Simplified Method of Testing Ice for Creep and Relaxation

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Abstract

Creep experiments usually consist of testing a series of specimens under different loads, which are, however, constant for each specimen. Generally a group of creep curves are obtained as a result of tests, and the curves are used to determine the relationship between the stress and the deformation or its rate and time.

The above method, however, is rather time-consuming. The authors suggest a method substantially simplifying and speeding up the tests and enabling them to be carried out on two specimens.

In essence, the method is as follows. The load on the specimen is transmitted through a dynamometer, where a predetermined initial tension is set up, after which the position of the dynamometer is fixed so that the height of the specimen and dynamometer is maintained constant during the course of the entire experiment. Under the action of the load from the dynamometer, plastic flow deformation develops in the specimen, which results in a decompression of the dynamometer, which is recorded. Thus, the test is reduced to a creep test at a variable stress, the stress variation being due to the specimen deformation.

The data on the stress variation in the dynamometer can be used to calculate the specimen creep characteristics.

If the given initial stress transmitted through the dynamometer to the specimen is close to failure stress, the stress drop can be regarded as a decrease in specimen strength with time, while the stress at a given moment may be considered as long-term strength.

I. Introduction

As is well known, the purpose of creep tests is to establish the relationship between deformation ε, stress σ and time t:

$$\varepsilon = \varphi(\sigma, t), \tag{1}$$

or between deformation rate $\xi = \mathrm{d}\varepsilon/\mathrm{d}t$, σ and t:

$$\xi = f(\sigma, t). \tag{2}$$

The tests are usually performed by subjecting a series of similar specimens with different loads, which are constant for each specimen. A group of creep curves are plotted from test results, and the resulting curves determine the form of the creep eq. (1). If it is necessary to establish the relationship between the deformation rate and the stress, as is usually done for ice, the creep curves are reconstructed into deformation rate vs. time curves which determine the form of eq. (2). The initial portion of the creep curve (with a diminishing deformation rate) is often neglected, and only the process of steady-state viscoplastic creep with a constant rate is considered

$$\xi = f(\sigma) . \tag{3}$$

Relaxation tests are performed by assigning to the specimens a certain deformation ε caused by the initial stress σ_0; the time variation of the stress σ necessary to maintain the constancy of the given deformation $\varepsilon_0=\text{const.}$ is determined by

$$\sigma = f(\sigma_0, t) . \tag{4}$$

If the given initial stress σ_0 is close to the ultimate strength of ice, the relation (4) can be conventionally regarded as the equation of decrease in the strength of ice with time.

Since both the relaxation and the creep processes depend on the development of plastic deformation with time, eqs. (1) and (4) are bound by a definite relationship. In its general form, the rheological state equation can be written by

$$\phi(\sigma, \varepsilon, t) = 0 \text{ or } \phi(\sigma, \xi, t) = 0 . \tag{5}$$

The relationship between the creep and the relaxation equations is established by proceeding from a particular hypothesis on which the various creep theories depend.

Determination of the rheological characteristics according to the accepted method involves time- and labour-consuming experiments and requires a great number of similar specimens. To speed up and simplify the experiments, S. S. Vyalov* proposed a new method of testing for the creep and relaxation of frozen soils, ice, rocks, etc., with the aid of a dynamometric device designed by V. F. Ermakov.

II. Essential Features of Proposed Method

The load on the specimen is applied through a dynamometer by tensioning to a given initial stress σ_0. The building-up of specimen creep deformations results in the decompression of the dynamometer which is recorded on the indicating scale of the same device. The data on the weakening of the initially preset stress with time can be used to determine the creep and relaxation characteristics. Dynamometric tests can be regarded as tests for creep under the action of a time-variant stress or as a relaxation tests at varying deformations, the variations of stress and deformation being interrelated. The proposed method is suitable for tests with different types of loading (compression, tension, shear), and although we shall further discuss a device for compression testing, the key diagram of the device and the mode of data processing remains valid for any type of test.

