Physical Studies on Deposited Snow. III.*
Mechanical Properties (2).

by

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II. Mechanical Properties of Deposited Snow
(continued from previous paper)

§ 9. Break-down of snow by a falling body.

In the previous paper—Physical Studies on Deposited Snow II, Mechanical Properties (1)—the authors studied principally the mechanical behaviour of snow when it was acted upon by a force not so large as to break it down. But the studies in that paper were not wholly confined to the cases of small force. In one experiment a slowly increasing load was applied on the snow cover in which case the snow cover intermittently broke down each time the increasing load reached a certain value. In the following sections from §9 to §14 the break-down of snow caused by a heavy body dropped from some height upon the surface of the snow will be dealt with. Such a sort of break-down must have some common character with the above mentioned intermittent break-down of snow under a slowly increasing load.

Attention was mainly directed to the measurement of the resisting force of the snow to which the falling body was subjected during its fall within the snow. After several preliminary experiments (*1*) (*2*), the authors came to use the experimental equipment shown schematically in Fig. 1 (*3*) (*4*). In this figure A represents a solid cylinder made of lead (diameter: 6.6 cm, weight: 2 kg or 5 kg) which is the heavy body to be dropped onto the block of snow. To the bottom of A is attached a steel ring B on whose inner and outer surfaces are cemented strain gauges W. A hollow cylinder C made of thin iron plate fits loosely at its upper part to the lower part of lead cylinder A. The steel ring and the hollow cylinder together weigh 150 gr. When the combination ABC is dropped onto the block of snow, the snow exerts resisting forces to the bottom of hollow cylinder C. Steel ring B is then compressed in the vertical direction and the electric resistance of the fine wires within the strain gauges W is changed in proprotion to the resisting force of

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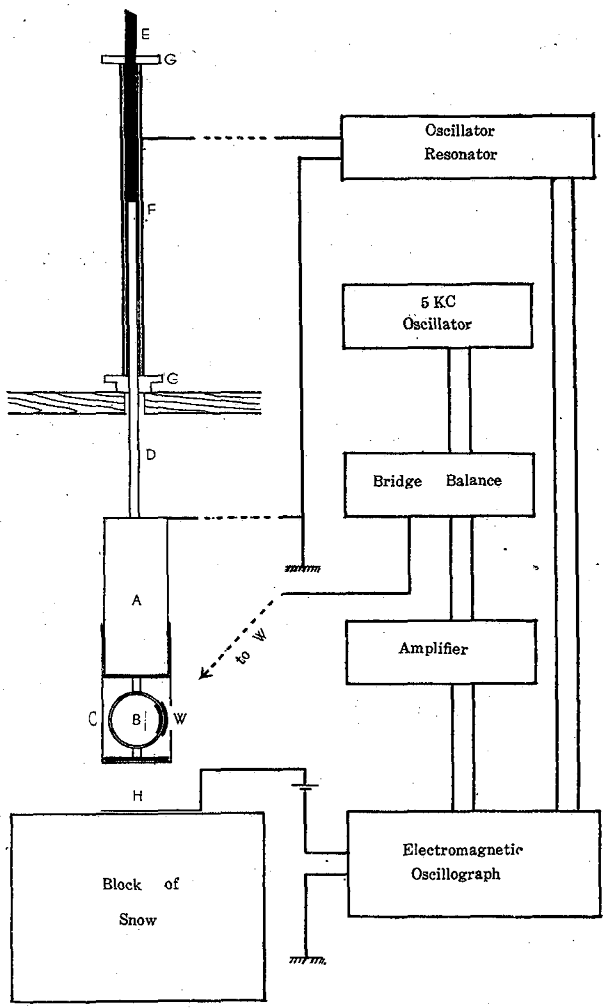


Fig. 1 Experimental equipment for measuring the resisting force of snow against a body falling onto it.

snow. The change in electric resistance of W is, after having been magnified by the bridge balance and the amplifier, recorded by the electromagnetic oscillograph. The resisting force of snow appears as soon as the bottom of hollow cylinder C touches the surface of snow block and lasts for a few tenths of a second until the falling system ABC is stopped after it has penetrated some distance into the snow. Copper wire H laid on the snow surface makes an electric closed circuit in conjunction with the falling system ABC at the instant the bottom of hollow cylinder C touches the snow surface. This instant is then indicated on the oscillogram by the appearance of electric current in the closed circuit.

As shown in Fig. 1, lead cylinder A has a long metal rod D firmly screwed on its top which rod in turn carries a long ebonite rod E attached to its top. F is a hollow cylinder made of plastic with metal collars GG at its ends. As falling system ABC goes down metal rod D and ebonite rod E slide down through the plastic cylinder F guided by the metal collars GG, which keeps the falling system from turning aside or from being tilted. But this device is of one more use. The outer surface of the plastic cylinder F is lined with a metal plate which together with metal rod D forms an electric condenser. The capacity of this condenser is proportional to the length of the part of metal rod D which is within cylinder F. Therefore the changing position of the falling system can be continually indicated on the oscillogram by recording on it the change of capacity of the above noted condenser while the system is falling.

Fig. 2 (a) is an example of an oscillogram. Time goes on from left to right in this figure. Wavy curve R represents the resisting force of snow of which the magnitude can be read on the inscribed scale of kgs. The instant α, at which the

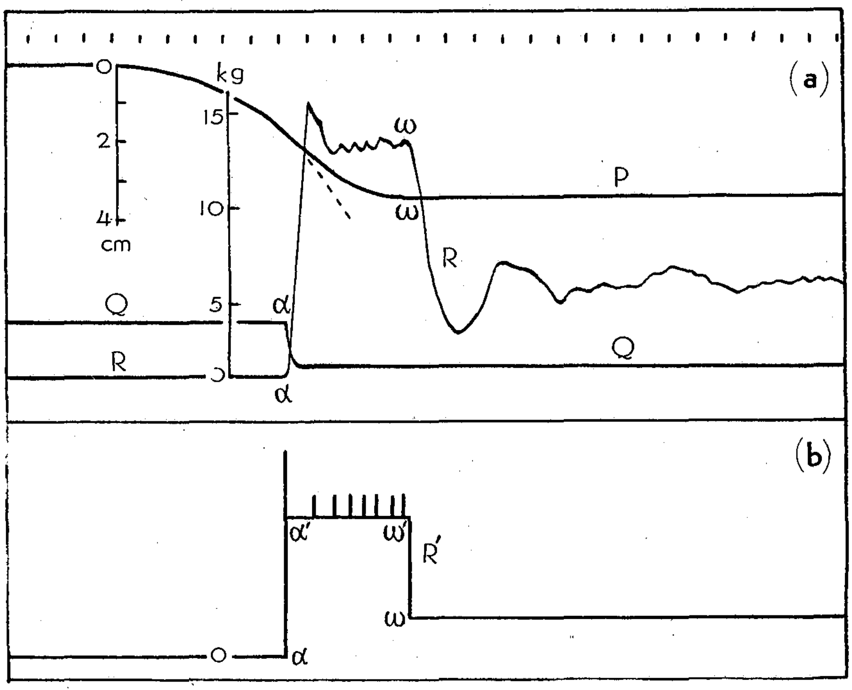


Fig. 2 (a) Result of experiment made on a block of snow so laid that the snow layers stood upright. P: position of falling body, Q: contact of falling body with snow is shown by the shift of this line at point α, R: resisting force of snow, ω: at this point falling body was stopped by the snow.
(b) curve of resisting force of snow revised from the experimental curve R of (a).

bottom of hollow cylinder C shown in Fig. 1 touched the snow surface is indicated by a shift on the straight line Q running horizontally near the bottom of the figure. This shift in the straight line was caused by the contact of cylinder C with copper wire H laid on the snow surface. One sees curve R begin to rise simultaneously with this shift. Curve P sloping down from left to right is the curve of position of the falling system. At point ω, curve P ceases to change becoming horizontally straight thereafter, which shows that the falling system was stopped by the snow at that time point. This is in accord with the fact that curve R representing the resisting force of snow begins to drop at the same instant. The broken line branching off from curve P at time point α shows the position which the falling system would have taken had there been no snow block. Each interval between two adjacent points standing in a line along the top edge of the figure represents 0.01 sec.

The falling system is an oscillatory one because it consists of two masses—lead cylinder A and hollow cylinder C—connected by an elastic spring—steel ring B. The measurement of the resisting force of snow is based on this very oscillatory character of the system. But it is an inevitable defect common to all the oscillatory measuring systems that such a system cannot represent truly a force which undergoes a rapid fine change within the period of free oscillation of the system; rapid fine changes taking place within the period tend to be smoothed out or averaged. Therefore it is desirable to use a falling system having as short a period of free oscillation as possible in order to obtain the true representation of the force. In the case of the present falling system, there are two periods of free oscillation distinctly different according to the circumstances.

(1) *During the fall of the system.* The free oscillation of the whole system is nothing but the alternate elongation and contraction in the vertical direction of the steel ring B connecting heavy lead cylinder A and light hollow cylinder C. Let the center of gravity of the whole system, cylinder A and cylinder C be denoted by G, G_A and G_C respectively. When the system oscillates freely (the system can execute a free oscillation while it is falling, being subjected to no external force other than that of gravity) G_A and G_C move relative to G, but the displacement of G_A relative to G is always much smaller than that of G_C since the mass of cylinder A is much larger than that of cylinder C. In other words G_A is practically standing still in relation to G and only G_C oscillates relative to G. Therefore the free oscillation of the whole system during its fall can be replaced by the free oscillation of cylinder C relative to cylinder A which is now fixed in space. Since the mass of hollow cylinder C is small the period of this oscillation is short; it was found by observation to be 0.006 sec.

(2) *When the system is stopped by the snow.* Hollow cylinder C appears as if it were fixed in space since it cannot move being held by the snow. Then the free oscillation of the whole system can take place only as a result of the motion

of heavy cylinder A. In this case the period of free oscillation of the system turns out to be much longer than that of (*1*) since the mass of the oscillating body is large. It was found to be about 0.05 sec.

It is certainly an advantageous situation that the period of free oscillation in case (*1*) is shorter than that in case (*2*) for the resisting force of snow makes its appearance when the system is under the conditions of the former case. But the period of free oscillation in case (*1*) is still not sufficiently short for the true representation of the change in the resisting force of snow. Curve R of Fig. 2 (a) exhibits fine fluctuations taking place within periods shorter than 0.01 sec (= interval between two adjacent dots at the top of Fig. 2 (a)). This is nothing but an indication of the existence of some fine changes which cannot be followed by the measuring device here in use. Then some imagination must be used in order to draw out anything near to truth from curve R of the resisting force. The authors suppose that the true change of the resisting force can be represented somewhat as in curve R′ of Fig. 2 (b). The angular figure $aa'\omega\omega'$ of curve R′ represents the general trend of the change in the resisting force which is faithfully revealed on curve R. Vertical segments standing on top of the angular figure are impulsive forces which are supposed to have appeared at each trough of the zigzag of curve R. An impulsive force lasting only a small fraction of the period of free oscillation of the falling system will cause it to draw a heap-like curve having breadth nearly equal to the period of its free oscillation. The vertical segment of curve R′ corresponding to a trough of curve R represents the heap of curve R immediately following that trough. It is possible to determine by calculation the magnitude of the impulsive force from the shape of the heap shown on curve R. Such a calculation will be made on examples of oscillograms inserted in the later sections.

In Fig. 2 (a) curve R becomes wavy at a period of about 0.05 sec after time point ω—the time point when the falling system is stopped by the snow. This wave form of curve R is of course the result of free oscillation taking place in the falling system under the conditions of the above stated case of (*2*). The central line of the wave form coincides with the weight of the whole falling system, which indicates that the snow would continue to exert against the system a constant force equal in magnitude to its weight if the system were a single solid body.

After each experiment the block of snow was cut vertically through the centre of the hole made in it by the fall of the falling system. The cut surface was then sprayed with water coloured with ink; there appeared a 'spray figure' disclosing the change which had been produced in the structure of snow by the fallen system. The left illustration of Fig. 3 is an example of a spray figure. (This is not the one obtained at the experiment of Fig. 2). The region of compressed snow developed below the fallen system is shown by the crescent-shaped dark bands. The spray figure discloses also the stratified structure of the undeformed snow around the region of compressed snow by horizontal trains of dark spots. The right illustration

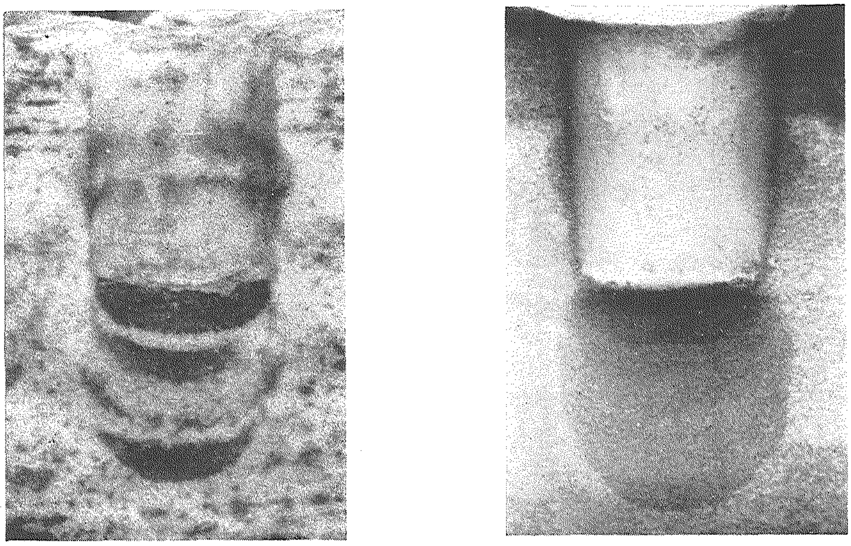


Fig. 3 Left : Spray figure on the cut surface of block of snow.
Right : Shade figure on the same block.

of Fig. 3 is the 'shade figure' of the same block of snow. The snow block was cut into a plate about 4 cm thick with one of its surfaces halving the hole made in the snow block. The shade figure can be seen on the plate by illuminating it from behind. The boundary of the region of compressed snow is shown more distinctly on the shade figure than on the spray figure, but the shade figure fails to disclose the inner structure of that region.

