th good in t -th period.

W_i^t ; shadow wage rate of i -th labour force in t -th period.

L_i^t ; labour input of i -th labour force in t -th period.

Here the Reader should pay attention to the following points.

- i) Q_i^t is not the expected price in t -th period but the accounting price in t -th period. Also W_i^t is not the expected wage rate in future but the shadow wage rate in future. As already mentioned, in the socialist economy the price of many fundamental consumer's goods is set at a low level and the existing wage rate is undervalued.
- ii) In the above-mentioned formula the discount rate does not appear explicitly. The discount factor is taken into account in the planning procedure at the macroeconomic level. Consequently the discount factor is reflected in the value of the accounting prices. This formula implies implicitly that a single discount rate is applied in the project evaluation in all industries.
- iii) Thirdly this formula does not regard the problem of financial supply of investment funds. In the real economy the investment funds are supplied by national budget, enterprise's own fund for investment, borrowing from foreign bank and so on. Therefore a different rate of discount is used in the financial calculation depending on the source of the investment fund. On the other hand, in the above-mentioned formula, the interest rate of international foreign currency market is reflected in the accounting prices of the tradable goods through the accounting foreign exchange rate. Also the scarceness of investment fund is expressed in the accounting price of capital goods. Summing up, we must distinguish the

financial calculation of the enterprise from the calculation of accounting net surplus in the project evaluation.

iv) In the model of this Thesis the conception of capital never appears explicitly, since we have assumed the full mobility of goods and resources. Therefore we can not obtain the capital rent from our model. This approach can be justifiable in the infinite and the long-term economic planning. But in the medium-term economic planning this problem becomes crucial one. So in the medium-term economic planning the Author recommends the use of the comprehensive approach which can handle with the scarceness of the capital good which has not full mobility. In the medium-term economic planning, for example, the accounting price of the steel which has full mobility should be different from the accounting price of the steel which is fixed in the production facilities.

v) The project which has a positive value of accounting net surplus(V) should be recommended to be adopted. Needless to say, the final decision may be influenced by the socio-political aspects. Therefore the first rule in the following form should not be interpreted as an absolute criterion. $V \geq 0$.

The project may have several variants which bring a positive value of V . For example, the hydro-electric generation plant with 100 Mwh has positive value of V' and the thermal-electric generation plant with 100 Mwh also has a positive value of V'' . Suppose that the optimal plan of the macroeconomic planning requires 100Mwh of electricity in the region concerned. Then the second project of thermal-electric generation plant should be recommended to be adopted, if $V' < V''$.

Here, however we must take into account the difference of operation periods, which is examined in VII-2.

The second rule of project evaluation is, therefore, that the project which has a higher accounting net surplus should be recommended to be adopted.

VII-2 Difference of operation periods

Firstly suppose that the hydro-electric generation plant (plant A) with 100 Mwh can operate during 8 years and the thermal-electric generation plant(plant B) with 100 Mwh can operate during 6 years. In this case the difference of operation periods can be easily solved. That is, plant A should be recommended to be adopted, if $V_A / 8 > V_B / 6$.

More generally

$$\text{MAX } V / T = \frac{(\text{accounting net surplus of the project})}{(\text{operation period})}$$

=: (accounting net surplus of the project per year)

The above-mentioned formula is not precise from a theoretical point of view. So the Author presents a more precise formula. Suppose that the project A can operate 8 years and the project B can operate 6 years. In this case we must compare the following two complexes of projects. 1) Three times of construction of the project A during 24 years. 2) Four times of construction of the project B during 24 years. Then we check the following criterion.

$$\sum_{t=1}^8 v_A^t + \sum_{t=9}^{16} v_A^t + \sum_{t=17}^{24} v_A^t > \sum_{t=1}^6 v_B^t + \sum_{t=7}^{12} v_B^t + \sum_{t=13}^{18} v_B^t + \sum_{t=19}^{24} v_B^t$$

Finally we must investigate the most important problem, i.e. the technological progress. In the above-mentioned case we have assumed that in future the same project will be constructed. In reality, however, always we can observe the technological progress. Now suppose that the project A can operate 12 years and the project B can operate 6 years. In this case the second construction of the project B may need a quite low level of the accounting value of inputs and, therefore, may bring a quite high level of the accounting net surplus owing to the technological progress. Therefore even when the above-mentioned formula recommends the construction of the project A, the construction of the project B may be favourable.

Fast construction and fast liquidation is a principal rule of the management under the technological progress.

VII-3 Relation between the final demand and the accounting price

In this Thesis the accounting price of the domestically produced good is determined mainly by the induced labour input. But here the Reader should notice the fact that the accounting price is also influenced by the structure of the final demand. The Author illustrates this fact in the following simple figure.

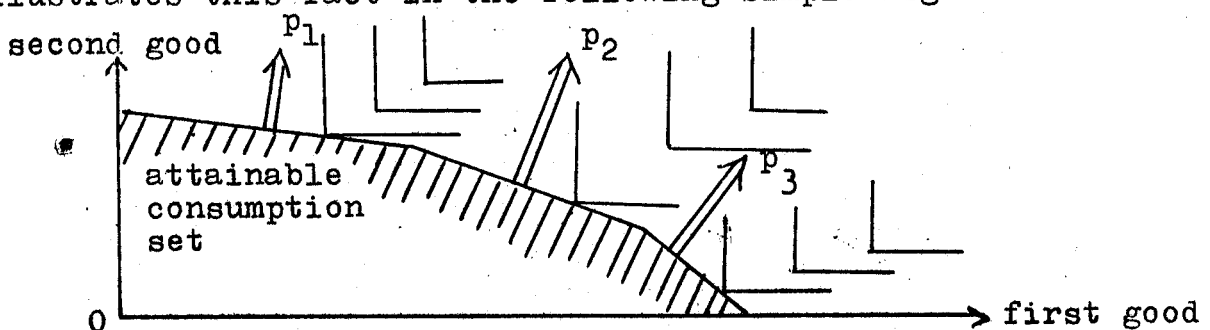


figure 1

Suppose now that the final demand for the first good increases relatively comparing to the final demand for the second good. Apparently from figure 1 the accounting price of the first good rises relatively comparing to the accounting price of the second good. Now let us set the accounting price of the second good at the level of unity. Then we can illustrate the following figure.

accounting price of
the first good

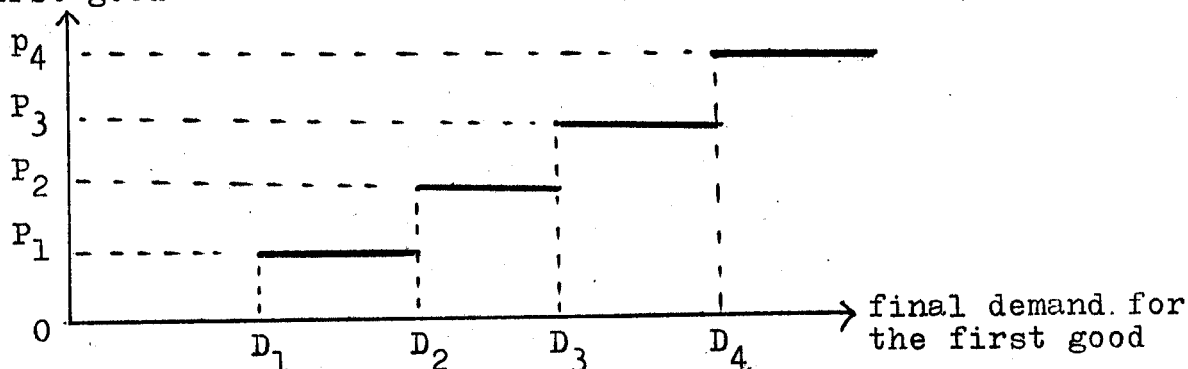


figure 2

From figure 2 the Reader can easily understand the fact that the method of this Thesis takes into consideration the market condition, i.e. the demand-supply relation, in the determination of the accounting prices.

VII-4 Necessity of quantitative adjustment and coordination between the macro level and the micro level

The most difficult element of the above-mentioned criterion for project evaluation is the non-determination of output level of the project. This difficulty concerns the linear production function at the micro level. For example, the first project with four mills of cement production has a two-times higher value of V than the second project with two mills of cement production, i.e. $V_1 = 2V_2$. In this case the enterprise can not determine the optimal output level. Now the Author investigates the method by which the production program of the enterprise is determined.

Firstly let us consider the most simple case where the industry contains only one factory. This supposition may apply to a certain kind of heavy industry in the socialist economy. The output level of the industry and consequently the output level of the factory is determined by the macroeconomic planning at the central level. In this case there is no choice about the output level of products at the enterprise level. But even in this simple case the

above-mentioned formula can be an effective criterion for project evaluation, since firstly the enterprise may have many kinds of variant with different technology and secondly there is a choice about the assortment of the final products. Therefore firstly the above-mentioned formula can be utilized for the choice of technology and consequently for the choice of structure of material inputs and labour inputs. Secondly the formula can be utilized for the determination of assortment of the final products. Let us consider a case of automobile industry. The macroeconomic plan determines the aggregate output level of the enterprise, when the automobile industry possesses only one factory for the passenger car production. Suppose that the macroeconomic plan designates the production program of 10 billion zloty's passenger car for export. Then the enterprise has to determine the structure of outputs, that is, whether the factory produces small number of expensive cars with a 1500ccm engine or large number of cheap cars with a 600ccm engine. For the determination of the assortment of outputs the above-mentioned formula can be helpful.

Next let us examine a more general case where the industry possesses many factories and the same product is produced by several factories. This supposition is very plausible. Since the project evaluation concerns the long-term planning and not the short-term planning, in this case the enterprise does not receive a priori a quantitative directive from the ministry level. At the first stage the enterprise should look for the project which has the highest value of V and determine the output level and assortment of products taking into account various kinds of restriction. This restriction concerns the resource constraint, final domestic demand for the product and the upper limit of the export of the product. Some restrictions are designated by the ministry but other restrictions are settled by the factory itself. The factory which can not find out any project with positive value of V should be liquidated in the long run. At the second stage the enterprise should check whether the domestic production of the particular good is more favourable than import of the good concerned. Cost of import is measured by the import dollar price multiplied by the accounting exchange rate. If the import cost of the product is less than the domestic production cost of the product, then the product should not be produced in the country. This is the third rule of project evaluation at the enterprise level.

Next the enterprises report their production plan to the mini-

stry. The proposition by the enterprises may not coincide with the optimal plan of the macroeconomic planning. Therefore the quantitative adjustment and coordination become indispensable.

This quantitative adjustment between the ministry level and the enterprise level is a really troublesome problem and this problem exceeds the scope of this Thesis. The Author does not present any concrete procedure for the quantitative adjustment.

But the Author would like to point out the following fact. In the long-term economic planning the condition of material balance is not a supreme premise. Material unbalance can be adjusted by production program in the medium-term planning or by international trade. It is sufficient in the long run that the material balance is maintained in every group of products. The long-term planning does not require any strict material balance of each particular good. Therefore the Author emphasizes the usefulness of the above-mentioned formula even when the quantitative adjustment is executed.

The ministry should respect the most profitable direction of development in each industry and each enterprise. The quantitative adjustment at the enterprise level should be restricted to the minimal limit in the long-term economic planning.

VII-5 External effects and systems approach

The cost-benefit analysis has been developed in the capitalist countries especially in the sphere of external effects. For example, the advocates of the cost-benefit analysis argue that the project evaluation of the construction of the second airport in London should be influenced by the transportation cost from the airport to the center of London and also influenced by the noise suffer of inhabitants around the new airport.

The word of "external effect" should be more widely interpreted as accompanied effects to the project. Let us consider the location of steel plant. The first variant is in the sea side district and the second is in the inland district. The project evaluation of steel plant concerns not only construction cost of the steel plant itself but also construction cost of transportation facilities. If the government locates the steel plant in the inland district, then the investment of rail road transportation between the steel plant and the port should be executed for the import of iron ore and export of steel products. If the government locates the steel plant in the sea side region, then the investment of rail road also

becomes necessary for the transportation of coal from the inland region to the sea side region. Furthermore the construction of new plant requires the housing construction for workers, institution for education and health service, park and recreation facilities etc.

These all costs should be taken into account in the project evaluation of the steel plant. Therefore the systems approach, which was explained in chapter VI, becomes powerful in the project evaluation at the enterprise level.

Needless to say, the external effects in a strict sense, e.g. air pollution and water pollution, are also included in the project evaluation, although the quantitative measurement of these external effects is quite difficult.

Finally the project evaluation may be influenced by the aspect of regional development and the socio-political aspects. Therefore the project evaluation should be done by a synthetic and comprehensive manner.

VII-6 Non-linearity of production function

Till now we have assumed that the enterprise has a linear production function. But in some industries we can witness the decreasing returns of output, especially in agriculture. Under the assumption of decreasing returns the above-mentioned formula of project evaluation turns into the following form.

$$V(Y) = \sum_{t=0}^T \sum_{i=1}^n Q_i^t Y_i^t - \sum_{t=0}^T \sum_{i=1}^n Q_i^t X_i^t(Y) - \sum_{t=0}^T \sum_{i=1}^L W_i^t L_i^t(Y)$$

where $Y =: (Y_1, Y_2, \dots, Y_n)$

We can illustrate the simplest case with one kind of output.

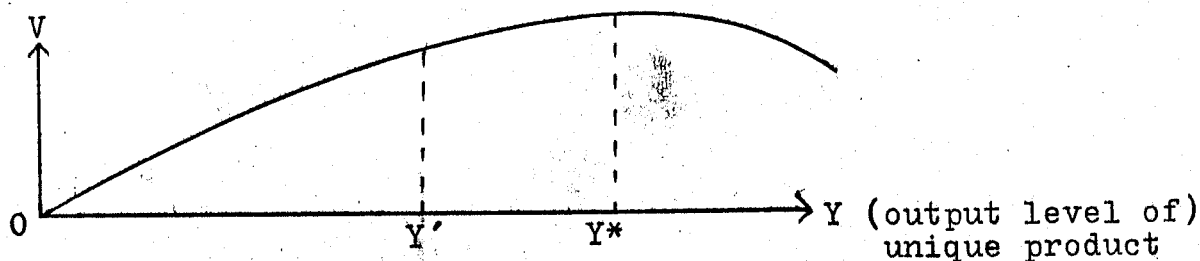


figure 3

Apparently the enterprise should determine the output level at Y^* . At the level of Y' the enterprise earns a positive value of V . But if the enterprise increases its output from Y' to Y^* , then the enterprise can receive a higher social net value.

Therefore the assumption of non-linearity brings the uniqueness of the optimal output level. The assumption of decreasing

returns of outputs, i.e. strict convexity of production set is a very convenient tool in economic analysis. But we can not confirm the existence of decreasing returns in developed industries from the experience of many countries especially in long run. Therefore the Author does not adopt the assumption of decreasing returns even at the enterprise level, though in agriculture the decreasing returns may exert a strong influence in output level. In the project evaluation in agriculture, therefore, we should take into account the existence of decreasing returns.

VII-7 Uncertainty

The most troublesome problem in project evaluation is uncertainty. For the project evaluation of coal industry the future international price of coal is crucial. The international price of coal in 1985 may be 100 dollars per ton or may be 120 dollars per ton.