A diagram of a compression test and the curves obtained from the test are presented in Fig. 1. A soil specimen 1 is loaded by a tensioning device 7 through a settlement plate 3 and a dynamometer 4. The dynamometer deformation is recorded on the indicating

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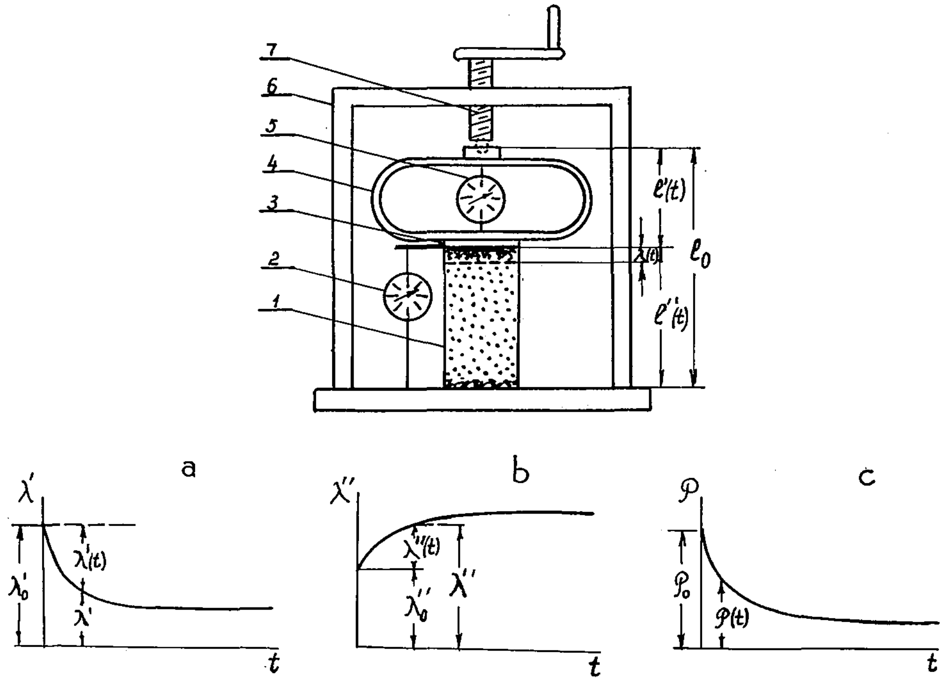

Fig. 1. Diagram of dynamometric device for determining rheological characteristics, and curves obtained from test

1, specimen; *2*, specimen deformation indicating scale; *3*, settlement plate; *4*, dynamometer; *5*, dynamometer indicating scale; *6*, frame; *7*, tensioning device

a: Dynamometer deformation curve, b: Specimen deformation curve, c: Dynamometer decompression curve

scale 5, and that of the soil specimen on the indicating scale 2. After assigning the initial load $\mathscr{P}_0$ the position of the dynamometer is fixed by securing the tensioning device 7 in frame 6. Although the height of the specimen l'' and the height of the dynamometer l' varies in the course of the test, their total height remains constant

$$l_0 = l'(t) + l''(t) = \text{const.} \tag{6}$$

When the initial load $\mathscr{P}_0$ is assigned, the initial deformation of the dynamometer $\lambda_0 = l' - l'_0$ occurs, where l' and l'_0 are the heights of the dynamometer before and after the application of the load, respectively. This deformation decreases as the dynamometer decompresses (Fig. 1 a)

$$\lambda' = \lambda''_0 + \lambda''(t)\,. \tag{7}$$

Quite similarly, when the load is applied, an initial deformation of the specimen, $\lambda''_0 = l'' - l''_0$ occurs, which builds up under the action of the load transmitted by the dynamometer (Fig. 1 b)

$$\lambda'' = \lambda''_0 + \lambda''(t)\,. \tag{8}$$

In this case (ignoring the calibration corrections) the deformation of dynamometer decompression should be equal to the deformation of specimen compression

$$\lambda'(t) = \lambda''(t) = \lambda(t)\,. \tag{9}$$

As a result of dynamometer decompression, the given initial stress therein, $\mathscr{P}_0$, will drop in accordance with a certain law, $\mathscr{P}(t)$ (Fig. 1 c).

