Title	Specific Heat and Heat of Fusion of Sea Ice
Author(s)	ONO, Nobuo; 小野, 延雄
Description	International Conference on Low Temperature Science. I. Conference on Physics of Snow and Ice, II. Conference on Cryobiology. (August, 14-19, 1966, Sapporo, Japan)
Citation	Physics of Snow and Ice : proceedings, 1(1), 599-610
Issue Date	1967
Doc URL	https://hdl.handle.net/2115/20328
Type	departmental bulletin paper
File Information	1_p599-610.pdf

Specific Heat and Heat of Fusion of Sea Ice*

Nobuo ONO
小 野 延 雄
The Institute of Low Temperature Science
Hokkaido University, Sapporo, Japan

Abstract

The specific heat and heat of fusion of sea ice in the temperature range between the freezing point and $-8°C$, computed from fairly accurate equations, were illustrated in figures and in tables.

These quantities are defined on the basis of thermal and phase equilibrium conditions between the brine and the surrounding pure ice. From this point of view, simple preliminary experiments were undertaken to determine whether the equilibrium is nearly always maintained or not. It was found from the experiments that a fairly large temperature difference between the brine and the surrounding pure ice appears quickly with a rapid change of ice temperature and then disappears gradually.

I. Introduction

The sea ice around Hokkaido has a relatively high temperature, that is to say it is of the so-called warm sea ice type, and is influenced by the diurnal change of air temperature. For example, young sea ice at Mombetsu harbour in February is $25 \sim 30$ cm in thickness, and the air temperature changes from about $-15°C$ in the early morning to nearly zero degrees in the afternoon, so that the temperature profile in the ice sheet changes over a wide range within a day.

It is usually considered that the brine in the sea ice has an equilibrium concentration in relation to its temperature. When the sea ice temperature rises from θ to θ', the brine in the sea ice is diluted to equilibrium concentration corresponding to θ' by the melting of some of the pure ice at the ice-brine interface. Therefore, in the case of warm sea ice, the brine volume and its concentration also change over wide ranges within a day.

The specific heat and heat of fusion of sea ice are generally defined according to the above equilibrium considerations, that is to say the heat required to raise the temperature of sea ice from θ to θ' is composed of the total sum of heat required to raise the temperature of pure ice and that of brine and to melt some of the pure ice.

In this paper, the specific heat and heat of fusion of sea ice were computed with reasonable accuracy in a temperature range between the freezing point and $-8°C$, and were illustrated in figures and in tables.

These quantities may be applied only in the thermal and phase equilibrium conditions. In the case of a sudden rise of the sea ice temperature, at first a thin melted

---

* Contribution No. 789 from the Institute of Low Temperature Science.

water layer will appear at the ice-brine interface. The melted water has a lower concentration and a higher equilibrium temperature than those of the brine, and the brine pocket is too small to make convection in it. The melted layer is considered to be concentrated by the diffusion of salt only. Therefore, it is assumed that the melting of ice at the ice-brine interface will occur gradually and it takes a fairly long time to reach the equilibrium state between the brine and the surrounding pure ice. Accordingly, in this case, a non-equilibrium state may be expected in the sea ice.

As far as we know, observations on the thermal behaviour of brine in the non-equilibrium state with the surrounding ice have not been made as yet. Preliminary experiments were undertaken to determine whether the equilibrium between the brine and the surrounding ice is nearly always maintained or not.

II. Specific Heat and Heat of Fusion of Sea Ice in $0>\theta>-8°C$

II. 1. *Specific heat of sea ice in* $0>\theta>-8°C$

The specific heat of sea ice of salinity S at temperature θ may be written as

$$c_{\theta S}=m_{i\theta}\,c_{i\theta}+m_{b\theta}\,c_{b\theta}+\lambda_\theta\frac{\mathrm{d}m_{b\theta}}{\mathrm{d}\theta}\,, \tag{1}$$

where $m_{i\theta}$ and $m_{b\theta}$ are the masses of pure ice and of brine in one gram of sea ice at θ respectively, $c_{i\theta}$ and $c_{b\theta}$ are respectively the specific heat of pure ice and of brine at θ, and λ_θ, the latent heat of pure ice at θ, is given by

$$\lambda_\theta=\lambda_0+(c_{w\theta}-c_{i\theta})\,\theta\,, \tag{2}$$

here λ_0 is the latent heat of pure ice at 0°C, and $c_{w\theta}$ is the specific heat of pure water at θ.

The salinity of brine at θ is written as

$$S_{b\theta}=\frac{S}{m_{b\theta}}\,,$$

so that, eq. (1) can be transformed as

$$c_{\theta S}=c_{i\theta}\left(1-\frac{S}{S_{b\theta}}\right)+c_{b\theta}\frac{S}{S_{b\theta}}-\lambda_\theta\frac{S}{S_{b\theta}^2}\frac{\mathrm{d}S_{b\theta}}{\mathrm{d}\theta}\,,$$

or

$$c_{\theta S}=c_{i\theta}+f(\theta)\cdot S\,. \tag{3}$$

The values of specific heat of sea ice were computed and were illustrated in tables by Malmgren (1927), Schwerdtfeger (1963), Nazintsev (1964) and Pounder (1965). The proportional factor $f(\theta)$ can be determined as the difference between $c_{\theta S}$ for $S=1‰$ (or $(n+1)‰$) and that of $0‰$ ($n‰$) at a constant temperature, and was found from their tables as inversely proportional to the square of temperature. Thus,

$$c_{\theta S}=c_{i\theta}+g(\theta)\frac{S}{\theta^2}\,. \tag{4}$$

The values of $g(\theta)$ obtained from their tables are summarized in Table 1. It is found from the table that $g(\theta)$ does not depend on the temperature, and mean values are nearly the same as 4.1 to 4.5. Then we have