Fig. 4 is the sketch of the spray figure obtained in the experiment of Fig. 2. In this experiment the snow block was not laid in the original position which it had had in the snow cover but was placed sideways in such a way that the snow layers composing it stood upright. The vertical broken lines of Fig. 4 show the boundaries between the snow layers. The resisting force of snow appears in simplest form when the experiment is done on the snow block laid in this position. In its natural position the snow block gives a very intricate resisting force as will be shown in the later sections.

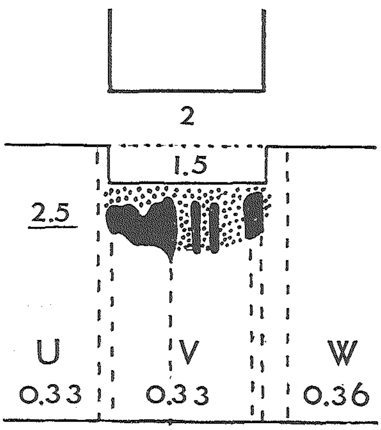


Fig. 4 Sketch of spray figure in the experiment of Fig. 2.

The figures 2 and 1.5 in the upper part of Fig. 4 indicate respectively in units of cm the height above the snow surface of the bottom of the falling system before it is let fall and the depth by which it penetrates into the snow. Underlined figure 2.5 is the thickness of the region of compressed snow. Letters U, V, W are the terms attached by the authors to the snow layers. The decimal fractions below the terms of snow layers show their snow densities.

§10. The simplest case of break-down of snow by a falling body.

The snow cover is composed of many layers and their properties differ from one layer to the next owing to the difference in the conditions under which they have lain since their formation. Therefore a block of snow cut out of the snow cover for use in experiment has properties variable in the vertical direction. That variability will exert complicatedly changing resisting force against the falling body when placed in its original position. But when the block is so laid that its layers stand upright the snow structure is uniform in the direction of fall of the falling body and the resisting force of snow appears in a simple manner as shown in Fig. 2 of the previous section. However, the resisting force is never uniform in spite of the uniformity of the snow structure, being accompanied by intermittent impulsive forces superposed on the constantly acting one. It was thought that such impulsive forces may have arisen from the frictional force acting between the region of compressed snow below the falling body and the snow mass surrounding the region. But the following experiment shows that this is not the case.

The authors surrounded a pillar of snow of the same thickness as the falling system with a sheet of soft and tough paper on its cylindrical side surface and let the system fall onto it. The paper kept the snow pillar from being burst sideways but it did not interfere with its being compressed. It is clear that no frictional force was present here since there was no mass of snow surrounding the developed region of compressed snow. But the intermittent impulsive forces still appeared in this case as shown by curves R, R′ of Fig. 5, where R is the observed curve of the resisting force of the snow pillar while R′ is the one revised from R in the sense explained in the previous section. In the case illustrated in Fig. 5 the continually acting force increased gradually in contrast to the case of Fig. 2 where it was kept constant

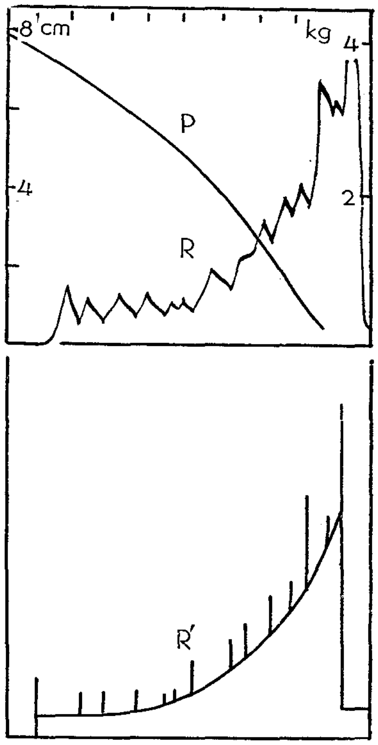
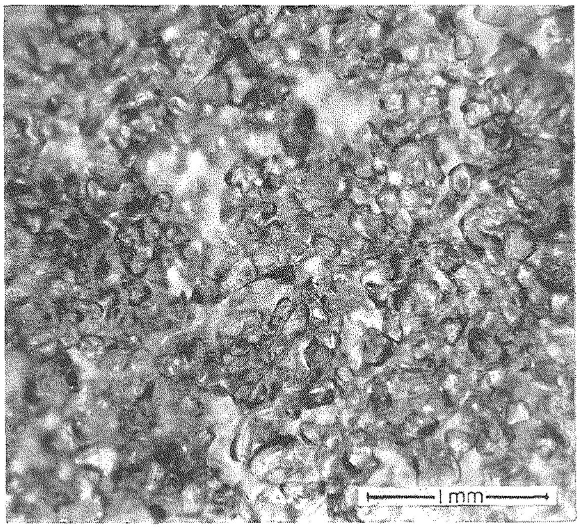
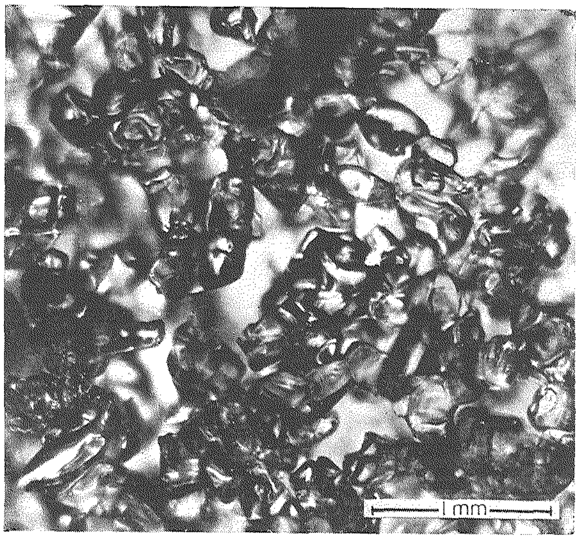


Fig. 5 Result of experiment on a pillar of snow protected on its side surface by a sheet of soft and tough paper.

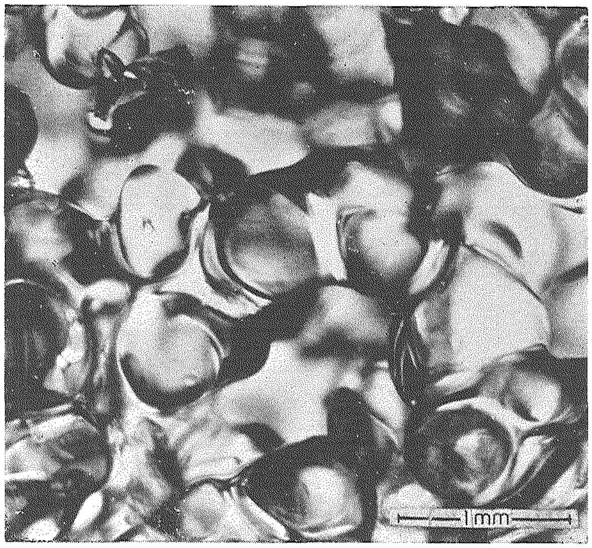


(a)



(b)

Fig. 6 Structure of deposited snow.
(a) Compact snow in early stage.
(b) Matured compact snow.



(c)

Fig. 6 Structure of deposited snow.
(c) Old compact snow.

from the beginning to the end. The frictional force seems to have influence on the continually acting force. But the appearance of the impulsive forces in both the cases proves that they had their causes not in the friction but in the mechanism of compression of snow itself.

The authors advance a theory below which explains the appearance of intermittent impulsive forces in the resisting force of the snow of uniform structure (5). The snow is, excepting newly fallen snow, composed of short rods of ice connected with one another so as to construct three dimensional net works. One can see the actual net works in the photographs of Fig. 6. They were photographed by a microscope on snow samples which had been cut by a razor blade into a sheet as thin as 2 mm. Now for the sake of simplicity let the net work be represented by an irregular mesh such as shown in A of Fig.. 7.

Four schematic drawings C_1, C_2, C_3, C_4 of Fig. 7 show the advancing front of the region of compressed snow which develops below the falling system as it descends into the snow. It may well be supposed that some of the ice rods composing the snow located near the advancing front of compressed snow are damaged in some way, for example by being cracked, although they are still keeping their original forms. The black areas in the four figures indicate the region where the damages to ice rods have taken place. In the state of C_1 only the ice rods immediately close to the front of compressed snow have been damaged. A damaged rod, being deprived of ability to withstand the force coming from the falling body, shifts the

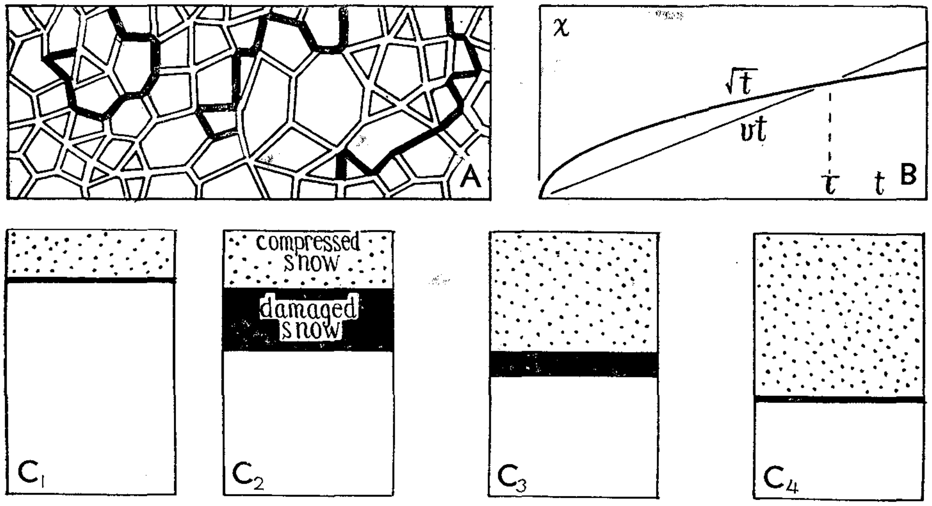


Fig. 7 A. Model of the structure of snow. Dark rods show damaged ice rods.
B. Change in position x of the front of damaged snow is shown by curve $\sqrt{t}$ while that of the front of compressed snow by curve vt. They cross each other at point $t=\tau$.
C_1, C_2, C_3, C_4. Development of the regions of damaged and compressed snow.

force it should have to bear towards its neighbours. But this load is not equally passed on to the neighbours and it will occur that one among them is damaged by being loaded over the limit of its endurance. In this way the damage will be transmitted step by step from a rod to one of its neighbours. But the damage is not restricted to transmission in the downward direction. Each ice rod has several neighbours meeting at its ends and the load may be shifted from the damaged rod to any of the neighbours, even to the one lying above it causing damage to it. Which of the neighbours is actually damaged depends on such factors as their respective strengths, their positions relative to the ice rods lying nearby which are distributed in the space at random; it cannot be definitely determined and it is a problem of stochastical nature. Sketch A of Fig. 7 indicates in a schematical fashion such a damage transmission in the networks of the ice rods.

The ice rod, if it should be damaged, cannot be damaged exactly at the instant the force is applied; there is needed certain time before it is damaged. This phenomenon was shown by PRESTON (6) with glass rods and was called by him the phenomenon of 'static fatigue'. (A description of static fatigue is given in the previous paper 'Physical Studies on Deposited Snow, II, p. 45). In this way the damage is transmitted along the ice rods step by step with certain time intervals between the steps; also, the direction of transmission is entirely non-predetermined at each step. Then the transmission of damage resembles formally the Brownian motion of a solute molecule which is diffusing into the mass of solvent. Therefore the damaged region in the state C_1 of Fig. 7 which is still at the initial stage of its development can be thought to extend downwards according to the law of diffusion as shown in the subsequent figures C_2, C_3, C_4. Generally the front of the region of solvent permeated by the diffusing solute molecules at first proceeds rapidly but its speed of advance comes to be slowed down as time goes on. If the distance by which the front has advanced in time t is denoted by x, there holds the relation

$$x \text{ is proportional to } \sqrt{t}\,. \qquad (1)$$

The damaged region of snow here in question will also develop according to relation (1). But the front of compressed snow, being pushed down by the massive falling body, proceeds downwards almost at a constant speed. Therefore, starting from the state of C_1 of Fig. 7, at first the front of the damaged region advances faster than that of the compressed snow and there develops a wide region of damaged ice rods (the state of C_2). But the region is constantly being destroyed at its upper part by the pursuing front of compressed snow and comes to be lessened in extent as the advancing speed of its front is slowed down (the state of C_3). Then the state of C_4 arrives in which the front of compressed snow catches up with that of the damaged region.