For a number of materials for instance, rock and frozen soils, dynamometer decompression will continue until an equilibrium is established between the stress transmitted through the dynamometer and the internal resistance forces of the specimen. Accordingly, the final value of the load, $\mathscr{P}_\infty$, may be regarded (provided that the initial load $\mathscr{P}_0$ was close to a failure load) as an ultimate long-term strength or ultimate flow. If, on the other hand, the decompression of the dynamometer continues indefinitely and the final stress tends to zero, this will mean that the ultimate long-term strength or the ultimate flow (in the Bingham sense) of the test material is equal or close to zero. This assumption would evidently be valid for ice, and this can be checked with the above described device.

The design of the device is given in Fig. 2. The device is meant for compression tests of cylindrical specimens of two diameters, 35.7 and 45.2 mm, and 10 and 16 cm^2 in cross section, respectively. The height of the specimens is 80 and 100 mm respectively, *i.e.* the height to diameter ratio is 2.2. The maximum compression stress is 1 500 kg. The loading of the specimen can be done mechanically, manually or by a combined method. To allow for self deformations of the device, it should be calibrated and the corresponding correction should be introducted into the measurement results

$$\lambda''(t) = \lambda_0'' + \lambda'(t) + \Delta\lambda(t), \qquad (10)$$

where $\Delta\lambda(t)$ is the device deformation, as determined by calibration.

III. Determining Rheological Characteristics

Now we will discuss the operating conditions of the device. As can be seen from Fig. 1, the load $\mathscr{P}$ is taken up by the dynamometer and specimen connected in series. The dynamometer deformation is described by the Hooke law

$$\mathscr{P} = E\lambda', \qquad (11)$$

where E is the deformation modulus of the dynamometer, and the deformation of the specimen obeys a creep law which is unknown so far eq. (5). In the simplest form

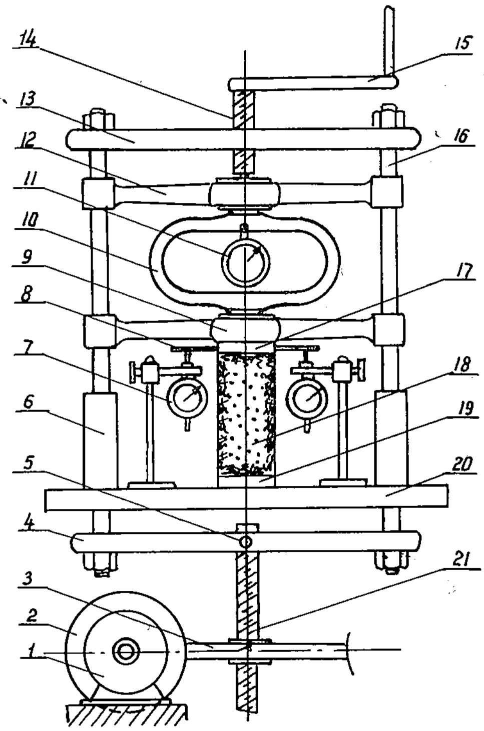

Fig. 2. Design of dynamometric device
1, electric motor; *2* and *3*, gear transmission; *4* and *13*, lower and upper transverse pull rods; *5*, stop screw; *6*, guiding bush; *7*, specimen deformation indicating scale; *8*, support for specimen deformation indicating scale; *9* and *12*, lower and upper movable guides; *10*, dynamometer; *11*, dynamometer deformation indicating scale; *14*, upper loading screw; *15*, tap wrench for manual loading; *16*, longitudinal pull rod; *17* and *19*, upper and lower cylindrical settlement plates; *18*, ice specimen; *20*, bearing plate; *21*, lower loading screw

this law can be written thus

$$\lambda'' = f(\mathscr{P})\,F(t)\,, \tag{12}$$

where $f(\mathscr{P})$ is a function describing the relationship between the load $\mathscr{P}$ and the specimen deformation λ'', and $F(t)$ is the creep function characterizing the building-up of deformation with time; at $t=0$ we have $F(0)=1$. The operating conditions of the device are described by the equalities

$$\mathscr{P} = \mathscr{P}' = \mathscr{P}'' \text{ and } \lambda_0 = \lambda' + \lambda'' = \text{const.} \tag{13}$$