Table 1. Values of $g(\theta)$ in $0>\theta>-8°\text{C}$

Temp. °C	Malmgren 1927	Schwerdtfeger 1963	Nazintsev 1964	Pounder 1965
−1	4.193		3.96	
−2	4.132	4.52	4.10	4.32
−3	4.059		4.19	
−4	3.984	4.64	4.22	4.32
−5	3.900		4.20	
−6	4.176	4.32	4.07	4.32
−7	4.263		4.02	
−8	4.096	4.48	4.03	4.48
Mean	4.100	4.49	4.10	4.36

$$c_{\theta S}=c_{i\theta}+K\frac{S}{\theta^2}\,. \tag{5}$$

Untersteiner (1961) first pointed out this fact and established a practical formula using Malmgren's value:

$$c_{\theta S}=0.5+4.1\frac{S}{\theta^2}\,.$$

In fact, eq. (5) can be derived from eq. (1) as follows (Ono 1966):

Consider that $m_{b\theta}$ grams of brine in eq. (1) are composed of $m_{w\theta}$ grams of pure water and m_s grams of salt. It was shown by Lyman and Fleming (1940) and Assur (1958) that, in the temperature range between zero and −8°C, where $Na_2SO_4 \cdot 10H_2O$ starts to precipitate, the freezing point of brine decreases linearly with the increasing mass ratio $z_\theta=m_s/m_{w\theta}$ (not the salinity of brine $S_{b\theta}=m_s/(m_s+m_{w\theta})$):

$$\theta=-\alpha z_\theta \quad (0>\theta>-8°\text{C})\,. \tag{6}$$

Then,

$$m_{w\theta}=-\alpha\frac{m_s}{\theta}\,,\qquad m_{b\theta}=m_s-\alpha\frac{m_s}{\theta}\,, \tag{7a}$$

$$m_{i\theta}=1-m_s+\alpha\frac{m_s}{\theta}\,, \tag{7b}$$

and

$$\frac{dm_{b\theta}}{d\theta}=\alpha\frac{m_s}{\theta^2}\,. \tag{7c}$$

The specific heat of pure ice, of pure water, and of brine in equilibrium with the ice can be closely approximated by the linear function of θ:

$$c_{i\theta}=c_{i0}+\beta\theta\,, \tag{8a}$$

$$c_{w\theta}=c_{w0}+\gamma\theta\,, \tag{8b}$$

$$c_{b\theta}=c_{w0}+\delta\theta\,. \tag{8c}$$

The notation c_{i0} and c_{w0} are used to present the specific heat of pure ice and of pure water at 0°C respectively, and the constants β, γ and δ were given by Nazintsev (1964) and Pounder (1965).

Table 2. Specific heat of sea ice (cal/g·°C)

