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Electrical Properties of Sea Ice*

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Abstract

In order to investigate the relation between the geometrical structure of brine cells in sea ice and its electrical properties, measurements of the permittivity (ϵ') and the conductivity (σ) of sea ice were carried out in an audio frequency range (100 c/s~50 kc/s) and in various temperature ranges ($-5\sim-70^\circ\text{C}$).

Two anomalous dispersions in the permittivity due to a different mechanism were observed and extremely large values of permittivity (in the order of 10^6 at 100 c/s) were obtained in the first anomalous dispersion range. Grouping due to a phase change of brine in sea ice related to temperature was observed in the permittivity and the conductivity corresponding to the precipitation point of sodium chloride and the eutectic point. No remarkable difference was found between the parallel and the vertical application of the electrical field to the brine cells in sea ice. From the obtained results, the arrangements of brine cells are considered, from an electrical view point, to be not isolated like a cylinder or layer but connected like a three dimensional network. A high conductivity was observed in a high temperature range and this is in good agreement with that which was obtained by the present author by the electrical sounding method using the Wenner's four electrode configuration *in situ*.

I. Introduction

Recently, attempts to determine the thickness of sea ice or glacial ice using electrical sounding method which is widely adopted for geophysical prospecting and also measurements of radio propagation over snow-covered ground and so forth in the field of radio engineering in cold and polar regions, have made by some authors (Dichtel, 1951; Hatherton, 1960; Ozawa, 1958; Yoshino, 1961). From such practical usages, an urgent demand for more accurate information on the electrical properties of natural occurring ice has been made. The present author applied the electrical sounding method to determine the thickness of sea ice on the Okhotsk Sea coast in 1960-63 and obtained reasonable results (Fujino, 1960, 1963). The obtained results showed that sea ice-sea water system can be analysed in much the same manner as in ordinary layered structure in spite of the fact that this system contains a complicated structure of sea ice. The electrical layered structure in sea ice is expected from the apparent structure. In general, two different structures are usually observed in sea ice, the granular ice which is observed in the surface layer of the sea ice, and the mosaic ice which composes the main part of the sea ice. The granular ice is composed of almost spherical ice crystals

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Fig. 1 A. Crystal structure of granular layer in sea ice. Perpendicular section against freezing direction

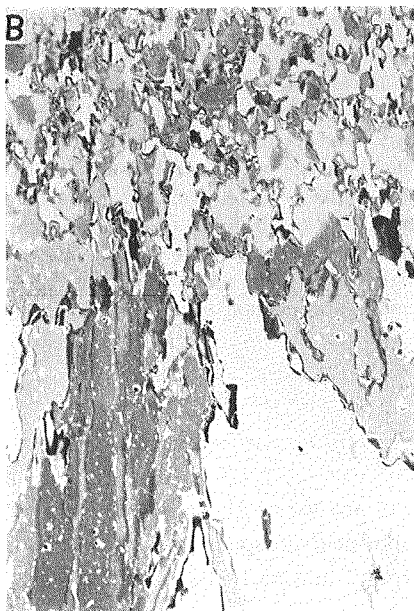
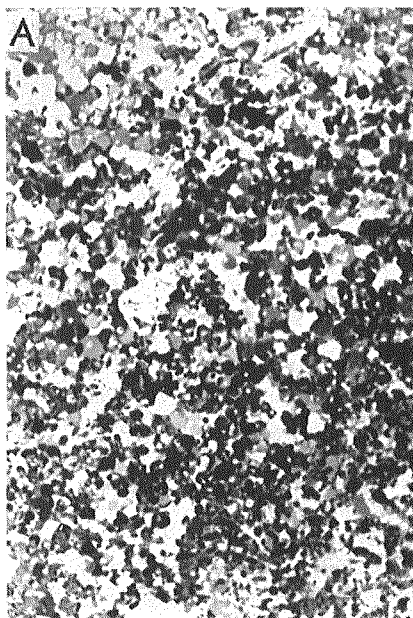
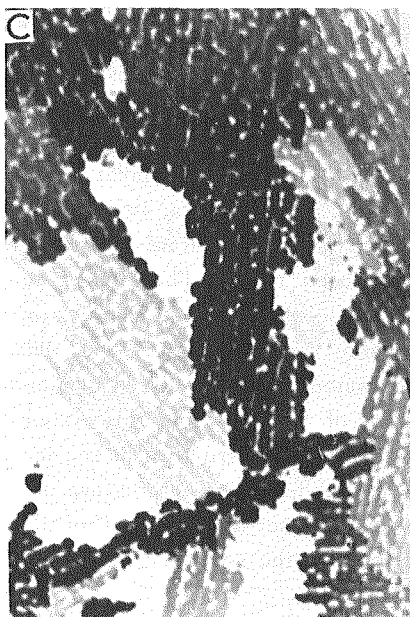