Hence, taking into account eqs. (11) and (12), we obtain

$$F(t) = \frac{\lambda_0 - \mathscr{P}/E}{f(\mathscr{P})} = \frac{\lambda''}{f(\mathscr{P})}\,. \tag{14}$$

In this expression, λ'' is the specimen deformation developing with time, described by the curve of Fig. 1 b, $\mathscr{P}$ is the time-variant dynamometer stress described by the curve of Fig. 1 c, and $f(\mathscr{P})$ is a certain function which is also determined experimentally. The most widespread form of this function for ice is

$$f(\mathscr{P}) = B\mathscr{P}^n\,. \tag{15}$$

Then the expression (14) will assume the form

$$F(t) = \frac{\lambda_0 - \mathscr{P}/E}{B\mathscr{P}^n} = \frac{\lambda''}{B\mathscr{P}^n}\,, \tag{16}$$

where the parameters B and n are calculated from the test data (this will be discussed below). The following units are adopted in these expressions: $\mathscr{P}$ in kg, λ in cm, B in cm/kg^n, E in kg/cm. For conversion to true units we should assume: $\sigma = \mathscr{P}/\omega$ in kg/cm^2, $\varepsilon = \lambda''/h$ a dimensionless value, $\bar{B} = B(\omega^n/h)$ in cm^2/kg, where ω is the specimen cross section, and h its height. The final deformation equation determined experimentally will be written thus:

$$\varepsilon = \bar{B}\sigma^n F(t)\,. \tag{17}$$

If we are to express the deformation law in terms of rates, eq. (18) will take the form

$$\xi = \bar{\eta}\sigma^n \Omega(t)\,, \tag{18}$$

where $\xi = \mathrm{d}\varepsilon/\mathrm{d}t$ is the ice deformation rate, $\bar{\eta}$ and n are parameters determined from the experiment (see below), and $\Omega(t)$ is the time function determined by the expression

$$\Omega(t) = \frac{v''}{\eta\mathscr{P}^n}\,. \tag{19}$$

The above expression is derived in a similar manner to eq. (16), the only difference being that the specimen deformation λ'' is replaced by its rate $v'' = \mathrm{d}\lambda''/\mathrm{d}t$; the value of the parameter η included in the equation is bound with the parameter $\bar{\eta}$ from eq. (18) by the relation $\bar{\eta} = \eta(\omega^n/h)$.

If we consider exclusively a steady-state flow with a constant velocity, the function $\Omega(t) = \text{const.}$ (for instance, $\Omega(t) = 1$) and eq. (18) takes the form

$$\xi = \bar{\eta}\sigma^n\,. \tag{20}$$

The relaxation equation can be obtained directly from eq. (17).

The parameters of the deformation equations discussed above are determined in the following manner.

1. The device is loaded with an initial load $\mathscr{P}_0$ for 5 to 10 seconds (manually or mechanically); in the course of loading it is desirable to measure the specimen deformations with the aid of a recorder in order to obtain a load vs. deformation curve in rapid loading. At the end of loading the initial dynamometer deformation λ_0' and the initial specimen deformation λ_0'' are recorded.

2. After loading the device and fixing the position of the dynamometer, the straightening (deformation) of the dynamometer which is calculated from eq. (7) is observed. These data are used to compute the weakening of the stress in the dynamometer

$$\mathscr{P}(t) = \lambda' E ,$$

which is reflected by the graph of Fig. 3 a. Simultaneously, the specimen deformation is recorded, which is calculated from eq. (8). The accuracy of the experimental results is verified by seeing whether the condition of eq. (9) is met when eq. (10) is taken into account. The specimen deformation is described by the graph of Fig. 3 b.