Up to just the state of C_4 the front of compressed snow proceeds in the damaged region which cannot offer any large resisting force owing to its weakened structure. The continual resisting force appearing in this time is that one represented by the angular figure of Fig. 2 (b). At the state of C_4 the front of compressed snow comes to be confronted by the strong undamaged snow and must anew damage some of ice rods of the undamaged snow which are in direct contact with it. At this instant the snow exerts an impulsive force against the compressed snow which the authors suppose is one of the above stated intermittent impulsive forces observed in the experiment. Then the state of snow is returned to state C_1 and the same course of process as described above will be repeated. Such a process from C_1 to C_4 is repeated again and again and the impulsive force appears intermittently at the end of each cycle.

Let the sequence of damaged ice rods which are indicated in A of Fig. 7 by dark rods be called 'damaged path'. Then the tip of the damaged path will move like a particle subjected to Brownian motion. Therefore, if coordinate x is taken downwards from the front of compressed snow in state C_1 and time t is counted from the instant of this state, the probability with which the tip of a damaged path will be found in the range from x to $x+dx$ at time t is given by

$$W(x, t)\, dx = \frac{1}{(\pi Dt)^{\frac{1}{2}}} \exp(-x^2/4Dt)\, dx\,, \qquad (2)$$

$$D = l^2/2\theta\,, \qquad (3)$$

where l is the mean length of ice rods while θ is the time interval between occurrence of successive damages to two adjoining ice rods. Let it be supposed that N damaged paths start at state C_1. Of course the tips of N damaged paths come to be distributed non-uniformly as time goes on, their density of distribution becoming smaller as x is increased. Now let X_q be such a value of x that Nq tips have x greater than X_q at time t, where q is a number smaller than unity. Then any stratum of height dx in the region with x smaller than X_q is traversed by more than Nq damaged paths, that is, if the cross-sectional area of the compressed snow is denoted by A, the density of damaged ice rods is greater than $(Nq)/(lA)$ anywhere in this region. It is expected that the density is *much* greater than this value since the damaged paths wander about, up and down, traversing many times stratum dx located in this region. For this reason this region will be identified with the previously noted damaged region through which the front of compressed snow can advance under the small continual resisting force. X_q is related to Nq by the relation

$$Nq = \int_{X_q}^{\infty} N \cdot W(x, t)\, dx \,, \tag{4}$$

which yields

$$\Phi\left(\frac{X_q}{2\sqrt{Dt}}\right) = 1-q \tag{5}$$

where Φ is the error integral $\Phi(x) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^x \exp(-s^2)\, ds$.

Equation (5) gives

$$X_q = \sqrt{t} \cdot 2\sqrt{D}\, \Phi^{-1}(1-q) \tag{6}$$

which shows that the front of the damaged region advances in proportion to $\sqrt{t}$ as was noted in the preceding paragraph. The authors cannot find any manner by which the value of q is to be determined so they put $\Phi^{-1}(1-q)$ equal to unity for the sake of convenience. This is the same thing as putting $q=0.15$ since $\Phi^{-1}(0.85)=1$.

Let the advancing speed of the front of compressed snow and the time interval between two successive impulsive forces be denoted by v and τ respectively, then the following equation is deduced from equation (6)

$$v\tau = \sqrt{\tau} \cdot 2\sqrt{D} \,,$$

which yields by the aid of equation (3)

$$\theta = 2l^2/v^2\tau \,. \tag{7}$$

In the case of experiment of Fig. 2, the front of compressed snow advanced by 4 cm in 0.04 sec as seen from Fig. 2 and Fig. 4. This gives $v = 100$ cm/sec. Fig. 2 shows that τ is nearly equal to 0.005 sec, while l, the mean length of ice rods composing the snow, was found by microscopic observation to be about 0.05 cm.

By the use of these values for v, τ, l equation (7) yields

$$\theta = 10^{-4}\,\text{sec}.$$

The distance by which the front of compressed snow descends between two successive impulsive forces is found to be about 0.5 cm, that is, a distance ten times as large as the mean length of the ice rods.

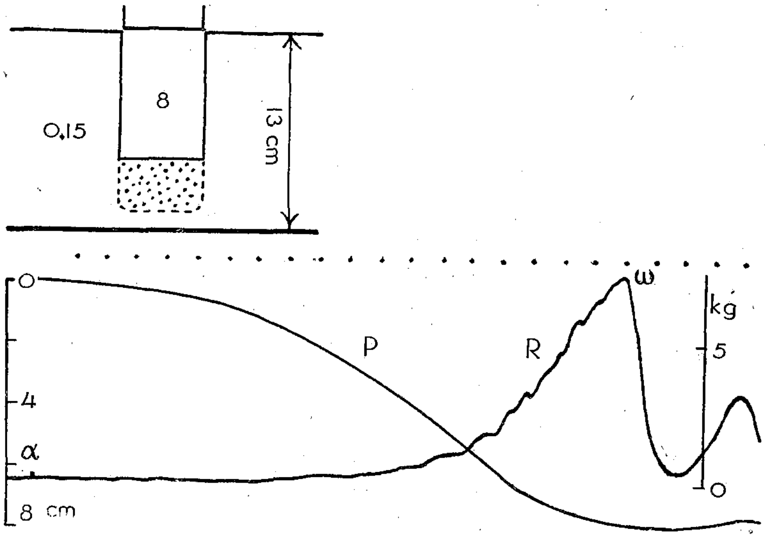


Fig. 8 Soft snow. Density: 0.15 gr/cm³. Temperature: −2.6°C. Experiment was done on a block of soft snow laid in the original position it had had in the snow cover. Absence of structure in the region of compressed snow shows that the snow was uniform in the vertical direction.

It is better to think of the soft snow as merely an assemblage of individual snow crystals weakly linked with one another than to think of it as possessing the above described structure made of ice rods. But, if it is desired to conceive of the structure of soft snow within the limit of the above described one, it may be permissible to say that the soft snow is made of very weak ice rods connected together to form a three dimensional net work. Then, since the intermittent impulsive forces of the resisting force of snow should arise from damage to the ice rods, these forces will be absent or very weak, if present at all, in the case of soft snow. Fig. 8 is a record of resisting force of soft snow of the density 0.15 gr/cm³. The falling system of the weight of 2 kg was let fall at time point α into the snow from just above its surface. The falling system was stopped by the snow after having sunk into it 8 cm within 0.2 sec, when the front of compressed snow reached the level 1 cm above the base of the snow. As seen from Fig. 8 the resisting force is almost absent in the first half of the process. In the second half it gradually increases and at the same time it becomes wavy, which is the indication of the presence of impulsive forces. But the height of the waves is very low as compared with that in the case of Fig. 2 in which case the ice rods were strong. (Note the difference of the scale of resisting force between the cases illustrated in Fig. 2 and Fig. 8).

It is rather a strange matter that the boundary of the compressed snow developed under the falling system is so distinct as shown in the shade figure of Fig. 3. It would be more natural to expect that the compressed snow is most densely packed just below the falling system and its density gradually decreases down- and out-wards to coincide with that of the undeformed snow without any distinct boundary between the two. Indeed such a distribution of snow density is realised in the case of wet granular snow as will be described in paragraph (*2*) of the next section. The distinctness of the boundary in the case of dry compact snow can be explained by the above described manner of development of the damaged region in the net work of ice rods. The damaged region cannot be much enlarged before its front is overtaken by that of the compressed snow as seen from the numerical result shown above. Therefore the distance between the edges of the compressed snow and of the undamaged snow is always kept within a small limit which circumstance by itself will not allow the boundary between them to become broad. Moreover the stop of fall of the falling system will tend to be realised when the conditions are as in state C_4 of Fig. 7 where the compressed snow and the undamaged snow are in direct contact with each other. The boundary must become very distinct in such a case.

§ 11. The resistance exhibited by stratified snow.

When the falling system is dropped on the block of snow so placed that the snow layers composing it lie horizontally just as in the snow cover from which it has been cut out, the resisting force displays changes much more complicated than those in the uniform snow described in the previous two sections. This is a matter of course since the strength of snow layers which the falling system encounters successively during its fall differs from one layer to another. But the mode of change in the resistance as shown on the oscillogram can be classified into the following four types (*3*). Of course there are many which are intermediate between any two of these types and some exceptions which cannot by any means be put into the framework of this classification.

A-type: A large resisting force lasting about 0.01 sec appears at the very first and then a long-lasting more or less constant resisting force follows with small fluctuations.

B-type: Resisting force fluctuates with large amplitude and the oscillogram of resisting force looks like a series of many steep peaks.

C-type: Resisting force increases step by step, forming two or three stages before it reaches the final value.

D-type: Resisting force increases gradually from nothing to a maximum value showing no stages different to the former case of C-type.

These types may be represented schematically by the figures shown in Fig. 9.

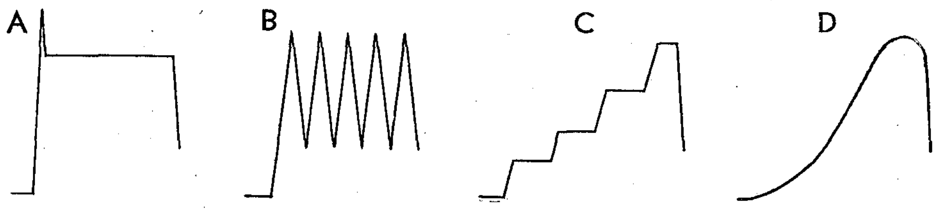


Fig. 9 Four types of resisting force of composite snow.

Explanation of each of the types

(*1*) *A-type*. This type was found generally on soft compact snow which was considerably uniform in its structure, that is, which showed no distinct stratifications. Curve R of Fig. 10 is an example of this type in the conditional sense described below. The spray and shade figures shown in Fig. 3 of the preceding section § 9 are those of the snow block used in the experiment illustrated in Fig. 10. The snow block was not uniform but consisted of two layers, a thin surface layer and a main one of large thickness, as shown clearly in the shade figure of Fig. 3. The surface layer was fine grained being composed of ice rods as short as 0.2 mm while the structure of the main one was coarse being composed of ice rods about 1 mm long.

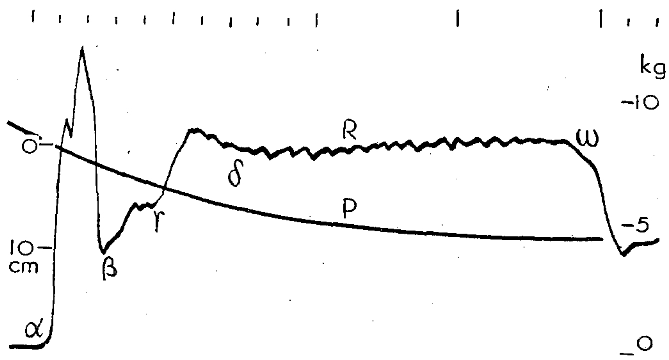


Fig. 10 A-type. Compact snow. −3.7°C. Spray and shade figures are shown in Fig. 3 of § 9. Density of snow: 0.23 gr/cm^3. Thickness of the compressed snow: 7.5 cm.

Two peaks of curve R between points α and β at the left of Fig. 10 represent in reality two large impulsive forces which the falling system must exert on snow to damage and destroy the net work of ice rods of the surface layer. They must have resisted the falling system most strongly because none of them had yet been harmed. The uppermost dark crescent-shaped band in both the spray and shade figures of Fig. 3 is the surface layer compressed by the falling system.

The light band below next to the uppermost dark one in the spray figure will correspond to the range of small force between points β and γ of curve R of Fig. 10, while the second dark band below the light one corresponds to the heap-shaped part lying between points γ and δ of the curve. Remaining part of the compressed snow must be produced by the resisting force represented by the horizontal part from point δ to point ω of curve R. The series of many small heaps in this part of the curve must arise from the intermittent impulsive forces described in the previous section. It may seem contradictory that the lower part of the region of compressed snow does not appear uniform in the spray figure in spite of the uniformity of the resisting force corresponding to this part. But in the shade figure this part of the region of compressed snow seems entirely uniform. Therefore the lower part of the compressed snow must not in reality be so non-uniform as it appears in the spray figure. Indeed it is an inherent defect of the spray figure to be too sensitive to the slight structural change produced in snow while it loses entirely the power to distinguish the grade of deformation of snow when it exceeds a certain low limit.

Curve R of Fig. 10 can be represented by the schematical figure A of Fig. 9 as a whole, but the same applies also to the part of curve R lying to the right of point γ. It will be better to define the A-type resistance on the basis of the physical nature of the snow rather than merely on the basis of the apparent form of the curve of resisting force. On this account only the part to the right of point γ (Fig. 10) of curve R should be considered to belong to A-type; the whole of the curve is composite in the sense that the large impulsive forces were added merely by chance to the head of curve of A-type.

Even non-uniform compact snow tended to exhibit this A-type resistance when the falling system was let fall parallel to the direction of layers as already described in § 9. An example of this case is shown by Fig. 2 of § 9.

