The Relaxation of Stresses in Ice

K. F. VOITKOVSKY

К. Ф. Войтковский

Permafrost Institute of the Siberian Branch of Academy of Sciences of the U.S.S.R. Yakutsk, U.S.S.R.

Abstract

The rate of stress relaxation in ice is determined by its structure, its temperature and the magnitude of the initial stress applied. The rate depends also upon the time interval between the moment of shear stress application and the time of initiation of the stress relaxation process. Theoretical and experimental investigations showed that this time interval is of substantial importance, though usually little or no attention has been paid to it. When the deformation of ice is held constant, the shorter the above mentioned time interval is, the more rapidly the resistance of ice decreases, and, when the deformation of ice is increased by creep, the longer the time of the preliminary creep is, the more slowly the stress relaxation occurs. For example, a series of experiments gave the following results. Three identical ice cylinders of random structure were compressed at $-3°C$ by the same initial load $\sigma_i = 7\,kg/cm^2$. The increase in deformation was stopped in the respective cylinders for 6 minutes, 1.4 hours and 8 hours after the application of the initial load. The time required for the stress in ice to be reduced to half the initial value was found to be 6,58 and 102 minutes for the first, the second and the third cylinders. Thus the stress relaxation in the first cylinder the stress relaxation occurred 17 times as rapidly as in the third cylinder.

I. Introduction

If a force applied to ice is of such a nature as to cause shear stress, the ice undergoes not only elastic deformation but also an plastic one. Therefore, if the deformation is prevented to increase at a given moment and is held constant thereafter, the elastic deformation begins immediately to decrease at the expense of the corresponding increase in the plastic deformation, and, with the decrease in the elastic deformation, the force of resistance in the ice decreases.

Stress has for a long time been believed to decrease exponentially, as eq. (1):

$$\sigma_t = \sigma_\lambda + (\sigma_i - \sigma_\lambda)\, e^{-t/\alpha}, \qquad (1)$$

where

σ_i: stress at the initial moment,
σ_λ: stress at the elastic limit,
α: relaxation time.

But this formula was found to be not highly in appropriate for expressing the stress relaxation in ice, because the point of view from which the formula was derived is unacceptable for the description of ice deformation.

II. The Correlation between the Rate of Relaxation of Stresses in Ice and the Preliminary Creep

Experimental investigations show that the deformation caused in ice by a load keeps changing noticeably if the load is continually changed, and, even after the load change is stopped, the subsequent deformation fails to occur in the same manner; it depends not only upon the magnitude and the duration of the initial load but also upon those of the loads which acted with a varying intensity upon the ice in succession. From these facts it may well be assumed that the speed of stress relaxation in ice should depend upon the time during which the ice suffered deformation before the relaxation begins.

Let the above situation be considered with respect to uniaxial compression of ice. For a load of a constant magnitude, the relative compressive strain induced by the load can be expressed by the empirical formula (2):

$$\varepsilon_t \approx \frac{\sigma}{E} - k\sigma^2 t\left(1+\frac{a\,t_0}{1+at}\right), \tag{2}$$

where

- σ: compressive stress, kg/cm^2,
- E: elastic modulus of ice, kg/cm^2,
- t: time elapsed from the beginning of loading in hours,
- $k=\dfrac{K}{(1+|\theta|)5.2}$,
- K: coefficient characterising the steady rate of creep of ice (for ice of random structure, $K\approx 3\times 10^{-5}$ cm$^4\cdot$degree/kg$^2\cdot$hr),
- θ: temperature of ice in °C,
- a and t_0: empirical coefficients, characterising the initial stage of creep which slows down with time.

When the compressive stress is changed from σ_i to σ_1 at the time t_i hours after the moment of primary loading, it is shown, under some assumptions, that the subsequent change in relative compression can be expressed by the following equation:

$$\varepsilon_t = \varepsilon_{(\sigma_i, t_i+t)} - (\varepsilon_{\sigma_i, t} - \varepsilon_{\sigma_1, t}), \tag{3}$$

where

- $\varepsilon_{(\sigma_i, t_i+t)}$: the deformation which the ice would have at t (time elapsed from the moment of changing the stress), if it were subjected to stress σ_i continually after the initial loading.
- $\varepsilon_{\sigma_i, t}$: the deformation which the ice would have at t, if it were subjected to σ_i continually after the moment of changing the stress.
- $\varepsilon_{\sigma_1, t}$: the deformation which the ice would have at t, if it were subjected to σ_1 continually after the moment of changing the stress.

