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Studies of the Mechanical Properties of Sea Ice X

The Flexural Strength of Small Sea Ice Beams*

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Abstract

The flexural strength of small beams of sea ice was measured by a bending method in which each beam was bent at a constant speed by an electric motor. The force applied to the beam was measured by an electric load-cell and was recorded with an oscillograph. The beams were cut from ice blocks which were taken from a sea ice sheet, in such a way as to make its wide plane parallel to the surface of ice sheet. The span of the beam was 24 cm, the width was 5 cm, the thickness was 2 cm.

The measurement was made at various temperatures between $-3\sim-30^{\circ}\text{C}$ and at several speeds of bending. By changing the bending speed, the increasing rate of applied stress was varied between $0.05\sim 4\text{ kg/cm}^2\cdot\text{sec}$. The obtained flexural strength was $9\sim 13\text{ kg/cm}^2$.

The results indicate that the flexural strength of sea ice beam depends largely upon the increasing rate of stress and the temperature.

I. Introduction

The strength of both natural and artificial sea ice, has been studied by many authors, however, little attention has been paid to the relation between the strength and the increasing rate of applied force.

Sea ice has a complex structure composed of pure ice, liquid brine and air inclusions. It is well known that the sea ice is a visco-elastic substance from a mechanical view point. Therefore, the strength of sea ice should depend upon the increasing rate of applied force or stress. In the study of the flexural strength of small beams of sea ice at -3°C in the laboratory, the author (Tabata, 1960) demonstrated that the strength of sea ice depends greatly upon the increasing rate of applied stress. When the stress rate is larger than $3\text{ kg/cm}^2\cdot\text{sec}$, sea ice deforms elastically and the strength increases with the stress rate, and for a lesser stress rate than the above, sea ice shows a plastic deformation and the strength slightly increases with the decreasing of the stress rate. Peyton (1963) measured the strength of artificially made sea ice by changing the loading rate at various temperatures and found that the strength increases with increasing the loading rate. In the investigation of the strength of sea ice by a cantilever method (Tabata, 1963, 1964, 1965, 1966, 1967), it was found that a similar tendency of stress rate dependence of the strength of sea ice existed. In these papers the average temperature of sea ice ranged between -1.0 to -2.5°C , and the critical value of the stress rate for elastic deformation was approximately $1\text{ kg/cm}^2\cdot\text{sec}$.

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With the lowering of the temperature, sea ice becomes hard and the strength increases. Therefore, it may easily be considered that the boundary value of the stress rate which classifies the elastic and the plastic state may be changed by the temperature. The principal objective of the present study is to determine the variation of this mechanical behaviour with temperature. To reach this objective, the tensile strength of small rectangular sea ice beams cut from natural sea ice sheet was tested. The data was obtained with various increasing rates of applied stress between 0.05 and 4 kg/cm²-sec at temperatures between -3 and -30°C. In addition, some measurements were made at -42 and -54°C.

II. Method of Measurement

Specimen. Several blocks of sea ice were cut from "young sea ice" of 25 cm in thickness in the coast of Okhotsk Sea in Hokkaido and transported to the low temperature room of the institute. The small rectangular ice beams were cut from the block with a band saw in such a way as to make its wide plane parallel to the surface of natural ice sheet. The size of the beam is 30~40 cm in length, about 5 cm in width and about 2 cm in thickness. To make the surfaces of the sample smooth the beam was placed on a jig and planed carefully with a carpenter plane.

After planing, the size of the beam was measured by a vernier calliper and the weight was also measured. The density of the beam was obtained from its weight and volume which was calculated from the size. The vertical structure of the beam was fairly uniform throughout in thickness due to the limited thickness of the samples.

Temperature. A small hole was drilled at the end part of the beam. A thermometer was placed in the hole and the temperature was measured just before the measurement. All the beams were kept in a well controlled thermostat for more than 3 hours. The measurement was carried out at the following temperatures; -3, -5.8, -9, -13.5, -19.6 and -29.5°C. In some cases, the test was made at temperatures of -42 and -54°C.

Equipment for bending. The flexural strength measurements were performed using

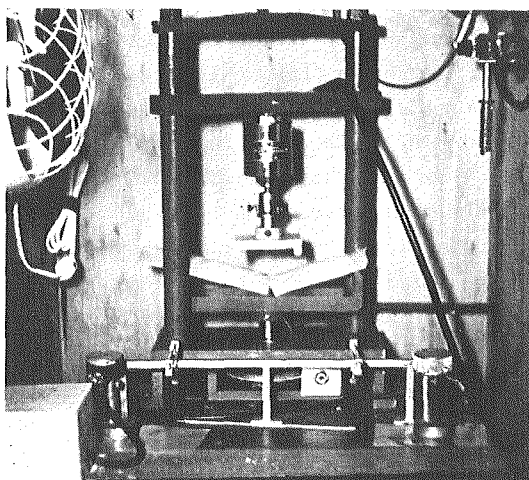


Fig. 1. Testing apparatus

a flexural strength meter which was constructed in the machine shop of the institute as shown in Fig. 1. In this apparatus, the lower supporting end was moved vertically at constant speed by an electric motor. The speed of moving can be regulated by means of changing several gear wheels from 1×10^{-4} to 2×10^{-1} cm/sec. The distance of two supporting ends is 24 cm and that of the upper loading ends is 8 cm.

The force applied to the beam was measured with an electric load cell which was connected to a recording apparatus. The total capacity of the load cell was 100 kg. A strength meter was also placed within a thermostat and its temperature was kept to that of the measurement.

At a given constant ascending speed of the supporting end, the beam is moved upward and is bent downward until it is broken. After 5~8 beams were tested at a certain rising speed, the speed was changed and again several beams were broken. By changing the rising speed of the beam, the increasing rate of applied load (or force) is also changed. With changing the increasing rates of load the increasing rate of the tensile stress in the lowermost surface of the beam also changed. From this view point, the quantity "increasing rate of stress" or "stress rate" $\dot{\sigma}$ is used in the present paper. After a series of measurements which were composed of several tests of various rising speed performed at a certain temperature, the test temperature was changed. The increasing rate of maximum tensile stress at the lower surface of the beam was changed from 0.05 to 4 kg/cm²-sec and, in some cases, it was increased to 8 kg/cm²-sec.