Fig. 1 B. Crystal structure of sea ice. Parallel section against freezing direction. Upper part is the granular and lower part is the mosaic

Fig. 1 C. Crystal structure of mosaic layer in sea ice. Perpendicular section against freezing direction



and brine cells, which are connected with each other in the form of a three dimensional network. The mosaic ice is composed of thin ice platelets and cylindrical shaped or film like layered brine cells, which are arranged in parallel along the ice platelets (Figs. 1 A, 1 B, 1 C). From these structures, it may easily be expected that these two layers may have different electrical properties and also may have remarkable dielectrical properties.

The dielectrical properties of pure ice have been investigated by many workers from a view point of molecular structure of ice, because ice has a large dielectric constant and is known as a typical polar dielectric material (Smyth, 1932; Murphy, 1934; Auty, 1952; Powles, 1952; Gränicher, 1957). While, in regard to the natural occurring ice, such as snow, sea ice, glacial ice and lake ice, few measurements were made by some authors (Kuroiwa, 1951; Cumming, 1952). According to their measurements, such natural occurring ice always contains many impurities and the effects of such impurities make their behaviours different and more complex from that of pure ice.

As mentioned above, sea ice is composed of ice crystals and brine of electrolyte and is considered to be one of the most dielectrically complex materials because of large value of salinity and intricated structures.

The present author made the measurements of the dielectric properties of sea ice in a cold room and investigated its relations to its geometrical structure together with its temperature and frequency dependence as a dielectric mixture of ice and brine. The main purpose of this paper is to clarify dielectric properties of sea ice in relation to above mentioned parameters.

II. The Experimental Procedure

The most generally used method for measuring a complex dielectric constant, characterising the dielectric properties of material, consists of the measurement of the capacitance C_0 of an empty condenser and the capacitance C and the resistance R of the condenser filled with the material in test. Then the quantities of the complex dielectric constant of the material are found to stand in the following relations with C_0 , C_p and R_p (in parallel connection): The permittivity ϵ' of the material is given by

$$\epsilon' = C_p/C_0.$$

The loss factor ϵ'' of the material is also given by

$$\epsilon'' = 1/\omega C_0 R_p,$$

where C_p is the equivalent parallel capacitance and R_p is the equivalent parallel resistance. Instead of the loss factor, in this paper, the conductivity which is given by the reciprocal of R_p is adopted for arranging the results.

In general, if the material in test is a solid, the condenser is made by the specimen, forming of a slab or disk, and two thin metal electrodes, attaching tightly on the two parallel surfaces of the specimen and C_0 is calculated geometrically. But such a method can not be applied for sea ice because the drainage of brine occurs easily during the measurement and the character of sea ice changes by the draining of brine. To avoid this trouble a cell-type condenser was adopted in this measurement. The cell was made of acrylite with an inner size of $18 \times 18 \times 3$ cm and, inside of the cell, a pair of chrome

plated copper conducting plates were set without guard rings. The specimen was cut out from a large block, reformed to a square plate as the same size as the cell. After putting the specimen into the cell, brine at freezing point was poured into the cell, so that the specimen was cemented with the electrodes completely. Then the cell was set in a refrigerator at -5°C . After finishing the measurements of the whole frequency range at a given temperature, temperature of the refrigerator was then lowered to the next measuring temperature. Before the measurement, considering the phase lag of temperature between the specimen and the refrigerator, the cell was kept at this temperature, which was regulated within $\pm 0.2^{\circ}\text{C}$, for more than 5 hours. The measuring temperature was changed step by step over the entire temperature range with the specimen.

The measurements of the equivalent parallel capacitance C_p and the equivalent parallel resistance R_p were made using an a-c bridge in the frequency range of 100 c/s to 50 kc/s and in temperature range of -5 to -70°C .