These three deformations are obtained from formula (2).

In Fig. 1 $\varepsilon_{(\sigma_i, t_i+t)}$, $\varepsilon_{\sigma_i, t}$ and $\varepsilon_{\sigma_1, t}$ are shown by broken lines and ε_t by a full line. From this figure it is seen that, if $\sigma_1 < \sigma_i$, ε_t begins to decrease at time t_i from ε_i, the relative

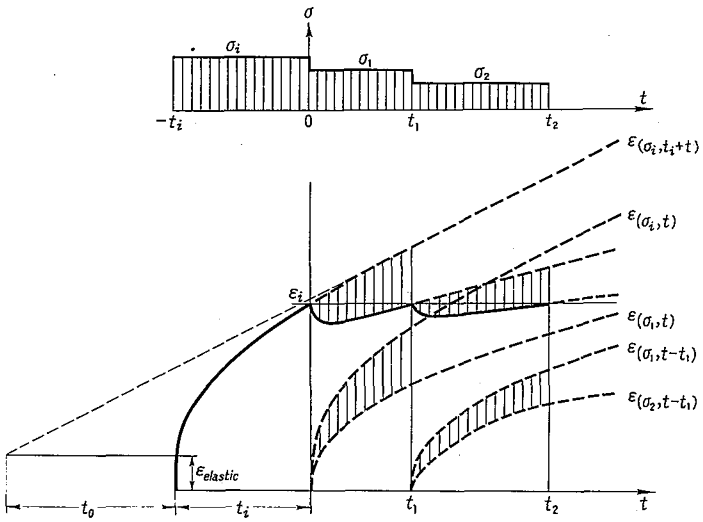

Fig. 1. Diagram of the computed value of ice strain with a change in stresses

compressive strain which ε_t attains at t_1, soon reverts to an increase and reaches ε_i again at time t_1. Time t_1 can be obtained from equation $\varepsilon_t=\varepsilon_i$, which is, by the use of formula (3), written as:

$$k\sigma_i^2 t_0'(\zeta_{t_i+t_1}-\zeta_{t_i})-\frac{\sigma_i-\sigma_1}{E}-k(\sigma_i^2-\sigma_1^2)(t_1+t_0''\zeta_{t_1})=0\,, \qquad (4)$$

where

$$\zeta_{t_i}=\frac{at_i}{1+at_i}\,;\quad \zeta_{t_1}=\frac{at_1}{1+at_1}\,;\quad \zeta_{t_i+t_1}=\frac{a(t_i+t_1)}{1+a(t_i+t_1)}\,,$$

and t_0' and t_0'' are the values which the empirical coefficient t_0 in formula (2) takes respectively in the stage of initial loading and after the load is changed. $t_0'=30$—100 hours. t_0'' is different according as the load is increased or decreased; in the former case $t_0''=5$—30 hours and in the latter $t_0''=5$—10 hours.

To simplify calculations, let it be assumed that the stress relaxation proceeds in such a way that the stress is reduced by 10% of its initial value σ_i everytime the strain is restored to the value ε_i. Thus $\sigma_1=0.9\,\sigma_i$; $\sigma_2=0.8\,\sigma_i$; $\sigma_3=0.7\,\sigma_i$ etc.

If σ_1 is put equal to $0.9\,\sigma_i$ in eq. (4), it gives

$$kE\sigma_i[0.81t_1+t_0'(\zeta_{t_i+t_1}-\zeta_{t_i})-0.19t_0''\zeta_{t_1}]=0.1\,, \qquad (5)$$

from which t_1 is to be determined.

Once t_1 is obtained, time t_2 at which the strain restores the value ε_i after the second reduction in stress can be determined by the equation

$$kE\sigma_i[0.64t_2+0.17t_1+t_0'(\zeta_{t_i+t_1}-\zeta_{t_i})-t_0''(0.19\zeta_{t_2}+0.17\zeta_{t_2-t_1})]=0.2\,. \qquad (6)$$

The value of t_2 obtained here should be counted from the moment of the first reduction in stress. In a similar way t_3, t_4, t_5 etc. can be calculated.