In order to handle the uncertainty some theories have been explored by western economists, that is, the two-parameters approach (E-V approach) and the theory of portfolio selection. Both approaches assume the preference of decision maker to the uncertainty and consequently possess a subjective character. But here a serious problem arises. That is, every expectation must have a subjective character. One manager may expect the international price of coal in 1985 to be 100 dollars with 50%'s probability and the other manager may expect it to be 100 dollars with 30%'s probability. In the theory of uncertainty we can not exclude the subjective approach.¹⁾

The portfolio approach assumes the possibility of decentralization of risk. This approach is useful in decision making for stock investment. One can buy 500 stocks of FordMotors and 300 stocks of General Motors and 200 stocks of Clyster Motors. On the other hand in the case of industrial investment the government can not construct 50% of A project, 30% of B project and 20% of C project. If the government reduces the A project to the half level of the original program, then the reduced project may produce less than half of products of original program. If the government constructs 20% of C project, then this project brings no output.

The practically applicable method is only the maximization of expected value.²⁾

In order to explain the maximization of expected value let us consider once more the case of the coal industry. If the price of

coal is 100 dollars, the coal mine in A-district brings 80 million zloty of social net surplus and also suppose that if the price of coal is 120 dollars, then this coal mine brings 160 million zloty of social net surplus. Suppose that the subjective probability of 100 dollars is 50% and that of 120 dollars is 50%. Then the expected value of social net surplus (V) of the coal mine in A-district is calculated as follows. $E(V_A) = 80 \times 0.5 + 160 \times 0.5 = 120$.

The coal mine in B-district brings 90 million zloty of social net surplus in the case of 100 dollars of the international price and it brings 140 million zloty of social net surplus in the case of 120 dollars of the international price. The expected value of social net value (V) of the coal mine in B-district is as follows. $E(V_B) = 90 \times 0.5 + 140 \times 0.5 = 115$

Therefore in this case the coal mine should be constructed in A-district, since $E(V_A) > E(V_B)$.

Uncertainty is a very delicate problem. Information about uncertainty is always huge and can not be accumulated at the central level. Only the enterprise level can handle the information about uncertainty. Therefore decision making about uncertainty should be made exclusively at the enterprise level.

FOOTNOTES TO CHAPTER VII

- 1) See. W.Sadowski(50), especially chapter 5.
- 2) Sadowski(50) mentions that the maximization of expected value is useful even under subjective expectation.

CHAPTER VIII CONCLUDING REMARKS

In this chapter the Author presents a very short summary of contributions of this Thesis.

The contributions of this Thesis — if they exist — consist in the following points.

1. The Author investigated the multi-level structure of economic planning. From the viewpoint of time horizon the Author investigated three kinds of economic planning; i.e. the medium-term, the long-term and the infinite economic planning. As apparent from the models of this Thesis, these three economic plannings possess a similar structure. In the medium-term economic planning, however, it is necessary to take into consideration of frozen capital, which this Thesis manages unsufficiently. Therefore the method of this Thesis may face some difficulties in the medium-term economic planning. But it must be remembered that the main theme of this Thesis, as the title of the Thesis shows, consists in the long-term economic planning.

From the viewpoint of decision making the Author assumes only three stages of decision making; i.e. the socio-political process, the central planning level and the enterprise level. Of course, this is a strong simplification. In this Thesis many intermediate economic organizations are excluded. The Reader should bear in his/her mind this simplification.

2. In this Thesis the Author tried to connect the investment decision with the macroeconomic planning in a consistent manner. Till now many economists investigated the macroeconomic planning. And also many economists investigated the investment problem. But, according to the Author's impression, not so many economists considered this problem of consistent connexion of the macroeconomic planning and the investment decision.

3. The Author incorporated the international trade into the consumption-turnpike model. This approach has been hardly tried by the economists in capitalist countries. Needless to say, the international trade is a very important element in the modern socialist economy.

4. The Author examined the optimal level of the foreign currency debt. Under a certain assumption the Author proved that the optimal level of the foreign currency debt should decrease to zero in the long run. Here the word of the "long run" should not be interpreted as 5 years or 10 years. On the contrary we should interpret the "long run" as 20 years or more. This proposition was derived from the turnpike property.

5. The Author deduced some principles on determination of the accounting prices distinguishing the imported good, the exportable good and the domestically produced good. Here the Author does not repeat these principles, since they were presented in chapters V and VI.

6. The Author presented in chapter VI some decomposition principles on accounting prices. There the Author systematically decomposed the accounting index at the industry level into the accounting price at the enterprise level. In this procedure such guiding parameters play an important role as the shadow wage rate, the shadow exchange rate and so on.

The enterprise can calculate its own accounting prices using these guiding parameters and observing its own condition of production activity. Production activity is influenced by many factors, e.g. the international price of the product in future. Such a minute information can not be accumulated at the central level. Only the inferior level can manage these minute and specific informations. Therefore the method of this Thesis — the Author hopes — can be applicable and helpful in practice.

7. The Author proved the infinite turnpike property under weaker assumptions than that of Gale(13). That is, the strict convexity of technology set is not assumed in this Thesis. But, on the other hand, the Author introduced some strong assumptions; i.e. the fixed structure of final demand and the assumption V-2-3-6-6. Therefore the Author considers that this point is not a so great contribution to the purely mathematical economics.

MATHEMATICAL APPENDIX TO CHAPTER IV

M-IV-2-3

Here the Author proves the lemma IV-2-3-A in IV-2-3.

Lemma IV-2-3-A $\|T\|$ is a nonempty, compact and convex subset in R^{2n} .

Proof.

Let $x_t = 0$, then it is apparent that $\|T\|$ is nonempty. Apparently $\|T\|$ is compact, since $0 \leq v x_t \leq 1$.

Suppose that $(a_t^1, b_t^1) \in \|T\|$ and $(a_t^2, b_t^2) \in \|T\|$.

$$\text{Then } \lambda a_t^1 + (1 - \lambda) a_t^2 = (A+B)(\lambda x_t^1 + (1 - \lambda) x_t^2)$$

$$\lambda b_t^1 + (1 - \lambda) b_t^2 = \frac{(1+B)}{(1+q)} (\lambda x_t^1 + (1 - \lambda) x_t^2)$$

Since $0 \leq \lambda v x_t^1 + (1 - \lambda) v x_t^2 \leq 1$, we can obtain

$$0 \leq v(\lambda x_t^1 + (1 - \lambda) x_t^2) \leq 1$$

So we have proved the convexity.

Q.E.D.

MATHEMATICAL APPENDIX TO CHAPTER V

M-V-1

In this section the Author presents the dual formulation of the comprehensive approach of V-1-3.

Firstly the following notations are defined.

- I; (n,n) unit matrix.
 O_{nm} ; (n,m) zero matrix.
 O_{1n} ; n-dimensional row vector whose all elements are zero.
 O_{nl} ; n-dimensional column vector whose all elements are zero.
 X_t ; n-dimensional column vector of X_{it} .
 J ; T-dimensional column vector of J_t .
 P ; T-dimensional column vector of P_t .
 N ; (T-1)-dimensional column vector of $(N_2, N_3, N_4, \dots, N_T)$.
 F_t ; n-dimensional column vector of F_{it} .
 L ; T-dimensional column vector of L_t .
 g ; T-dimensional column vector: $g' = (N_1, 0, 0, 0, \dots, 0)$.
 h ; (T-1)-dimensional column vector: $h' = ((1-d_1)N_1, 0, 0, 0, \dots, 0)$.
 v ; n-dimensional row vector of v_i .
 b ; n-dimensional row vector of b_i .

V_t ; (T,n) matrix. B_t ; (T,n) matrix.

$$V_t = \begin{pmatrix} O_{1n} \\ O_{1n} \\ \vdots \\ O_{1n} \\ v \\ O_{1n} \\ \vdots \\ O_{1n} \end{pmatrix} \quad \leftarrow \begin{matrix} t\text{-th} \\ \text{element} \end{matrix}$$

$$B_t = \begin{pmatrix} O_{1n} \\ O_{1n} \\ \vdots \\ O_{1n} \\ b \\ O_{1n} \\ \vdots \\ O_{1n} \end{pmatrix} \quad \leftarrow \begin{matrix} t\text{-th} \\ \text{element} \end{matrix}$$

- b_{tt} ; n-dimensional column vector, i-th element of which is $b_{i,tt}$.
 D_t ; (n,T) matrix: $D_t = (O_{nl}, \dots, O_{nl}, b_{tt}, b_{t,t+1}, \dots, b_{t,T})$.
 f_t ; n-dimensional column vector of f_{it} .
 Z_t ; (n,T) matrix: $Z_t = (O_{nl}, O_{nl}, \dots, O_{nl}, \overset{\uparrow}{f_t}, O_{nl}, \dots, O_{nl})$.
 $\uparrow$
t-th element

W ; (T,T) matrix. W is a diagonal matrix, i-th diagonal element of which is $-1/(1+w)^i$.

S ; (T,T-1) matrix. $S = \begin{pmatrix} O_{1,T-1} \\ -I_{T-1} \end{pmatrix}$

K ; (T-1,T) matrix: $K = (-I_{T-1}, O_{t-1,1})$

Q; (T-1,T-1) matrix.

$$Q = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -(1-d_2) & 1 & 0 & \dots & 0 & 0 \\ 0 & -(1-d_3) & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & -(1-d_{T-1}) & 1 \end{bmatrix}$$

R; T-dimensional column vector: $R' = (P_0, P_0, P_0, \dots, P_0)$.

Now we can formulate the constraints (2),(3),(4),(5) and (6) of V-1-3 as follows.

$$\begin{bmatrix} -I+A & 0_{nn} & \dots & 0_{nn} & D_1 & Z_1 & 0_{n,T-1} \\ 0_{nn} & -I+A & \dots & 0_{nn} & D_2 & Z_2 & 0_{n,T-1} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0_{nn} & 0_{nn} & \dots & -I+A & D_T & Z_T & 0_{n,T-1} \\ 0_{Tn} & 0_{Tn} & \dots & 0_{Tn} & 0_{TT} & W & 0_{T,T-1} \\ V_1 & V_2 & \dots & V_T & 0_{TT} & 0_{TT} & 0_{T,T-1} \\ B_1 & B_2 & \dots & B_T & 0_{TT} & 0_{TT} & S \\ 0_{T-1,n} & 0_{T-1,n} & \dots & 0_{T-1,n} & K & 0_{T-1,T} & Q \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_T \\ J \\ P \\ N \end{bmatrix} \leq \begin{bmatrix} -F_1 \\ -F_2 \\ \vdots \\ -F_T \\ -R \\ L \\ g \\ h \end{bmatrix}$$

Under the above-mentioned constraints the CPB maximizes the objective function.

$$\text{MAX } F = \sum_{t=1}^T P_t$$

Next let us consider the dual problem. Firstly we define the following notations.

w_{it}^F ; dual price of i-th product in t-th period.

w_t^F ; n-dimensional column vector of w_{it}^F .

w_t^R ; dual price of consumption of the lower limit in t-th period.

w_t^R ; T-dimensional column vector of w_t^R .

w_t^L ; dual price of labour force in t-th period.

w_t^L ; T-dimensional column vector of w_t^L .

w_t^g ; dual price of capital in t-th period.

w_t^g ; T-dimensional column vector of w_t^g .

w_t^h ; dual price of investment in (t-1)-th period.

w_t^h ; (T-1)-dimensional column vector of w_t^h .

E_{T1} ; T-dimensional column vector of unity: $E_{T1} = (1, 1, 1, \dots, 1)$.

A' ; transpose matrix of A. L' ; transpose vector of L.

dual problem

$$\text{MIN } G = \sum_{t=1}^T \sum_{i=1}^n w_{it}^F (-F_{it}) + \sum_{t=1}^T w_t^R (-P_0) + \sum_{t=1}^T w_t^L L_t + w_1^G N_1 + w_2^h (1-d_1) N_1 \dots (7)$$

subject to

$$\begin{bmatrix} -I+A' & 0_{nn} & \dots & 0_{nn} & 0_{nT} & V'_1 & B'_1 & 0_{n,T-1} \\ 0_{nn} & -I+A' & \dots & 0_{nn} & 0_{nT} & V'_2 & B'_2 & 0_{n,T-1} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{nn} & 0_{nn} & \dots & -I+A' & 0_{nT} & V'_T & B'_T & 0_{n,T-1} \\ D'_1 & D'_2 & \dots & D'_T & 0_{TT} & 0_{TT} & 0_{TT} & K' \\ Z'_1 & Z'_2 & \dots & Z'_T & W' & 0_{TT} & 0_{TT} & 0_{T,T-1} \\ 0_{T-1,n} & 0_{t-1,n} & \dots & 0_{T-1,n} & 0_{T-1,T} & 0_{T-1,T} & S' & Q' \end{bmatrix} \begin{bmatrix} w_1^F \\ w_2^F \\ \vdots \\ w_T^F \\ w_T^R \\ w_t^L \\ w_1^G \\ w_2^h \end{bmatrix} \geq \begin{bmatrix} 0_{n1} \\ 0_{n1} \\ \vdots \\ 0_{n1} \\ 0_{T1} \\ E_{T1} \\ 0_{T-1,1} \end{bmatrix} \dots (8)$$

Finally we examine the property of the dual prices. By the dual theorem of linear programming we can obtain the following relations.

$$F^* = G^* \dots (9)$$

$$F \leq F^* \dots (10)$$

where * denotes the optimal program.

Then $F^* = G^* = 0$ and $F - G^* \leq 0$. That is,

$$\begin{aligned} \sum_{t=1}^T P_t + \sum_{t=1}^T \sum_{i=1}^n w_{it}^F F_{it} + \sum_{t=1}^T w_t^R P_0 \\ \leq \sum_{t=1}^T w_t^L L_t + w_1^G N_1 + w_2^h (1-d_1) N_1 \dots (11) \end{aligned}$$

We can interpret the left hand side of (11) as the benefit and the right hand side as the cost. Therefore we obtain the following relation.

$$\left(\begin{array}{l} \text{net benefit of} \\ \text{the optimal} \\ \text{program} \end{array} \right) = \left(\begin{array}{l} \text{benefit of} \\ \text{the optimal} \\ \text{program} \end{array} \right) - \left(\begin{array}{l} \text{cost of the} \\ \text{optimal} \\ \text{program} \end{array} \right) = 0$$

$$\left(\begin{array}{l} \text{net benefit of} \\ \text{non-optimal} \\ \text{program} \end{array} \right) = \left(\begin{array}{l} \text{benefit of} \\ \text{non-optimal} \\ \text{program} \end{array} \right) - \left(\begin{array}{l} \text{cost of} \\ \text{non-optimal} \\ \text{program} \end{array} \right) \leq 0$$

M-V-2-3-2

Here the Author presents the proof of two important propositions of V-2-3-2.

Proposition V-2-3-2-A Index of the welfare standard (3) is a continuous concave function.

Proof

In the proof of this and the following proposition c' and c'' denote the n -dimensional vector, i -th element of which is c'_i and c''_i respectively.

$$\begin{aligned}
 & \theta U(c') + (1-\theta)U(c'') \\
 &= \theta \min_i \left(\frac{c'_1}{f_1}, \frac{c'_2}{f_2}, \dots, \frac{c'_n}{f_n} \right) + (1-\theta) \min_i \left(\frac{c''_1}{f_1}, \frac{c''_2}{f_2}, \dots, \frac{c''_n}{f_n} \right) \\
 &= \min_i \left(\frac{\theta c'_1}{f_1}, \frac{\theta c'_2}{f_2}, \dots, \frac{\theta c'_n}{f_n} \right) + \min_i \left(\frac{(1-\theta)c''_1}{f_1}, \frac{(1-\theta)c''_2}{f_2}, \dots, \frac{(1-\theta)c''_n}{f_n} \right) \\
 &\leq \min_i \left(\frac{\theta c'_1 + (1-\theta)c''_1}{f_1}, \frac{\theta c'_2 + (1-\theta)c''_2}{f_2}, \dots, \frac{\theta c'_n + (1-\theta)c''_n}{f_n} \right) \\
 &= U(\theta c' + (1-\theta)c'') \quad \text{Q.E.D.}
 \end{aligned}$$

Proposition V-2-3-2-B Linearly approximated index of the welfare standard (4) is a continuous concave function.