When arranging the data in accordance with the relations (18) and (20) the graph of Fig. 3 b is reconstructed into the graph 3 c where the deformation rate v'' is plotted as the ordinate.

3. The experiment is repeated with identical specimens loaded with a different value of the initial load $\mathscr{P}_0$; the data of this experiment serve to plot a second set of curves on the graph of Fig. 3.

4. The parameter B and n of eq. (16) are determined from the graph of Fig. 3 in accordance with the expressions

$$n = \frac{\ln \dfrac{\mathscr{P}_2(t)}{\mathscr{P}_1(t)}}{\ln \dfrac{\lambda_2''}{\lambda_1''}} ; \qquad B = \frac{\lambda_1''}{[\mathscr{P}_1(t)]^n} = \frac{\lambda_2''}{[\mathscr{P}_2(t)]^n} . \tag{21}$$

Here $\mathscr{P}_1$ and λ_1'', $\mathscr{P}_2$ and λ_2'' are the values of the stresses in the dynamometer and the respective specimen deformations at an arbitrary moment of time in two simultaneous experiments at different initial loads $\mathscr{P}_{0(1)}$ and $\mathscr{P}_{0(2)}$ (Fig. 3). To check the reliability of the determination of n and B, the calculations by eq. (21) should be performed for several moments of time (the values of n and B should coincide).

5. The creep function $F(t)$ is determined on the basis of the graphs 3 a and 3 b.

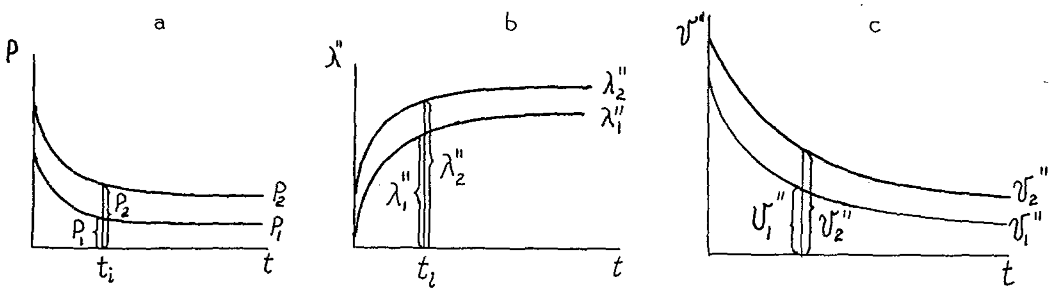

Fig. 3. Experimental data

To this end, the values of the function are calculated from eq. (16) and a graph is plotted. Thus the analytical expression of the function $F(t)$ is found. The correctness of the results obtained is ascertained by the coincidence of the values of $F(t)$ for two simultaneous tests at different $\mathscr{P}_0$. The final form of the deformation equation is given by eq. (17).

6. When arranging the experimental data in accordance with eqs. (18) and (20), the values of the parameters η and n are determined from the graphs of Fig. 3 c, by analogy with **4**

$$n=\frac{\ln\dfrac{\mathscr{P}_2(t)}{\mathscr{P}_1(t)}}{\ln\dfrac{v_2''}{v_2''}};\qquad \eta=\frac{v_1''}{[\mathscr{P}(t)]^n}=\frac{v_2''}{[\mathscr{P}_2(t)]^n};\tag{22}$$

the function $\Omega(t)$ is determined according to eq. (19) by analogy with **5**. The constancy of the value of $\Omega(t)=\text{const.}$ indicated that a steady-state flow has been established since a given moment.