Temp.	Salinity S (‰)											
−°C	0	1	2	3	4	5	6	7	8	9	10	11
0.1	0.51	431.7										
0.2	0.51	108.3	216.1	323.9								
0.3	0.50	48.41	96.3	144.2	192.1	240.0						
0.4	0.50	27.45	54.40	81.3	108.3	135.2	162.2	189.1				
0.5	0.50	17.75	34.99	52.24	69.49	86.7	104.0	121.2	138.5	155.7		
0.6	0.50	12.48	24.46	36.43	48.41	60.39	72.36	84.34	96.3	108.3	120.3	132.3
0.7	0.50	9.30	18.10	26.90	35.70	44.50	53.30	62.10	70.90	79.70	88.50	97.29
0.8	0.50	7.24	13.98	20.72	27.46	34.20	40.93	47.67	54.41	61.15	67.89	74.62
0.9	0.50	5.83	11.15	16.48	21.80	27.12	32.45	37.77	43.10	48.42	53.74	59.07
1.0	0.50	4.81	9.12	13.44	17.75	22.06	26.37	30.68	34.99	39.30	43.61	47.92
1.1	0.50	4.06	7.62	11.18	14.75	18.31	21.87	25.42	28.99	32.55	36.11	39.67
1.2	0.50	3.49	6.48	9.48	12.47	15.46	18.45	21.44	24.43	27.42	30.41	33.40
1.3	0.50	3.05	5.61	8.16	10.71	13.26	15.81	18.36	20.92	23.47	26.02	28.57
1.4	0.50	2.70	4.90	7.10	9.29	11.49	13.69	15.89	18.09	20.29	22.48	24.68
1.5	0.50	2.42	4.33	6.24	8.16	10.07	11.98	13.90	15.81	17.72	19.64	21.55
1.6	0.50	2.19	3.87	5.56	7.24	8.93	10.61	12.30	13.98	15.67	17.35	19.04
1.7	0.50	1.99	3.48	4.98	6.47	7.96	9.45	10.94	12.43	13.92	15.41	16.90
1.8	0.50	1.83	3.17	4.50	5.83	7.16	8.49	9.82	11.15	12.49	13.82	15.15
1.9	0.50	1.70	2.89	4.08	5.28	6.47	7.66	8.86	10.05	11.24	12.44	13.63
2.0	0.50	1.58	2.66	3.73	4.81	5.89	6.97	8.04	9.12	10.20	11.27	12.35
2.1	0.50	1.48	2.46	3.44	4.41	5.39	6.37	7.35	8.33	9.30	10.28	11.26
2.2	0.50	1.39	2.28	3.18	4.07	4.96	5.85	6.74	7.64	8.53	9.42	10.31
2.3	0.50	1.32	2.13	2.94	3.76	4.57	5.39	6.20	7.02	7.83	8.64	9.46
2.4	0.50	1.25	2.00	2.75	3.50	4.25	5.00	5.75	6.50	7.25	8.00	8.75
2.5	0.50	1.19	1.88	2.57	3.26	3.95	4.64	5.33	6.01	6.70	7.39	8.08
2.6	0.50	1.14	1.78	2.41	3.05	3.69	4.33	4.96	5.60	6.24	6.88	7.51
2.7	0.50	1.09	1.68	2.27	2.86	3.45	4.04	4.63	5.22	5.81	6.40	6.99
2.8	0.50	1.05	1.60	2.15	2.71	3.26	3.81	4.36	4.91	5.46	6.01	6.56
2.9	0.50	1.01	1.53	2.04	2.55	3.06	3.58	4.09	4.60	5.11	5.63	6.14
3.0	0.50	0.98	1.46	1.93	2.41	2.89	3.37	3.85	4.32	4.80	5.28	5.76
3.2	0.50	0.92	1.34	1.76	2.18	2.60	3.02	3.44	3.86	4.28	4.71	5.13
3.4	0.50	0.87	1.24	1.62	1.99	2.36	2.73	3.11	3.48	3.85	4.22	4.59
3.6	0.50	0.83	1.16	1.50	1.83	2.16	2.49	2.82	3.16	3.49	3.82	4.15
3.8	0.50	0.80	1.09	1.39	1.69	1.99	2.29	2.59	2.88	3.18	3.48	3.78
4.0	0.50	0.77	1.04	1.30	1.57	1.84	2.11	2.38	2.65	2.92	3.19	3.46
4.2	0.50	0.74	0.98	1.23	1.47	1.72	1.96	2.20	2.45	2.69	2.94	3.18
4.4	0.50	0.72	0.94	1.16	1.39	1.61	1.83	2.05	2.28	2.50	2.72	2.94
4.6	0.50	0.70	0.90	1.11	1.31	1.51	1.72	1.92	2.12	2.33	2.53	2.73
4.8	0.50	0.68	0.87	1.06	1.24	1.43	1.62	1.80	1.99	2.17	2.36	2.55
5.0	0.50	0.67	0.84	1.01	1.18	1.36	1.53	1.70	1.87	2.04	2.22	2.39
5.2	0.50	0.65	0.81	0.97	1.13	1.29	1.45	1.61	1.77	1.93	2.09	2.24
5.4	0.50	0.64	0.80	0.94	1.08	1.23	1.38	1.53	1.67	1.82	1.97	2.12
5.6	0.50	0.63	0.77	0.91	1.04	1.18	1.32	1.45	1.59	1.73	1.86	2.00
5.8	0.50	0.62	0.75	0.88	1.01	1.13	1.26	1.39	1.52	1.64	1.77	1.90
6.0	0.49	0.61	0.73	0.85	0.97	1.09	1.21	1.33	1.45	1.57	1.69	1.81
6.2	0.49	0.61	0.72	0.83	0.94	1.05	1.16	1.27	1.39	1.50	1.61	1.72
6.4	0.49	0.60	0.70	0.81	0.91	1.02	1.12	1.23	1.33	1.43	1.54	1.64
6.6	0.49	0.59	0.69	0.79	0.89	0.99	1.08	1.18	1.28	1.38	1.48	1.58
6.8	0.49	0.59	0.68	0.77	0.86	0.96	1.05	1.14	1.23	1.33	1.42	1.51
7.0	0.49	0.58	0.67	0.75	0.84	0.93	1.02	1.10	1.19	1.28	1.37	1.45
7.2	0.49	0.57	0.66	0.74	0.82	0.91	0.99	1.07	1.15	1.24	1.32	1.40
7.4	0.49	0.57	0.65	0.73	0.81	0.88	0.96	1.04	1.12	1.20	1.28	1.35
7.6	0.49	0.57	0.64	0.71	0.79	0.86	0.94	1.01	1.08	1.16	1.23	1.31
7.8	0.49	0.56	0.63	0.70	0.77	0.84	0.91	0.98	1.05	1.12	1.19	1.26
8.0	0.49	0.56	0.62	0.69	0.76	0.83	0.89	0.96	1.03	1.09	1.16	1.23