Proof

$$\begin{aligned}
 & \theta U(c') + (1-\theta)U(c'') \\
 &= \min_i \left(\theta \frac{c'_i - k_i}{f_i} \right) + \theta \sum_{i=1}^n k_i \\
 &\quad + \min_i \left((1-\theta) \frac{c''_i - k_i}{f_i} \right) + (1-\theta) \sum_{i=1}^n k_i \\
 &\leq \min_i \left(\frac{\theta c'_i + (1-\theta)c''_i - \theta k_i - (1-\theta)k_i}{f_i} \right) + \sum_{i=1}^n k_i \\
 &= \min_i \left(\frac{\theta c'_i + (1-\theta)c''_i - k_i}{f_i} \right) + \sum_{i=1}^n k_i \\
 &= U(\theta c' + (1-\theta)c'') \quad \text{Q.E.D.}
 \end{aligned}$$

M-V-2-3-5

Here the Author formulates the abstract model of the infinite economic planning and proves some properties. Firstly the Author formulates the whole system of the model from (1) to (5) of V-2-3-5 using matrix and vector.

Constraints (1) and (3) can be summed up in the following manner.

$$\begin{pmatrix} A+B & I_n & 0_{nn} & 0_{nl} \\ 0_{ln} & -d^{e'} & 0_{ln} & -1 \end{pmatrix} \begin{pmatrix} X_t \\ E_t \\ M_t \\ Z_t \end{pmatrix} + \begin{pmatrix} C_t \\ S_t \end{pmatrix} \leq \begin{pmatrix} I_{n+B} & 0_{nn} & I_n & 0_{nl} \\ 0_{ln} & 0_{ln} & -(1+r)d^{m'} & -(1+r) \end{pmatrix} \begin{pmatrix} X_{t-1} \\ E_{t-1} \\ M_{t-1} \\ Z_{t-1} \end{pmatrix} \dots\dots\dots(6)$$

where I_n ; (n,n) unit matrix.

0_{nn} ; (n,n) zero matrix.

0_{nl} ; n -dimensional column zero vector.

0_{ln} ; n -dimensional row zero vector.

$d^{e'}$; n -dimensional row vector of d_i^e .

$d^{m'}$; n -dimensional row vector of d_i^m .

S_t ; surplus of foreign currency in t -period, which is scalar and measured by dollar.

S_t is a kind of slack variable. We interpret S_t as an amount of the foreign currency which is endowed to the developing countries or the International Red Cross.

Now we introduce the following new notations.

$$x_{it} =: X_{it} / (1+s)^t(1+q)^t$$

$$e_{it} =: E_{it} / (1+s)^t(1+q)^t$$

$$m_{it} =: M_{it} / (1+s)^t(1+q)^t$$

$$z_t =: Z_t / (1+s)^t(1+q)^t$$

$$c_{it} =: C_{it} / (1+s)^t(1+q)^t$$

$$s_t =: S_t / (1+s)^t(1+q)^t$$

Using these notations the constraints (1),(2),(3),(4) and (5) of V-2-3-5 turn into the following form.

$$A x_t + e_t + c_t + B(x_t - \frac{x_{t-1}}{(1+s)(1+q)}) \leq \frac{x_{t-1}}{(1+s)(1+q)} + \frac{m_{t-1}}{(1+s)(1+q)} \dots\dots\dots(1')$$

$$\sum_{i=1}^n v_{io} x_{it} \leq L_0 \quad \dots\dots\dots(2')$$

$$\sum_{i=1}^n d_i^e e_{it} + z_t \geq (1+r) \sum_{i=1}^n \frac{d_i^m m_{i,t-1}}{(1+s)(1+q)} + \frac{(1+r) z_{t-1}}{(1+s)(1+q)} \quad \dots\dots\dots(3')$$

$$\left. \begin{aligned} 0 &\leq e_{it} \leq h_{io}^e \\ 0 &\leq m_{it} \leq h_{io}^m \end{aligned} \right\} \dots\dots\dots(4')$$

$$0 \leq z_t \leq H_0 \quad \dots\dots\dots(5')$$

Now we define the notation (w) as follows. $w = (1+s)(1+q)$.
So the constraints (1') and (3') can be formulated as (6').

$$\begin{pmatrix} A+B & I_n & 0_{nn} & 0_{nl} \\ 0_{ln} & -d^e & 0_{ln} & -1 \end{pmatrix} \begin{pmatrix} x_t \\ e_t \\ m_t \\ z_t \end{pmatrix} + \begin{pmatrix} c_t \\ s_t \end{pmatrix} \leq \begin{pmatrix} I_n+B & 0_{nn} & I_n & 0_{nl} \\ 0_{ln} & 0_{ln} & -(1+r)d^m & -(1+r) \end{pmatrix} \begin{pmatrix} x_{t-1} / w \\ e_{t-1} / w \\ m_{t-1} / w \\ z_{t-1} / w \end{pmatrix} \quad \dots\dots(6')$$

The constraint (6') can be written as follows using new notations.

$$R \begin{pmatrix} x_t \\ e_t \\ m_t \\ z_t \end{pmatrix} + \begin{pmatrix} c_t \\ s_t \end{pmatrix} \leq Q \begin{pmatrix} x_{t-1} / w \\ e_{t-1} / w \\ m_{t-1} / w \\ z_{t-1} / w \end{pmatrix} \quad \dots\dots\dots(6'')$$

where (R) and (Q) are $(n+1, 3n+1)$ matrices, whose elements are explained by (6'). Now we introduce the further new notations.

$$\hat{x}_t' =: (x_t', e_t', m_t', z_t')$$

$$\hat{x}_t =: R \hat{x}_t'$$

$$\hat{y}_t =: Q \hat{x}_t' \frac{1}{w}$$

$$\hat{c}' =: (c_t', s_t')$$

So the constraint (6'') turns into the following form.

$$\hat{x}_t + \hat{c}_t \leq \hat{y}_{t-1} \quad \dots\dots\dots(6''')$$

Using these notations we can define the production set // T// as follows.

$$// T// =: \left\{ (\hat{x}_t, \hat{y}_t) \left| \begin{aligned} \hat{x}_t &= R \hat{x}_t' \\ \hat{y}_t &= Q \frac{\hat{x}_t'}{w} \\ \sum_{i=1}^n v_{io} x_{it} &\leq L_0 \\ 0 &\leq e_{it} \leq h_{io}^e \\ 0 &\leq m_{it} \leq h_{io}^m \\ 0 &\leq z_t \leq H_0 \end{aligned} \right. \right\}$$

Property V-2-3-5-A // T// is a compact, convex and nonempty set.

Proof

Compactness of // T// is apparent, because the following set // T'// is a closed and bounded set.

$$// T'// =: \left\{ (x'_t, e'_t, m'_t, z'_t) \mid \begin{array}{l} \sum_{i=1}^n v_{io} x_{it} \leq L_o \\ 0 \leq e_{it} \leq h_{io}^e \\ 0 \leq m_{it} \leq h_{io}^m \\ 0 \leq z_t \leq H_o \end{array} \right\}$$

Now suppose that $(x_t^1, y_t^1) \in // T//$ and $(x_t^2, y_t^2) \in // T//$. Then $(\theta x_t^1 + (1-\theta)x_t^2, \theta y_t^1 + (1-\theta)y_t^2) \in // T//$, because

$$\begin{aligned} \sum_{i=1}^n v_{io} (\theta x_{it}^1 + (1-\theta)x_{it}^2) &\leq \theta L_o + (1-\theta)L_o = L_o \\ 0 &\leq \theta e_{it}^1 + (1-\theta)e_{it}^2 \leq h_{io}^e \\ 0 &\leq \theta m_{it}^1 + (1-\theta)m_{it}^2 \leq h_{io}^m \\ 0 &\leq \theta z_t^1 + (1-\theta)z_t^2 \leq H_o \end{aligned}$$

Therefore // T// is convex.

Finally let us suppose that $x_t=0, e_t=0, m_t=0$ and $z_t=0$. That is, let us suppose a situation with no economic activity. This activity apparently belongs to the set // T//. Therefore // T// is nonempty. Q.E.D.

Now we define the preference field of the infinite economic planning. In order to define the preference field $// C// \subset R_{n+1}$, we define $// C'// \subset R_n$. // C'// is the set of admissible consumption vectors in R_n . Here R_n denotes a n -dimensional Euclidean space. The conception of the admissible consumption field is explained already in V-2-3-2. Now we must assume the existence of satiation point of consumption. This assumption is required only from a mathematical viewpoint. For example, one person can not consume 1000 kg of meat per year, or one person never purchases 10 television sets in a year and so on. So we may assume that // C'// is bounded from above. We can set this upper limit as great as we want. Also we define // S// as follows; $// S// =: \{s \mid 0 \leq s \leq \bar{s}\}$. ($\bar{s}$) can be interpreted as the upper limit of the foreign currency gift for the developing countries. We can imagine any large limit of $\bar{s}$ and any large satiating consumption limit, so the boundedness of // C'//

and $\parallel S \parallel$ is acceptable from the economic point of view. The preference field $\parallel C \parallel$ is defined as follows; $\parallel C \parallel =: \parallel C' \parallel \times \parallel S \parallel$, where $\times$ denotes the Cartesian product.

Since the admissible consumption field has been well defined in V-2-3-2, we obtain the following property, proof of which is selfevident from the definition of the admissible consumption field.

Property V-2-3-5-B $\parallel C \parallel$ is nonempty, compact and convex.

The initial condition of the infinite economic planning is given exogeneously. That is, M_{-1} , Z_{-1} and X_{-1} are already determined historically. So we have the historically determined data of M_{1980} , Z_{1980} and X_{1980} . The initial condition (R_0) is defined as follows.

$$R_0 = Q \begin{pmatrix} x_{-1} / w \\ e_{-1} / w \\ m_{-1} / w \\ z_{-1} / w \end{pmatrix}$$

Now we can formulate the infinite economic planning as a von Neumann economy model. Most important theorems of this Thesis will be directly derived from the following formulation as a von Neumann economy model.

The infinite economic planning

$$\text{MAX} \quad \sum_{t=0}^{\infty} \left(\min_i \left(\frac{c_{it} - k_i}{f_i} \right) + \sum_{i=1}^n k_i \right) = \sum_{t=0}^{\infty} \hat{F}(\hat{c}_t) \quad \dots\dots\dots(7')$$

$$\text{s.t.} \quad (\hat{x}_t, \hat{y}_t) \in \parallel T \parallel \quad ; (t=0,1,2,\dots, \infty) \quad \dots\dots\dots(8)$$

$$\hat{c}_t \in \parallel C \parallel \quad ; (t=0,1,2,\dots, \infty) \quad \dots\dots\dots(9)$$

$$\hat{x}_0 + \hat{c}_0 \leq R_0 \quad \dots\dots\dots(10)$$

$$\hat{x}_t + \hat{c}_t \leq \hat{y}_{t-1} \quad ; (t=1,2,3,\dots, \infty) \quad \dots\dots\dots(11)$$

Continuity and concaveness of the objective function (7') can be easily proved in the same manner as that of the proposition V-2-3-2-A.

M-v-2-3-6

Here the Author mathematically investigates the optimal stationary program of the infinite economic planning and proves its uniqueness. Firstly we define the optimal stationary program.

Optimal stationary program

$$\begin{aligned} \text{MAX.} \quad F(c) &= \min_i \left(\frac{c_i - k_i}{f_i} \right) + \sum_{i=1}^n k_i \\ \text{s.t.} \quad x + c &\leq y \\ (x, y) &\in // T // \\ c &\in // C // \end{aligned}$$

where x , y and c are $(n+1)$ -dimensional vector.

This definition is very abstract, so it is better to define once more the optimal stationary program more concretely. This concrete version of the optimal stationary program is formulated by the relations (12), (13), (14), (15) and (16) of the text (V-2-3-6).

Now we consider the lowest level of the admissible consumption ($\underline{c}$). That is, ($\underline{c}$) is the lower limit of consumption which is admissible from the social point of view. ($\underline{c}$) is illustrated in the figure 12.

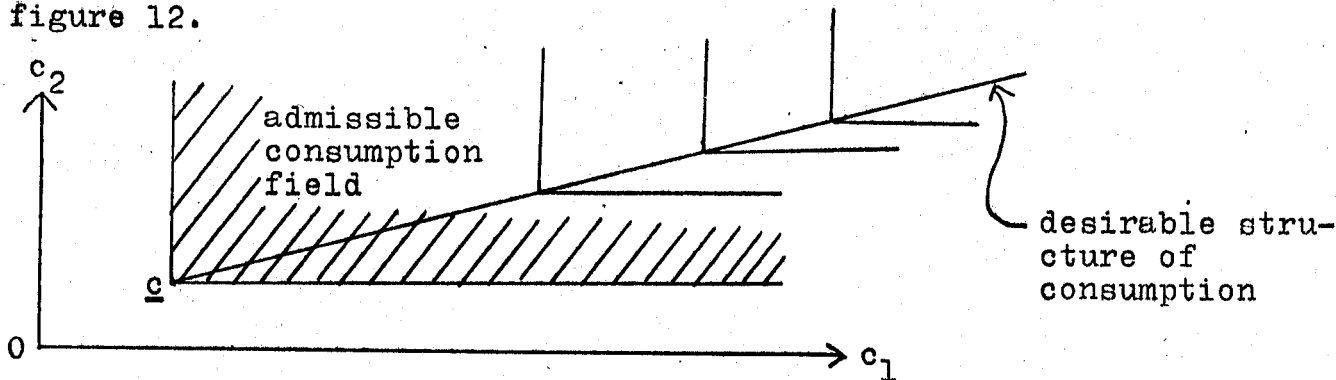


figure 12

Here we introduce the following assumption.

Assumption V-2-3-6-A There exists $(\underline{x}, \underline{e}, \underline{m}, \underline{z})$ which satisfies the following relation (17) and all constraint conditions (13), (14), (15) and (16) and also satisfies the condition (14) with strict inequality. $A \underline{x} + \underline{e} + \underline{c} + B(\underline{x} - \frac{\underline{x}}{(1+s)(1+q)}) < \frac{\underline{x}}{(1+s)(1+q)} + \frac{\underline{m}}{(1+s)(1+q)}$ (17)

Property V-2-3-6-A Under the assumption V-2-3-6-A there exists an optimal stationary program.

Proof Proof is very easy and therefore omitted. The Reader can refer, for example, the Theorem 7.B.3 by Takayama(54), p.584.

In the optimal stationary program, $s^* = 0$, i.e. the international donation should be zero. (Hereafter (*) denotes the optimal stationary program). This statement is apparent, since the international donation has to decrease the level of import or has to increase the level of export and in turn decrease the level of consumption. Here the Author introduces the following assumption.

Assumption V-2-3-6-B $(1+r) > (1+s)(1+q)$. That is,
 $(1 + \frac{\text{international bank rate}}{\text{bank rate}}) > (1 + \frac{\text{rate of population growth}}{\text{rate of technological progress}})(1 + \frac{\text{rate of technological progress}}{\text{rate of technological progress}})$

Under this assumption we can prove that $z^* = 0$. Since $\frac{1+r}{(1+s)(1+q)} z > z$, from (14) we can understand that a positive level of (z) inevitably has to decrease the import or increase the export and in turn has to decrease the consumption level. Therefore,
 $z^* = 0$ (18)

Now we can prove the following property.