(2) *B-type.* Wet granular snow was apt to yield an oscillogram of the resisting force of the type shown in Fig. 11. The region of compressed snow could not be distinguished from the undeformed snow in spite of spraying the surface of the snow block cut into halves. It was shown in the previous paper (kinematographic pictures on Pl. VIII of 'Physical studies on deposited snow, II') that the wet snow deformed like gelatine gel when pressed statically and there appeared no distinct boundary of the region of compressed snow. Therefore it can be said that neither the slowly acting statical force nor the short lasting large force can develop a distinctly limited region of compressed snow in the wet snow. This circumstance may be caused by such an inherent structure of wet granular snow as that it consists of ice granules linked with one another by weak viscous forces.

Since the wet granular snow subjected to the experiment of Fig. 11 was uniform

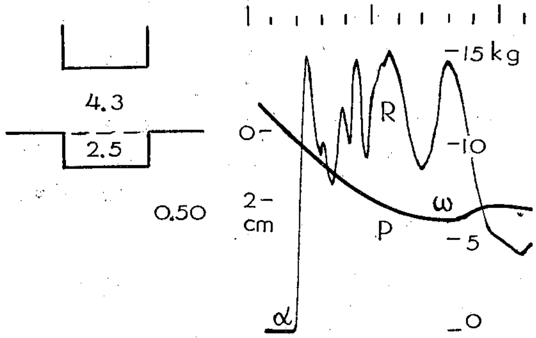


Fig. 11 B-type. Wet granular snow.

in density there is no reason to attribute the appearance of the large peaks of curve R to the stratification of snow. As noted in pp. 56–58 of the previous paper the fluidity of wet snow is great and the stress produced in it tends to be relaxed with considerable rapidness. When the falling system falls on the snow a rapidly enlarging strain appears in it and the stress would also be enlarged in proportion to the strain if there were no relaxation in the stress. But as a matter of fact the relaxation of stress existent in the wet snow to a high degree retards the increase of stress with the result that much time is needed for the stress to reach the value of break-down of snow. This stress of snow is the very cause of its resistance and in wet granular snow the latter turns out to increase more slowly than in the case of dry snow. After the wet snow is broken down by the attainment of stress to the break-down value accompanied by the release of stress, the stress begins afresh to increase again due to the strain which is kept increasing incessantly by the falling system. Repetition of such a process will be the cause of successive appearance of high and broad peaks (as compared to the case of dry snow) in the curve of resisting force of wet granular snow.

In the case of statical compression of wet snow described in the previous paper, it was cracked when the pressing body descended by 10 cm into the snow. This descent occurred in $t_1=7$ sec. Therefore the wet snow was broken down when it was strained to a value proportional to 10 cm under the condition that the strain was enlarged at a speed proportional to $v_1=10/7=1.4$ cm/sec. In the case of experiment of Fig. 11 the falling system descended into the snow to a depth of 2.5 cm in the time 0.07 sec, that is, at the speed of $v_2=2.5/0.07=36$ cm/sec. Four large peaks of the resisting force oscillogram indicate that the wet snow was broken down every time it was strained up to a value proportional to $2.5/4=0.6$ cm, the time interval between two successive breakdowns being $t_2=0.07/4=0.02$ sec.

Now let the wet snow be represented by the model of Maxwell widely used in rheology. If the model of Maxwell begins to be strained at a constant rate $\dot{e}$, the stress s increases after the equation

$$s = E\dot{e}\tau\left[1-\exp(-t/\tau)\right], \qquad (1)$$

where E is the elastic constant of the model while τ is the time of relaxation, that is, the coefficient of viscosity η of the model divided by E. It is now assumed that the break-down of snow occurred in both the cases described in the previous paragraph when the stress s reached the same value at times t_1 and t_2 respectively. Then, under one more assumption that E and τ were of the same values in both cases, there should hold the following relation

$$v_1\left[1-\exp(-t_1/\tau)\right] = v_2\left[1-\exp(-t_2/\tau)\right], \qquad (2)$$

since $\dot{e}$ was proportional to v_1 and v_2 respectively in each case. If the relaxation time τ is put equal to 0.5 sec, the above equation (2) is satisfied by the use of the numerical values given in the previous paragraph for v_1, t_1, v_2, t_2, the left and right sides of the equation being equal to 1.4 and 1.44 respectively. The authors have not yet succeeded to determine the time of relaxation of wet snow on account of experimental difficulties but they think that 0.5 sec is a reasonable value for that time. The time of relaxation of dry snow was found to be of the order of 4~15 min as shown in § 2 (c) of the previous paper.

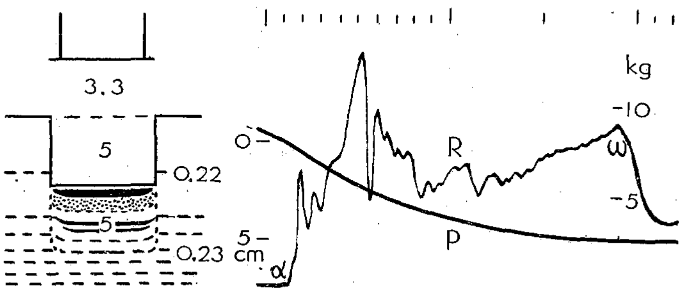


Fig. 12 B type. Compact snow. −3.8°C. A disc of diameter 7.6 cm was attached to the bottom of the falling system.

Compact snow tended also to belong to this type when it showed marked stratifications. An example of this case is shown in Fig. 12. Although the curves in Figs. 11 and 12 resemble each other in that they both show large peaks, one can find a large difference in their character. The peaks of Fig. 11 are simple in form. But each of the peaks of Fig. 12 looks like curve R of Fig. 2 with some modification. Each of the snow layers, being uniform by itself, seems to act individually one after another upon the falling system with a resisting force belonging to A-type with the result that the whole oscillogram of the resisting force looks like a succession of many large peaks. Therefore the cause is quite different in the two cases of Fig. 11 and Fig. 12.

(3) *C-type.* Compact snow was apt to show this type. Curve R of Fig. 13 is an example. The resisting force rises to the flat maximum at point ω with two

stages at points β and γ on its way. At the flat maximum and at the two stages the curve is wavy, which suggests that each of its three parts between points α and β, β and γ, γ and ω represents individually the resisting force of A-type which was brought about by each of the separate uniform snow layers composing the snow. The general tendency of the curve to rise with increasing depth of the falling system will be due to the fact that the snow layer became stronger as it

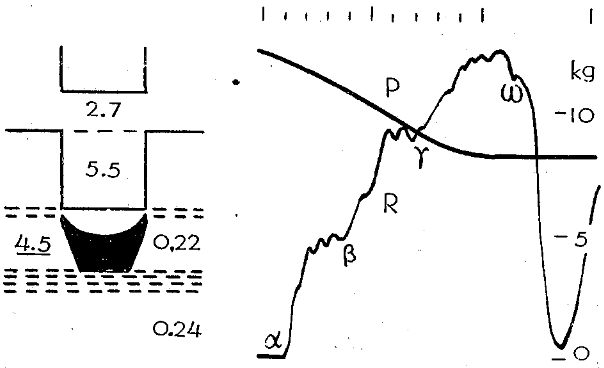


Fig. 13 C-type. Compact snow. −2.6°C.

lay deeper in the snow so that larger force was needed in order to break it down. Indeed, as shown in the sketch of the spray figure annexed to Fig. 13, the lower part of the region of compressed snow was subjected to much more intense compression than its upper part. The upper part of the compressed snow printed light in the spray figure will correspond to the first stage from point α to point β of curve R while the remaining lower part printed dark will have been produced by the compressive force which lasted from point β to point ω.

It is noted that in some cases it was impossible to point out such a correspondence between the spray figure and the curve of resisting force. There might be some other factors which caused the stages to appear in the rising curve of resisting force.

Fig. 14 shows another example belonging to C-type. Curve R of the resisting

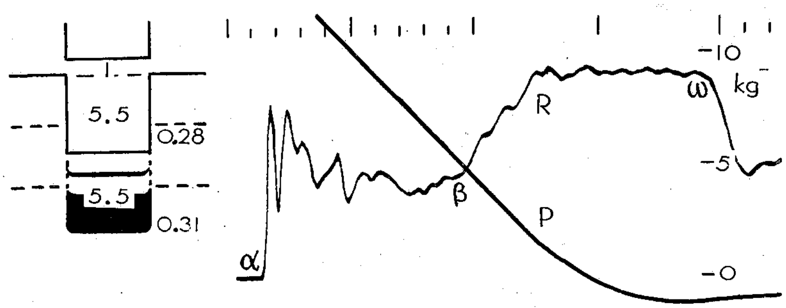


Fig. 14 C-type. Compact snow. −2.7°C.

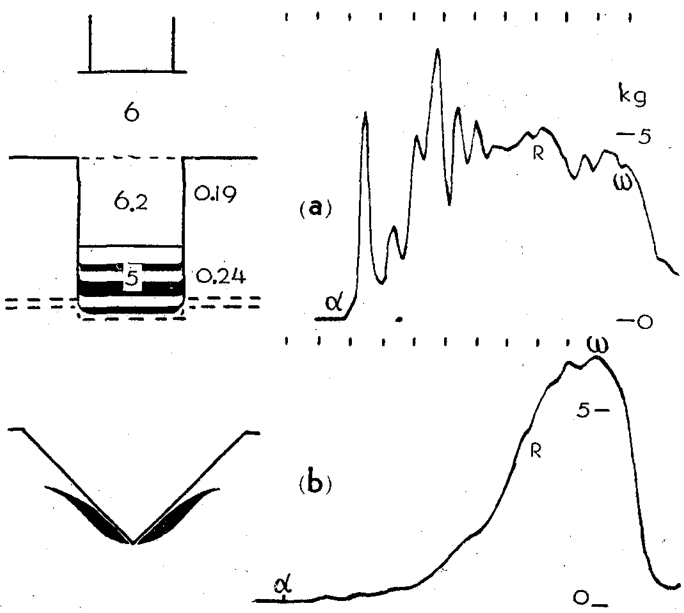


Fig. 15 Soft compact snow. −5.8°C.
(a) falling system with flat bottom. B-type.
(b) falling system with conical bottom. D-type.

force of snow is composed of two parts, the first part between points α and β and the second one between points β and ω. The first part seems to belong B-type rather than to A-type while the second part clearly is of type A.

(4) *D-type*. Soft fresh snow was likely to show this type of oscillogram of which an example is curve R of Fig. 8 at the end of the previous section. The snow crystals composing the soft fresh snow are connected with one another so weakly that the falling system falls with no resistance in the first stage of the fall. But as the mass of snow compressed by the falling system is accumulated below it the undeformed snow surrounding the compressed snow begins to act upon the system with increasing resisting force through the latter. It is thought that the increase of resisting force is due to the enlargement of side surface of the region of compressed snow through which the undeformed snow surrounding it exerts frictional force.

Dry granular snow showed a type intermediate between C and D.

When the flat bottom of the falling system was replaced by a conical one with its apex downwards, the resisting force of D-type was found also on stratified compact snow. In the case of soft compact snow the curve of resisting force was perfectly of this type as shown in Fig. 15 (b), while the same snow presented the curve of B-type shown in Fig. 15 (a) when struck by the falling system with flat bottom. In the case of hard compact snow the curve was of D-type in general

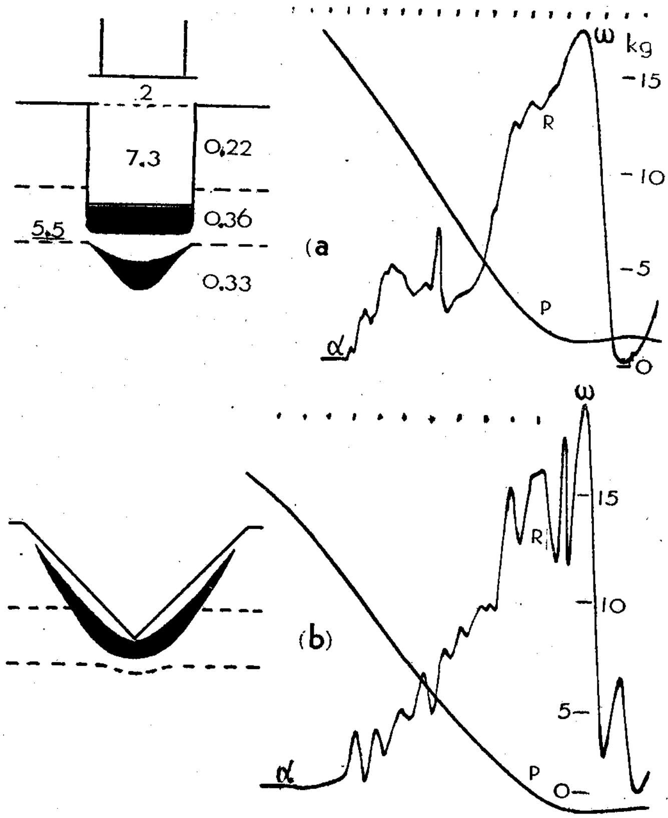


Fig. 16 Hard compact snow which is somewhat wet.
(a) falling system with flat bottom. C-type.
(b) falling system with conical bottom. D-type with the character of B-type.

tendency but it was often accompanied by marked fluctuations as shown by the lower curve of Fig. 16. The upper curve of the same figure which was obtained on the same snow in the case of the falling system with flat bottom belongs to C-type.