The flexural strength σ_c can be obtained from

$$\sigma_c = \frac{24F_c}{bh^2}, \quad (1)$$

where F_c is the load at failure, 24 cm is the length of span of the beam, b is the width and h is the thickness of the beam.

Young's modulus. Prior to the strength measurement, Young's modulus of the beam was measured. Small and thin iron plates were mounted by freezing to the both ends of the lower surface of the beam. Two electro-magnets were placed just under these iron plates and then the ice beam was forced to vibrate laterally. Thus the resonance frequency of the beam was measured. This method was used by Kuroiwa (1954), Tabata (1958) and others to obtain Young's modulus of ice and of sea ice.

The Young's modulus E_v is obtained from

$$E_v = 0.946 \rho \frac{l^4}{h^2} f^2, \quad (2)$$

where, f is the resonance frequency of the ice beam, ρ is the density of ice, h and l is the thickness and the length of the beam respectively. Young's modulus E_v obtained from this method is named as *vibration Young's modulus*.

The amount of deflection δ of the beam at the loaded point is represented by

$$\begin{aligned} \delta &= \frac{5}{324} \frac{F}{E_b} \frac{l^3}{I_z} \\ &= 2.56 \frac{F}{E_b} \frac{1}{bh^3} \times 10^{-3}, \end{aligned}$$

where l is the length of the span and is 24 cm, F is the force applied to the beam and

E_b is the Young's modulus, and named as *bending Young's modulus*. The ice beam is always bent with a constant speed during the strength measurement. Therefore, if the ice is an elastic substance, the force which is exerted on the ice beam is expected to increase linearly.

The bending Young's modulus then be computed from

$$E_b = k \frac{F}{\delta} \frac{1}{bh^3}, \quad (3)$$

where k is a constant.

Crystallographic structure. To determine the crystallographic structure, polarized photographs of the broken segments were taken. Though the thickness of the beam was too thick (about 2 cm) to take a clear polarized photograph, the structure and size of grain was observed satisfactorily. Examples of the photograph are shown in Fig. 2. The grain size of all samples was almost similar to those seen in the figure.

Salinity. After taking the polarized photograph, the broken segment was stored in a tightly sealed polyethylen bag, and then melted. The chlorinity of melted water was measured using Mohr's silver-nitrate method. The salinity was then calculated from the chlorinity using Knudsen's formula which is widely used by oceanographers.

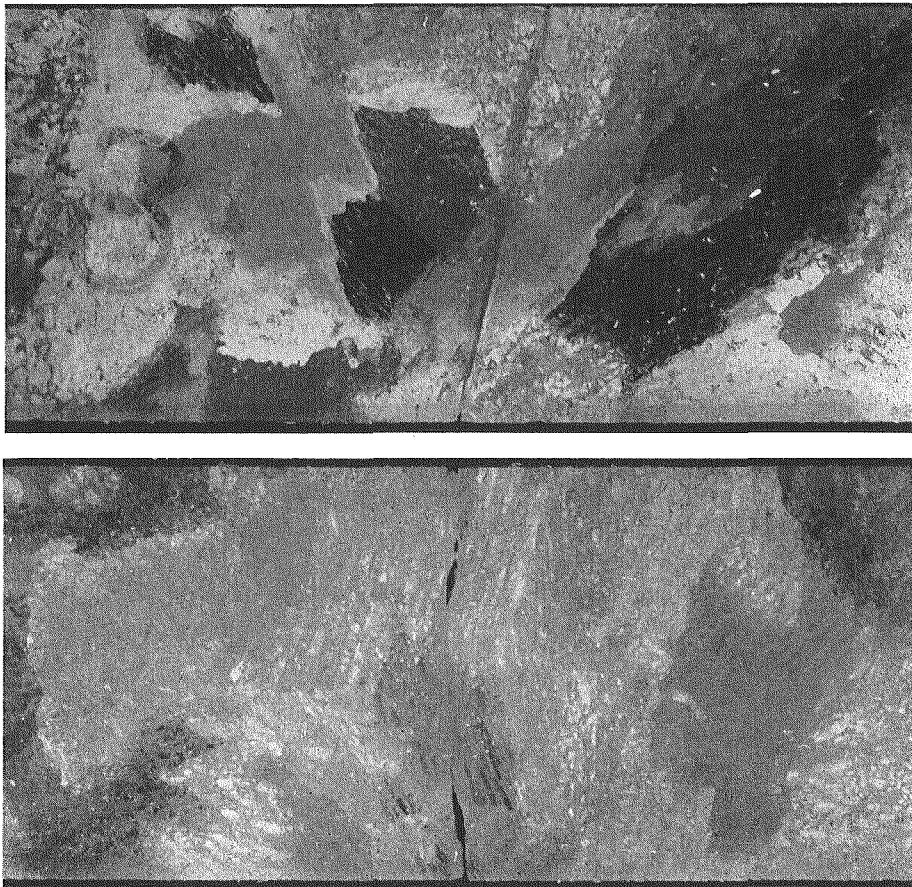


Fig. 2. Polarized photographs of broken beams

III. Results of Measurement

Figure 3 is examples of the recorded curve. In Fig. 3(a), since the exerted force increasing linearly, the ice beams were deformed elastically and broken. On the contrary