Property V-2-3-6-B The optimal stationary program satisfies the desirable structure of consumption; i.e.

$$(c_1^* - k_1)/f_1 = (c_2^* - k_2)/f_2 = \dots = (c_n^* - k_n)/f_n = \xi.$$

Proof Suppose that $(c_1^* - k_1)/f_1 > \xi$. Then we have c_1^{**} such that $c_1^{**} < c_1^*$ and $(c_1^{**} - k_1)/f_1 = \xi$. The CPB can export the amount of $(c_1^{**} - c_1^*)$ and earn the amount of $d_1^e(c_1^{**} - c_1^*)$. Then the CPB can import all kinds of consumption good by the amount of $\Delta c_i > 0$. Therefore there exists a more desirable program. This is a contradiction. Q.E.D.

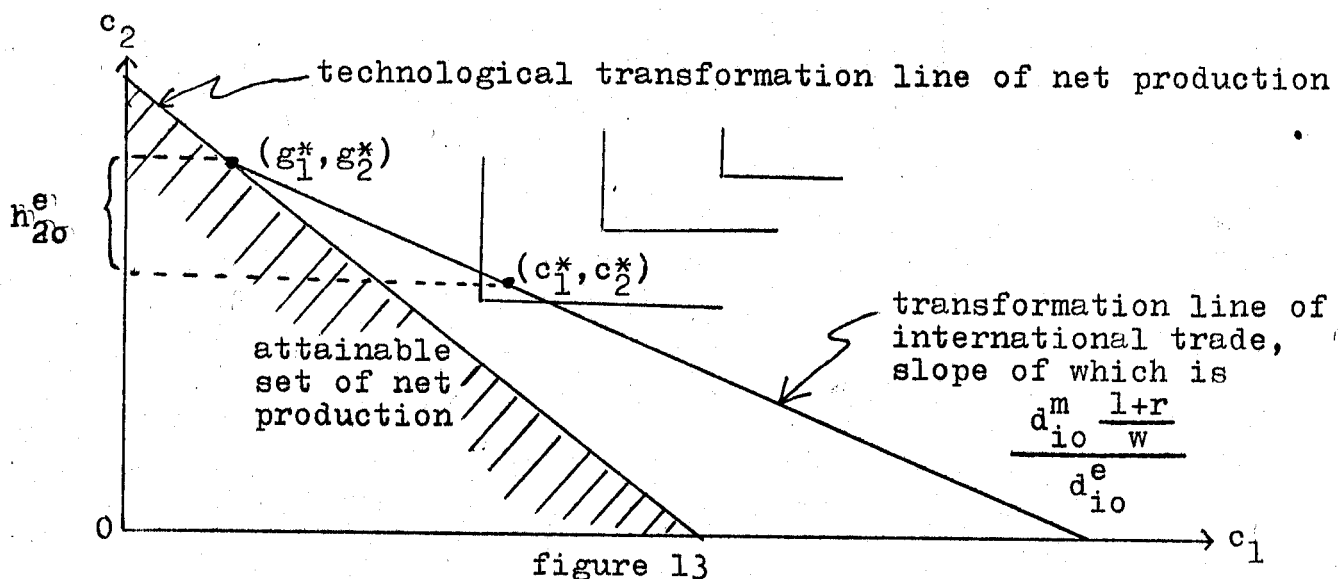
Property V-2-3-6-B means the following property.

Property V-2-3-6-C Consumption program of the optimal stationary program is unique.

Proof By the property V-2-3-6-B, $(c_i^* - k_i)/f_i = \xi; (i=1,2,\dots,n)$. Suppose that there exists another solution. The second solution also must satisfy the property V-2-3-6-B. So we obtain $(c_i^{**} - k_i)/f_i = \zeta; (i=1,2,\dots,n)$ and $\zeta \neq \xi$. The objective function $F(c)$ requires that $\zeta = \xi$. This is a contradiction. Q.E.D.

This property assures the uniqueness of consumption program of the optimal stationary program, however does not guarantee the uniqueness of production program. The uniqueness of production program plays a very important role in the convergence property of

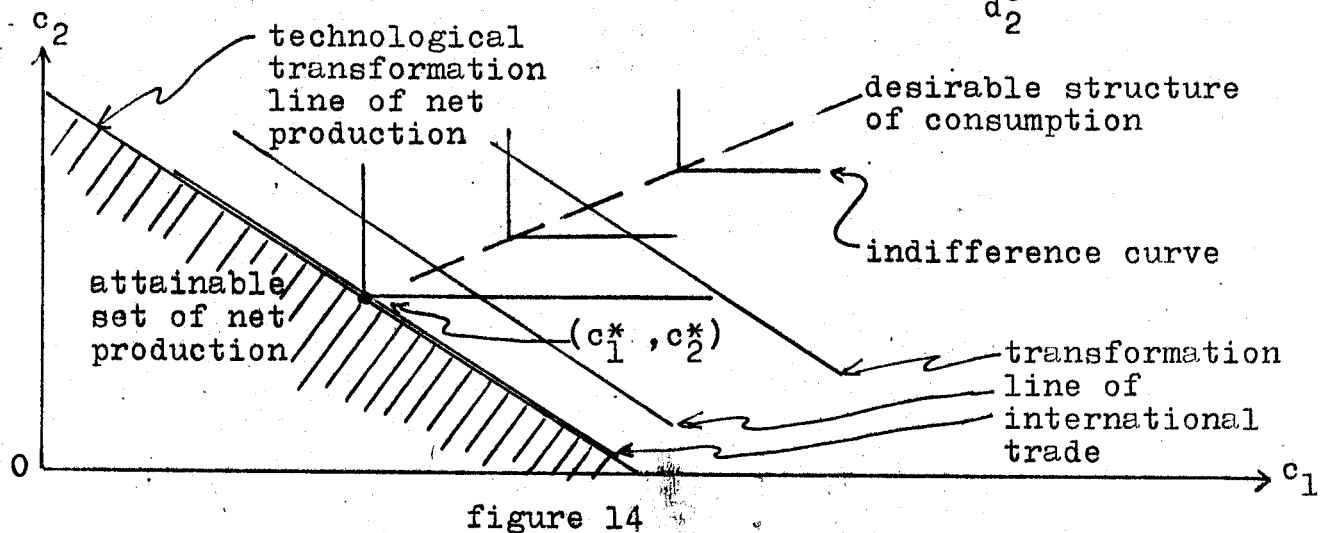
the optimal plan in the infinite economic planning.



In the case of figure 13 the optimal production program is also unique. The optimal production program of net production should be the point (g_1^*, g_2^*) and the government exports the 2-nd good and imports the 1-st good. Trade balance is as follows;

$$d_2^e(g_2^* - c_2^*) = \frac{d_1^m(1+r)}{(1+s)(1+q)}(c_1^* - g_1^*) .$$

Therefore the slope of the transformation line of international trade is $\frac{d_1^m \frac{1+r}{w}}{d_2^e}$.



In the case of figure 14 unfortunately the optimal program of net production is not unique. Since the proof of uniqueness is rather complicated, here the Author presented a non-rigorous explanation by the help of illustration. Rigorous investigation is now presented.

Uniqueness of the optimal stationary program of production

In the optimal stationary program the material balance condition (12) of V-2-3-6 is always maintained with equality.

$$A x + e + c + B(x - \frac{x}{w}) = \frac{x}{w} + \frac{m}{w} \quad \dots\dots\dots(12')$$

where $w =: (1+s)(1+q)$

We can transform the equation (12') as follows.

$$(\frac{I}{w} - A - B + \frac{B}{w}) x = e + c - \frac{m}{w}$$

$$\text{Or } J x = g \quad \dots\dots\dots(19)$$

where $J =: (\frac{I}{w} - A - B + \frac{B}{w})$ and $g =: e + c - \frac{m}{w}$.

About the matrix (J) we introduce the following assumption.

Assumption V-2-3-6-D J is non-singular, that is, J^{-1} exists.

Suppose that the CPB intends to increase the net production of j-th industry by marginal one unit. Then the gross output vector must change and the required labour input has to exceed the labour supply (L_0).

$$x^*(j) =: J^{-1} \begin{bmatrix} g_1^* \\ g_2^* \\ \vdots \\ g_{j-1}^* \\ g_j^* + 1 \\ g_{j+1}^* \\ \vdots \\ g_n^* \end{bmatrix} \quad \text{and} \quad \sum_{i=1}^n v_{i0} x_i^*(j) > L_0$$

We can interpret the difference $(\sum_{i=1}^n v_{i0} x_i^*(j) - L_0)$ as an index of scarceness of j-th industry's products. This difference can be calculated also in the following manner.

$$x^*(j) = J^{-1} g^* + J^{-1} h_j = x^* + J^{-1} h_j$$

where h_j is n-dimensional column vector, j-th element of which is unity and other elements of which are zero. $h_j =: (0, 0, \dots, 0, 1, 0, \dots, 0)$

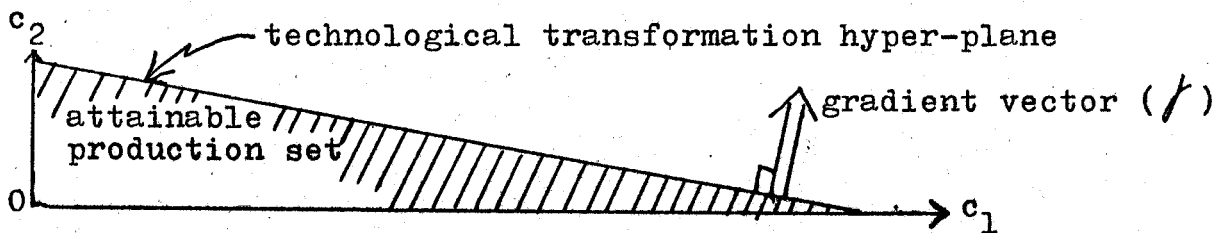
$$\begin{aligned} \sum_{i=1}^n v_{i0} x_i^*(j) - L_0 &= v'_0 (x^* + J^{-1} h_j) - L_0 \\ &= L_0 + v'_0 J^{-1} h_j - L_0 \\ &= v'_0 J^{-1} h_j \end{aligned}$$

where $v'_0 =: (v_{10}, v_{20}, \dots, v_{n0})$

Then we can articulate the gradient vector (f) of the technological transformation hyper-plane of net production.

$$f' = (v'_0 J^{-1} h_1, v'_0 J^{-1} h_2, \dots, v'_0 J^{-1} h_n)$$

$$=: (f_1, f_2, \dots, f_n)$$



We can express the technological transformation hyper-plane as follows. $\gamma'g = \underline{M} = \text{constant}$.

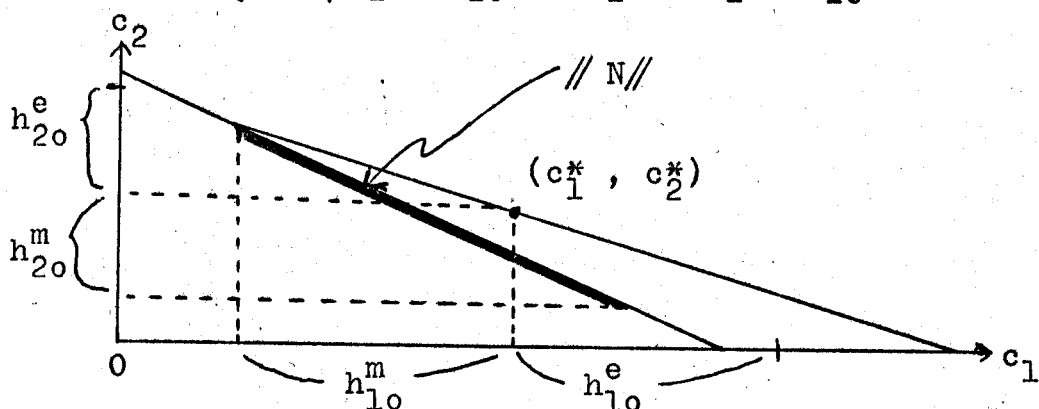
Here one question arises; i.e. are all points on the technological transformation hyper-plane the attainable program? For the simplicity the Author introduces the following assumption.

Assumption V-2-3-6-E Net production should not be negative; i.e. $g_i \geq 0$; $(i=1,2,\dots,n)$.

In the following parts of the argument this assumption will be omitted.

In order to answer the above-mentioned question, now, we define the institutional attainable set of net production $//N//$.

$$//N// =: \left\{ g \mid \begin{array}{l} \gamma'g = \underline{M} \\ g_i \geq 0 ; (i=1,2,\dots,n) \\ c_1^* - h_{10}^m \leq g_i \leq c_1^* + h_{10}^e ; (i=1,2,\dots,n) \end{array} \right\}$$



The set $//N//$ is illustrated in the above figure.

Next we define the international value of net production $(V(g))$. Also here the international trade price is defined as follows.

$$(\text{international trade price}) =: d_i = \begin{cases} d_i^e & , \text{ if } c_1^* < g_i \\ d_i^m \frac{(1+r)}{w} & , \text{ if } c_1^* > g_i \end{cases}$$

$$\begin{aligned} (\text{international value of net production}) &=: V(g) \\ &= \sum_{i=1}^n d_i (g_i - c_1^*) + (\text{value of } c^*) \\ &= \sum_{i=1}^n d_i (g_i - c_1^*) + \underline{V} \end{aligned}$$

where $\underline{V}$ is treated as a constant term.

Now we can transform the original optimal problem into the following reduced problem.

$$\begin{aligned} \text{MAX} \quad & V(g) \\ \text{s.t.} \quad & g \in //N// \end{aligned} \quad \dots\dots\dots(20)$$

Apparently the solution of the reduced problem is identical to the solution of the original optimal stationary problem.

In order to prove the uniqueness of the optimal stationary program we examine the following two different cases, i.e. (i) and (ii).

(i) Firstly suppose that there are two optimal programs of production, i.e. g^1 and g^2 . Also suppose that the program g^1 contains the export of k-th product and the program g^2 contains the import of k-th product. Now we imagine a combination of two programs.

$$g^3 =: \theta g^1 + (1-\theta)g^2$$

$$\begin{aligned} V(g^3) &= \sum_{i=1}^n d_i^3(\theta g_i^1 + (1-\theta)g_i^2 - c_i^*) + \underline{V} \\ &= \theta \sum_{i=1}^n d_i^3(g_i^1 - c_i^*) + (1-\theta) \sum_{i=1}^n d_i^3(g_i^2 - c_i^*) + \theta \underline{V} + (1-\theta)\underline{V} \end{aligned}$$

We can set θ to a very small number, then g^3 also contains the import of k-th product. Therefore $d_i^3 = d_i^2$; $(i=1,2,\dots,n)$. That is, g^3 belongs to the same "phase" as g^2 .

$$\begin{aligned} V(g^3) &= \theta \sum_{i=1}^n d_i^2(g_i^1 - c_i^*) + (1-\theta) \sum_{i=1}^n d_i^2(g_i^2 - c_i^*) + \theta \underline{V} + (1-\theta)\underline{V} \\ &= \theta \sum_{i=1}^n d_i^2(g_i^1 - c_i^*) + \theta \underline{V} + (1-\theta)V^* \end{aligned}$$

Now let us remember that $d_i^e < d_i^m \frac{1+r}{w}$ and $d_k^1 = d_k^e < d_k^m \frac{1+r}{w} = d_k^2 = d_k^3$.

$$\text{Then } \sum_{i=1}^n d_i^2(g_i^1 - c_i^*) > \sum_{i=1}^n d_i^1(g_i^1 - c_i^*)$$

$$\begin{aligned} \text{Therefore, } V(g^3) &> \theta \sum_{i=1}^n d_i^1(g_i^1 - c_i^*) + \theta \underline{V} + (1-\theta)V^* \\ &= \theta V^* + (1-\theta)V^* \\ &= V^* \end{aligned}$$

This is a contradiction, since $g^3 \in // N //$.