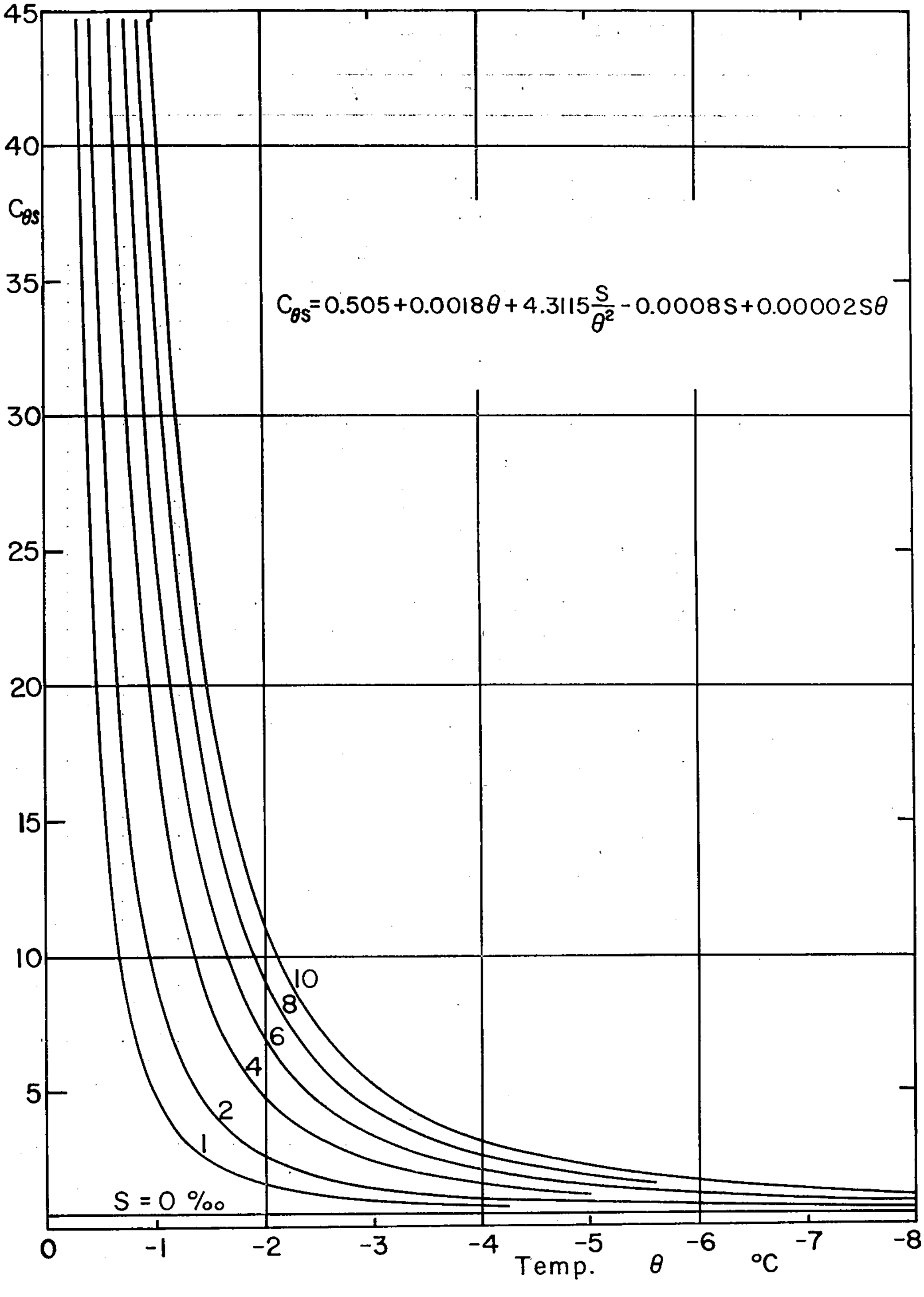

Fig. 1. Specific heat of sea ice (cal/g·°C)

Table 3. Heat of fusion of sea ice (cal/g)

Temp.	Salinity S (‰)												
−°C	0	1	2	3	4	5	6	7	8	9	10	11	12
0.1	79.7	36.6											
0.2	79.8	58.2	36.6	15.0									
0.3	79.8	65.4	51.0	36.6	22.2	7.8							
0.4	79.9	69.1	58.3	47.4	36.6	25.8	15.0	4.2					
0.5	79.9	71.3	62.6	54.0	45.3	36.7	28.0	19.3	10.7	2.0			
0.6	80.0	72.8	65.5	58.3	51.1	43.9	36.7	29.4	22.2	15.0	7.8	0.6	
0.7	80.0	73.8	67.6	61.4	55.3	49.1	42.9	36.7	30.5	24.3	18.1	11.9	5.7
0.8	80.1	74.7	69.2	63.8	58.4	53.0	47.5	42.1	36.7	31.3	25.8	20.4	15.0
0.9	80.1	75.3	70.5	65.7	60.8	56.0	51.2	46.4	41.5	36.7	31.9	27.1	22.2
1.0	80.2	75.9	71.5	67.2	62.8	58.5	54.2	49.8	45.5	41.1	36.8	32.5	28.1
1.1	80.2	76.3	72.3	68.4	64.5	60.5	56.6	52.6	48.7	44.7	40.8	36.8	32.9
1.2	80.3	76.7	73.1	69.4	65.8	62.2	58.6	50.0	51.3	47.7	44.1	40.5	36.9
1.3	80.3	77.0	73.7	70.3	67.0	63.6	60.3	56.9	53.6	50.2	46.9	43.5	40.2
1.4	80.4	77.3	74.2	71.1	68.0	64.9	61.7	58.6	55.5	52.4	49.3	46.2	43.1
1.5	80.4	77.5	74.6	71.7	68.8	65.9	63.0	60.1	57.2	54.3	51.4	48.5	45.6
1.6	80.5	77.8	75.0	72.3	69.6	66.9	64.1	61.4	58.7	56.0	53.2	50.5	47.8
1.7	80.5	78.0	75.4	72.8	70.3	67.7	65.1	62.6	60.0	57.4	54.9	52.3	49.7
1.8	80.6	78.2	75.7	73.3	70.9	68.4	66.0	63.6	61.2	58.7	56.3	53.9	51.4
1.9	80.6	78.3	76.0	73.7	71.4	69.1	66.8	64.5	62.2	59.9	57.6	55.3	53.0
2.0	80.7	78.5	76.3	74.1	71.9	69.7	67.5	65.4	63.2	61.0	58.8	56.6	54.4
2.1	80.7	78.7	76.6	74.5	72.4	70.3	68.2	66.1	64.0	62.0	59.9	57.8	55.7
2.2	80.8	78.8	76.8	74.8	72.8	70.8	68.8	66.8	64.8	62.8	60.8	58.8	56.8
2.3	80.8	78.9	77.0	75.1	73.2	71.3	69.4	67.5	65.5	63.6	61.7	59.8	57.9
2.4	80.9	79.1	77.2	75.4	73.6	71.8	69.9	68.1	66.3	64.4	62.6	60.8	59.0
2.5	80.9	79.2	77.4	75.7	73.9	72.2	70.4	68.7	66.9	65.1	63.4	61.6	59.9
2.6	81.0	79.3	77.6	75.9	74.2	72.5	70.8	69.2	67.5	65.8	64.1	62.4	60.7
2.7	81.0	79.4	77.8	76.2	74.5	72.9	71.3	69.7	68.0	66.4	64.8	63.1	61.5