The region of compressed snow develops in two different ways. It develops mainly sideways in the case of soft compact snow as indicated in Fig. 15 (b) and (a) of Fig. 17 while it does mainly downwards in the case of hard compact snow as shown in Fig. 16 (b) and (b) of Fig. 17. As noted above there appeared fluctuations in the resisting force of hard compact snow whereas they are absent in that of soft compact snow. This difference must arise from whether there is the downward development of compressed snow or not, because there always appear fluctuations on the resisting force of all sorts of compact snow when the falling system falls onto it with flat bottom making the region of compressed snow extend only in the downward direction.

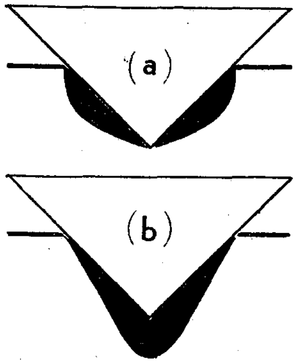


Fig. 17 Two types of compressed snow developed by falling cone.
(**a**) in the case of soft compact snow. (**b**) in the case of hard compact snow.

M. KURODA developed a method of determining the hardness of snow by dropping a wooden cone with a metal apex attached (weight: 1 kg, angle of apex: 90°) onto the snow from such a level that the apex of the cone is located 10 cm high above the surface of snow (7). The hardness of snow is represented by the height (=10 cm) of the cone minus the depth by which the apex of the cone sinks in the snow, that is, by the height of the flat top surface of the cone above the surface of snow. The above stated circumstance that a cone dropped onto snow is subjected to resisting force which undergoes similar change regardless of the structure of the snow may be considered as a guaranty for the adequacy of KURODA's method.

§ 12. The relation between the resisting force and the stratification of the snow.

The form of the curve of resisting force of snow is so intricate that it is not always possible to establish a reliable correspondence between the curve and the stratified structure of snow. But when there are some layers in the snow which are stronger than those lying above and below them, they resist the falling body with a large force causing prominent peaks to appear on the curve of resisting force. In such cases it is possible to determine with some degree of confidence which layer was broken down to give rise to a given part of the curve of the resisting force on the basis of the positions which the peaks take on the curve.

The snow used in the experiment illustrated in Fig. 18 (a) was composed of five layers J7 to J19 from below upward as shown in the left portion of that figure. The snow layers are here named after the date on which they were formed, for example, J7 means a layer deposited on January 7th. The uppermost layer J19 is soft, J15 and J13 are compact and J7 is a thick layer of granular snow. J7′ is a thin crust sheet formed by the metamorphosis of the uppermost part of J7. The figures standing below the letter *W* on the edge of the figure represent the statical strength (kg) or the maximum load-supporting strength of the respective snow layers. The statical strength was determined in the following way. When it was to be determined on some certain layer, for example on layer J13, the two layers J19 and J15 lying above it were removed and a disc of the same diameter as that of the falling system was placed on the exposed surface of layer J13. Small weights were put on the disc one after another until the disc sank into the snow by breaking it down abruptly. The total of the small weights at this instant

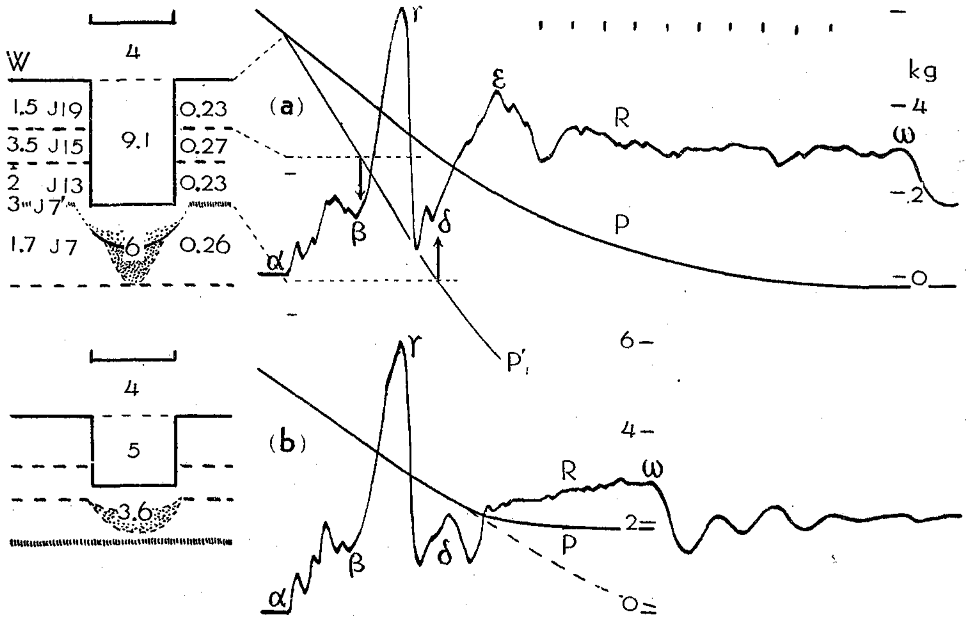


Fig. 18 Resisting force of snow containing two strong layers J 15 and J 7'. −2.3°C. (a) Curve P' shows the position of the front of compressed snow developing below the bottom of falling system of which the position is given by curve P. The early part of curve R ranging from point α to point δ belongs to C-type while the remainder from point δ to point ω belongs to A-type.
(b) The result of experiment performed on the same snow in such a way that the falling system would be stopped before it would begin to break down the layer J 7'. The second peak of curve R is absent in this case indicating that this peak was caused in the case of (a) by the break-down of layer J 7'.

added to the weight of the disc was defined as the statical strength W of that layer.

In the case of Fig. 18 (a) the statical strength W of layers J 15 and J 7' is larger than that of the ones adjacent to them. Therefore the prominent rises in the oscillogram R of resisting force shown in Fig. 18 (a) ranging from point β to point γ and from point δ to point ε must have been brought about by the break-down of these strong layers. The reason for assigning the beginnings of break-down of the layers in question to points β and δ lies in the following. As shown in the figures inserted in the previous section, the height h of the compressed snow developed below the falling system is nearly equal to the depth D by which the system has sunk into the snow. (It was noted on p. 63 of the previous paper that h was found to be 1.5 to 4 times as large as D in the experiment described there. But the speed of the falling body was much slower in that experiment than in the present one. Such a difference in the experimental conditions may be the cause of difference in the ratio of h to D). Curve P′ in Fig. 18 (a) is drawn so as to represent the depth twice as far below the snow surface as the bottom of the falling system of which the position is given by curve P. Therefore curve P′ indicates, though not very accurately, the position of the front of compressed snow. The horizontal dotted lines indicate the levels of the top of the strong layers J 15 and J 7′ on the scale of depth used in the oscillogram. The dotted lines meet curve P′ at the time points β and δ, which indicates that the front of compressed snow reached at these time points the tops of the layers in question to cause the commencement of their break-down.

In order to give more solid basis to the supposition that the two peaks of curve R of Fig. 18 (a) are due to the break-down of layers J 15 and J 7′, the following experiment was perfomed. To the top of the ebonite rod of the falling system (see Fig. 1 of § 9) was attached a string in such a way that the bottom of the falling system would be positioned at the level half way between the snow surface and the top of layer J 7′ when it was suspended by the string. If the falling system is dropped under such conditions its descent will be stopped at the above stated level. The compressed snow will just reach layer J 7′ without breaking it down whereas it can get through layer J 15. Then the curve of resisting force will show only the first peak γ due to the break-down of the latter layer. Curve R shown in Fig. 18 (b) is the resisting force obtained by such an experiment. Up to point δ it is the same in form as curve R of Fig. 18 (a) giving the first peak γ. But the second peak ε present in (a) is entirely lacking in this case proving that peak is in reality caused by the break-down of layer J 7′. The sketch of the section of the snow block on the left side of Fig. 18 (b) indicates that the compressed snow had not certainly broken down layer J 7′. In reality the string was cut by the falling system when it was extended to its full length, which is shown on curve R by the deep trough immediately to the right of point δ. Thereafter the falling system continued for some time to descend at a much reduced speed until it was stopped at point ω.

Returning to curve R of Fig. 18 (a), its first part from point α to point β due to the break-down of layer J 19 belongs to A-type with small fluctuations as described in the previous section. The heap in front of point β must have arisen from the second compression which occurred on the compressed snow when its development was temporally arrested by the strong layer J 15 before it began to give way at point β. The rise of curve R starting at this point is not so steep but that it can be supposed to be caused by a single impulsive force. It seems that layer J 15 endured elastically up to point γ and then suddenly gave way as a whole. The low resisting force following point γ will be due to the break-down of the weak layer J 13. The crust sheet J 7′, being an almost perfect solid sheet of ice, would have perhaps resisted the falling system with its elasticity until it was broken down at point ε. The thick granular layer J 7 exerted for a rather long time (from point ε to point ω) a nearly constant resisting force to the falling system which had now been slowed down in its speed of descent.

If the falling system is dropped from different heights H above the snow surface, the system will be stopped after having sunk into the snow by different depths D. The higher H is, the deeper D becomes with the result that the snow gives way down to the deeper lying layers. Therefore, if from the result of an experiment performed with $H=H_1$ is subtracted that of another experiment performed with $H=H_2$ which is smaller than H_1, the remainder can be considered to have resulted from the break-down of the deep layers which occurred only in the first experiment. Application of such a procedure to a series of experiments conducted with different H will yield the correspondence between the respective parts of the curve of resisting force and the layers which gave rise to them.

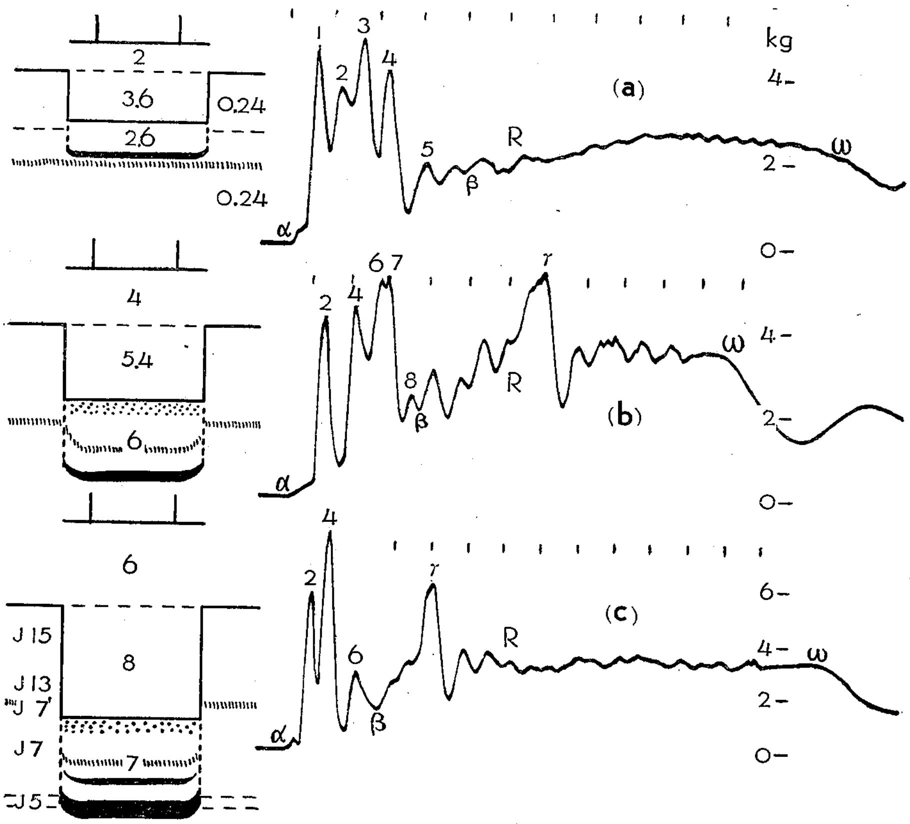


Fig. 19 The falling system was dropped from different height H above the snow surface. $-2.5°C$.
(a): $H=2$ cm, (b): $H=4$ cm, (c): $H=6$ cm. Points β indicate the instants when the front of compressed snow arrived at the bottom of the uppermost layer J15. The peak γ appearing in curves R of (b), (c) is due to the break-down of the crust sheet J7'.

Fig. 19 illustrates the results of three experiments made on the same snow but with different H, that is, with $H=2$ cm, 4 cm and 6 cm. They were made on the same snow as used in the experiment of Fig. 18 when the snow cover built up to layer J 15. Let the experiments performed with $H=2$ cm, 4 cm, 6 cm be called (a), (b), (c) respectively. In the case of (a) the development of compressed snow was stopped at just above crust sheet J 7′. But this sheet was passed through by the compressed snow in the cases of (b) and (c) and the curve R of the resisting force came to show the large peak γ which is lacking in the case of (a). Therefore this peak γ must be due to the break-down of the crust sheet.