(ii) Next suppose that g^1 and g^2 belong to the same "phase". In other words these two programs contain the export and the import of the same kind of products. In this case we need the following assumption V-2-3-6-C.

Assumption V-2-3-6-C The slope of the technological transformation line of net production is always different from the ratio of the international trade terms.

The Author will present this assumption rigorously once more later.

It is apparent that $V(g^3) = V^*$. Under this assumption, however, we can choose Δg such that $\Delta g = (0, 0, \dots, 0, \Delta g_i, 0, \dots, 0, \Delta g_j, 0, \dots, 0)$,

$\delta_i \Delta g_i + \gamma_j \Delta g_j > 0$ and $g^3 + \Delta g \in //N//$. Therefore under this assumption we can find Δg such that $V(g^3 + \Delta g) > V(g^3) = V^*$. Then $(g^3 + \Delta g)$ should be an optimal program and g^1 and g^2 are not optimal. This is a contradiction.

So we have proved the following property.

Property V-2-3-6-D Optimal stationary program of production is unique.

Assumption V-2-3-6-C is quite plausible, because the technological transformation line(hyper-plane) is determined by the domestic technology. But, on the other hand, the international trade terms are determined by the world economic situation, especially by the production technology in the world. The international trade terms are also influenced by the demand conditions in the world market. Therefore we may consider that the technological transformation ratio of the domestic production is different from the international trade terms.

The Author presents here once more the assumption V-2-3-6-C in a mathematical form.

Assumption V-2-3-6-C (mathematical version)

$$\frac{\delta_i}{\delta_j} \neq \frac{d_i^e}{d_j^m \frac{(1+r)}{w}} \quad \text{for any combination of } i \text{ and } j.$$

$$\text{and } \frac{\delta_i}{\delta_j} \neq \frac{d_i^m \frac{(1+r)}{w}}{d_j^e} \quad \text{for any combination of } i \text{ and } j.$$

Summing up we have proved the following theorem.

Theorem V-2-3-6 Under assumptions V-2-3-6-C, D and E there exists an unique optimal stationary program of production.

M-V-2-3-7

In this subsection the Author presents the convergence property of the optimal infinite program to the optimal stationary program.

Theorem V-2-3-7

The optimal program $(c_t^{**}, s_t^{**}, x_t^{**}, e_t^{**}, m_t^{**}, z_t^{**})$ converges to the optimal stationary program $(c^*, s^*, x^*, e^*, m^*, z^*)$, when $t \rightarrow \infty$.

Proof

Firstly we define the following notations.

$$\hat{u}_t^{**} = \begin{pmatrix} x_t^{**} \\ e_t^{**} \\ m_t^{**} \\ z_t^{**} \end{pmatrix}, \quad \hat{c}_t^{**} = \begin{pmatrix} c_t^{**} \\ s_t^{**} \end{pmatrix}, \quad \hat{x}_t^{**} = R \hat{u}_t^{**}, \quad \hat{y}_t^{**} = Q \frac{\hat{u}_t^{**}}{w}$$

where the matrices (R) and (Q) were defined in (6'') in M-V-2-3-5.

Definition of the price vector associated with the optimal stationary program was given in IV-2-3. The price vector associated with the optimal stationary program is denoted by p^* .

Now we define the following function.

$$\varphi(\hat{x}_t^{**}, \hat{y}_t^{**}, \hat{c}_t^{**}) =: \hat{F}(\hat{c}_t^{**}) + p^*(\hat{y}_t^{**} - \hat{x}_t^{**} - \hat{c}_t^{**})$$

where, as defined in M-V-2-3-5, $\hat{F}(\hat{c}_t^{**}) =: F(c_t^{**})$.

We have already proved the uniqueness of the optimal stationary program. As already proved in the theorem IV-2-3-C, the optimal stationary program $(\hat{x}^*, \hat{y}^*, \hat{c}^*)$ is an unique program of $(\hat{x}, \hat{y}) \in // T //$ and $\hat{c} \in // C //$ which maximizes the function φ . That is, the optimal stationary program $(\hat{x}^*, \hat{y}^*, \hat{c}^*)$ maximizes the function φ .

Apparently from the theorem IV-2-3-C, $\varphi(\hat{x}^*, \hat{y}^*, \hat{c}^*) = \hat{F}(\hat{c}^*)$. The optimal program $(\hat{x}_t^{**}, \hat{y}_t^{**}, \hat{c}_t^{**})$ is, needless to say, an eligible program. (Definition of eligible program is given, for example, in the textbook by Takayama(54), p.589.) Therefore,

$$\varphi(\hat{x}_t^{**}, \hat{y}_t^{**}, \hat{c}_t^{**}) = \hat{F}(\hat{c}^*) - u_t \quad \text{and} \quad u_t \rightarrow 0, \quad \text{when } t \rightarrow \infty.$$

Therefore $\varphi(\hat{x}_t^{**}, \hat{y}_t^{**}, \hat{c}_t^{**}) \rightarrow \varphi(\hat{x}^*, \hat{y}^*, \hat{c}^*)$, when $t \rightarrow \infty$.

Now suppose that $\hat{c}_t^{**} \rightarrow \hat{c}^{***}$, when $t \rightarrow \infty$ and also suppose that $\hat{c}^{***} \neq \hat{c}^*$. Since $\hat{c}_t^{**}$ is the optimal program, $\hat{c}_t^{**}$ should satisfy the condition of the desirable structure of consumption. Also since $\hat{c}_t^{**}$ is the optimal program, $\hat{x}_t^{**} + \hat{c}_t^{**} - \hat{y}_t^{**} = 0$; ($t=0, 1, 2, \dots, \infty$).

If $\hat{F}(\hat{c}_t^{**}) \longrightarrow \hat{F}(\hat{c}^{***}) < \hat{F}(\hat{c}^*)$, then $\hat{F}(\hat{c}_t^{**}) < \hat{F}(\hat{c}^*)$; $t \geq T'$.

So $\sum_{t=0}^{\infty} (\hat{F}(\hat{c}_t^{**}) - \hat{F}(\hat{c}^*)) \longrightarrow -\infty$.

This is a contradiction to the eligibility of $\hat{c}_t^{**}$.

Therefore $\hat{F}(\hat{c}_t^{**}) \longrightarrow \hat{F}(\hat{c}^*)$, when $t \longrightarrow \infty$.

Under the condition of the desirable structure of consumption this relation means that $\hat{c}_t^{**} \longrightarrow \hat{c}^*$.

As already proved in the precedent subsection, a consumption program determines an unique program of production.

So $(\hat{x}_t^{**}, \hat{y}_t^{**}) \longrightarrow (\hat{x}^*, \hat{y}^*)$, when $t \longrightarrow \infty$.

That is, $\hat{c}_t^{**} \longrightarrow \hat{c}^*$, $\hat{s}_t^{**} \longrightarrow \hat{s}^*$, $\hat{x}_t^{**} \longrightarrow \hat{x}^*$, $\hat{e}_t^{**} \longrightarrow \hat{e}^*$, $\hat{m}_t^{**} \longrightarrow \hat{m}^*$ and $\hat{z}_t^{**} \longrightarrow \hat{z}^*$, when $t \longrightarrow \infty$. Q.E.D.

M-V-2-3-9

In this subsection the Author proves some properties of V-2-3-9 on the accounting prices of the infinite economic planning.

The optimal stationary program was already defined in V-2-3-6 and M-V-2-3-6, however it is convenient to rewrite the problem in the following manner.

$$\begin{aligned} \text{MAX} \quad & \hat{F}(\hat{c}) \\ \text{s.t.} \quad & \hat{x} + \hat{c} \leq \hat{y} \\ & \hat{c} \in //C// \quad \text{and} \quad (\hat{x}, \hat{y}) \in //T//. \end{aligned} \quad \dots\dots\dots(5)$$

where the notations $\hat{F}(\cdot)$, $\hat{x}$, $\hat{y}$, $\hat{c}$, $//C//$ and $//T//$ were defined in the mathematical appendix (M-V-2-3-5).

Now the Author proves the following important property.

Property V-2-3-9-A Under the assumption V-2-3-6-A there exists a $(n+1)$ -dimensional price vector $\hat{p}^* \geq 0$ such that

$$(i) \quad \hat{F}(\hat{c}) + \hat{p}^*(\hat{y} - \hat{x} - \hat{c}) \leq \hat{F}(\hat{c}^*) + \hat{p}^*(\hat{y}^* - \hat{x}^* - \hat{c}^*)$$

for all $(\hat{x}, \hat{y}) \in //T//$ and $\hat{c} \in //C//$,

$$(ii) \quad \text{and} \quad \hat{p}^*(\hat{y}^* - \hat{x}^* - \hat{c}^*) = 0, \quad \hat{y}^* - \hat{x}^* - \hat{c}^* \geq 0.$$

Proof Assumption V-2-3-6-A means the Slater condition. So we can directly apply the Kuhn-Tucker theorem, since $//T//$ and $//C//$ are convex set, $\hat{F}(\hat{c})$ and the constraint (5) are concave function and finally we already proved the existence of the optimal stationary

stationary program $(\hat{x}^*, \hat{y}^*, \hat{c}^*)$.

Q.E.D.

From the property V-2-3-9-A we can directly obtain the following property.

Property V-2-3-9-B Under the assumption V-2-3-6-A there exists a n-dimensional price vector p^* such that

$$(i) \quad F(c) + p^* \left(\frac{1}{w} (I_n + B)x + \frac{1}{w} m - (A+B)x - e - c \right) \\ \leq F(c^*) + p^* \left(\frac{1}{w} (I_n + B)x^* + \frac{1}{w} m^* - (A+B)x^* - e^* - c^* \right)$$

for all $c \in //C//$ and (x, e, m, z) which satisfy the constraints (12), (13), (14), (15) and (16) of V-2-3-6.

$$(ii) \quad p^* \left(\frac{1}{w} (I_n + B)x^* + \frac{1}{w} m^* - (A+B)x^* - e^* - c^* \right) = 0 \\ \text{and } \frac{1}{w} (I_n + B)x^* + \frac{1}{w} m^* - (A+B)x^* - e^* - c^* \geq 0$$

Now the Author presents an economic interpretation of property V-2-3-9-B. From the property V-2-3-9-B we can easily obtain the following properties.

Property V-2-3-9-C If $F(c) > F(c^*)$, then

$$p^* \left(\frac{1}{w} (I_n + B)x + \frac{1}{w} m - (A+B)x - e - c \right) < 0 \quad \dots\dots\dots(1)$$

Property V-2-3-9-D

$$p^* \left(\frac{1}{w} (I_n + B)x^* + \frac{1}{w} m^* - (A+B)x^* - e^* - c^* \right) = 0 \quad \dots\dots\dots(2)$$

M-V-2-3-10

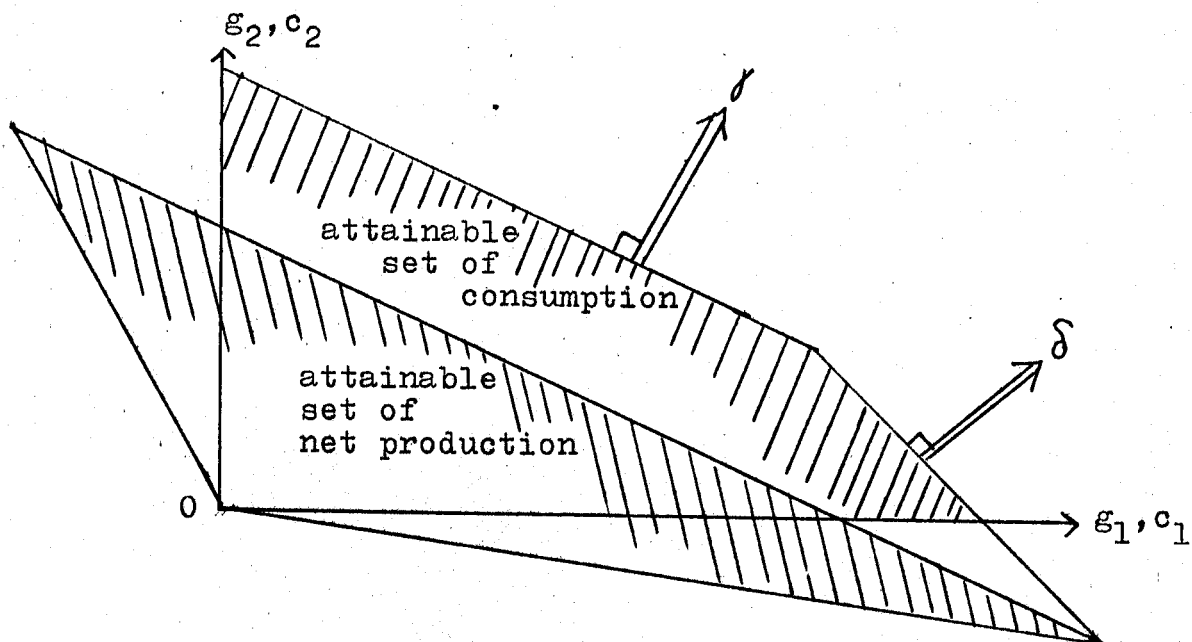
Here we investigate the possibility of negative net production. Apparently the gross output(x) cannot be negative even with the international trade. Therefore the attainable set of net production is bounded.

// A(g) // denotes the attainable set of net production; i.e.

$$// A(g) // =: \{ g \mid g = Jx, \quad v_0' x \leq L_0 \}$$

Needless to say, some elements of net production vector (g) can be negative. But, apparently from the following figure, we need no modification to the procedures for the calculation of accounting prices.

We can illustrate the set // A(g) // as follows.



So the procedure 1 and 2, and the conclusion 2 are also valid without any modification. The procedure 1 is transformed as follows.

Procedure 1'

- (i) If i -th industry produces positive gross output, then the accounting price is γ_i .
- (ii) If i -th industry does not produce gross output and therefore the final demand is satisfied by import, then the accounting price is δ_i ;

$$\delta_i =: \gamma_j \frac{(1+r)d_i^m}{d_j^e}$$

where j denotes the marginal exporting industry.

M-V-2-4

In this subsection the Author investigates mathematically the simplified long-term economic planning and proves some theorems.

Firstly we transform the whole system of the simplified long-term economic planning model (i.e. from (1) to (9) of the text V-2-4-3) into the following system using "per capita" variables.

$$\text{MAX} \sum_{t=0}^T F(c_t) = \sum_{t=0}^T \left(\min\left(\frac{c_{i,t} - k_i}{f_i}\right) + \sum_{i=1}^n k_i \right) \dots\dots\dots(1)$$

$$\text{s.t.} \quad \hat{x}_t + \hat{c}_t \leq \hat{y}_{t-1}; (t=1, 2, \dots, T) \dots\dots\dots(10)$$

$$\hat{x}_0 + \hat{c}_0 \leq \underline{K}_0 \quad \dots\dots\dots(11)$$

$$\hat{y}_T \geq \underline{K}_T \quad \dots\dots\dots(12)$$

$$(\hat{x}_t, \hat{y}_t) \in // T // \quad ; (t=0, 1, 2, \dots, T) \quad \dots\dots\dots(13)$$

$$\hat{c}_t \in // C // \quad ; (t=0, 1, 2, \dots, T) \quad \dots\dots\dots(14)$$

where

$$\underline{K}_0 =: \begin{pmatrix} \underline{X}_1 + B \underline{X}_1 + \underline{M}_1 \\ \vdots \\ \underline{X}_n + B \underline{X}_n + \underline{M}_n \\ -(1+r) \sum_{i=1}^n d_i^m \underline{M}_i - (1+r) \underline{Z} \end{pmatrix}$$

$$\underline{K}_T =: \begin{pmatrix} X_{1,T}^* / w^{T+1} + B(X_{1,T}^* / w^{T+1}) + M_{1,T}^* / w^{T+1} \\ \vdots \\ X_{n,T}^* / w^{T+1} + B(X_{n,T}^* / w^{T+1}) + M_{n,T}^* / w^{T+1} \\ -(1+r) \sum_{i=1}^n d_i^m (M_{i,T}^* / w^{T+1}) - (1+r) Z_T^* / w^{T+1} \end{pmatrix}$$

That is, $\underline{K}_0$ denotes the initial condition vector. $\underline{K}_T$ denotes the final condition vector in "per capita" term.