The numerical results shown below are obtained from the calculation made with $\sigma_i=7$ kg/cm², $E=45\times10^3$ kg/cm², $K=3\times10^{-5}$, $\theta=-3$°C, $k=0.144\times10^{-5}$, $a=0.5$, $t_0'=30$

hours and $t_0'' = 5$ hours. If it is assumed that $t_i = 0.1$ hour, that is to say, relaxation begins 0.1 hour after the application of the initial load, it turns out that $t_1=0.015$, $t_2=0.032$, $t_3=0.050$, $t_4=0.07$ and $t_5=0.095$ hour. In case when t_i is taken as 10 hours, the results are $t_1=0.25$, $t_2=0.8$, $t_3=2.0$, $t_4=3.4$ and $t_5=4.9$ hours.

It is seen from the above calculations that the force of resistance shown by the ice kept at a constant deformation decreases the more quickly, the shorter the time interval between the moments of applying the initial load and of initiating the relaxation, namely, the longer the preliminary creep lasts, the more slowly the stress relaxes.

In order to examine such theoretical results as above, experimental investigations were made on the stress relaxation of compressed ice. Two kinds of ice were used in the experiments: ice of random structure···ice composed of randomly oriented crystals, and ice of uniform structure···ice composed of crystals whose optic axes are uniformly oriented in a given direction.

III. Experiments

The samples composed of ice of random structure were prepared by freezing at −3°C mixtures of shaved ice and water in a metallic cylinder. After removal from the cylinder, both ends of the samples of ice were made even and then were kept at a constant temperature for not less than 24 hours. The samples thus made were dull white in appearance, containing a considerable number of small air bubbles. The density of the ice was about 0.85 g/cm³. The samples were about 100 mm long and 71.5 mm in diameter with a cross-sectional area of 40 cm².

Ice of uniform structure was made in the following manner. Boiled water was cooled and frozen in a flat box which was heated from both sides and from the bottom. In the middle part of the ice plate thus frozen, optic axes of crystals were strictly in a vertical direction. The samples of ice for experimental use were cut out from the ice plates in the form of an eight-angled prism about 100 mm long. The cut was so made that the optic axes of crystals were either parallel or perpendicular to the axes of the prisms.

The compressing device consisted of a rocker and special poles. The rocker provided the necessary load to the sample of ice. When the sample was compressed to some extent, the rocker came to rest on the poles and a part of the load on the sample began to be transferred to the poles.

That part of the load which remained to be bestowed on the sample of ice was indicated by a tensometer gauge attached to a duralumin tube which was placed between the rocker and the sample. In some cases a highly sensitive intensifier was connected to the gauge so that the load on the sample might be recorded automatically by an oscillograph. But this method of using the tensometer gauge could not successfully be applied for fixing the deformation of the sample at a constant value with sufficent accuracy. Everytime the load on the sample was reduced, the duralumin tube expanded elastically to give the sample an additional deformation amounting to about 0.1 mm ($\varepsilon_{add} = 0.001$). This additional creep was superposed on the process of relaxation, with the result that the speed of relaxation became a little smaller than it should have been.

Table 1. Stress relaxation in ice of random structure, −3°C

Sample No.	Before relaxation			During relaxation			
	Initial stress σ_i kg/cm²	Time of deformation t_i hr	Initial deformation $\varepsilon_i \times 10^3$	Time of half relaxation hr	Duration of the experiment hr	Additional deformation $\varepsilon_{add} \times 10^3$	Stress at the end of the experiment
1	7	0.067	1.3	0.6	48.0	1.5	0.12
2	7	1.8	7.9	3.0	93.6	1.9	0.25
3	7	2.6	7.0	1.6	42.0	1.0	1.1
4	7	4.5	11.2	5.0	116.5	2.9	0.8
5	7	28.0	68.0	5.7	58.7	2.2	1.25
6	6	0.25	0.5	2.3	53.0	1.1	0.25
7	6	1.5	0.8	8.3	46.3	0.9	1.0
8	6	10.7	1.0	5.0	38.0	1.7	0.7
9	6	24.5	1.1	8.7	37.5	3.0	0.15
10	6	36.0	1.2	12.0	131.0	1.3	0.75
11	5	1.0	0.6	4.9	47.8	0.8	0.3
12	5	3.75	4.0	8.3	50.2	1.1	0.9
13	5	24.1	13.0	6.7	46.5	1.0	0.5
14	5	30.7	12.0	16.7	38.1	1.4	1.15
15	5	33.6	20.5	11.6	92.4	1.1	0.4
16	5	47.0	14.4	8.3	97.0	1.1	0.3
17	4	45.6	12.0	12.5	31.9	1.6	0.6
18	4	47.8	13.6	10.7	30.7	1.2	0.7
19	4	146.6	13.5	18.5	62.0	0.6	0.8

The main results obtained by the above method are shown in Table 1.