The position curve P (not drawn in Fig. 19 so as not to confuse the figures) indicated that the points marked by β on the curves R correspond to the arrival of the front of compressed snow at the bottom of the uppermost layer J 15. Therefore the train of high peaks in the first portion of curves R up to point β must have appeared while layer J 15 was giving way. The peaks are very steep and some of the troughs interposed between them are very deep, which suggests that the resisting force here consisted of separate impulsive forces acting at each of the troughs with no continually acting force (cf. § 10). Supposing that such was the case, the magnitude of the impulsive forces can be computed in the following way.

Let z denote the amount of compression which was caused by the resisting force of snow on the steel ring B of the falling system (see Fig. 1 of § 9). The amount of compression, z, is proportional to the resisting force and the former can be obtained easily from the latter by mutiplying it with a proportional constant. The curve in A of Fig. 20 represents a train of peaks of the resisting force expressed in terms of z while the vertical segments p_1, p_2, p_3, represent by the magnitude of impulse the impulsive forces giving rise to them. Now let it be assumed that the time rate of change in z, that is dz/dt, have values V_1 and V_2 respectively just before and just after the time point of the first trough. Then the magnitude of impulsive force p_2 which has caused the second peak is given by

$$p_2 = m(V_2 - V_1),$$

where m is the mass of the hollow cylinder C constituting the lowest part of the falling system. The values of V_1 and V_2 can be calculated from Z_1, Z_2, z_2, the heights of the peaks and that of the bottom of the trough shown in A of Fig. 20, by the aid of a formula expressing the damped free oscillation of the hollow cylinder C. In the same way any other of the impulsive forces can be computed.

The magnitude of impulsive forces which caused the peaks in the first part of curves R in (a), (b), (c) of Fig. 19 is shown in the respective figures (a), (b), (c) in the right half of Fig. 20. The peaks of curve R and the impulsive forces which caused them are indicated by the same numerical figures in both Figs. 19 and 20. The horizontal coordinate of Fig. 20 represents the subsidence depth D of the

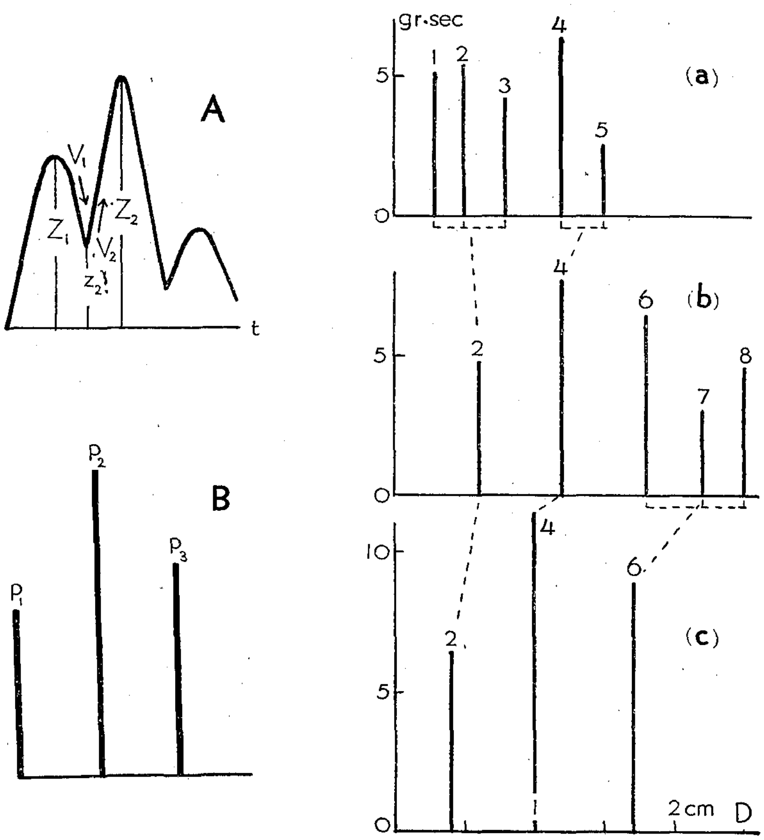


Fig. 20 The appearance of a train of peaks in the curve of resisting force such as shown in **A** is the result of the action of impulsive forces p_1, p_2, p_3 represented by the vertical segments in **B**. The magnitude of p_2 can be computed when such lengths Z_1, z_2, Z_2 as shown in **A** have been determined experimentally.
(**a**), (**b**), (**c**): The impulsive forces which gave rise to the peaks located in the early part of curves R of Fig. 19. The ordinate D is the depth of the bottom of the falling system below the surface of snow.

falling system determined from the position curve P (not drawn in the copies of oscillograms in Fig. 19).

In Figs. 19 and 20, in case (a) having the least value of H when the speed of the falling system was smallest five impulsive forces from 1 to 5 appeared while the system was sinking $D \doteq 1.5$ cm into the snow. In case (b) in which the speed of the falling system was larger than in (a) the impulsive forces appearing in the range of D less than 1.5 cm are only two in number in contrast to the five impulsive forces of (a). Judging from the relative positions of the segments representing the impulsive forces in (a) and (b) of Fig. 20, it may well be supposed that the three impulsive forces 1, 2, 3 and the two 4, 5 in the case of (a) coalesced respectively into single impulsive forces 2 and 4 in the case of (b). The remaining three 6, 7, 8 of the impulsive forces appearing in (b) seem to have united themselves into the single one 6 in (c) in which case the falling system got through the layer J15 at a speed which was the largest among the three cases.

Such a tendency of the impulsive forces to coalesce with the increase of speed of the falling system will be understood if they are supposed to have made their appearance by the mechanism discussed in the preceding section, § 10. According to that mechanism the impulsive force should appear every time the front of compressed snow developing below the falling system catches up with the front of the region of damaged ice rods. Therefore, if the speed of the front of compressed snow is increased by the increase of that of the falling system, the time needed for the front of compressed snow to overtake that of damaged snow is shortened with the result that the impulsive forces occurring successively tend to coalesce. It should follow conversely from the same reasoning that the impulsive forces come to be separated from each other distinctly as the speed of falling system becomes reduced with the approach of the stop of the fall of falling system, although they will come at the same time to be attended with danger of not being distinguishable because their magnitude becomes small. Appearance of the train of small heaps on the later part of the oscillograms R of resisting force in Fig. 19 can be explained in such a way.

§ 13. The stop of the fall of the falling system.

As seen from the oscillograms inserted in the previous sections the resisting force of snow usually continues to have a nearly constant value in the later stage of its action to drop finally to the value equal to the weight of the falling system simultaneously with the arresting of its fall. The constancy of the resisting force may well be understood if the snow layer, which is now being intruded upon by the compressed snow developing below the falling system, resists it with the yield stress W'. (Strictly W' is the yield stress multiplied by the horizontal sectional area of the compressed snow. But for the sake of simplicity W' will be called merely the yield stress). Since the falling velocity of the falling system is much reduced in the later stages of its fall the yielding speed of the layer in question must be very small, which circumstance then results in the approach of the yield stress W' of that layer to its statical yield stress W which is a constant quantity proper to that layer. W' may fluctuate but it must be held at a nearly constant value since it is always restricted near to the value of constant W. Then, if the mass of the falling system is denoted by M, its falling velocity v will be given by the equation of motion

$$M \frac{dv}{dt} = -(W' - Mg), \tag{1}$$

which shows that the falling velocity is decelerated at a constant rate since the quantity contained in the parentheses on the right side of the equation is a positive constant.

When the falling velocity v is reduced to zero the snow layer ceases to yield and begins to act upon the falling system with the elastic force F in place of the yield stress W'; the equation of motion of the falling system is then changed into

$$M\frac{dv}{dt} = -(F-Mg)\,. \tag{2}$$

The essential difference between equations (1) and (2) lies in that W' in (1) is constant while F in (2) can change in the range of values from W' to zero. The velocity v is not held at the value zero but changes its sign, that is, the falling system begins to be raised by the elastic force of the snow. Thereafter the situation becomes such that the system is placed on an elastic base with the result that the system will continue to perform oscillation up and down together with the snow layer. But the internal friction of the snow layer will rapidly cause attenuation of the oscillatory motion and the falling system will very soon come to a standstill. Indeed the attenuation of the free oscillation of snow is rapid especially when the temperature is not much below 0°C; this is shown in the figures on Pl. II of the previous paper.

The above mentioned free oscillation of the falling system is so weak that it can usually not be detected distinctly in the curve P of the oscillogram which gives the position of the falling system. (The free oscillation here in question should not be confused with the free oscillation of the lead cylinder A relative to the hollow cylinder C of the falling system (cf. § 9). The latter oscillation appears with a large amplitude on the oscillogram R of resisting force after the falling system is stopped. The former is the oscillation of the center of gravity of the whole falling system and may be seen on curve P representing the position of the system.) One will see in Fig. 16 (a) of § 11 a small rise in curve P after the time point ω. This is nothing but the sign of the very first part of the free oscillation.

Fig. 21 shows the result of four experiments; the three different marks connected by a vertical line belong to one and the same experiment. The triangular marks indicate the final value of the resisting force of snow just before it begins to decline as a result of the stop of the falling system, that is, the yield strength W' of the snow layer which is the deepest to be broken down by the falling system. The light circles represent the statical yield strength W of that layer in the original state. W is the same as the statical strength defined in the previous section § 12 and was actually determined by the means described there. W always stands above W' as shown in Fig. 20. But the difference between the two is not large and it will be permissible to say that W' is near to W as noted above. If the snow yields in the last stage of its break-down by the mechanism described in § 10, it follows naturally that W′ is less than W, because then W' represents nothing but the yield

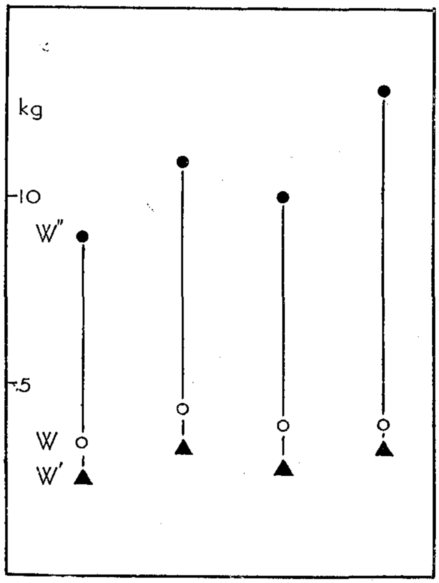


Fig. 21

Light circle, W: the statical strength in its original state of the deepest snow layer which is broken down by the falling system.

Triangle, W': the yield stress (=yield strength) of the same snow layer while it is giving way under the falling system.

Dark circle, W'': the statical strength of the bottom of the hole made by the falling system in the snow.

strength of the snow in the state where some of the ice rods composing its network have already been damaged.

If the bottom of the hole made by the falling system is pressed down by an increasing load, the bottom gives way when the load reaches a certain value W'' which is much larger than W' as well as W. W'' is the statical strength of the bottom of the hole; it is represented in Fig. 21 by the dark circles. But that very bottom stopped the falling system by resisting it with the yield stress W' not coming up even to the half of W''. The strength of the bottom in that case was W'. Such a marked increase from W' to W'' in the strength of the bottom of the hole seems to be caused by the fact that the compressed snow developed below the falling system has been arrested to a standstill. It is generally known about the phenomenon of solid friction that the force needed to cause a body which has been standing still to begin to move is much lager than that necessary to keep it moving. A similar phenomenon seems to have occurred in the present cases. The lower oscillogram of resisting force R in (**b**) of Fig. 22 was obtained when the falling system was dropped onto the bottom of the hole which it had made before by the preceding fall giving the upper oscillogram (**a**). The exceedingly high peak at the head of oscillogram (**b**) represents the large impulsive force which was needed to start the movement of the compressed snow. Once the compressed snow has begun to move, that is, after the high peak at the head the resisting force was reduced to a small value; that value of the resisting force continued to the end of the process.

As stated above in §9 a block of snow so placed that the layers composing it stand upright resists the falling system falling onto it with a resisting force belonging to A-type (cf. §11). The resisting force of A-type reaches a peak at the beginning and then a nearly constant force which follows it lasts for a rather long time. Therefore the time mean value $\bar{R}$ of the resisting force turns out to be equal to the magnitude of that long-lasting constant force which represents at the same time the yield strength W' of the snow. It will be expected that the larger the

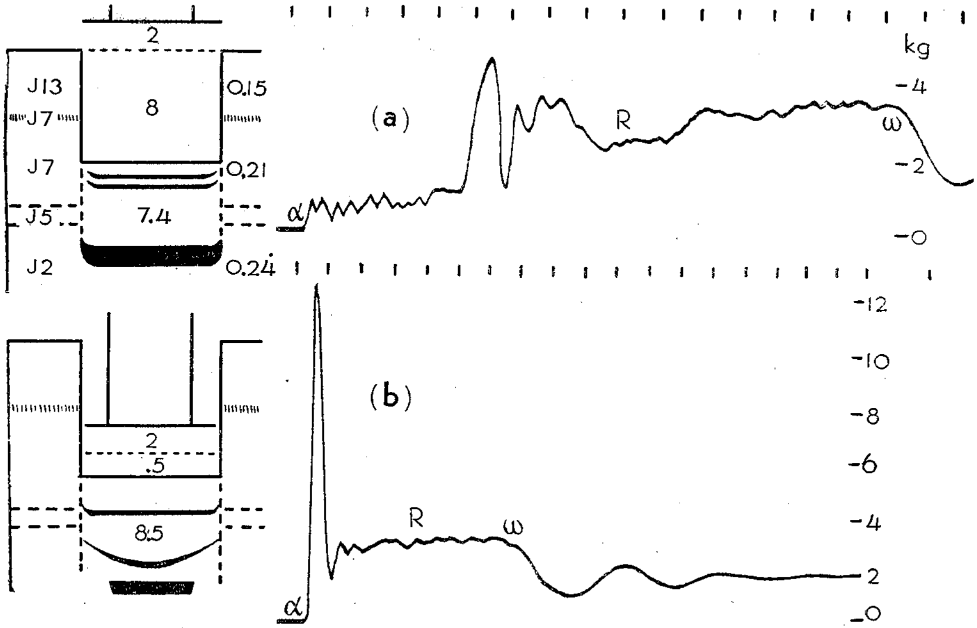


Fig. 22 The falling system subjected to resisting force as shown by oscillogram (a) made a hole 8 cm deep in the snow. The system was then dropped again from 2 cm above the bottom of the hole and the lower oscillogram (b) of resisting force was obtained.

density ρ of the snow is, the larger is $\bar{R}$, and indeed such a relationship is found actually to hold as indicated by the light circles in Fig. 23. The dark circles in the same figure represent the density of the snow after it had been compressed by the falling system. The broken line connecting the light and drak circles indicates merely that the two belong to one and the same sample of snow.