Theorem V-2-4-3

Under the assumption V-2-4-3-A there exists an optimal program of the long-term economic planning.

Proof

Attainable set $(\hat{x}_t, \hat{y}_t, \hat{c}_t)$ is compact, since $// T //$ and $// C //$ are compact. The objective function (1) is a continuous function. Therefore by the Weierstrass's theorem there exists an optimal program. Q.E.D.

Firstly we define the attainable program in the simplified long-term economic planning.

Definition of attainable program

$(x_t^0, e_t^0, m_t^0, z_t^0, c_t^0, s_t^0)$ is an attainable program, if

$$a) \begin{pmatrix} A+B & I_n & 0_{nn} & 0_{nl} \\ 0_{ln} & -d^e & 0_{ln} & -1 \end{pmatrix} \begin{pmatrix} x_t^0 \\ e_t^0 \\ m_t^0 \\ z_t^0 \end{pmatrix} + \begin{pmatrix} c_t^0 \\ s_t^0 \end{pmatrix} \leq \begin{pmatrix} I_n + B & 0_{nn} & I_n & 0_{nl} \\ 0_{ln} & 0_{ln} & -(1+r)d^m & -(1+r) \end{pmatrix} \begin{pmatrix} x_{t-1}^0 / w \\ e_{t-1}^0 / w \\ m_{t-1}^0 / w \\ z_{t-1}^0 / w \end{pmatrix}$$

$$\text{where } d^{e'} = (d_1^e, d_2^e, \dots, d_n^e) ; (t=1, 2, \dots, T)$$

$$d^m = (d_1^m, d_2^m, \dots, d_n^m)$$

$$b) v_{i0}' x_t^o \leq L_0 ; (t=0, 1, 2, \dots, T)$$

$$c) 0 \leq e_{i,t}^o \leq h_{i0}^e ; (i=1, 2, \dots, n \text{ and } t=0, 1, 2, \dots, T)$$

$$d) 0 \leq m_{i,t}^o \leq h_{i0}^m ; (i=1, 2, \dots, n \text{ and } t=0, 1, 2, \dots, T)$$

$$e) 0 \leq z_t^o \leq H_0 ; (t=0, 1, 2, \dots, T)$$

f) (initial condition)

$$R \begin{pmatrix} x_0^o \\ e_0^o \\ m_0^o \\ z_0^o \end{pmatrix} + \begin{pmatrix} c_0^o \\ s_0^o \end{pmatrix} \leq \begin{pmatrix} \underline{X} + B \underline{X} + \underline{M} \\ -(1+r)d^{e'} \underline{M} - (1+r)\underline{Z} \end{pmatrix}$$

where the matrices (R) and (Q) were defined in (6'') in M-V-2-3-5. That is, R is the first matrix of the condition a).

Q is the second matrix of the condition a).

$\underline{X}$; n-dimensional column vector of X_i .

$\underline{M}$; n-dimensional column vector of M_i .

$\underline{Z}$; n-dimensional column vector of Z_i .

$$g) R \begin{pmatrix} x_T^o \\ e_T^o \\ m_T^o \\ z_T^o \end{pmatrix} \geq Q \begin{pmatrix} x_T^* \\ e_T^* \\ m_T^* \\ z_T^* \end{pmatrix}$$

// A// denotes the set of attainable programs.

Now we can define the competitive price vector in the simplified long-term economic planning.

Definition of competitive price vector

If there exists a price vector $p_t ; (t=0, 1, 2, \dots, T, T+1)$,

$p_t \geq 0$, $p_t \in R_{n+1}$, which satisfies the following conditions,

then this price vector is called the competitive price vector

associated with an attainable program $(c_t^o, s_t^o, x_t^o, e_t^o, m_t^o, z_t^o)$.

$p_t' = (p_{1,t}, p_{2,t}, \dots, p_{n,t}, p_{z,t})$, That is, p_t is a column

vector and p_t' is a row vector.

$$(i) \quad F(c_t^0) - F(c_t) \geq p_t'(c_t^0 - c_t) \quad \dots\dots\dots(15)$$

for all $c_t \in //C//$; $(t=0,1,2,\dots,T)$

$$(ii) \quad p_{t+1}' \begin{pmatrix} (I+B)x_t^0/w & + & m_t^0/w \\ -(1+r)d^m m_t^0/w & - & (1+r)z_t^0/w \end{pmatrix} - p_t' \begin{pmatrix} (A+B)x_t^0 & + & e_t^0 \\ -d^e e_t^0 & - & z_t^0 \end{pmatrix} \\ \geq p_{t+1}' \begin{pmatrix} (I+B)x_t/w & + & m_t/w \\ -(1+r)d^m m_t/w & - & (1+r)z_t/w \end{pmatrix} - p_t' \begin{pmatrix} (A+B)x_t & + & e_t \\ -d^e e_t & - & z_t \end{pmatrix}$$

for any attainable program (x_t, e_t, m_t, z_t) ; $(t=0,1,2,\dots,T)$.
 $\dots\dots\dots(16)$

$$(iii) \quad p_0' \left(\underline{K}_0 - \begin{pmatrix} c_0^0 \\ s_0^0 \end{pmatrix} - \begin{pmatrix} (A+B)x_0^0 & + & e_0^0 \\ -d^e e_0^0 & - & z_0^0 \end{pmatrix} \right) = 0 \quad \dots\dots\dots(17)$$

$$p_t' \left(\begin{pmatrix} \frac{1}{w}(I+B)x_{t-1}^0 & + & \frac{1}{w}m_{t-1}^0 \\ -\frac{(1+r)}{w}d^m m_{t-1}^0 & - & \frac{(1+r)}{w}z_{t-1}^0 \end{pmatrix} - \begin{pmatrix} c_t^0 \\ s_t^0 \end{pmatrix} - \begin{pmatrix} (A+B)x_t^0 & + & e_t^0 \\ -d^e e_t^0 & - & z_t^0 \end{pmatrix} \right) = 0 \\ ;(t=0,1,2,\dots,T) \quad \dots\dots\dots(18)$$

$$(iv) \quad p_{T+1}' \left(\begin{pmatrix} \frac{1}{w}(I+B)x_T^0 & + & \frac{1}{w}m_T^0 \\ -\frac{(1+r)}{w}d^m m_T^0 & - & \frac{(1+r)}{w}z_T^0 \end{pmatrix} - \underline{K}_T \right) = 0 \quad \dots\dots\dots(19)$$

This definition is rather complicated, so we rewrite this definition using the notations $(\hat{x}_t, \hat{y}_t, \hat{c}_t)$, which were defined in the precedent subsection M-V-2-3-5.

Rewritten definition of competitive price vector

If there exists a price vector p_t ; $(t=0,1,2,\dots,T,T+1)$,
 $p_t \geq 0$ and $p_t \in R_{n+1}$, which satisfies the following condition,
 then this price vector is called the competitive price vector
 associated with an attainable program $(\hat{x}_t^0, \hat{y}_t^0, \hat{c}_t^0)$.

$$(i) \quad \hat{F}(\hat{c}_t^0) - \hat{F}(\hat{c}_t) \geq p_t'(\hat{c}_t^0 - \hat{c}_t) \quad \dots\dots\dots(15')$$

for all $\hat{c}_t \in //C//$; $(t=0,1,2,\dots,T)$

$$(ii) \quad p_{t+1}' \hat{y}_t^0 - p_t' \hat{x}_t^0 \geq p_{t+1}' \hat{y}_t - p_t' \hat{x}_t \quad \dots\dots\dots(16')$$

for all $(\hat{x}_t, \hat{y}_t) \in //T//$; $(t=0,1,2,\dots,T)$

$$(iii) \quad p_0' (\underline{K}_0 - \hat{c}_0^0 - \hat{x}_0^0) = 0 \quad \dots\dots\dots(17')$$

$$p_t' (\hat{y}_{t-1}^0 - \hat{c}_t^0 - \hat{x}_t^0) = 0 ;(t=1,2,\dots,T) \quad \dots\dots\dots(18')$$

$$(iv) \quad p_{T+1}' (\hat{y}_T^0 - \underline{K}_T) = 0 \quad \dots\dots\dots(19')$$

A program $(\hat{x}_t^o, \hat{y}_t^o, \hat{c}_t^o)$ is called the competitive program, if $(\hat{x}_t^o, \hat{y}_t^o, \hat{c}_t^o)$ satisfies (15'), (16'), (17'), (18') and (19').

Now the Author gives an economic interpretation of the definition of the competitive prices.

(i) Condition (15') means that

if $\hat{F}(\hat{c}_t) \geq \hat{F}(\hat{c}_t^o)$, then $p_t' \hat{c}_t^o \leq p_t' \hat{c}_t$; $(t=0, 1, 2, \dots, T)$

Also if $p_t' \hat{c}_t^o \geq p_t' \hat{c}_t$, then $\hat{F}(\hat{c}_t^o) \leq \hat{F}(\hat{c}_t)$; $(t=0, 1, 2, \dots, T)$

These relations mean the maximization of the objective function in t -th period within the budget constraint.

(ii) From (18') we obtain the following relation, since $p_t \geq 0$ and $\hat{x}_t + \hat{c}_t \leq \hat{y}_{t-1}$.

$$0 = p_t' (\hat{x}_t^o + \hat{c}_t^o - \hat{y}_{t-1}^o) \geq p_t' (\hat{x}_t + \hat{c}_t - \hat{y}_{t-1}) \quad \dots\dots\dots(20)$$

for all attainable program $(\hat{x}_t, \hat{y}_t, \hat{c}_t)$.

From (16') we obtain the following relation.

$$\sum_{t=0}^T (p_{t+1}' \hat{y}_t^o - p_t' \hat{x}_t^o) \geq \sum_{t=0}^T (p_{t+1}' \hat{y}_t - p_t' \hat{x}_t) \quad \dots\dots\dots(21)$$

Also from (20) we obtain the following relation.

$$\sum_{t=0}^T (p_t' \hat{x}_t^o - p_t' \hat{y}_{t-1}^o + p_t' \hat{c}_t^o) \geq \sum_{t=0}^T (p_t' \hat{x}_t - p_t' \hat{y}_{t-1} + p_t' \hat{c}_t) \quad \dots\dots\dots(22)$$

From (12) and (19') we obtain the following relation.

$$0 = p_{T+1}' (\hat{y}_T^o - \underline{K}_T) \leq p_{T+1}' (\hat{y}_T - \underline{K}_T)$$

Therefore $p_{T+1}' \hat{y}_T^o \leq p_{T+1}' \hat{y}_T \quad \dots\dots\dots(23)$

From (21), (22) and (23) we can obtain the following relation.

$$- p_0' \hat{y}_{-1}^o + \sum_{t=0}^T p_t' \hat{c}_t^o \geq - p_0' \hat{y}_{-1} + \sum_{t=0}^T p_t' \hat{c}_t$$

Recalling that $\hat{y}_{-1}^o = \hat{y}_{-1} = \underline{K}_0$ we can obtain

$$\sum_{t=0}^T p_t' \hat{c}_t^o \geq \sum_{t=0}^T p_t' \hat{c}_t \quad \dots\dots\dots(24)$$

for all attainable $\hat{c}_t$.

(24) means that the competitive program maximizes the total value of consumption. Consequently (16') means that the competitive program maximizes the total value of net output, i.e. the total value of consumption. In other words the competitive program maximizes its "profit".

(iii) (17') and (18') indicate the following relation.

$$\left(\begin{array}{l} \text{value of initial} \\ \text{stock of the com-} \\ \text{petitive program} \\ \text{in the beginning} \\ \text{of } t\text{-th period} \end{array} \right) = \left(\begin{array}{l} \text{value of consump-} \\ \text{tion of the comp-} \\ \text{etitive program} \\ \text{in } t\text{-th period} \end{array} \right) + \left(\begin{array}{l} \text{value of output except} \\ \text{consumption of the} \\ \text{competitive program} \\ \text{in } t\text{-th period} \end{array} \right)$$

(iv) (19') means that the value of the final stock of the competitive program in T -th period should be equal to the value of the final stock which is determined by the optimal stationary program of the infinite economic planning.

(17') and (19') also mean that

$$\text{if } \hat{y}_{i,t-1}^0 - \hat{c}_{i,t}^0 - \hat{x}_{i,t}^0 > 0, \text{ then } p_{i,t} = 0 \dots\dots\dots(25)$$

$$, \text{ since } p_t \geq 0 \text{ and } \hat{y}_{t-1} \geq \hat{c}_t + \hat{x}_t.$$

(25) means that if an excess supply exists, then the price of the product is zero.

Theorem V-2-4-4-A

If an attainable program $(\hat{x}_t^0, \hat{y}_t^0, \hat{c}_t^0)$ is competitive, then it is an optimal program of the simplified long-term economic planning.

Proof

From the conditions (15') and (17'),

$$\begin{aligned} \hat{F}(\hat{c}_0^0) - \hat{F}(\hat{c}_0) &\geq p'_0(\hat{c}_0^0 - \hat{c}_0) = p'_0(\underline{K}_0 - \hat{x}_0^0) - p'_0 \hat{c}_0 \\ &\geq p'_0(\underline{K}_0 - \hat{x}_0^0) - p'_0(\underline{K}_0 - \hat{x}_0) = p'_0(\hat{x}_0 - \hat{x}_0^0) \end{aligned} \dots\dots\dots(26)$$

From the conditions (15') and (18'),

$$\begin{aligned} \hat{F}(\hat{c}_t^0) - \hat{F}(\hat{c}_t) &\geq p'_t(\hat{c}_t^0 - \hat{c}_t) \\ &\geq p'_t(\hat{y}_{t-1}^0 - \hat{x}_t^0) - p'_t(\hat{y}_{t-1} - \hat{x}_t) \quad ; (t=1, 2, \dots, T) \end{aligned} \dots\dots\dots(27)$$

From (26) and (27) we obtain

$$\begin{aligned} \sum_{t=0}^T \hat{F}(\hat{c}_t^0) - \sum_{t=0}^T \hat{F}(\hat{c}_t) &\geq \\ \sum_{t=0}^{T-1} (p'_{t+1} \hat{y}_t^0 - p'_t \hat{x}_t^0) - \sum_{t=0}^{T-1} (p'_{t+1} \hat{y}_t - p'_t \hat{x}_t) - p'_T \hat{x}_T^0 + p'_T \hat{x}_T \end{aligned} \dots\dots\dots(28)$$

From (12) and (19'),

$$0 = p'_{T+1}(\hat{y}_T^0 - \underline{K}_T) \leq p'_{T+1}(\hat{y}_T - \underline{K}_T) \dots\dots\dots(29)$$

So from (28) and (29) we can obtain

$$\begin{aligned} \sum_{t=0}^T \hat{F}(\hat{c}_t^0) - \sum_{t=0}^T \hat{F}(\hat{c}_t) &\geq \sum_{t=0}^T (p'_{t+1} \hat{y}_t^0 - p'_t \hat{x}_t^0 - (p'_{t+1} \hat{y}_t - p'_t \hat{x}_t)) \end{aligned} \dots\dots\dots(30)$$

From (16') and (30) we obtain

$$\sum_{t=0}^T \hat{F}(\hat{c}_t^0) \geq \sum_{t=0}^T \hat{F}(\hat{c}_t)$$

Q.E.D.