In order to eliminate the additional creep during the stress relaxation, another series of experiments were conducted by unloading the sample in such a way that the load diminished stepwise as shown in Fig. 1. The deformation of the sample was controlled by means of an indicator of watch type which detected a displacement as small as 0.002 mm. In addition to that, a miniature electric switch attached to the sample lit a signal lamp by coming in contact at the moment when the sample recovered the given initial deformation ε_i after a reduction in the load. Each time the lamp was lit, the stress in the sample was diminished by 0.2~0.5 kg/cm² by reducing the weight on the rocker. At the beginning of relaxation, however, especially when the preliminary plastic deformation was very small, it was necessary to diminish the stress by an amount of 1 kg/cm² at a time. In this way the initial deformation ε_i of the sample could be maintained within the limits of 0.01 mm, or of 0.0001 in terms of relative strain. The experimental results are given in Table 2.

IV. Interpretation

Considerable variations are found among the speeds of stress relaxation, although they were speeds determined in the experiments conducted under the same conditions.

This is due to the fact that the samples used were not of the same nature; their density was determined after the compression and was found to vary from one sample to another. It was also noticed that the initial deformation of samples showed, as a rule, somewhat greater values than those calculated by formula (2). It may be assumed that the compression caused, along with the creep deformation, disarrangement and recrystallization of the ice crystals composing the samples. As a result their porosity would have been partially decreased due to condensation of ice. Of course such a statement needs to be checked by further confirmation.

Although the experimental data are somewhat scattered as mentioned above, they confirm fully the theoretical result that the duration of the preliminary deformation renders a considerable influence upon the speed of stress relaxation. This situation will be seen from the graphs in Figs. 2 and 3.

The fact that the preliminary creep of ice diminishes the speed of the subsequent stress relaxation is a result naturally deduced from the laws of deformation. It is known that the elastic deformation occurring in samples of ice at the application of load is exclusively determined by the elastic deformations which the ice crystals composing the samples undergo. The elastic deformations in the ice crystals cause internal stresses to