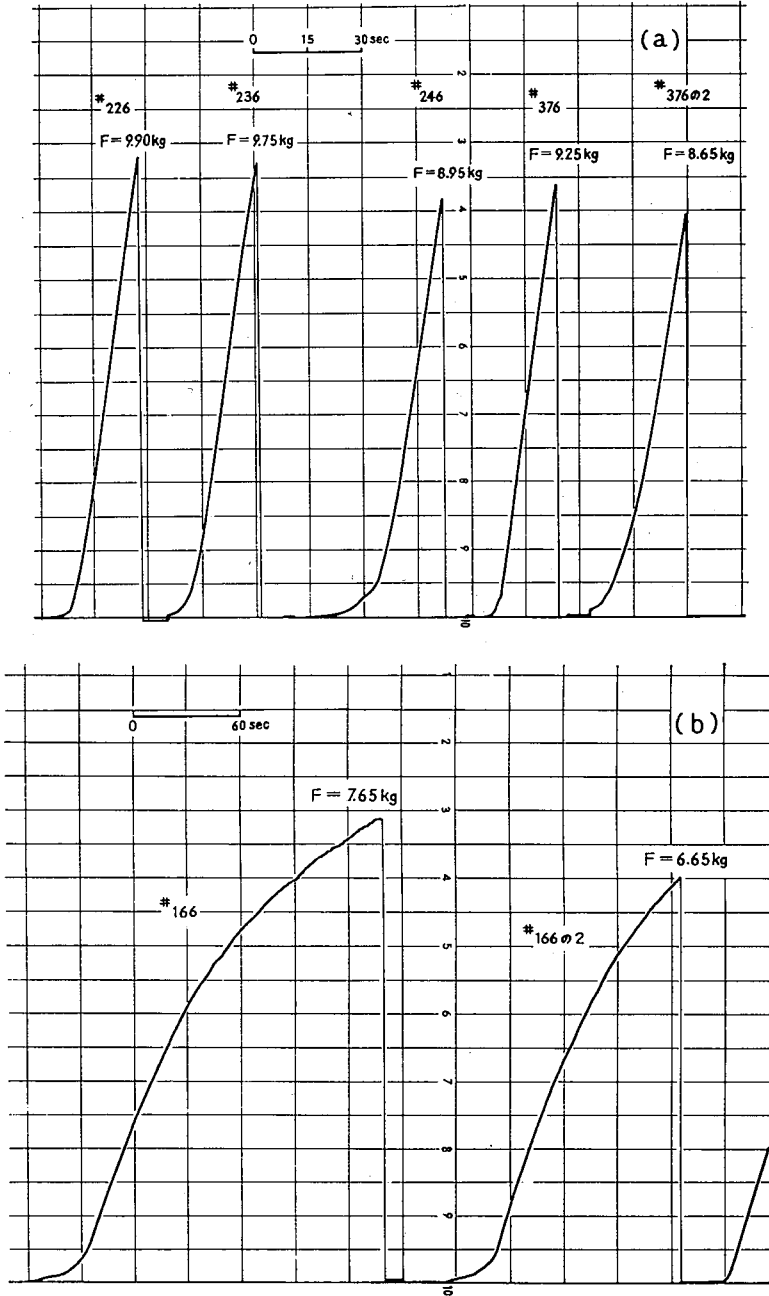


Fig. 3. Examples of the record of increasing in the force applied on the ice beam during the bending

in Fig. 3(b), the beams were deformed plastically and broken.

The failure of the beam always occurred between the upper two loading ends.

Strength, stress rate and temperature

The long period between the sawing out of ice blocks and the measurement made the density and salinity of ice at the time of the measurement relatively small because the liquid brine dripped from the sea ice. The density of tested ice was 0.77~0.87 and the salinity was 1.0~1.4‰. An extremely small density (0.77) ice was obtained from the lowermost, so-called, skelton layer.

An example of the obtained relation between the flexural strength σ_c and the increasing rate of the maximum tensile stress $\dot{\sigma}$ in the lower surface of the beam at -9°C is shown in Fig. 4(a). Each point is the average of 6 to 8 measurements. The density of the ice is listed in the figure.

From the figure, it is obvious that if the stress rate $\dot{\sigma}$ is small the strength decreases with the increasing of the stress rate, and for larger stress rates the strength increases with the stress rate.

Figure 4(b) shows the relation between stress rate $\dot{\sigma}$ and the ratios which indicate how many beams were broken brittly in the test at a given stress rate. Here 0% means all beams broke plastically. In the figure, elastic and plastic regions in which all beams break brittly or plastically respectively are well identified. When $\dot{\sigma}$ is larger than $0.6\text{ kg/cm}^2\text{-sec}$ all beams are bent elastically and broke brittly. Therefore, this region may be called the *elastic region*. On the contrary, when $\dot{\sigma}$ is less than $0.2\text{ kg/cm}^2\text{-sec}$

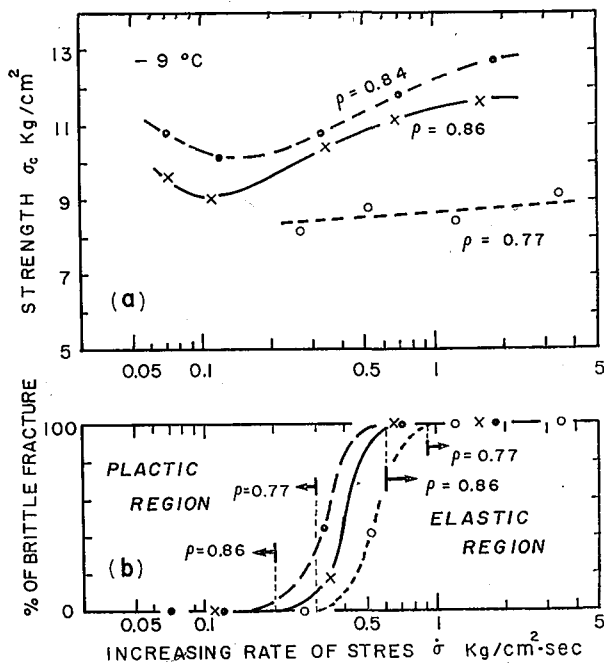


Fig. 4. (a) Relation between the increasing rate of stress $\dot{\sigma}$ and the strength σ_c at -9°C .
 (b) The ratio showing the number of beams fractured brittly in the test at a given stress rate at -9°C

all beams, except these with very low density ice, showed a plastic deformation and broke. This may be called the *plastic region*. When the value of $\dot{\sigma}$ lay between these two critical values, the circumstance became complicated, that is, even among several measurements to obtain one point in the figure some beams broke brittly while the remaining beams broke plastically. This region may be called the *mixed region*.

The pattern of the grain distribution such as size and shape etc., in the beam is not always the same in all beams, and size, shape and distribution of brine pockets or air spaces within the ice beam also differ from each other. Therefore, the pattern of the stress concentration within the ice beam due to the bending may not be similar in all of the beams. It seems that this is the reason why both brittle and plastic fractures were observed in the mixed region even at a given stress rate.

As is obvious from the figure, the critical values of $\dot{\sigma}$ for elastic or plastic region differ with the density of ice, in other words, the critical values of $\dot{\sigma}$ for brittle or plastic fracture increase with the lowering of the density of ice. Such an analysis was also applied to the data obtained at -5.8 , -13.5 , -19.6 and -29.5°C and is summarized as Figs. 5 and 6.