Theorem V-2-4-4-B

Under the assumption V-2-4-3-A (Slater's condition) if an attainable program $(\hat{x}_t^*, \hat{y}_t^*, \hat{c}_t^*)$ is an optimal program, then it is a competitive program. That is, there exists a competitive price vector which associates with the optimal program.

Proof

This proof is done by the Takayama's manner (54), p.582.

// T// and // C// are convex set. $\sum_{t=0}^T \hat{F}(\hat{c}_t)$ is a concave function, since $\hat{F}(\hat{c}_t)$ is a concave function. By the Kuhn-Tucker theorem we can obtain the following relation.

$$\begin{aligned} & \sum_{t=0}^T \hat{F}(\hat{c}_t^*) + p_0'(\underline{K}_0 - \hat{x}_0^* - \hat{c}_0^*) + \sum_{t=1}^T p_t'(\hat{y}_{t-1}^* - \hat{x}_t^* - \hat{c}_t^*) + p_{T+1}'(\hat{y}_T^* - \underline{K}_T) \\ & \geq \sum_{t=0}^T \hat{F}(\hat{c}_t) + p_0'(\underline{K}_0 - \hat{x}_0 - \hat{c}_0) + \sum_{t=1}^T p_t'(\hat{y}_{t-1} - \hat{x}_t - \hat{c}_t) + p_{T+1}'(\hat{y}_T - \underline{K}_T) \\ & \text{for all } (\hat{x}_t, \hat{y}_t, \hat{c}_t) \in // A// \quad \dots\dots\dots(31) \end{aligned}$$

and

$$p_0'(\underline{K}_0 - \hat{x}_0^* - \hat{c}_0^*) + \sum_{t=1}^T p_t'(\hat{y}_{t-1}^* - \hat{x}_t^* - \hat{c}_t^*) + p_{T+1}'(\hat{y}_T^* - \underline{K}_T) = 0 \quad \dots\dots(32)$$

From the definition of the attainable program we obtain

$$p_t \geq 0 ; (t=0, 1, \dots, T, T+1)$$

$$\underline{K}_0 - \hat{x}_0^* - \hat{c}_0^* \geq 0$$

$$\hat{y}_{t-1}^* - \hat{x}_t^* - \hat{c}_t^* \geq 0 ; (t=1, 2, \dots, T)$$

$$\hat{y}_T^* \geq \underline{K}_T$$

Therefore from (32) we obtain the following conditions (iii) and (iv) of the competitive price. That is,

$$(iii) \quad p_0'(\underline{K}_0 - \hat{x}_0^* - \hat{c}_0^*) = 0 \quad \text{and} \quad p_t'(\hat{y}_{t-1}^* - \hat{x}_t^* - \hat{c}_t^*) = 0 ; (t=1, 2, \dots, T)$$

$$(iv) \quad p_{T+1}'(\hat{y}_T^* - \underline{K}_T) = 0$$

Next we set $(\hat{x}_t, \hat{y}_t, \hat{c}_t)$ as follows.

$$\hat{x}_t = \hat{x}_t^* ; (t=0, 1, \dots, T). \quad \hat{y}_t = \hat{y}_t^* ; (t=0, 1, \dots, T).$$

$$\hat{c}_t = \hat{c}_t^* ; (t=0, 1, 2, \dots, t'-1, t'+1, \dots, T).$$

Then from (31) we obtain the following condition (i).

$$(i) \quad \hat{F}(\hat{c}_t^*) - \hat{F}(\hat{c}_t) \geq p_t'(\hat{c}_t^* - \hat{c}_t)$$

Next we set

$$\hat{c}_t = \hat{c}_t^* ; (t=0,1,\dots,T), \quad \hat{x}_T = \hat{x}_T^* \text{ and } \hat{y}_T = \hat{y}_T^* .$$

Then from (31) we obtain the following relation.

$$\sum_{t=0}^{T-1} p'_{t+1} (\hat{y}_t^* - \hat{y}_t) \geq \sum_{t=0}^{T-1} p'_t (\hat{x}_t^* - \hat{x}_t) \quad \dots\dots\dots(33)$$

$$\text{Now we set } \hat{x}_t = \hat{x}_t^* ; (t=0,1,\dots, t'-1, t'+1, \dots, T-1)$$

$$\hat{y}_t = \hat{y}_t^* ; (t=0,1,\dots, t'-1, t'+1, \dots, T-1)$$

Then from (33) we obtain

$$p'_{t+1} \hat{y}_t^* - p'_t \hat{x}_t^* \geq p'_{t+1} \hat{y}_t - p'_t \hat{x}_t ; (t=0,1,\dots, T-1) \quad \dots\dots(34)$$

Finally in (31) we set

$$\hat{c}_t = \hat{c}_t^* ; (t=0,1,\dots, T)$$

$$\hat{x}_t = \hat{x}_t^* ; (t=0,1,\dots, T-1)$$

$$\hat{y}_t = \hat{y}_t^* ; (t=0,1,\dots, T-1) . \quad \text{Then we obtain}$$

$$- p'_T \hat{x}_T^* + p'_{T+1} \hat{y}_T^* \geq - p'_T \hat{x}_T + p'_{T+1} \hat{y}_T \quad \dots\dots\dots(35)$$

The relations (34) and (35) constitute the following condition (ii) of the competitive program.

$$(ii) \quad p'_{t+1} \hat{y}_t^* - p'_t \hat{x}_t^* \geq p'_{t+1} \hat{y}_t - p'_t \hat{x}_t ; (t=0,1,\dots, T)$$

Q.E.D.

M-V-2-4-6-A

The Author presents here a more abstract and formal definition of the admissible consumption field.

We can take into account the change of the level of the collective consumption by supposing a different admissible consumption field from period to period. $\parallel C'_t \parallel$ denotes the admissible consumption field in t-th period. $\parallel C' \parallel$ denotes the admissible consumption field of the whole system.

$$\parallel C' \parallel =: \parallel C'_0 \parallel \times \parallel C'_1 \parallel \times \parallel C'_2 \parallel \times \dots\dots\dots \times \parallel C'_T \parallel$$

$\parallel C' \parallel$ is a compact and convex set, since $\parallel C'_t \parallel$ is compact and convex. Compactness of $\parallel C'_t \parallel$ is assumed by supposing the satiation point of consumption.

M-V-2-4-6-B

Here we discuss the treatment of complicated time lag of investment.

We can incorporate the complicated time lag of investment into our model without any theoretical difficulty.

Firstly we define the following notations.

B_t ; the matrix of total capital stock, which is necessary to produce one unit of product in t -th period.

B_t^{-1} ; the matrix of investment, which is necessary one period earlier. That is, $b_{i,j}^{-1}(1981)$ denotes the investment of i -th product in j -th industry in 1980 in order to produce additional one unit of j -th product in 1981.

B_t^{-2} ; the matrix of investment, which is necessary two periods earlier.

B_t^{-3} ; the matrix of investment, which is necessary three periods earlier.

Now we investigate the case of three-periods time lag. If Reader wants, he can construct a more general model. By definition,

$$B_t = B_t^{-1} + B_t^{-2} + B_t^{-3}$$

The condition of material balance (3) of M-V-2-4-7 becomes as follows.

$$A_t X_t + E_t^R + E_t^D + C_t + B_t^{-1} X_t + B_t^{-2} X_{t+1} + B_t^{-3} X_{t+2} - B_t X_{t-1} \leq X_{t-1} + M_{t-1}^R + M_{t-1}^D$$

Finally we must add some initial conditions and final conditions.

$$X_{-1} = \underline{X}_{-1}, X_{-2} = \underline{X}_{-2}, X_{-3} = \underline{X}_{-3},$$

$$\dot{X}_{T+1} = \underline{X}_{T+1}^*, X_{T+2} = \underline{X}_{T+2}^*, X_{T+3} = \underline{X}_{T+3}^*.$$

Then we can construct the model of an enlarged long-term optimal planning with complicated time lag of investment.

M-V-2-4-7

In this subsection the Author formulates the model of the enlarged long-term optimal planning and proves some theorems.

model

$$\text{MAX } F = \sum_{t=0}^T F_t(c_t) = \sum_{t=0}^T \left(\min_i \left(\frac{c_{i,t} - k_{i,t}}{f_{i,t}} \right) + \sum_{i=1}^n k_{i,t} \right)$$

.....(1)

$$\text{where } c_{i,t} = \frac{C_{i,t}}{(1+u)^t Q_t} =: \frac{C_{i,t}}{d_t} \dots\dots\dots(2)$$

$$d_t = (1+u)^t Q_t$$

subject to

$$A_t X_t + E_t^R + D_t^D + C_t + B_t(X_t - X_{t-1}) \leq X_{t-1} + M_{t-1}^R + M_{t-1}^D \dots\dots\dots(3)$$

;(t=0,1,...,T)

Condition (3) turns into as follows.

$$A_t x_t + e_t^R + e_t^D + c_t + B_t(x_t - \frac{Q_{t-1}}{Q_t(1+u)} x_{t-1}) \leq \frac{Q_{t-1}}{Q_t(1+u)} x_{t-1} + \frac{Q_{t-1}}{Q_t(1+u)} (m_{t-1}^R + m_{t-1}^D) \dots\dots\dots(3')$$

;(t=0,1,...,T)

$$\text{where } x_t = \frac{X_t}{Q_t(1+u)^t} = X_t/d_t \dots\dots\dots(3)$$

;(t=0,1,...,T)

x_t is a n-dimensional column vector.

$$c_t = C_t/d_t \dots\dots\dots(3)$$

;(t=0,1,...,T) (n-dimensional column vector)

$$e_t^R = E_t^R/d_t \dots\dots\dots(3)$$

;(t=0,1,...,T) (n-dimensional column vector)

$$e_t^D = E_t^D/d_t \dots\dots\dots(3)$$

;(t=0,1,...,T) (n-dimensional column vector)

$$m_t^R = M_t^R/d_t \dots\dots\dots(3)$$

;(t=0,1,...,T) (n-dimensional column vector)

$$m_t^D = M_t^D/d_t \dots\dots\dots(3)$$

;(t=0,1,...,T) (n-dimensional column vector)

$$\sum_{i=1}^n v_{i,t} X_{i,t} \leq L_t \dots\dots\dots(4)$$

;(t=0,1,...,T)

where L_t denotes the amount of labour force in t-th period.

$$\sum_{i=1}^n v_{i,t} x_{i,t} \leq L_t/d_t \dots\dots\dots(4')$$

;(t=0,1,...,T)

$$\sum_{i=1}^n d_{i,t}^R E_{i,t}^R + z_t^R \geq (1+r_t^R) \sum_{i=1}^n d_{i,t}^{mR} M_{i,t-1}^R + (1+r_t^R) z_{t-1}^R \dots\dots\dots(5)$$

;(t=0,1,...,T)

The condition (5) turns into the following "per capita" form.

$$\sum_{i=1}^n d_{i,t}^R e_{i,t}^R + z_t^R \geq (1+r_t^R) \frac{Q_{t-1}}{Q_t(1+u)} (\sum_{i=1}^n d_{i,t-1}^{mR} m_{i,t-1}^R + z_{t-1}^R) \dots\dots\dots(5')$$

;(t=0,1,...,T)

$$\text{where } z_t^R = Z_t^R/d_t \dots\dots\dots(5)$$

;(t=0,1,...,T) z_t^R is a scalar.

$$z_t^D = Z_t^D/d_t \dots\dots\dots(5)$$

;(t=0,1,...,T) z_t^D is a scalar.

Similarly we obtain the following condition.

$$\sum_{i=1}^n d_{i,t}^D e_{i,t}^D + z_t^D \geq (1+r_t^D) \frac{Q_{t-1}}{Q_t(1+u)} (\sum_{i=1}^n d_{i,t-1}^{mD} m_{i,t-1}^D + z_{t-1}^D) \dots\dots\dots(6')$$

;(t=0,1,...,T)

$$0 \leq E_{i,t}^R \leq h_{i,t}^R \dots\dots\dots(7)$$

;(t=0,1,...,T. i=1,2,...,n)

$$\text{So } 0 \leq e_{i,t}^R \leq h_{i,t}^{eR}/d_t \quad ; (t=0,1,\dots,T, i=1,2,\dots,n) \dots\dots\dots(7')$$

Similarly

$$0 \leq e_{i,t}^D \leq h_{i,t}^{eD}/d_t \quad ; (t=0,1,\dots,T, i=1,2,\dots,n) \dots\dots\dots(8')$$

$$0 \leq m_{i,t}^R \leq h_{i,t}^{mR}/d_t \quad ; (t=0,1,\dots,T, i=1,2,\dots,n) \dots\dots\dots(9')$$

$$0 \leq m_{i,t}^D \leq h_{i,t}^{mD}/d_t \quad ; (t=0,1,\dots,T, i=1,2,\dots,n) \dots\dots\dots(10')$$

$$0 \leq z_t^R \leq H_t^R/d_t \quad ; (t=0,1,\dots,T) \dots\dots\dots(11')$$

$$0 \leq z_t^D \leq H_t^D/d_t \quad ; (t=0,1,\dots,T) \dots\dots\dots(12')$$

Available resource restricts the level of output. That is,

$$0 \leq x_{i,t} \leq g_{i,t} \dots\dots\dots(13)$$

$$\text{or } 0 \leq x_{i,t} \leq g_{i,t}/d_t \dots\dots\dots(13')$$

Initial conditions are as follows.

$$x_{-1} = \underline{x}, \quad e_{-1}^R = \underline{e}^R, \quad e_{-1}^D = \underline{e}^D, \quad m_{-1}^R = \underline{m}^R, \quad m_{-1}^D = \underline{m}^D \\ z_{-1}^R = \underline{z}^R, \quad z_{-1}^D = \underline{z}^D \dots\dots\dots(14')$$

Final conditions are as follows.

$$x_T + B_T x_T + m_T^R + m_T^D \geq \underline{x}_T^* + B_T \underline{x}_T^* + \underline{m}_T^* \dots\dots\dots(15')$$

and

$$\sum_{i=1}^n d_{i,T}^{mR} m_{i,T}^R + \sum_{i=1}^n d_{i,T}^{mD} m_{i,T}^D + z_T^R + z_T^D \leq \sum_{i=1}^n d_i^m \underline{m}_T^* + \underline{z}_T^* =: \underline{D}_T^* \dots\dots\dots(16')$$

where $\underline{m}_T^*$, $\underline{z}_T^*$ and $\underline{x}_T^*$ are the solution of the optimal stationary program of the infinite economic planning. If $(1+r) > (1+s)(1+q)$, then $\underline{z}_T^* = 0$.

Now we have defined the whole model of the enlarged long-term optimal planning. Here we present some preliminary properties.

Property V-2-4-7-A The objective function F is a concave function.

Property V-2-4-7-B All constraints, i.e. (3'), (4'), (5'), (6'), (7'), (8'), (9'), (10'), (11'), (12'), (13'), (14'), (15') and (16') are concave functions.

Property V-2-4-7-C The set of $(x_t, e_t^R, e_t^D, m_t^R, m_t^D, z_t^R, z_t^D)$ which satisfies (4'), (7'), (8'), (9'), (10'), (11'), (12'), (13') and (14') is a compact and convex set. We call this set $//A^E//$.

Next we rewrite the model as follows.