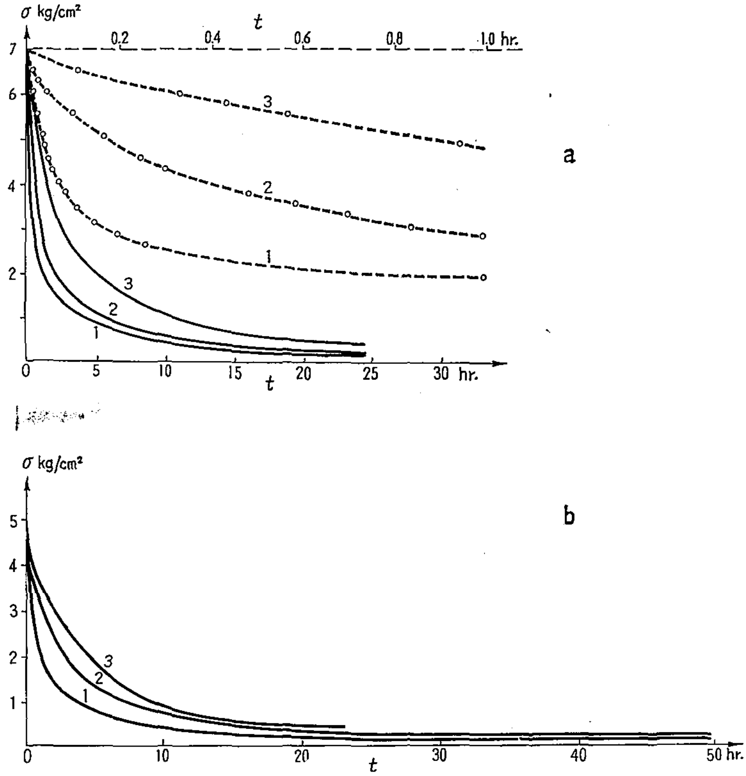

Fig. 2. Stress relaxation in ice samples of random structure

a. $\sigma_i=7$ kg/cm^2: 1-exper. No. 20, $t_i=0.1$ hr; 2-exper. No. 22, $t_i=1.4$ hr; 3-exper. No. 23, $t_i=$ 8.0 hr
b. $\sigma_i=5$ kg/cm^2: 1-exper. No. 31, $t_i=1.3$ hr; 2-exper. No. 32, $t_i=7.4$ hr; 3-exper. No. 33, $t_i=19.1$ hr

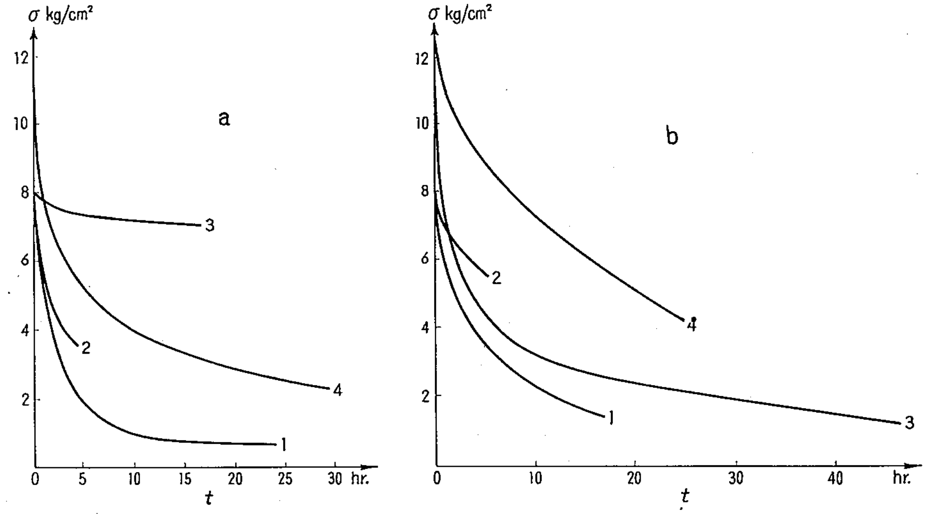

Fig. 3. Stress relaxation in ice samples of oriented structure

a. Under compression perpendicular to the direction of the crystal axes:
1-exper. No. 49, $\sigma_i=8$ kg/cm², $t_i=0.52$ hr; 3-exper. No. 48, $\sigma_i=8$ kg/cm², $t_i=47.0$ hr
2-exper. No. 50, $\sigma_i=8$ kg/cm², $t_i=18.0$ hr; 4-exper. No. 46, $\sigma_i=12$ kg/cm², $t_i=0.67$ hr

b. Under compression in the direction of the crystal axes:
1-exper. No. 43, $\sigma_i=8$ kg/cm², $t_i=2.5$ hr; 3-exper. No. 41, $\sigma_i=12.5$ kg/cm², $t_i=0.17$ hr
2-exper. No. 44, $\sigma_i=8$ kg/cm², $t_i=18.5$ hr; 4-exper. No. 42, $\sigma_i=12.5$ kg/cm², $t_i=27.1$ hr

appear within the crystals and strengthen the contacts between them. Due to the internal stresses the ice crystals themselves suffer plastic deformation, relative displacement and, in some cases, destruction. In this way plastic deformation starts immediately after the elastic deformation which occurs instantly, and tends to redistribute and equalize the internal stresses.

Resistance of the ice sample against the load is increased by the disarrangement of ice crystals, with the result that the sample becomes stiffer and the speed of its deformation is reduced. In the stage of unloading, the elastic deformation accumulated in the ice crystals causes the elementary plates, of which the ice crystals are composed, to move in the direction opposite to that in which they were displaced in the initial deformation. At the same time the crystals are themselves displaced in such a way that they tend to restore their initial relative positions. As a result the ice sample continues expanding for some time even after the instant elastic expansion is finished at the moment of unloading. Naturally, the more compacted the ice crystals become in the stage of primary creep, the less the reverse creep will be in the stage of unloading. Generally creep proceeds quickly in its initial stage. These provide an explanation for the experimental results in which the reverse creep proceeds quickly for a short time after the unloading, and, when the deformation is held constant, the stress in the ice sample relaxes at a high speed.