Since the strength of sea ice is dependent on its density, Fig. 5 was written using only the data obtained from the ice having a density of 0.86 to 0.87. Each point is the average of 6 to 8 measurements. It is obvious that the strength is dependent on both temperature and stress rate. At a given stress rate, the strength increases with the lowering of the temperature and at a certain temperature, the strength, as is also seen in natural sea ice sheets (Tabata, 1966, 1967), increases with the increase of stress rate.

Temperature dependence of the strength which is obtained from Fig. 5, however,

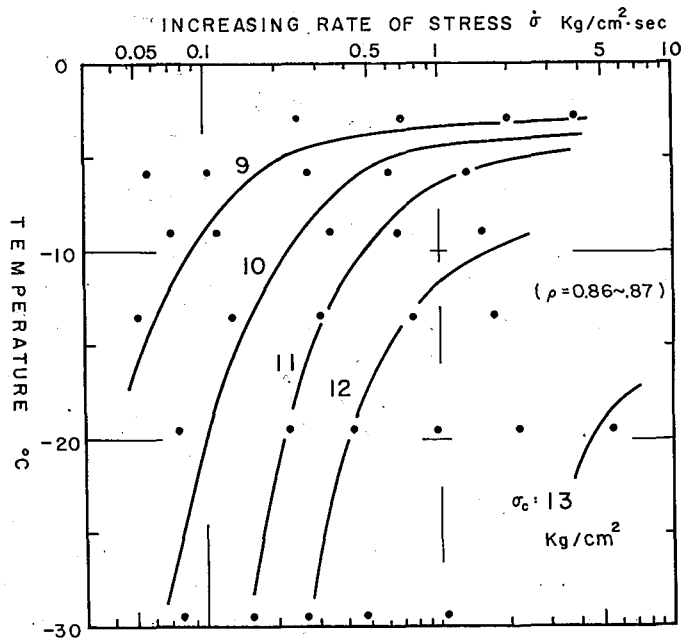


Fig. 5. The strength of sea ice σ_c in relation to the temperature and the increasing rate of stress $\dot{\sigma}$

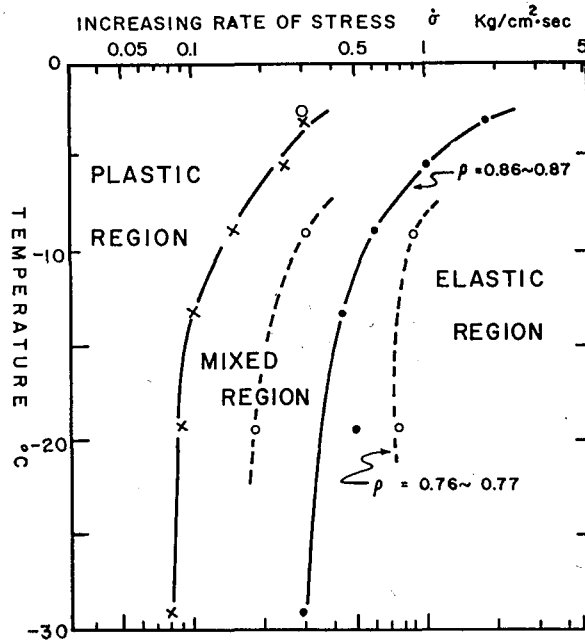


Fig. 6. The border lines among the elastic, the mixed and the plastic region in relation to the temperature and the increasing rate of stress $\dot{\sigma}$

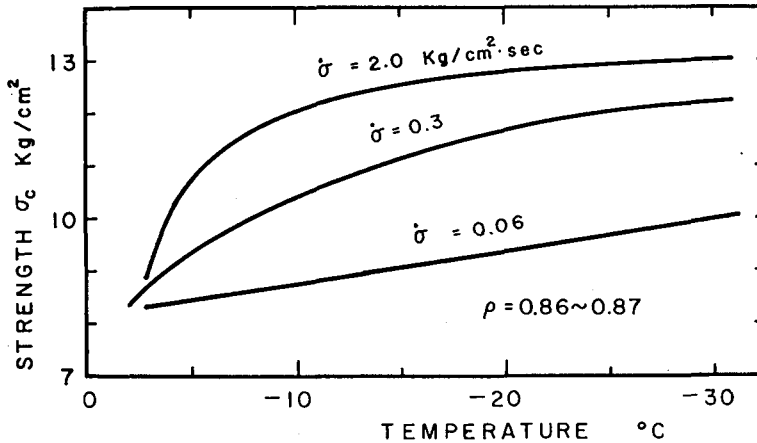


Fig. 7. Temperature dependence of the strength in various increasing rates of stress $\dot{\sigma}$

somewhat differs with the stress rate, and is shown as a parameter of the stress rate in Fig. 7. In the case of a very small stress rate, less than 0.1 kg/cm²·sec, the increase of strength with the lowering of the temperature is very gentle. If the rate is larger than approximately 1 kg/cm²·sec, the strength increases rapidly with the lowering of the temperature in a relatively high temperature range and then increases gradually with the lowering of the temperature. If the stress rate lies between the above two critical values,

approximately 0.1 to 1 kg/cm²·sec, the strength increases moderately with the lowering of the temperature.

The sudden increasing of the strength of sea ice was expected at -8.2°C and at -22.9°C at which the solid salts precipitate from the brine. This was emphasized by Assur (1958) but is not recognized in Figs. 5 and 7. This may be caused by a low salinity of the ice tested.

Both the temperature and the stress rate dependence of the elastic and the plastic regions are summarized as Fig. 6. The density of ice used is 0.86 to 0.87. The solid lines indicate the border lines between the three regions, plastic, mixed and elastic. To draw these solid lines, data obtained at -3°C (Tabata, 1960) was also used. It is clear that with the lowering of the temperature, the least stress rate which is sufficient to lead to an elastic deformation decreases rapidly. The range of the stress rate of mixed region becomes remarkably narrow with the lowering of the temperature, in other words, it can be said that sea ice can be treated as an elastic substance when the temperature is very low.

The dotted lines in the figure are the same characteristic lines in ice samples with a density of 0.76 to 0.77. It is also clear that the values of the critical stress rate shows a tendency to increase with the decrease in density. The sea ice of density 0.89 was also tested at -2°C . The critical value of the stress rate between the elastic and mixed region was 0.5 kg/cm²·sec. This result also supports the above mentioned tendency.