2.8	81.1	79.5	77.9	76.4	74.8	73.2	71.7	70.1	68.5	67.0	65.4	63.8	62.2
2.9	81.1	79.6	78.1	76.6	75.1	73.5	72.0	70.5	69.0	67.5	65.9	64.4	62.9
3.0	81.2	79.7	78.3	76.8	75.3	73.9	72.4	70.9	69.4	68.0	66.5	65.0	63.6
3.2	81.3	79.9	78.5	77.1	75.8	74.4	73.0	71.6	70.2	68.8	67.5	66.1	64.7
3.4	81.4	80.1	78.8	77.5	76.2	74.9	73.6	72.3	71.0	69.7	68.4	67.1	65.8
3.6	81.5	80.3	79.0	77.8	76.6	75.3	74.1	72.8	71.6	70.4	69.1	67.9	66.7
3.8	81.6	80.4	79.3	78.1	76.9	75.7	74.6	73.4	72.2	71.0	69.9	68.7	67.5
4.0	81.7	80.6	79.5	78.3	77.2	76.1	75.0	73.9	72.8	71.6	70.5	69.4	68.3
4.2	81.8	80.7	79.6	78.6	77.5	76.4	75.4	74.3	73.2	72.2	71.1	70.0	69.0
4.4	81.9	80.9	79.8	78.8	77.8	76.8	75.8	74.7	73.7	72.7	71.7	70.6	69.6
4.6	82.0	81.0	80.0	79.0	78.1	77.1	76.1	75.1	74.1	73.2	72.2	71.2	70.2
4.8	82.1	81.1	80.2	79.3	78.3	77.4	76.4	75.5	74.5	73.6	72.7	71.7	70.8
5.0	82.2	81.3	80.4	79.5	78.6	77.7	76.7	75.8	74.9	74.0	73.1	72.2	71.3
5.2	82.3	81.4	80.5	79.7	78.8	77.9	77.0	76.2	75.3	74.4	73.5	72.7	71.8
5.4	82.4	81.5	80.7	79.8	79.0	78.2	77.3	76.5	75.6	74.8	73.9	73.1	72.2
5.6	82.5	81.7	80.8	80.0	79.2	78.4	77.5	76.7	75.9	75.1	74.3	73.4	72.6
5.8	82.6	81.8	81.0	80.2	79.4	78.6	77.8	77.0	76.2	75.4	74.6	73.9	73.1
6.0	82.7	81.9	81.1	80.4	79.6	78.8	78.0	77.3	76.5	75.7	75.0	74.2	73.4
6.2	82.8	82.0	81.3	80.5	79.8	79.0	78.3	77.5	76.8	76.0	75.3	74.5	73.8
6.4	82.9	82.1	81.4	80.7	80.0	79.2	78.5	77.8	77.0	76.3	75.6	74.9	74.1
6.6	83.0	82.3	81.5	80.8	80.1	79.4	78.7	78.0	77.3	76.6	75.8	75.1	74.4
6.8	83.1	82.4	81.7	81.0	80.3	79.6	78.9	78.2	77.5	76.8	76.1	75.5	74.8
7.0	83.2	82.5	81.8	81.2	80.5	79.8	79.1	78.4	77.8	77.1	76.4	75.7	75.1
7.2	83.3	82.6	82.0	81.3	80.6	80.0	79.3	78.6	78.0	77.3	76.7	76.0	75.3
7.4	83.4	82.7	82.1	81.4	80.8	80.2	79.5	78.9	78.2	77.6	76.9	76.3	75.6
7.6	83.5	82.8	82.2	81.6	80.9	80.3	79.7	79.0	78.4	77.8	77.1	76.5	75.9
7.8	83.6	82.9	82.3	81.7	81.1	80.5	79.9	79.2	78.6	78.0	77.4	76.8	76.1
8.0	83.7	83.1	82.5	81.8	81.2	80.6	80.0	79.4	78.8	78.2	77.6	77.0	76.4