7. The weakening of the resistance should be determined, as stated above, by assigning an initial load $\mathscr{P}_0$ close to the ultimate strength of ice at rapid loading. The latter is established tentatively, by breaking down the specimen on the same device. The resistance σ_S at any moment t may be taken as approximately equal to the stress in the dynamometer, but with a correction for the latter's flexibility

$$\sigma_S=\sigma(t)\left\{\frac{\omega}{E\lambda_0''}[\sigma_0-\sigma(t)]+1\right\}^{-n},\tag{23}$$

where $\sigma(t)=\mathscr{P}(t)/\omega$ and $\sigma_0=\mathscr{P}_0/\omega$, ω is the cross section (cm^2) of the specimen, $\mathscr{P}_0$ and $\mathscr{P}(t)$ are the initial stress of the dynamometer and the stress at the moment t (in kg), respectively, λ_0'' is the initial specimen deformation (in cm), E is the dynamometer deformation modulus (in kg/cm).

IV. Some Experimental Data

Figures 4 and 5 exibit the initial data of some of the experiments. Figure 4 demonstrates the curve of decrease of stress in polycrystalline glacier ice ($\gamma=0.88$ g/cm^3) at a temperature of -8°C. The initial stress was $\sigma_0=10.6$ kg/cm^2, thus corresponding to about 16.4 % of the conventional momentary stress which is equal to 64.8 kg/cm^2. It can be seen from the figure that the stress decrease with time occurs rather smoothly, although the most intensive decrease takes place within the first five days. Thus, the ratio of the stress at a given moment to the initial stress $\sigma(t)/\sigma_0(100)$ (in percentage) was: 10 min after loading—94%, after 1 hr—87%, after 12hrs—65%, after 24hrs—57%, after 5 days—35%, after 15 days—22.8%, and, with a conventional deformation stabilization—after 60 days—10.9% (1.16 kg/cm^2). In another experiment using the same initial stress, but at a test temperature $\theta=-4$°C, the ratio $\sigma(t)/\sigma_0(100\%)$ was: after 10 min—91.5%, after 1 hr—81%, after 12 hrs—53%, after 24 hrs—44%, after 5 days—22.6% and after 15 days—12.7%. These examples show that an increase of temperature, from -8 to -4°C, led to a more intensive stress decrease with time and to a more rapid change of deformations. Thus, at -4°C stress decrease in 5 days was 77.4%, and in 15 days

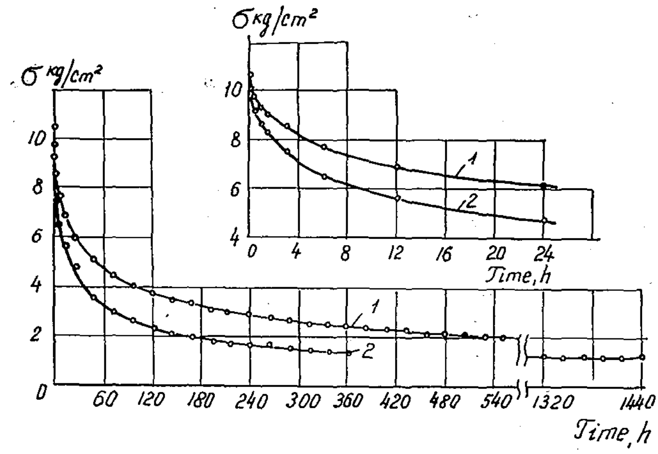

Fig. 4. Stress decrease curves. Polycrystalline ice at temperatures *1*, −8°C; *2*, −4°C