Strength of fresh water ice

To compare the strength of sea ice and that of fresh water ice, the strength of pure ice was also measured. Test specimens were made from commercial ice. The size of the specimen and the method of measurement were the same as that used in sea ice. The ice used was very clear and contained no visible flaws or air bubbles, and the density measured as 0.917. The ice consisted of columnar grains and *c*-axis was approximately parallel to the axis of the column. The specimen was sawed at its wide plane perpendicular to the axis of the column. Therefore, the orientation of the tensile stress was

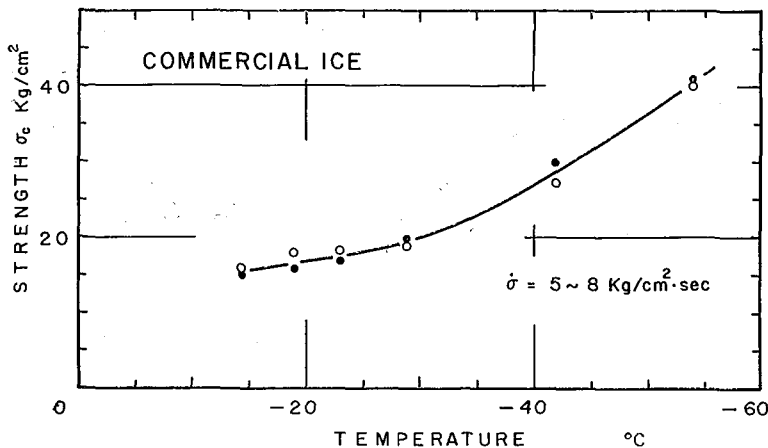


Fig. 8. Temperature dependence of pure ice beam in a stress rate $5 < \dot{\sigma} < 8 \text{ kg/cm}^2\cdot\text{sec}$

perpendicular to the direction of grain elongation and was approximately perpendicular to the optical c -axis. There were two grain sizes. The large one is about 1 cm^2 in area and the small one is about 0.4 cm^2 .

The strength measurement was carried out between -14 and -54°C and the results is shown in Fig. 8. The stress rate ranged from 5 to $8\text{ kg/cm}^2\text{-sec}$. The pure ice beams which have the same size as sea ice specimens but in which the orientation of grain elongation is parallel to its wide plane and directed towards its narrow width were also tested. The obtained results are also listed in the figure. The strengths were 15 to 40 kg/cm^2 in a temperature range of -14 to -54°C . A remarkable temperature dependence of the strength is obvious in the figure. A difference of the strength due to the grain size and the grain orientation was not seen. The tensile strength of pure ice has been measured by many authors, while some report higher values others report lower values, however, such a remarkable temperature dependence has not been reported.

Notch effect in the strength

Pure ice shows a strong temperature dependence in a high stress rate range ($5\sim 8\text{ kg/cm}^2\text{-sec}$). In the case of a large stress rate, the strength of sea ice in a temperature range less than -10°C is insensitive to the temperature change. As may be seen in Figs. 7 and 8, this temperature dependence is one of the most remarkable differences between sea ice and pure ice.

Weeks (1962) also measured the strength of pure ice. Concerning the strength of sea ice, Weeks explained that the strength of the ice-to-ice bonds between the brine pockets can be considered as temperature independent. The result in the present experiment shows, however, that pure ice strength itself has a strong temperature dependence in the strength. On the other hand, the strength of sea ice does not show a temperature dependence.

On the strength of sea ice it has been considered that, because of the presence of the brine pockets and air inclusions, the effective cross sectional area is reduced and these defects act as stress concentrators. The stress concentration factor due to a row of equally spaced circles or ellipses is estimated as three or somewhat above three. An initial failure will take place at the edge of the hole and then the failure surface will spread outward. The strength of sea ice, therefore, is reduced to one third of the strength with no hole ice. With the changing of the ice temperature, since the size of brine pocket changes, the area of minimum section also changes and, especially at a low temperature, solid salts deposit on the brine-ice interface. Owing to these effects, as was stated by Anderson and Weeks (1958) and Assur (1958), the strength of sea ice may have a complicated temperature dependence.

However the above mentioned considerations may be applied only to the strength of sea ice in natural conditions. In small sample tests, such as the present paper is concerned with and even in other methods, the sample must be cut or sliced. Now within sea ice, there are many brine pockets and air inclusions. Their dimensions and shape are miscellaneous and it is impossible to avoid the appearance of some openings of these holes along the edges of the specimen. It can be said that there are invariably some notches on the surfaces of small sea ice specimens. The dimension and shape of

these notches, depth, width and curvature at its top, is also miscellaneous. If a notched plate is stressed, the notches act as stress concentrators. Though the stress concentration factor of the notch depends upon its dimension and shape, it is not uncommon to reach a value larger than three or more (Neuber, 1937). Therefore, an initial failure will take place at the notch. Then, it is considered that the strength of small sea ice specimen may be affected by the notch effect.

Previous measurements of the tensile strength of sea ice and even of pure ice using small specimens show a large scattering in obtained values. One of the causes of a scattering in the strength value is the notch effect and it may well be a principal one.

The pure ice beam used in the present paper was prepared very carefully and its surfaces were glassy smooth. An increasing of hardness of ice with the lowering of the temperature is always noticed in the low temperature room. Therefore, the observed temperature dependence of ice is quite sustainable. On sea ice, contrary to Weeks's results, the strength of the ice-to-ice bonds between the brine pockets itself may also be temperature dependent. Small ice specimens are usually manufactured under a relatively high temperature condition and then lowered to the test temperature. Notches in sea ice may have little liquid brine especially in such a high temperature. Therefore, the notch effect in the strength may not be weakened with the lowering of the ice temperature. In the present experiment, salinity of the sea ice beam was small and, of course, observable notches were presented. The nonsensitive temperature dependence of sea ice shown in Fig. 6 may be attributed to the notch effect.