In §5 of the previous paper the load supporting strength of the snow cover was determined in relation to the subsidence depth D of a cylindrical body below the surface of snow. The load supporting strength at a given depth D_i was not represented by a single value but by a range of values lying between W_i^* and W_i, where W_i^* is smaller than W_i. The upper limit W_i of the range was determined by the maximum load which the bottom of the hole made by the cylindrical body sunk to the level D_i could support while the lower limit W_i^* was conventionally put equal to the maximum load W_{i-1} which the level D_{i-1} lying one step above level D_i could support. W_i was certainly equal to the force needful to give movement to the compressed snow developed below the bottom of the hole at level D_i by destroying the cohesion between the compressed snow and the snow mass surrounding it. Then W_i corresponds to the high peak at the head of curve R of

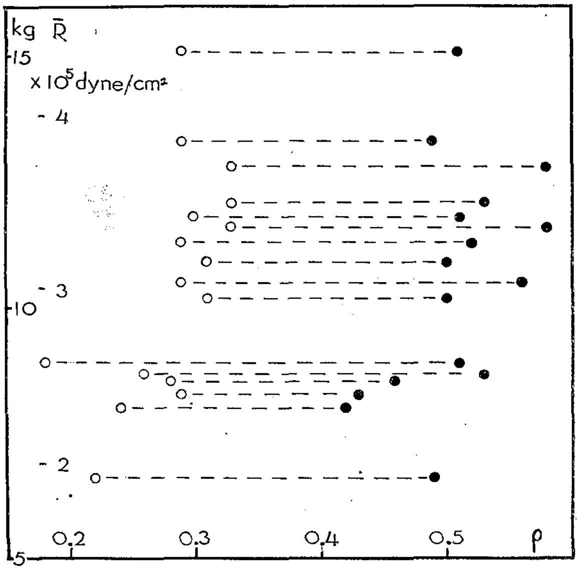


Fig. 23 $\bar{R}$: the mean value of the resisting force exerted against the falling system by a block of compact snow placed in such a way that the snow layers composing it stand upright. ρ: the density of snow. The density increases from the value represented by the light circle to that represented by the dark one due to the compression caused by the falling system. Temperature: $-2°C$ to $-5°C$.

Fig. 22 (b) of this paper. After the compressed snow had been started to move, the snow lying below the bottom of the compressed snow must have resisted the falling body with its yield stress W_i'. If this resistance W_i' were recorded on the oscillogram it would have given a long lasting force such as the one following the first peak of curve R of Fig. 22 (b). Then, from the physical point of view, it will be better to assign to the lower limit W_i^* of the range of load supporting strength the value W_i' rather than the above noted value W_{i-1}, although it is usually not practical to determine W_i' in the routine observation of the load supporting strength of snow cover.

§ 14. The softness of snow.

When the falling system (mass: M) is dropped from the height H above the snow surface and is stopped at the depth D below it, the gravity does the work $w=(H+D)Mg$. The energy needed for the break-down and compression of the snow must come from this work. The amount of the work w is enlarged as H is increased and the mass m^* of snow broken down and compressed by the falling system will be expected to increase with the increase of w. The mass m^* is the same as the mass of the compressed snow developed below the falling system and will be comprised to its half by the snow which has filled the space of the hole before it was made by the falling system, because the thickness of the region of compressed snow is nearly equal to the depth D of the hole as described above in §12. Then, if that mass of the snow which has filled the space of the hole is denoted by m, the ratio (m/w) will turn out to be a constant regardless of the amount of the work done w. Of course this ratio cannot be a constant common to all sorts of snow but can be such only for one and the same snow. Then, if its value is found to be larger on one snow than on another, the former may be said to be softer than the latter, because the snow broken down by a given magnitude of the work w is more in amount in the former case than in the latter. In this way the ratio (m/w) may be used as a measure of the softness of snow. On the same basis the reciprocal of the above ratio, that is (w/m), may be taken as a measure to represent the hardness of the snow. But the term 'hardness of snow' has since long ago been applied to the measure of hardness which is determined by the method of KURODA described in the previous section. KURODA's hardness and the reciprocal of the softness here in question have much in common with each other but the two are not the same in all points. Under such a circumstance the present authors adopt here the term 'softness' rather than 'hardness' in order not to confuse the expression.

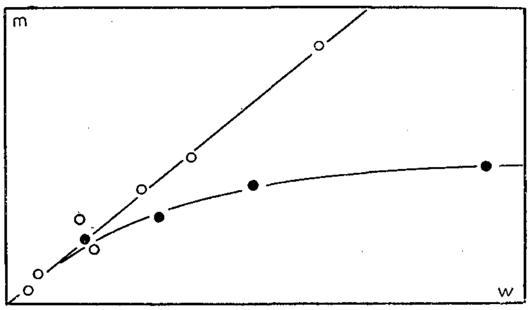


Fig. 24 m: the mass of snow which had filled the space of the hole before it was made by the falling system. w: the work done by the falling system to the snow.
Light circle: compact snow of density 0.31.
Dark circle: granular snow of density 0.57,

Fig. 24 shows the results of experiment in which the falling system was dropped from different heights onto a compact snow (density: 0.31) and on a granular snow (density: 0.57). In the case of the compact snow m was found to increase fairly well in proportion to w; the constancy of the ratio (m/w) holds good in this case. But in the case of granular snow the increasing rate of m was gradually lessened as w was increased, that is, the ratio (m/w) became smaller with the increase of w. Although such a non-constancy of the ratio lowers its value as a measure of softness of snow it will still be used here as such for the sake of practical convenience.

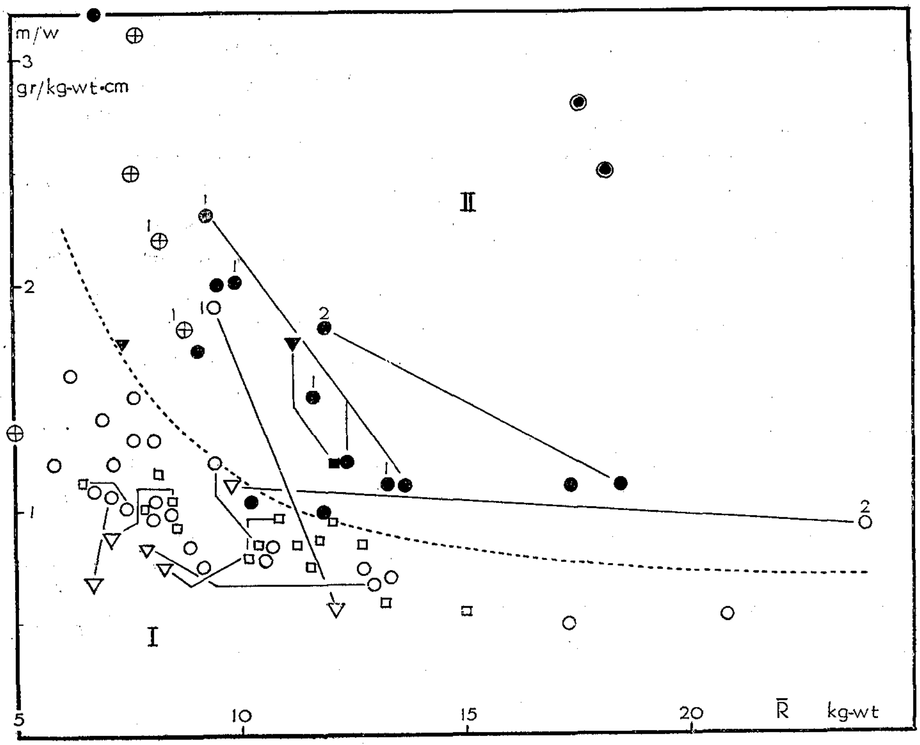


Fig. 25 *m/w*: measure of softness of snow. $\bar{R}$: time mean value of resisting force of snow. The meaning of the marks is given in the table inserted in the text.

Fig. 25 shows the relation between the ratio (m/w) and the time mean value $\bar{R}$ of the resisting force of snow. If T and T' denote respectively the time intervals from the beginning of the fall of the falling system and from its first contact with the snow surface to the instant of its arresting, $\bar{R}$ can be computed by the formula

$$\bar{R} = Mg(T/T'). \qquad (1)$$

The impulses imparted to the falling system by gravity and the resisting force of snow are given by $I_1 = MgT$ and $I_2 = \bar{R}T'$ respectively and both the impulses must be equal to each other according to the law of dynamics; then the equation $I_1 = I_2$ yields formula (1). T and T' can be determined from the oscillogram of the resisting force.

The value of m can be determined from the original thickness and the density of the snow layers located in the range from the snow surface down to the bottom of the hole made by the falling system.

Since $\bar{R}$ is the mean value of the force with which the snow resists the falling system as it forces its way into the snow, it may be said that the smaller $\bar{R}$ is, the softer is the snow. Indeed the decrease of $\overline{R}$ is accompanied by the increase of the ratio (m/w), a measure of softness, as shown in Fig. 25. Although the softness is in itself one of the mechanical properties of matter it is a rather vague conception being not uniquely defined from the physical point of view. Many different physical quantities can be adopted as the measure of softness and $\overline{R}$ too may be chosen as such. Moreover, the degree of smallness of $\overline{R}$ seems more suitable as the measure of softness than the ratio (m/w), because $\overline{R}$ has a straightforward physical meaning being a force the conception of which has direct bearing upon that of the softness. If there were found a simple devise to determine the time intervals T and T' experimentally, it would be better to choose $\overline{R}$ as the measure of softness of snow.

The different marks in Fig. 25 have respectively the significance shown in the following table. The same applies also to Figs. 26 and 27 below.

TABLE 1

		Newly deposited soft snow	Compact snow	Granular snow	Sand
Falling system with flat bottom; layers composing snow	lie horizontally	Cross in circle	Light circle	Dark circle	Double circle, inner circle dark
	stand upright		Light rectangle	Dark rectangle	
Falling system with conical bottom			Light triangle	Dark triangle	

Numerical figure 1 annexed to the mark shows that the snow was gathered together after it had been broken to fragments.
Numerical figure 2 annexed to the mark shows that the snow was wet by spraying with water.
The marks joined by a line indicate that they represent samples of the same snow subjected to different experimental procedures. All the data in Figs. 25, 26, 27 were obtained from the experiments performed with the falling system of weight 5 kg and diameter 6.6 cm in temperature range $-2°C \sim -5°C$.