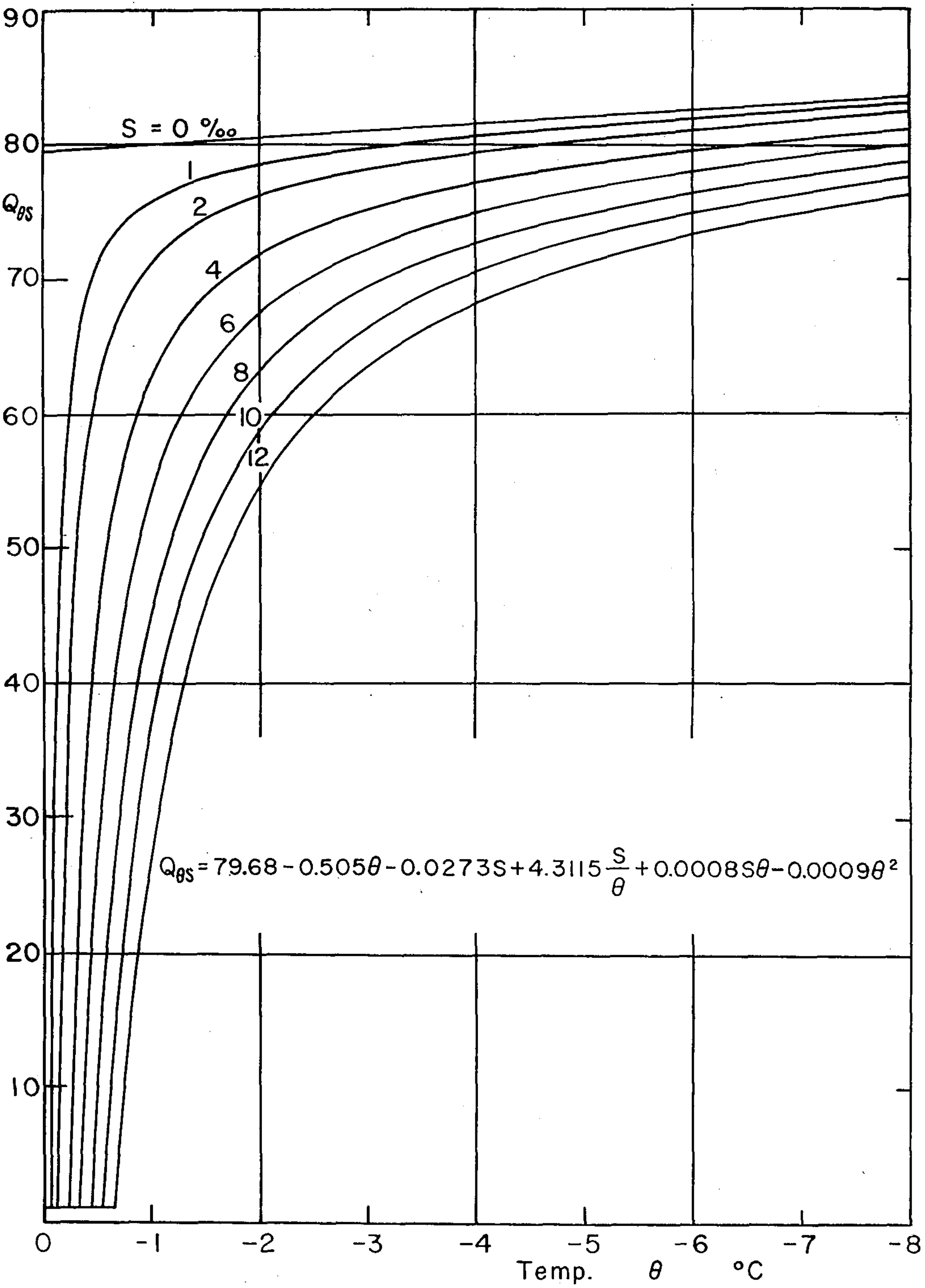

Fig. 2. Heat of fusion of sea ice (cal/g)

It is derived from eq. (1) using eqs. (2), (7a, b, c) and (8a, b, c) as

$$c_{\theta S}=c_{i0}+\beta\theta+\lambda_0\alpha\frac{m_s}{\theta^2}+(c_{w0}-c_{i0}+\alpha\gamma-\alpha\delta)\,m_s+(\delta-\beta)\,\theta m_s\,. \tag{9}$$

Using the following values: $c_{i0}=0.505$ cal/g°C, $c_{w0}=1.005$ cal/g°C, $\lambda_0=79.68$cal/g, $\alpha=54.11$ of Assur, $\beta=0.0018$, $\gamma=-0.002$, $\delta=0.022$ and $m_s=0.001\,S$ (the definition of the salinity of sea ice), eq. (9) becomes

$$c_{\theta S}=0.505+0.0018\theta+4.3115\frac{S}{\theta^2}-0.0008S+0.00002\theta S\,. \tag{10}$$

The values of $c_{\theta S}$ computed from eq. (10) were illustrated in Fig. 1 and Table 2.

In eq. (10), the fourth and the fifth terms in the right-side are negligibly small in comparison with the third term. Then we obtain

$$c_{\theta S}=0.505+0.0018\theta+4.3115\frac{S}{\theta^2}\,, \tag{11}$$

which is the same with eq. (5).

As is known in eq. (9), the constant K in eq. (5) is equal to $\lambda_0\alpha$, so that a slight scattering in the values of K in Table 1 may be caused by the use of different data on the equilibrium concentration of brine within ice at the temperature θ. If we use 52.41 of Lyman and Fleming's value for α, we obtain $K=4.176$.

II. 2. *Heat of fusion of sea ice in* $0>\theta>-8°C$

Integration of eq. (10) from θ to θ' yields the amount of heat q required to raise the temperature of one gram of sea ice from θ to θ':

$$q=0.505(\theta'-\theta)+4.3115\frac{S}{\theta\theta'}(\theta'-\theta)+0.0008S(\theta'-\theta)+(0.0009+0.00001S)(\theta'^2-\theta^2)\,. \tag{12}$$

In this equation, the third and the fourth terms are practically negligible in comparison with the first and the second terms in the temperature range of $0>\theta>-8°C$. Then we have

$$q=\left(0.505+4.3115\frac{S}{\theta\theta'}\right)(\theta'-\theta)\,, \tag{13}$$

which is nearly the same with the equation found in Untersteiner's paper.

As sea ice has no fixed temperature on its phase transition, the word "heat of fusion of sea ice of salinity S at temperature θ" is used to indicate the amount of heat required just to melt one gram of sea ice of S at θ.

By putting $m_{i\theta}=0$ in eq. (7b), the temperature of complete melting θ_m is obtained as

$$\theta_m=-\frac{\alpha m_s}{1-m_s}\simeq-\alpha m_s=-0.05411S\,, \tag{14}$$

because $m_s\ll1$.

The heat of fusion of sea ice $Q_{\theta S}$, that is, the integration of eq. (10) from θ to θ_m, is therefore obtained by substituting θ_m for θ' in eq. (12). Then, neglecting the higher-order terms of S, we have

$$Q_{\theta S}=79.68-0.505\theta-0.0273S+4.3115\frac{S}{\theta}+0.0008S\theta-0.0009\theta^2\,. \tag{15}$$

The values of $Q_{\theta S}$ calculated from eq. (15) are illustrated in Fig. 2 and Table 3.

With sufficient accuracy for practical purposes, neglecting the fifth and the sixth terms in the right-side of eq. (15), we obtain

$$Q_{\theta S}=79.68-0.505\theta-0.0273S+4.3115\,\frac{S}{\theta}\,. \tag{16}$$

III. Observation of Brine in Non-Equilibrium with Surrounding Ice

III. 1. *Experimental procedure*

Experiments were carried out in a low temperature laboratory at about −15°C.

To simplify the experiments, artificial brine pockets were made in the pure ice. Cylindrical ice specimens of about 7.5 cm in diameter and 7 cm in height were prepared from polycrystalline commercial pure ice. Four holes of 2 mm in diameter and 3 cm in depth were drilled into the specimen as shown in Fig. 3A. Copper-constantan thermocouples were inserted into each hole, then, pure water was put into the C and P holes. In order to make the artificial brine pockets in the ice specimen, concentrated sea water was poured into the B hole to about half of its depth. After the brine has frozen at a low temperature, pure water of about 0°C was put into the B hole and was then frozen, so that the brine was enclosed within the pure ice. The reference hole A was made occasionally as for brine or as for pure ice.