Vibration Young's modulus

The resonance frequency of the lateral vibration of the beam was measured and Young's modulus was obtained by using eq. (2). The measurements were carried out at -6 , -9 , -13 , -20 and -30°C . The obtained Young's modulus at -9°C was plotted against the density in Fig. 9. It is clear that Young's modulus decreases linearly

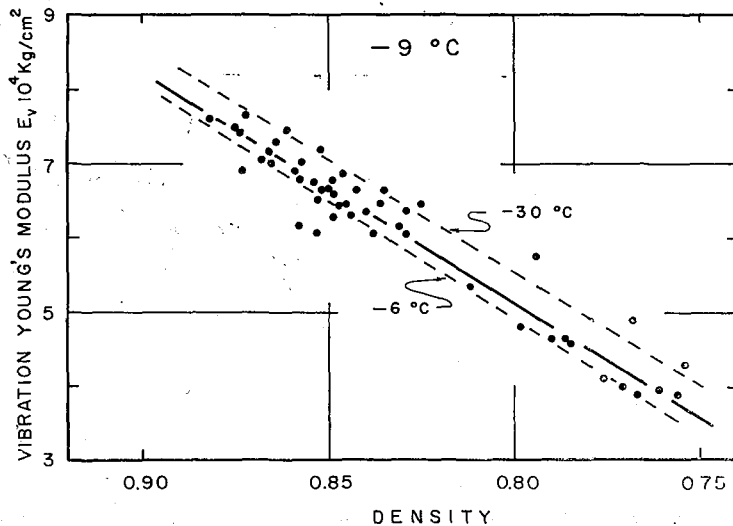


Fig. 9. Density dependence of Young's modulus obtained from lateral vibration method E_v in the temperature range -6 to -30°C .

with the decreasing in density. The relation between Young's modulus and density is represented by a solid line in the figure. This linear relation is also seen at -6°C and even at -30°C . Dotted lines in the figure are the relation at -6 and -30°C and they are represented by the following equations.

$$E_v(10^4 \text{ kg/cm}^2) = 8.7 - 30(0.98 - \rho) = 30\rho - 18.9 \text{ at } -6^{\circ}\text{C},$$

$$E_v(10^4 \text{ kg/cm}^2) = 9.2 - 30(0.92 - \rho) = 30\rho - 18.4 \text{ at } -30^{\circ}\text{C} (0.75 < \rho < 0.9).$$

As may be seen from these equations, Young's modulus increases only 0.5 kg/cm^2 with the temperature change from -6 to -30°C . The temperature dependence of vibration Young's modulus E_v is very small. On the contrary, the density dependence of E_v is very sensitive.

The linear relation between E_v and density ρ shown in the figure and the value of E_v are quite similar to those reported before by the author (Tabata, 1958). The value of E_v is also very close to that obtained by Brown (1963) with ultrasonic methods.

Bending Young's modulus

The relation between the obtained bending Young's modulus E_b and the increasing rate of applied stress $\dot{\sigma}$ at -9°C as a parameter of density is seen in Fig. 10. In the figure, it is obvious that E_b increases with $\dot{\sigma}$ and also that E_b decreases with the density. In Fig. 11, the relation between E_b and $\dot{\sigma}$ of sea ice having a density of 0.86 to 0.87 as a parameter of ice temperature is shown. Here we also notice that E_b increases with $\dot{\sigma}$ and also increases with the decrease in temperature. Figure 12 represents the temperature dependence of E_b in a given stress rate range $3 < \dot{\sigma} < 6 \text{ kg/cm}^2\text{-sec}$. The data obtained in a stress rate range $1 < \dot{\sigma} < 2.2 \text{ kg/cm}^2\text{-sec}$ is also shown in the figure and is identified in the legend. The temperature dependence of E_b is obvious. At -23°C , where solid $\text{NaCl}\cdot 2\text{H}_2\text{O}$ begins to precipitate, it seems that there is a sudden change of the temperature dependence, however, data is not sufficient to make clear this fact.

The relation between bending Young's modulus E_b and vibration Young's modulus E_v at -9°C is shown in Fig. 13. E_b is recognized to be as large as one third of E_v .

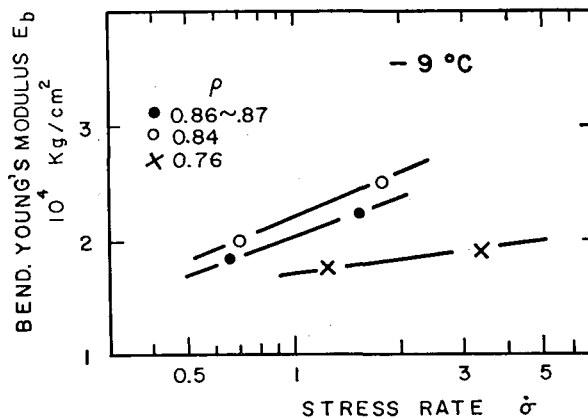


Fig. 10. The relation between the Young's modulus obtained from bending E_b and the increasing rate of stress at -9°C as a parameter of density ρ .

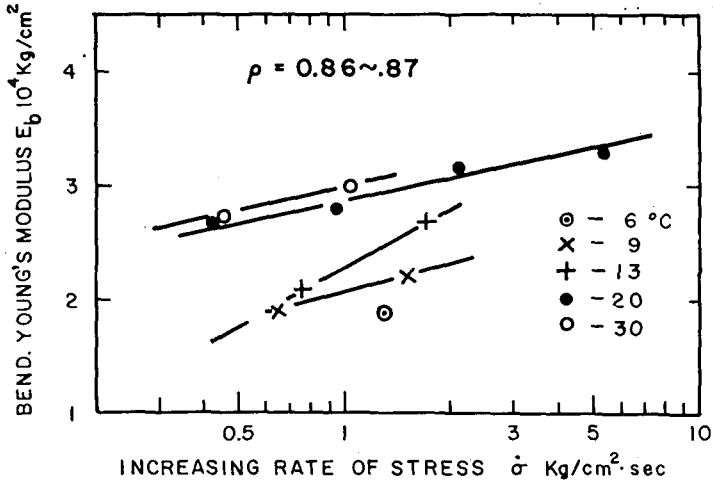


Fig. 11. The relation between the Young's modulus obtained from bending E_b and the increasing rate of stress at various temperatures