After finishing the measurements, the crystal structure of the specimen were examined by making thin sections and the salinity was measured by an electro-salinometer.

As mentioned above, since sea ice has a layered structure of granular ice and mosaic ice, measurements were made on the mosaic and the granular ice separately. In mosaic ice, specimens were obtained from natural sea ice collected from the Okhotsk Sea and artificial sea ice made from sea water in a cold room. Two kinds of applications of the electric field were made on mosaic ice, that is, the arrangement of brine cells is parallel to the applied field and that of brine cells is vertical to the field. In granular ice, since a suitable specimen

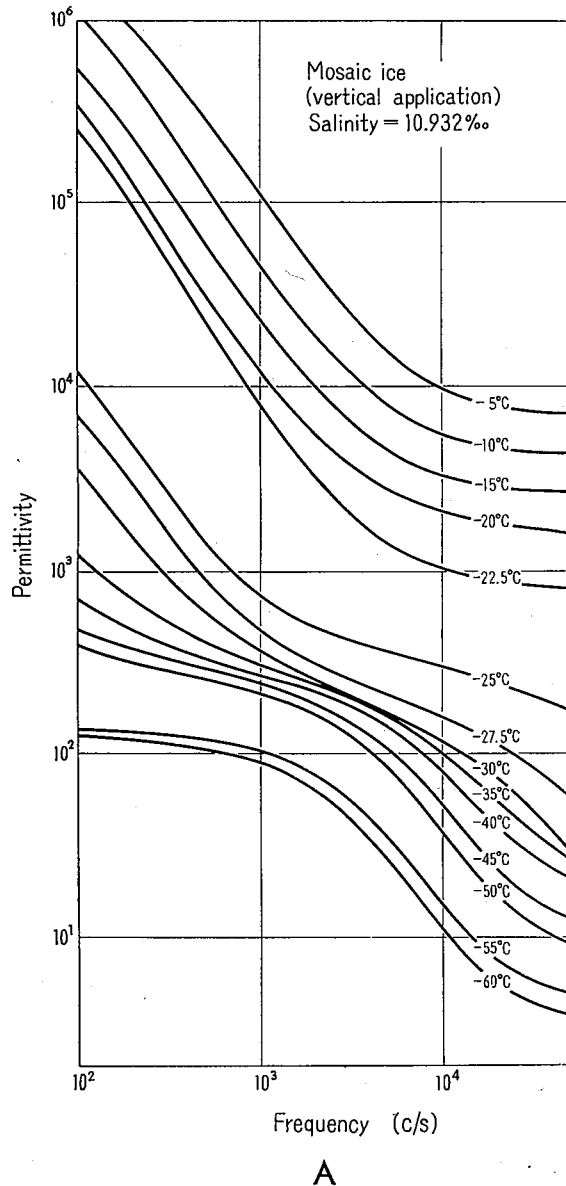
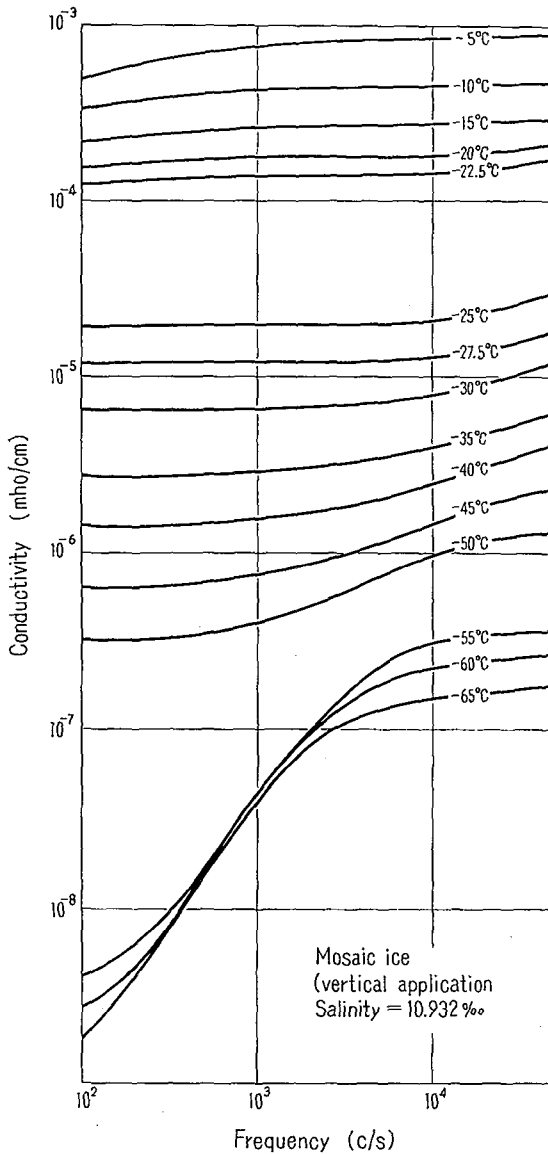