MAX F

$$\text{s.t. } (3'), (5'), (6'), (15') \text{ and } (16') \text{ and } (x_t, e_t^R, e_t^D, m_t^R, m_t^D, z_t^R, z_t^D) \in //A^E//.$$

Assumption V-2-4-7-A There exists $(x_t, e_t^R, e_t^D, m_t^R, m_t^D, z_t^R, z_t^D) \in // A^E //$ which satisfies conditions $(3'), (5'), (6'), (15')$ and $(16')$ with strict inequality.

Theorem V-2-4-7-A Under the assumption V-2-4-7-A of the mathematical appendix, there exists an optimal program of the enlarged long-term planning.

We can prove this theorem similarly as in the proof of the theorem V-2-4-3. Since there exists no difficulty, the Author would like to omit the proof.

Theorem V-2-4-7-B Under the assumption V-2-4-7-A of the mathematical appendix, if an attainable program $(x_t^*, e_t^{R*}, e_t^{D*}, m_t^{R*}, m_t^{D*}, z_t^{R*}, z_t^{D*})$ is an optimal program, then it is a competitive program. That is, there exists a competitive price vector which associates with the optimal program.

Proof

Firstly we define the following notations.

$$\hat{x}_t =: \begin{pmatrix} (A_t + B_t)x_t + e_t^R + e_t^D \\ - \sum_{i=1}^n d_{i,t}^{eR} e_{i,t}^R - z_t^R \\ - \sum_{i=1}^n d_{i,t}^{eD} e_{i,t}^D - z_t^D \end{pmatrix} \quad \hat{c}_t =: \begin{pmatrix} c_t \\ s_t^R \\ s_t^D \end{pmatrix}$$

$$\hat{y}_t =: \frac{Q_{t-1}}{Q_t(1+u)} \begin{pmatrix} (I + B) x_{t-1} + m_{t-1}^R + m_{t-1}^D \\ - (1+r_t^R) \sum_{i=1}^n d_{i,t-1}^{mR} m_{i,t-1}^R - (1+r_t^R) z_{t-1}^R \\ - (1+r_t^D) \sum_{i=1}^n d_{i,t-1}^{mD} m_{i,t-1}^D - (1+r_t^D) z_{t-1}^D \end{pmatrix}$$

That is, $\hat{x}_t, \hat{y}_t$ and $\hat{c}_t$ are $(n+2)$ -dimensional column vectors.

Henceafter we can execute the proof similarly as in the proof of the theorem V-2-4-4-B. Therefore the further parts of the proof are omitted.

Q.E.D.

M-V-2-6-2

In this subsection the Author presents the whole system of the medium-term optimal planning model.

The model

$$\begin{aligned} \text{MAX } G &= \sum_{t=0}^T \sum_{i=1}^n \frac{1}{(1+u)^t Q_t} (C_{i,t} - K_{i,t}) \\ &=: \sum_{t=0}^T \sum_{i=1}^n (c_{i,t} - k_{i,t}) \end{aligned} \quad \dots\dots\dots(1)$$

where

- $C_{i,t}$; the total value of consumption of i -th product in t -th period.
 $c_{i,t}$; "per capita" form of $C_{i,t}$.
 Q_t ; population in t -th period.
 $K_{i,t}$; the sum of the collective consumption and the minimal level of the private consumption.
 $k_{i,t}$; "per capita" form of $K_{i,t}$.
 u ; discount factor.

As already mentioned, the discount factor (u) is determined by the average rate of technological progress or the growth rate of "value" of labour. So the factor $(1+u)$ represents the scarceness of leisure in future.

Constraint conditions are as follows.

$$c_{i,t} \geq f_{i,t} \sum_{i=1}^n (c_{i,t} - k_{i,t}) \quad ; (t=0,1,\dots,T. \ i=1,2,\dots,n) \dots(2)$$

$$c_{i,t} \geq k_{i,t} \quad ; (t=0,1,\dots,T. \ i=1,2,\dots,n) \quad \dots\dots\dots(3)$$

Total material balance is as follows.

$$\begin{aligned} A_t^d X_t + A_t^m X_t + E_t^R + E_t^D + C_t + \sum_{u=0}^U B_t^{d,t+u} \max(0, (X_{t+u} - X_{t+u-1})) \\ + \sum_{u=0}^U B_t^{m,t+u} \max(0, (X_{t+u} - X_{t+u-1})) \leq X_t + M_t^R + M_t^D \\ ; (t=0,1,\dots,T) \end{aligned} \quad \dots\dots\dots(4)$$

where

$X_{T+1} = \underline{X}_{T+1}^*$. $X_{T+2} = \underline{X}_{T+2}^*$ and so on. These are the optimal solution of the long-term optimal planning.

$X_{-1} = \underline{X}_{-1}$; this is a historically given condition.

U ; the maximal investment cycle.

A_t^d ; the matrix of $a_{i,j,t}^d$. A_t^m ; matrix of $a_{i,j,t}^m$.

$B_t^{d,t+u}$; the matrix of $b_{i,j,t}^{d,t+u}$. $B_t^{m,t+u}$; the matrix of $b_{i,j,t}^{m,t+u}$.

E_t^R ; export vector to the ruble district

E_t^D ; export vector to the dollar district

M_t^R ; import vector from the ruble district
 M_t^D ; import vector from the dollar district

$\max(0, (X_{t+u} - X_{t+u-1}))$ means that i -th element of this vector is

$$\begin{cases} X_{i,t+u} - X_{i,t+u-1}, & \text{if } X_{i,t+u} \geq X_{i,t+u-1}. \\ 0, & \text{if } X_{i,t+u} < X_{i,t+u-1}. \end{cases}$$

Imported material balance is as follows.

$$A_t^m X_t + \sum_{u=0}^U B_t^{m,t+u} \max(0, (X_{t+u} - X_{t+u-1})) \leq M_t^R + M_t^D \quad ;(t=0,1,\dots,T) \quad \dots\dots\dots(5)$$

Labour resource constraint is as follows.

$$\sum_{j=1}^n v_{i,j,t} X_{j,t} \leq L_{i,t} \quad ;(t=0,1,\dots,T. i=1,2,\dots,n) \quad \dots\dots\dots(6)$$

where i denotes the kind of labour.

Foreign currency balances are as follows.

$$\sum_{i=1}^n d_{i,t}^{eR} E_{i,t}^R + Z_t^R \geq \sum_{i=1}^n d_{i,t}^{mR} M_{i,t}^R + (1+r_t^R) Z_{t-1}^R \quad ;(t=0,1,\dots,T) \quad \dots\dots\dots(7)$$

$$\sum_{i=1}^n d_{i,t}^{eD} E_{i,t}^D + Z_t^D \geq \sum_{i=1}^n d_{i,t}^{mD} M_{i,t}^D + (1+r_t^D) Z_{t-1}^D \quad ;(t=0,1,\dots,T) \quad \dots\dots\dots(8)$$

where

$Z_{-1}^R = \underline{Z}_{-1}^R$, $Z_{-1}^D = \underline{Z}_{-1}^D$. These are historically given conditions.

$Z_T^R = \underline{Z}_T^{R*}$, $Z_T^D = \underline{Z}_T^{D*}$. These are optimal solutions of the long-term optimal planning.

$$\left. \begin{array}{ll} 0 \leq E_{i,t}^R \leq h_{i,t}^{eR} & 0 \leq E_{i,t}^D \leq h_{i,t}^{eD} \\ 0 \leq M_{i,t}^R \leq h_{i,t}^{mR} & 0 \leq M_{i,t}^D \leq h_{i,t}^{mD} \\ 0 \leq Z_t^R \leq H_t^R & 0 \leq Z_t^D \leq H_t^D \end{array} \right\} \quad \dots\dots\dots(9)$$

The upper limit of output should be observed in the beginning periods because of the level of investment in the past. Also there is upper limit of output in some industries because of scarce resources. Of course, some $g_{i,t}$ may be infinite.

$$0 \leq X_{i,t} \leq g_{i,t} \quad ;(t=0,1,\dots,T. i=1,2,\dots,n) \quad \dots\dots\dots(10)$$

Now we have set up the model. We may plausibly assume that $X_{i,t+1} \geq X_{i,t}$, that is, the production level increases in all

industries and in all periods. Then we can obtain an usual linear programming model. The model is rather complicated and then the number of variables is large. But on the other hand the time horizon is short, i.e. 5 years. The Author considers that there is no difficulty of practical calculation.

As usually we can obtain the dual price from the dual model. This dual price is utilized as the accounting price of the medium-term planning. The Author does not present the dual model of the medium-term optimal planning, since the main subject of this Thesis is the infinite and the long-term optimal planning. .

MATHEMATICAL APPENDIX TO CHAPTER VI

M-VI-1-1-1

In the Soviet Union the Central Planning Board (Gosplan) indicated the recoupment period and the efficiency coefficient in 1961 as in the following table.

industry	recoupment period (T)	efficiency coefficient (E)
metallurgical industry	7	0.14
energy industry	7 - 10	0.14 - 0.1
coal industry	5	0.2
oil and gas industry	5	0.2
wooden industry	5	0.2
chemical industry	3 - 5	0.33 - 0.2
machinery industry	3 - 5	0.33 - 0.2
light industry	3 - 5	0.33 - 0.2
building industry	6	0.17
transport	10	0.1

Source. Gosplan(14), p.15

In the case of Poland the situation is shown in the following table. These recoupment periods are calculated under the condition that the plant is built in 1960 and lasts until 1980 and is to be liquidated in 1980.

industry	recoupment period of outlays for replacement of old installations in 1980
industry as a whole	3.1
generation of electricity and heat	8.0
fuel industry	3.9
ferrous metallurgy	3.6
non-ferrous metallurgy	4.4
engineering	1.5
chemical industry	3.6
building material industry	2.4
textile industry	1.8
food industry	4.4

Source. Rakowski(49), p.106.

M-VI-1-2-2

We compare each variant using the Fiszal's approach. Numerical calculation is executed by the following index.

$$E = \frac{K + \sum_{t=1}^{10} \frac{C}{(1+r)^t}}{\sum_{t=1}^{10} \frac{P}{(1+r)^t}}$$

The result is shown in the following table.

value of Fiszal's index

variant	interest rate				
	5%	8%	10%	12%	14%
1	1.537	1.587	1.623	1.66	1.698
2	1.388	1.447	1.488	1.53	1.574
3	1.353	1.421	1.470	1.52	1.570

M-VI-1-3-1-A

Firstly the Author introduces the following assumptions.

Assumption 1 Investment is executed only in the 0-th period; i.e.

$$K_0(i) > 0, K_1(i) = K_2(i) = K_3(i) = \dots = K_n(i) = 0$$

and in the 0-th period there is no operation; i.e. $P_0(i) = 0$

and $C_0(i) = 0$.

Assumption 2 Benefit of i-th variant is constant over time; i.e.

$$B_t(i) = P_t(i) - C_t(i) - K_t(i)$$

$$= P_t(i) - C_t(i) \quad (\text{for } t=1,2,\dots,n; \text{ because of assumption 1})$$

$$= \text{constant} \quad (\text{for } t=1,2,\dots,n)$$

Assumption 3 Operation lasts forever; i.e. $n = \infty$.

Assumption 4 The value of outcome of each variant is same and does not change over time; i.e.

$$P_t(1) = P_t(2) = \dots = P_t(m) = P, \text{ for } t=1,2,\dots,n.$$

and also $P_1(i) = P_2(i) = \dots = P_n(i) = P, \text{ for } i=1,2,\dots,m.$

That is, $P_t(i) = P \begin{pmatrix} t=1,2,\dots,n \\ i=1,2,\dots,m \end{pmatrix}$

Assumption 5 $E = r$.

Assumption 4 indicates the identity condition of the traditional criterion by Novozhilov. Assumption 5 means that the benefit of each period is discounted by the rate of the standard efficiency by Novozhilov. Under the assumption 4 the assumption 2 turns into the following assumption.

Assumption 2' $C_1(i) = C_2(i) = \dots = C_n(i) =: C(i)$, for $i=1,2,\dots, m$

Under these assumptions we can transform the equation(1) of VI-1-3-1 as follows.

$$\begin{aligned} DPV(i) &= -K_0(i) + \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (P - C_t(i)) \\ &= -K_0(i) + (P - C(i)) \sum_{t=1}^{\infty} \frac{1}{(1+E)^t} \\ &= -K_0(i) + \frac{P - C(i)}{E} \dots\dots\dots(2) \end{aligned}$$

Therefore the following criterion(3) of the difference approach turns into the criterion(4).

$$\text{MAX}_i DPV(i) \dots\dots\dots(3)$$

$$\text{MAX}_i (-E K_0(i) + P - C(i)) \dots\dots\dots(4)$$

The maximum criterion(4) is same as the following minimization criterion(5)

$$\text{MIN}_i (E K_0(i) + C(i) - P) \dots\dots\dots(5)$$

Because of the assumption 4 the criterion(5) turns into the following criterion.

$$\text{MIN}_i (C(i) + E K_0(i)) \dots\dots\dots(1')$$

The criterion(1') is the very criterion by Novozhilov.

Q.E.D.

M-VI-1-3-1-B

Let us consider whether the government should construct a new factory or not. Characteristics of the investment project is shown in the following table.

	0-th period	1-st period	2-nd period	3-rd period	4-th period
value of output	500	503	507	465	380
value of material inputs	100	96	87	66	58
value of labour input	200	213	231	232	231
investment outlays	700	0	0	0	0
scrap value of factory	0	0	0	0	290
standard efficiency	-	20%	20%	15%	15%
coefficient for conversion to the 0-th period	1.0	0.83	0.69	0.60	0.52

Here the coefficient for conversion to the 0-th period from the future period, e.g. the fourth period, is calculated as follows.

$$\left(\frac{1}{1+0.2}\right)\left(\frac{1}{1+0.2}\right)\left(\frac{1}{1+0.15}\right)\left(\frac{1}{1+0.15}\right) = 0.525099 \approx 0.52$$

The discounted present value of this project is calculated as follows.

$$\begin{aligned}
 DPV &= -700 + 500 - 100 - 200 \\
 &+ 0.83(503 - 96 - 213) \\
 &+ 0.69(507 - 87 - 231) \\
 &+ 0.60(465 - 66 - 232) \\
 &+ 0.54(380 - 58 - 231 + 290) \\
 &= 74.15 > 0
 \end{aligned}$$

In this case this project should be adopted according to the Kantorovich's criterion, because the discounted present value is positive.

M-VI-2-3

Property VI-2-3-A In a closed economy, which does not contain the international trade, the formula(1) and the formula (2) bring the same value, i.e. $V = V'$.

Proof

In a closed economy the macro-accounting index is given by the induced labour input (δ_i). As shown in chapter V, δ_i is obtained as follows.

$$\delta = v (I - A)^{-1} \quad \dots\dots\dots(5)$$

where δ ; n-dimensional row vector of δ_i .

v ; n-dimensional row vector of v_i .

v_i ; direct labour input in i-th sector.

We can obtain the following relation in the Leontief system, since $Ax + g = x$.

$$(I - A)^{-1} g = x \quad \dots\dots\dots(6)$$

where g ; n-dimensional column vector of net output (g_i).

x ; n-dimensional column vector of gross output(X_i).

From (5) and (6) we can obtain the following relation.

$$\begin{aligned} \delta g &= v (I - A)^{-1} g \quad \dots\dots\dots(7) \\ &= v x \\ &= L \end{aligned}$$

where L ; the total amount of labour input in the whole economy.

Therefore we can obtain the following equation.

$$V' = \frac{L}{\sum_{i=1}^n g_i} = \frac{\delta g}{\sum_{i=1}^n g_i} = \frac{\sum_{i=1}^n \delta_i g_i}{\sum_{i=1}^n g_i} = \frac{\sum_{i=1}^n P_i g_i}{\sum_{i=1}^n g_i} = V$$

Q.E.D.

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