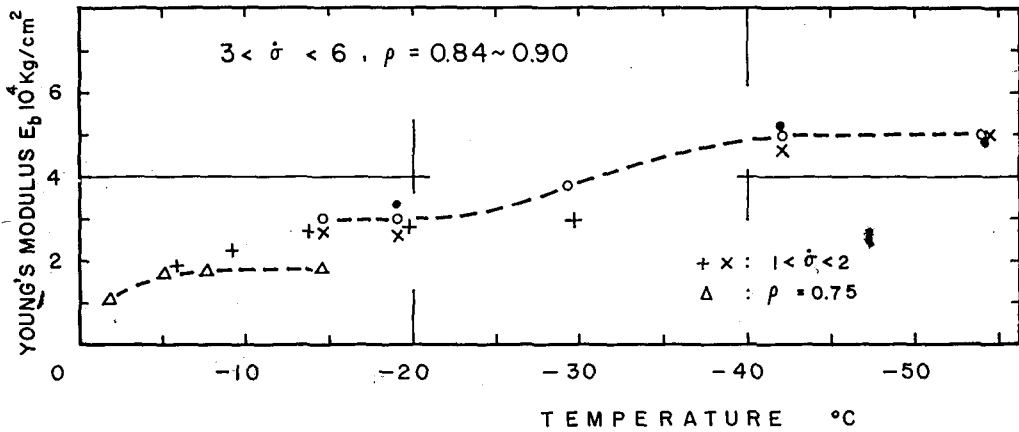


Fig. 12. Temperature dependence of Young's modulus in the stress rate 3 to 6 kg/cm²·sec

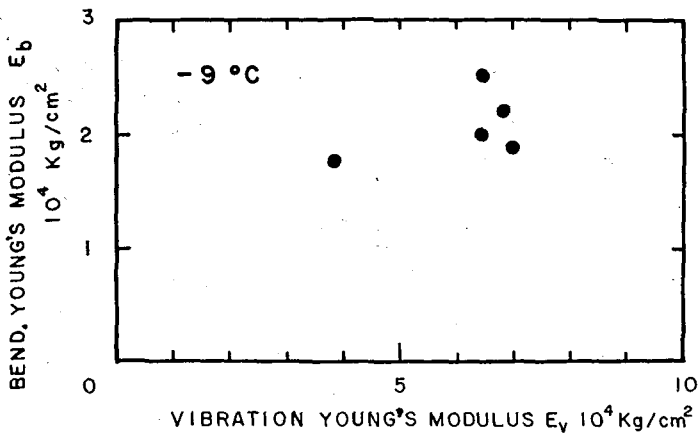


Fig. 13. Young's modulus obtained from bending E_b versus Young's modulus obtained from lateral vibration E_v

As was stated before, E_v is nonsensitive to the lowering of the temperature and, on the contrary, E_b is sensitive. Actually, at -30°C , the observed value of E_b has reached to a half magnitude of that of E_v .

Both the flexural strength σ_c and the bending Young's modulus E_b increases with the stress rate $\dot{\sigma}$. Therefore, it is expected that the larger strength might correspond to a larger Young's modulus. Figure 14 is the relation between the strength σ_c and the Young's modulus E_b and it confirms the above relation. This remarkable relation is also found in the strength of in-place strength measurement (Tabata, 1967).

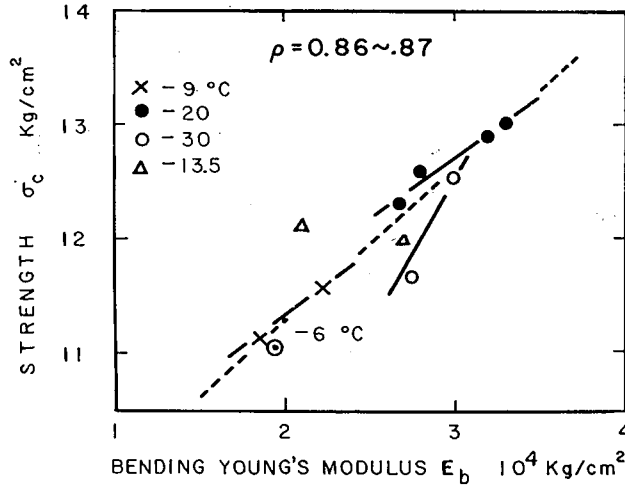


Fig. 14. Relation between the strength σ_c and the bending Young's modulus E_b at various temperatures

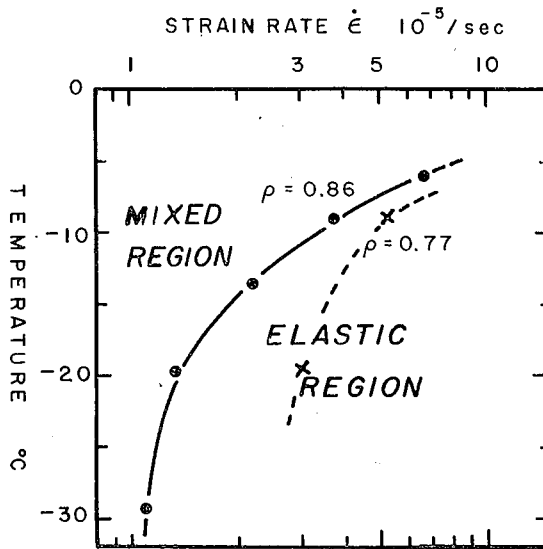


Fig. 15. The relation between the strain-rate $\dot{\epsilon}$ and the temperature in relation to the elastic region of fracture

Strain rate and strain

As was shown in Fig. 5, the behaviour of the bending varies from elastic to plastic with the stress rate. In the case of elastic bending, the bending Young's modulus was obtained and the relation between the stress rate and Young's modulus is shown in Fig. 11. The ratio of stress rate to Young's modulus is the strain rate $\dot{\epsilon}$. From Figs. 6 and 11, it is easy to obtain the minimum strain rate which leads to the elastic bending. Thus the abscissa of Fig. 6 can be rewritten with strain rate $\dot{\epsilon}$ and is shown in Fig. 15. It is obvious that the strain rate which leads to the elastic bending becomes remarkably small with the lowering of the temperature.

In Fig. 14, data obtained at temperatures -6 , -9 , -13.5 , -20 and -30°C are seen. It is surprising that the relation between the strength σ_c and the Young's modulus E_b seems to be temperature nonsensitive and may be represented by the dotted line in the figure. The ratio of strength to Young's modulus indicates the strain ϵ at the moment of failure. Therefore, it is concluded from the figure that the strain of the tested sea ice beam at the moment of failure is constant in the temperature range from -9 to -30°C and is obtained approximately as 0.4×10^{-3} .