Fig. 2. Dispersions of the permittivity (Fig. 2A) and the conductivity (Fig. 2B) of the mosaic ice. Brine cells are arranged vertical to the applied electric field

could not be obtained from natural sea ice, the specimen for the purpose was made artificially by soaking a snow block, which was collected from deposited city snow (density = 0.38), into brine at freezing point. For comparison, the same measurements were also made on pure ice which were made from distilled water in a cold room and on snow which was used to make granular ice.



B

III. Experimental Results

The permittivity and the conductivity which were obtained from C_p and R_p of each specimen are showed in Figs. 2, 3, 4, 5 and 6.

The behaviours of the permittivity and the conductivity related to frequency and to temperature of each specimen are summarized as follows:

1. SEA ICE

The permittivity and the conductivity of sea ice decrease with temperature. On the curves of the permittivity versus temperature and the conductivity versus temperature at the same frequency, two kinks are observed at the temperature of about -22.5 and -55°C (Figs. 7, 8 and 9). Considering from the behaviour of these curves in each temperature range divided by these two kinks, the curves of the permittivity versus frequency and the conductivity versus frequency may be divided into three groups, corresponding to the following temperature ranges; -5 to -22.5°C , -25 to -50°C , and less than -55°C . The characteristics of the permittivity and the conductivity of three groups are as follows:

i) Mosaic ice (Figs. 2 and 3)

The first group (-5 to -22.5°C).

In the permittivity, an extremely rapid decrease with increasing frequency is observed in low frequency range. We call this decrease "the first anomalous

dispersion" hereafter. The value of the permittivity (in the order of 10^6 at 100 c/s) decreases rapidly with increasing frequency. This rapid decrease continues up to about 5 kc/s and then, the rate of decrease becomes small. The first anomalous dispersion range shifts to the lower frequency side with decreasing temperature.

In the conductivity, a slight increase is observed at high frequency and at low temperature but the curves are almost flat in general.

The second group (-25 to -50°C). In the permittivity, the first anomalous dispersion is still observed. At the same time, the decrease with increasing frequency is observed in the high frequency range. We call this decrease "the second anomalous dispersion". The second anomalous dispersion gradually becomes remarkable with decreasing temperature, and also shifts to the lower frequency side with decreasing temperature.

In the conductivity, a slight increase which is observed in the high frequency range in the first group becomes more remarkable and shifts to the lower frequency side with decreasing temperature.

The third group (less than -55°C). In the permittivity, the first anomalous dispersion is not observed and the second anomalous dispersion becomes more remarkable. The second dispersion also shifts to the lower frequency side with decreasing temperature. In the conductivity, the frequency range of increase shifts to the lower frequency side and the dispersion becomes remarkable. The frequency range of dispersion in the conductivity also shifts to the lower frequency side with decreasing temperature as in the case of permittivity.