As creep of ice slows down gradually under a constant load, the possible reverse creep in the stage of unloading will also slow down and the speed of stress relaxation

Table 2. Stress relaxation in ice at −3°C

Sample No.	Density of ice g/cm³	Before relaxation: Initial stress σ_i kg/cm²	Before relaxation: Time of deformation t_i hr	Before relaxation: Initial deformation $\varepsilon_i \times 10^3$	During relaxation: Time of half relaxation hr	During relaxation: Duration of the experiment hr	During relaxation: Stress in the end of the experiment kg/cm²
1	2	3	4	5	6	7	8
I. Ice of random structure							
20	0.831	7.0	0.1	1.6	0.10	1.0	1.9
21	0.820	7.0	1.3	4.3	0.58	21.7	0.12
22	0.840	7.0	1.4	5.4	0.97	19.0	0.12
23	0.841	7.0	8.0	12.9	1.7	24.2	0.36
24	0.855	7.0	8.0	17.0	1.10	15.0	0.6
25	0.837	6.0	2.8	6.0	0.9	25.6	0.38
26	0.845	6.0	3.5	3.5	2.0	17.1	1.3
27	0.858	6.0	3.6	3.2	2.0	10.1	1.25
28	0.852	6.0	7.3	5.5	2.5	6.6	1.12
29	—	6.0	19.1	6.7	3.7	15.5	1.25
30	0.848	6.0	19.3	12.1	2.2	46.6	0.5
31	0.832	5.0	1.3	3.5	0.58	47.1	0.12
32	0.820	5.0	7.4	5.7	1.6	63.5	0.12
33	0.833	5.0	19.1	10.9	3.2	22.7	0.37
34	—	5.0	23.5	14.9	8.0	25.7	2.1
35	0.842	4.0	1.5	3.0	1.2	12.7	0.25
36	0.820	4.0	2.5	1.0	2.9	11.6	0.12
37	0.823	4.0	6.1	4.2	3.5	19.8	0.37
38	0.827	4.0	6.5	5.1	1.7	24.8	0.25
39	—	4.0	13.2	6.3	7.0	45.7	0.24
II. Ice of uniform structure compressed parallel to the optic axis							
40	0.893	21.5	0.13	3.2	0.44	2.0	5.5
41	0.900	12.5	0.17	2.8	1.7	47.0	0.9
42	0.900	12.5	27.1	6.2	15.3	27.1	4.3
43	0.903	8.0	2.5	3.7	3.8	17.5	1.4
44	0.904	8.0	18.5	0.5	—	5.5	5.2
III. Ice of uniform structure compressed perpendicular to the optic axis							
45	0.885	21.5	0.17	1.7	0.29	23.2	2.7
46	0.894	12.0	0.67	0.5	3.3	30.0	2.2
47	0.891	8.0	0.05	0.6	0.05	0.1	2.0
48	0.892	8.0	47.0	1.0	—	19.8	7.0
49	0.894	8.0	0.52	0.1	1.7	24.1	0.85
50	0.906	8.0	18.0	0.8	—	3.5	4.4

diminishes gradually with time.

In some of the experiments made on polycrystalline ice, relative deformation turned out to be considerably greater in the compressive experiments than in the extensive for the same value of compressive and extensive forces, in contradiction to the usual notions. It would be natural to consider that the relative deformation should be greater in extension rather than in compression, because the actual cross section of the ice sample would be narrowed by the extension due to the presence of air inclusions. In our opinion, the contradiction between such a consideration and the above mentioned fact may be dissolved by the introduction of the force of recrystallization. Compressive stresses promote, especially in the initial stage of deformation, the contraction of crystals, which results in the development of forces of recrystallization. The forces of recrystallization promote in their turn the contraction of crystals again and bring about a decrease in the porosity of ice. In case of extension, on the contrary, the forces of recrystallization prevent, to a certain degree, the deformations from growing.

References

1) VEINBERG, B. P. 1940 Led (The ice), Moscow, 524 pp.
2) VOITKOVSKY, K. F. 1960 Mekhanicheskie svoistva l'da (The mechanical properties of ice), Moscow, Izd. Akademii Nauk SSSR. 100 pp.