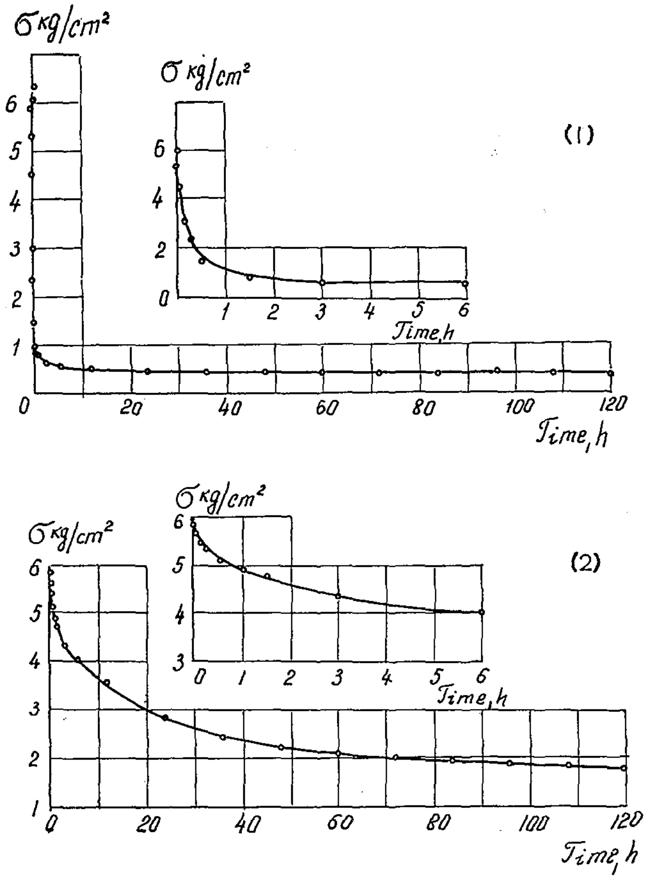

Fig. 5. Stress decrease curves. Lake ice at temperature −2°C, stress acts; (1) at an angle of 25∼30° to basal plane, (2) parallel to basal plane

87.3%, whereas at −8°C this stress decrease was achieved only after 15 and 55 days, respectively. Other experiments with polycrystalline ice at a temperature of −8°C, but at a lower initial stress, showed that a decrease in the initial stress brings about a decrease in the intensity of the stress drop with time. For instance, at $\sigma_0=6.0\,kg/cm^2$ the stress decrease after 5 days was only 35% of the initial predetermined stress, whereas at $\sigma_0=10.6\,kg/cm^2$ this decrease was 65%.

Experiments performed with lake ice ($\gamma=0.910\,g/cm^3$, $\theta=-2°C$) showed that the decrease in the ice strength with time is greatly affected by the direction of the applied stress relative to the basal planes of the crystals (the lake ice specimens, as a rule, consisted of one or more identically oriented crystals). Figure 5-1 displays the result of one of the experiments with lake ice ($\gamma=0.10\,g/cm^3$) at $\theta=-2°C$ and the initial stress $\sigma_0=6.26\,kg/cm^2$. In the experiment, the load applied to the specimen acted at an angle of 20 to 25° to the optical axis of the crystal (65 to 70° to the basal plane), and the specimen deformation occurred due to the shear action along the basal planes of the crystal. In this experiment the principal stress decrease took place within the first 30 min, during which the stress decreased by a factor of 6 and was equal to $\sigma(t)=1.04\,kg/cm^2$; during the first day the stress decreased by a factor of 14. Later on, the stress decrease occurred much slower and, on the whole, the stress was reduced about 20-fold relative to the initial value, coming down to as low as $\sigma(t)=0.34\,kg/cm^2$ (in another identical experiment the stress was reduced to $\sigma(t)=0.22\,kg/cm^2$ within the same time period).

In still another experiment with the same lake ice at $\theta=-2°C$ and initial stress $\sigma_0=5.8\,kg/cm^2$, where the applied stress acted along the basal planes and normally to the optical axes of the crystals, the stress decrease with time took place much slower than in the preceding case (Fig. 5-2), and during the first 30 min the stress was reduced by 12%, after 1 day—by 51%, and after 5 days by 70%. These examples indicate that the stress value at a given moment of time, $\sigma(t)$, depends on many factors: the structure of the ice, its temperature, the magnitude and the direction of the acting stress, etc. In all cases, however, the most intensive stress decrease occurs in the initial period after the load is applied, and a relatively insignificant change, during the later period of the tests.