The plane of Fig. 25 is divided into two domains; the lower domain I is occupied by the marks for compact snow and the upper domain II by those for soft and granular snow. The most important difference between the former and the latter snows consists in the degree of coherence between the ice elements composing them; in the former the ice elements, having the form of short rod, make up a strong net-work while in the latter the elements in the form of individual snow crystals or of ice granules are connected with one another by weak bonds. One will find in Fig. 25 a mark of compact snow (light circle) with a figure 1 annexed in domain II. This compact snow, having been once disjointed and being deprived of strong joints between its ice elements, has its position in domain II instead of domain I. Wetting of the compact snow also weakens the joints between the ice elements; the light circle with annexed 2 at the right extreme of Fig. 25 belongs to domain

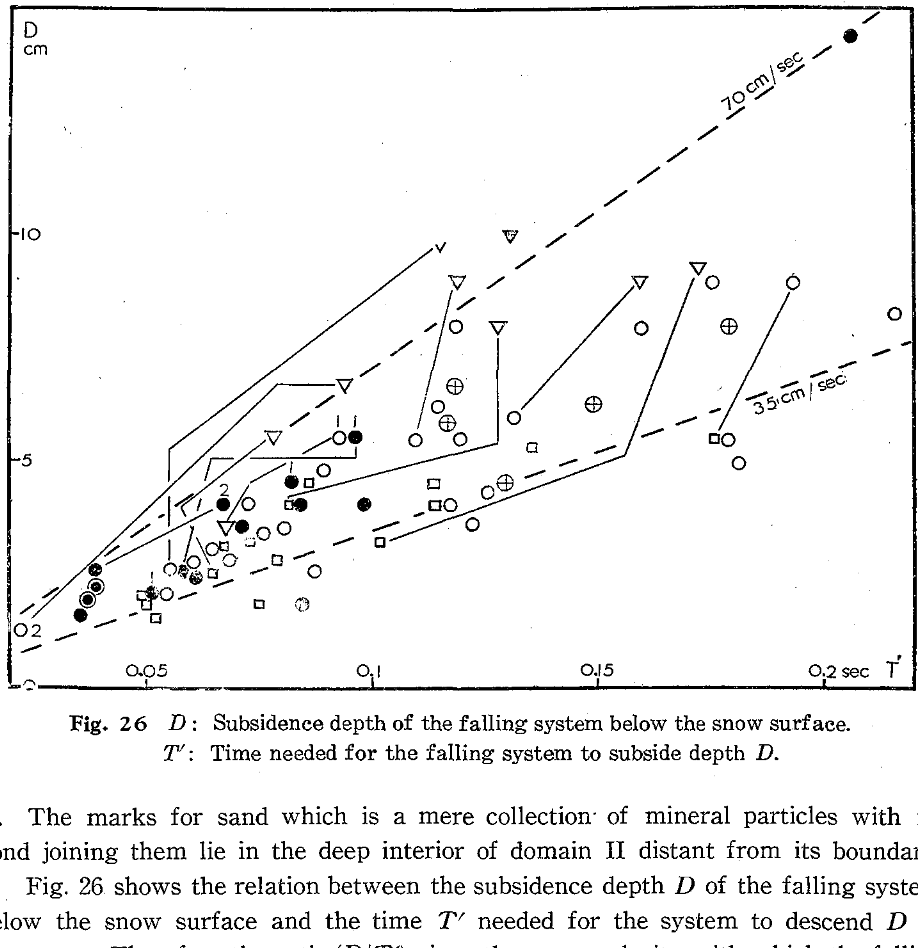


Fig. 26 D: Subsidence depth of the falling system below the snow surface.
T': Time needed for the falling system to subside depth D.

II. The marks for sand which is a mere collection of mineral particles with no bond joining them lie in the deep interior of domain II distant from its boundary.

Fig. 26 shows the relation between the subsidence depth D of the falling system below the snow surface and the time T' needed for the system to descend D in the snow. Therefore the ratio (D/T') gives the mean velocity with which the falling system fell in the snow subjected to the resisting force. Most of the marks in Fig. 26 lie in the domain bounded by two straight lines which correspond respectively to the mean velocities indicated by the annexed figures 35 cm/sec and 70 cm/sec. One sees from this figure that the mean velocity of the falling system is confined within a rather narrow range. Whether the bonds joining the ice elements of snow are strong or weak seems in this case to have no effect upon the mean velocity (D/T'). But the mean velocity is influenced by the shape of the bottom of the falling system; the conical bottom gave a larger velocity than the flat bottom did.

The measure of softness (m/w) is plotted against the density ρ of snow in Fig. 27. In the ranges of small and large densities the ratio (m/w) is large because the snow belonging to these ranges is soft or it is granular snow composed of weakly bonded ice elements. Two marks for soft snow are located in the region of large density; they represent soft snow which has been packed artificially to great density.

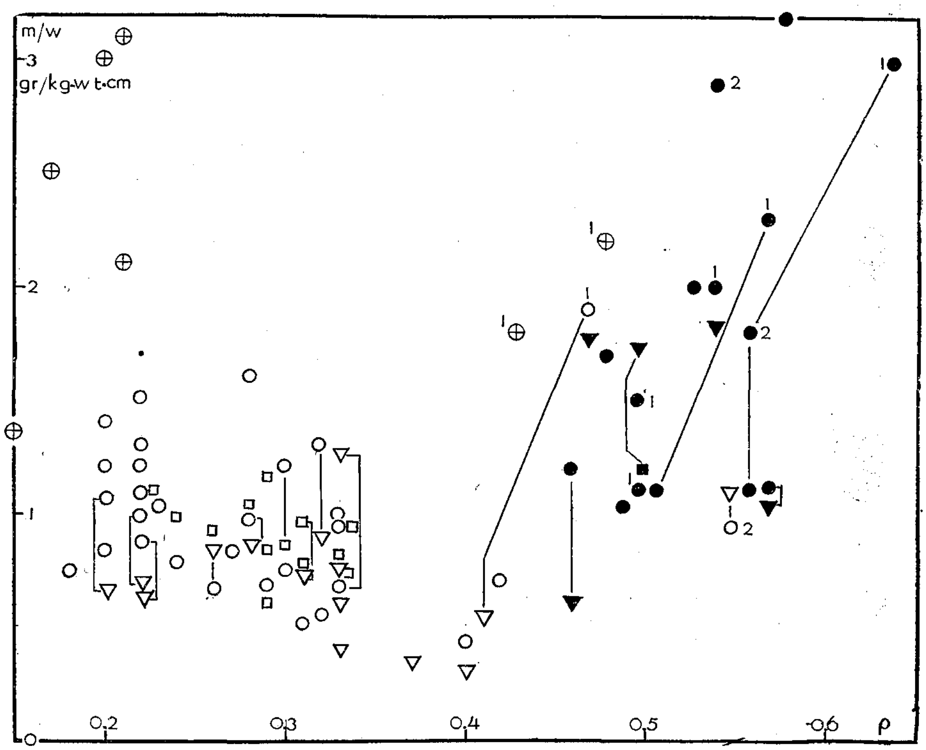


Fig. 27 *m/w*: measure of softness of snow.
ρ: density of snow.

The value of the ratio (*m/w*) is small lying in the range from 0.5 to 1.5 gr/kg-wt·cm in the case of compact snow with the medium density 0.2 to 0.4 gr/cm^3. It should be noticed that among nine pairs of experiments performed on the compact snow with flat and conical bottom of the falling system the relation large and small of the ratio (*m/w*) in the pair is not the same; among the nine pairs of marks joined by a line, the triangle lies above the circle in two pairs while the former is located below the latter in the remaining pairs.

Summary

This booklet contains a description of experimental research on the resistance offered by snow to a body falling into the snow.

§ 9. **Break-down of snow by a falling body.** A hollow cylinder made of thin iron plate (diameter: 6.6～10 cm) with closed bottom was fitted to the lower part of a solid lead cylinder (weight: 2 kg or 5 kg). Between the two a ring-formed steel spring was inserted so as to join them together into a single body. Strain gauges (very thin wires of an alloy with high electric resistance) were stretched along the

surfaces of the spring. When the whole system was dropped onto the snow surface from a height 0~6 cm above its surface, the resisting force of the snow on the bottom of the hollow cylinder compressed the ring-formed spring. The thin wires on its outer and inner surfaces were elongated and shortened due to the extension and contraction of each of the surfaces of the spring respectively, with the result that the electric resistance of the wires was changed in proportion to the magnitude of the resisting force. This change in the electric resistance was recorded by an electromagnetic oscillogram.

The falling system was arrested by the snow after it had penetrated some distance into the snow in a few tenths of a second.

The snow was then cut vertically through the center of the hole made by the falling system; the cut surface sprayed with coloured water showed the region of compressed snow developed below the falling system—spray figure. The region of compressed snow was displayed more distinctly when a plate about 4 cm thick cut out of the snow so as to halve the hole with one of its surfaces was examined by illumination from behind—shadow figure.

§10. The simplest case of break-down of snow by a falling body. The oscillogram of the resisting force of snow showed the simplest appearance in the case when the structure of the snow was uniform in the direction of fall of the falling body. The oscillogram showed a wavy segment laid almost horizontally. Judging from the precision limit of the measuring devise used in the experiment, the authors supposed that the actual resisting force shows a constant value superimposed by intermittent impulsive forces occurring at each trough of the waves of the oscillogram. A theory was advanced about the cause of such a sort of resisting force.

The snow was represented by an assemblage of short ice rods joined at their ends so as to make up a three dimensional net-work. With such a structure the snow resists the motion of the falling body by exerting an impulsive force on the bottom of the compressed snow developed below it, causing at the same time some of the ice rods immediately close to the bottom of the region of compressed snow to be damaged. A damaged ice rod then passes on damage to another ice rod near itself and this does the same to its neighbour in turn. In this manner the damage of the ice rod will propagate from one ice rod to the one near in just the same way as a solute molecule wanders about in the solution by Brownian motion. In this way a domain containing some number of damaged ice rods develops downwards from the bottom of compressed snow; its thickness x increases in proportion to $\sqrt{t}$, where t is the time counted from appearance of the impulsive force. The falling body receives a continual small resisting force while the bottom of the region of compressed snow is advancing through this damaged region. But such a situation can continue only for a certain time because the bottom of the compressed snow, proceeding downwards at a nearly constant speed, overtakes in a time τ the front of the damaged region which is advancing at a decreasing speed proportional to

$d\sqrt{l}/dt$. At this instant the bottom of the compressed snow encounters the non-damaged net-work of ice rods again to be acted upon by an impulsive force. The repetition of such a course will result in the appearance of a constant resisting force superimposed by a train of intermittent impulsive forces.

§ 11. **The resistance exhibited by stratified snow.** The oscillogram of the resisting force taken on the snow so stratified as it actually was in the natural snow cover showed much more complicated change than that described in the previous section. But the mode of the change can be classified into the following four types.

A-type: A large resisting force lasting about 0.01 sec appears at the very first and then a long-lasting more or less constant resisting force follows with small fluctuations.

B-type: Resisting force fluctuates with large amplitude and the oscillogram of resisting force looks like a series of many steep peaks.

C-type: Resisting force increases step by step, forming two or three stages before it reaches the final value.

D-type: Resisting force increases gradually from nothing to a maximum value showing no stages different to the former case of C-type.

Some discussion is presented on the cause of each of those types of resisting force.

§ 12. **The relation between the resisting force and the stratification of the snow.** Strength of the layers composing snow is different in each of them. The falling system will be met by a strong resistance when the front of the compressed snow developing below the falling system is advancing through a strong layer. In experiments each of the snow layers was loaded with an increasing weight having the same diameter as that of the falling system and the value of the weight at the instant when the snow layer began to give way was taken as the statical strength W of that layer. The oscillogram of the resisting force showed peaks of which the positions seemed to show a good correspondence with those of the layers in the stratification having large values of W. Whether such a correspondence between one peak and the strong layer causing it was true or not was examined in the following way. To the tip of the falling system was attached a string which would stop the fall of the falling system by suspending it just before the front of the compressed snow reached the top of the strong layer in question. The peak which had been supposed to be caused by that layer in the experiment performed before without the string actually vanished in this case, confirming the presupposition. On the basis of such a correspondence found between the peaks of the resisting force and the strong layers it could be determined which snow layer caused which part of the oscillogram of the resisting force.

Soft compact snow layer was apt to show a resisting force which graphed as a train of steep peaks. Under the assumption that each of the steep peaks was caused by an impulsive force the magnitude of the latter was determined by numerical calculation from the form of the train of peaks. Successive two or three of the impulsive forces tended to coalesce into a single one as the speed of the falling system was increased in accord with the theory advanced in § 10 in relation to the origin of such an impulsive force.

§ 13. The stop of the fall of the falling system. The resisting force of snow showed usually a nearly constant value W' about twice as large as the weight of the falling system for a rather long period in the later stage of its fall. Simultaneously with the arresting of the descent of the falling system the value of the resisting force declined to the same value as that of the weight of the falling system. W' is expected to be not far different from the statical strength W of the deepest snow layer to be broken down by the falling system because its speed must be much reduced when its descent is approaching the end. Indeed W' was found to be 70~80% of W.

The statical strength of the bottom of the hole made by the falling system showed a large value W'': 2.5 to 3 times as large as W. When the falling system was dropped on the bottom of the hole the oscillogram of the resisting force showed a very steep and high peak at first followed by a long lasting constant force having a value nearly equal to W'. The authors suppose that the undeformed snow mass surrounding the compressed snow below the bottom of the hole had enough time to adhere to the compressed snow while it was standing still after the first fall of the falling system had come to the end. The steep large peak of the resisting force appearing in the case of the second fall onto the bottom of the hole must have been the impulsive force needed to destroy the adherence between the above two snows. The realization of such an adherence can explain also the fact that the statical strength W of the bottom of the hole was found to have a large value.

§ 14. The softness of snow. The ratio (m/w), where m and w denote respectively the mass of snow which had filled the space of the hole before it was made by the falling system and the work done by gravity on the falling system during the whole course of its fall, was taken as a measure of the softness of snow. The time mean value $\bar{R}$ of the resisting force of snow which could be obtained from its oscillogram was found to be the smaller as the ratio (m/w) became larger—evidence for the adequacy of taking the ratio (m/w) for a measure of the softness. The value of (m/w) was found to lie in the range from 0.5 to 1.5 gr/kg-wt·cm in the case of compact snow. It was much larger on the soft snow and the granular snow.

The time T' from the instant of the first contact of the falling system with the snow to the instant of its arrest by the snow was plotted against the subsidence depth D of the falling system. The mean falling speed (D/T') of the system was found to be confined in a rather narrow range from 35 cm/sec to 70 cm/sec.

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"T. K." in the following is the abbreviation for "Teion-Kagaku" (Low Temperature Science), a scientific publication written in Jappanese, issued by the Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan.

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