Both the upper and lower surfaces of the ice cylinder were insulated thermally, and then, the ice cylinder was put into a copper vessel, which was set in a thermo-bath as shown in Fig. 3B.

Figure 3C shows the block diagram of the temperature recording system. Driving a rotary switch connected with four thermocouples, the temperature of a thermocouple is recorded for 10 seconds at 30 seconds interval.

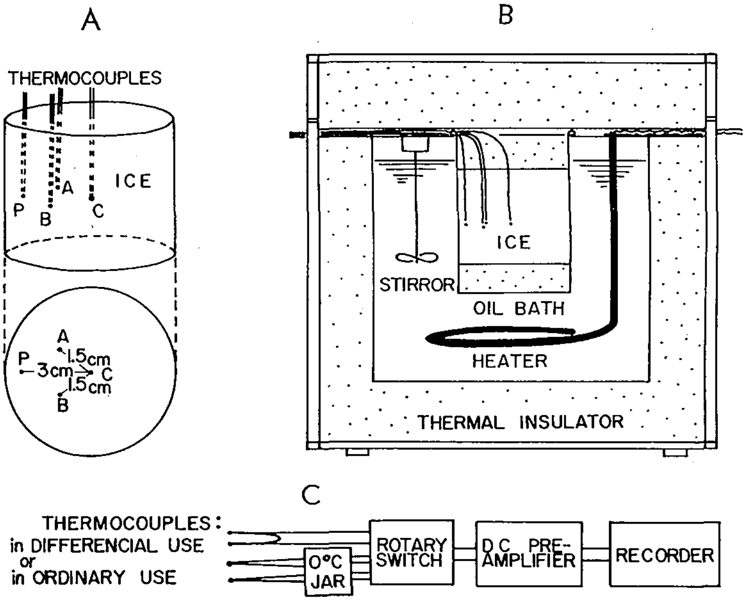

Fig. 3. Specimen and apparatus of experiment

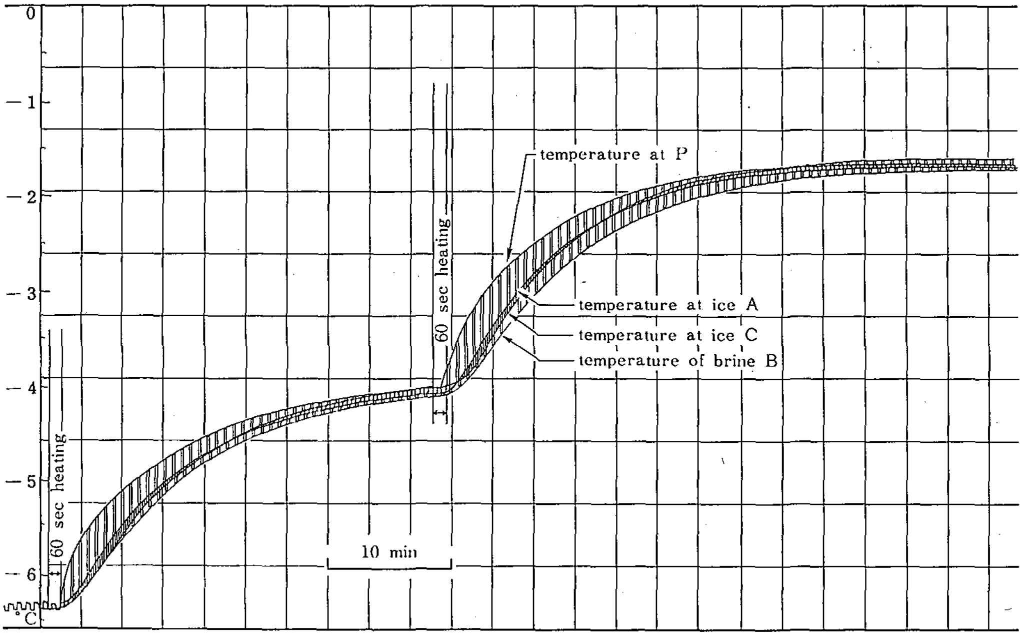

Fig. 4. A typical record of temperature-time curves

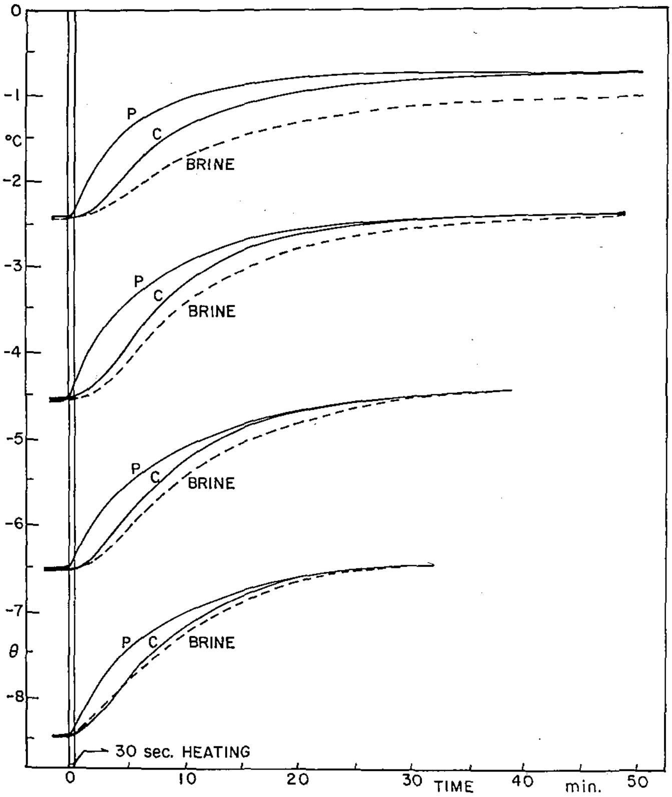

Fig. 5. Results of experiments

III. 2. *Results of experiments*

After the temperature of four thermocouples reached to nearly the same temperature and was considered to be in a stationary state, the oil bath was warmed by a short time heating, such as 30 to 60 seconds.