Strength and density

Anderson and Weeks (1958) pointed out that the strength of sea ice might be a function of the actual width of ice at the failure plane. Assur (1958) extended this work and showed that the strength of sea ice at high temperatures above -8.2°C depends upon the square root of the brine volume. Since then, many authors showed that the strength of sea ice decreases with the increasing of the brine volume and air inclusion.

Figure 16 represents the relation between the flexural strength and density obtained at -9°C and in the stress rate $1.2 \sim 1.8 \text{ kg/cm}^2 \cdot \text{sec}$.

Figure 17 was obtained by rewriting the abscissa in Fig. 16 with the summation of the relative volume of air inclusion and brine ν . The salinity of sea ice tested was very small, therefore, the value ν is mainly dependent on the relative volume of air inclusion, in other words, the density. In these figures, the tendency of the decrease in strength with the decrease of density or with the increase in the relative volume of air

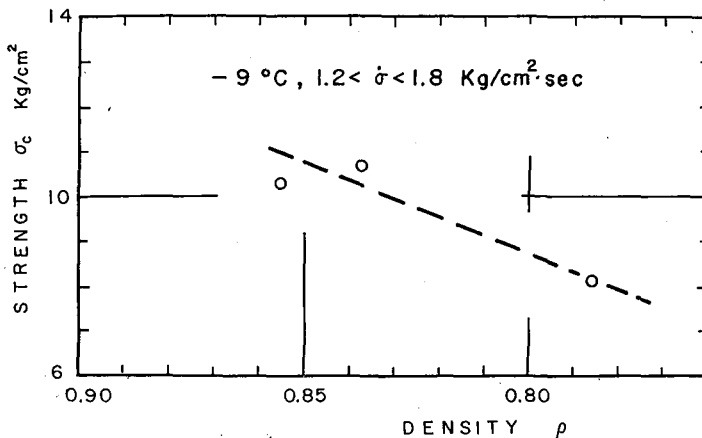


Fig. 16. Relation between the density and the strength

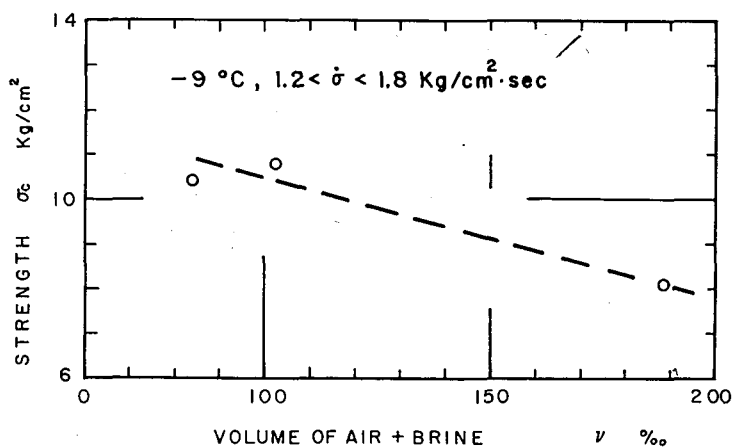


Fig. 17. Relation between the relative volume of air inclusion plus brine and the strength

inclusion and brine are clearly recognized. Because of a scattering in the values and an insufficient number of data, no quantitative relations were obtained.

Some measurements on the strength have been made on relatively high density sea ice beams such as 0.88~0.90 in a temperature range of -5 to -55°C . The results of the study will be written later.

References

- 1) ANDERSON, D. L. and WEEKS, W. F. 1958 A theoretical analysis of sea ice strength. *Trans. Amer. Geophys. Union*, **36**, 632-640.
- 2) ASSUR, A. 1958 Composition of sea ice and its strength. *In Arctic Sea Ice, Nat. Acad. Sci.—Nat. Res. Council U.S.A., Publ. 598*, 106-138.
- 3) BROWN, J. H. 1963 Elasticity and strength of sea ice. *In Ice and Snow* (W. D. KINGERY, ed.), M.I.T. Press, Cambridge, Mass., 79-106.
- 4) NEUBER, H. 1937 *Kerbspannungslehre*. Springer Pub. Co., Berlin.
- 5) PEYTON, H. R. 1963 Some mechanical properties of sea ice. *In Ice and Snow* (W. D. KINGERY, ed.), M.I.T. Press, Cambridge, Mass., 107-113.
- 6) POUNDER, E. R. and LANGLEBEN, M. P. 1964 Arctic sea ice of various ages. II. Elastic properties. *J. Glaciol.*, **5**, 99-105.
- 7) TABATA, T. 1958 Studies on mechanical properties of sea ice. II. Measurement of elastic modulus by the lateral vibration method. *Low Temp. Sci.*, **A 17**, 147-166.*
- 8) TABATA, T. 1960 Studies of the mechanical properties of sea ice. V. Measurement of flexural strength. *Low Temp. Sci.*, **A 19**, 187-201.*
- 9) TABATA, T. and FUJINO, K. 1964 Studies of the mechanical properties of sea ice. VII. Measurement of flexural strength *in situ* (1). *Low Temp. Sci.*, **A 22**, 147-154.*
- 10) TABATA, T. and FUJINO, K. 1965 Studies of the mechanical properties of sea ice. VIII. Measurement of flexural strength *in situ* (2). *Low Temp. Sci.*, **A 23**, 157-166.*
- 11) TABATA, T. 1966 Studies of the mechanical properties of sea ice. IX. Measurement of flexural strength *in situ* (3). *Low Temp. Sci.*, **A 24**, 259-268.*
- 12) TABATA, T. 1967 Studies of the mechanical properties of sea ice. XI. The flexural strength

of sea ice. *In* Physics of Snow and Ice, Part 1 (H. ÔURA, *ed.*), Inst. Low Temp. Sci., Sapporo, 539-550.

- 13) WEEKS, W. F. 1962 Tensile strength of NaCl ice. *J. Glaciol.*, **4**, 25-52.
- 14) YAMAJI, K. and KUROIWA, D. 1954 Study of elastic and viscous properties of snow by the vibration method. I. *Low Temp. Sci.*, **A 13**, 49-57.*

* In Japanese with English summary.