Figure 4 shows a typical record of temperature-time curves. This figure seems to be complicated due to the fact that it includes four temperature curves in a one-pen record. The temperature of brine rises with some time-lag to that of the center C, which is the farthest point from the heat source of the oil bath.

The results of measurements at various temperatures are summarized in Fig. 5. It is found from the figure that the time-lag increases with the raising of initial temperature, and in the case of near melting temperature, the temperature difference between the brine and the surrounding pure ice are observed over a fairly long time, such as two hours or more.

Figure 6 shows the record of the temperature differnce between the ice C and the brine B, or between the brine B and the ice A. Such difference appears slightly after the heating, and reaches a maximum such as 1°C or more within several minutes, and then decreases gradually with the lapse of time.

Based on the above preliminary experiments, although the artificial brine pockets were much larger than the natural brine cells in the sea ice, it may be concluded that

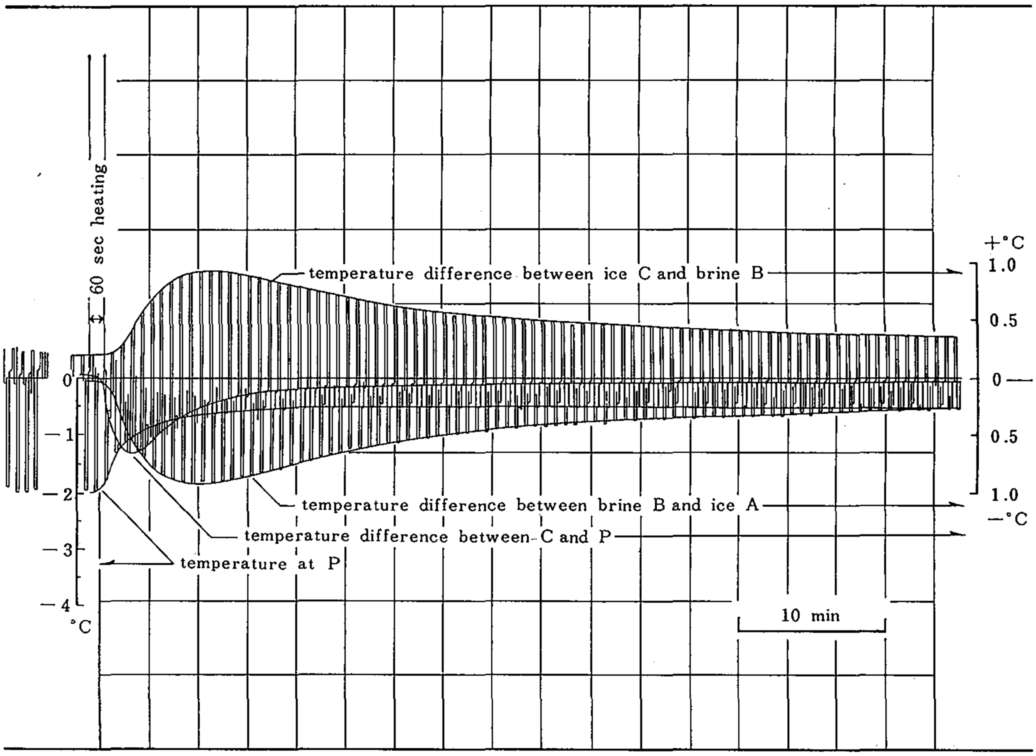

Fig. 6. A typical record of the temperature difference between ice and brine

the brine in the sea ice is not in an equilibrium state with the surrounding pure ice when the temperature of sea ice changes rapidly. This non-equilibrium state between the brine and the surrounding ice may be maintained over a fairly long time especially near the melting temperature.

Therefore, the values of specific heat of sea ice must be applied carefully for the thermal studies on the non-stationary process, such as the thermal diffusion in the warm sea ice.

Moreover, in the case of measurements of the specific heat of sea ice, it is necessary to have all of the ice and the brine reach the same temperature exactly. In other words, as it takes a fairly long time to reach the thermal equilibrium between the ice and the brine, the calorimeter for measuring specific heat of sea ice must be insulated completely.

References

1) ASSUR, A. 1958 Composition of sea ice and its tensile strength. *In* Arctic Sea Ice, *Nat. Acad. Sci.-Nat. Res. Council U.S.A., Publ.* **598**, 106-138.

2) LYMAN, J. and FLEMING, R. H. 1940 Composition of sea water. *J. Marine Res.*, **3**, 134-146.

3) MALMGREN, F. 1927 On the Properties of Sea Ice, The Norwegian North Polar Expedition with the "Maud" 1918-1925, Sci. Res., **1**, 5, 67 pp.

4) NAZINTSEV, Yu, L. 1964 Nekotorye dannye k raschetu teplovykh Svoistv morskogo l'da. (Some data on the calculation of thermal properties of sea ice.) *Trudy Arkt. i Antarkt. Inst.*, **267**, 31-47.

5) ONO, N. 1966 Thermal properties of sea ice. III. On the specific heat of sea ice. *Low Temp. Sci.*, **A 24**, 249-258. (In Japanese with English summary).

6) POUNDER, E. R. 1965 Physics of Ice, Pergamon Press. Oxford, 151 pp.

7) SCHWERDTFEGER, P. 1963 The thermal properties of sea ice. *J. Glaciol.*, **4**, 789-807.

8) UNTERSTINER, N. 1961 On the mass and heat budget of Arctic sea ice. *Arch. Meteorol. Geophys. Bioklim.*, **A 12**, 151-182.