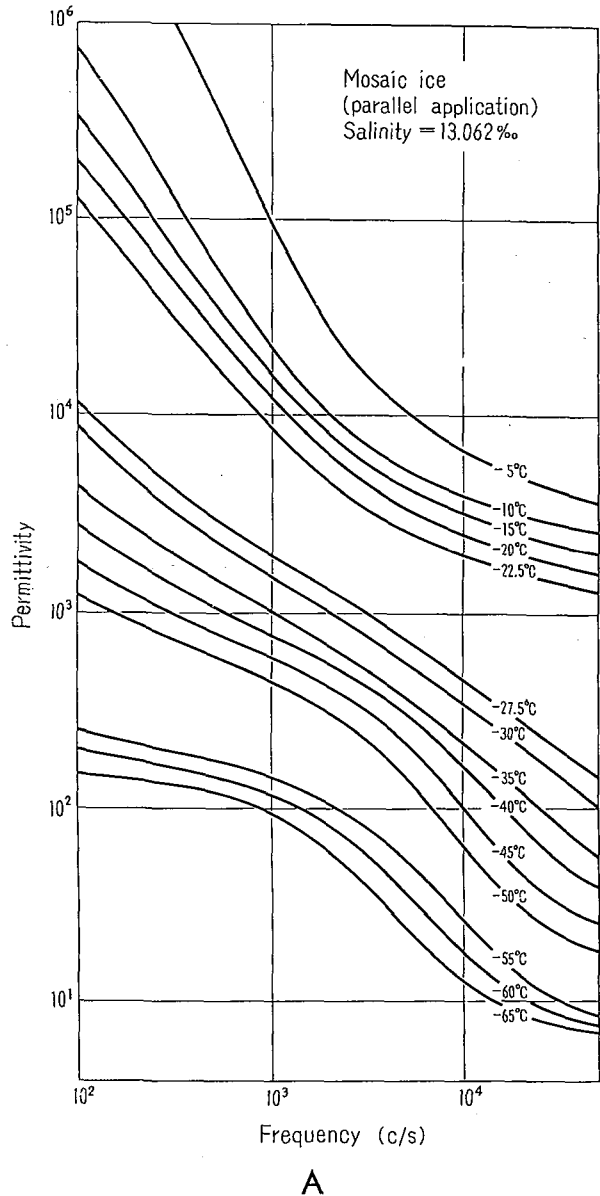
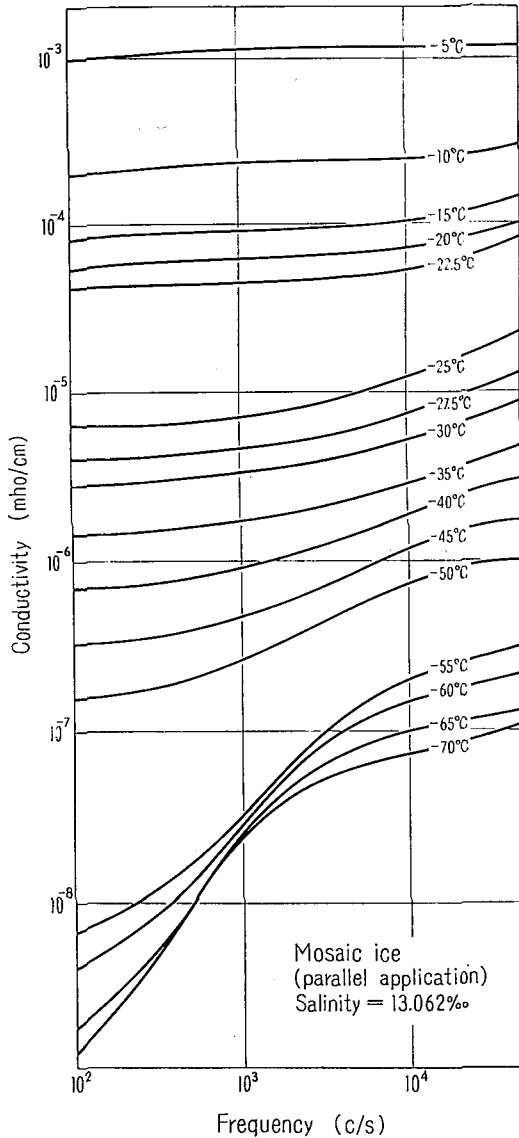


Fig. 3. Dispersions of the permittivity (Fig. 3A) and the conductivity (Fig. 3B) of the mosaic ice. Brine cells are arranged parallel to the applied electric field

ii) *Granular ice (Fig. 4)*

In spite of the large difference of the structure, the behaviour of the permittivity and the conductivity is similar to that of mosaic ice.

2. PURE ICE AND SNOW (FIGS. 5 and 6)



B

The frequency and temperature dependences of pure ice and snow were quite similar to the results obtained by various authors. The grouping related to temperature which is observed in sea ice is not observed in pure ice and snow, but the behaviour of the permittivity and the conductivity in pure ice and snow is quite similar to that of the third group of sea ice. In the permittivity, the anomalous dispersion is observed in the high frequency range but, in the low frequency range, such a dispersion as the first anomalous dispersion in sea ice is not observed in pure ice and snow. The frequency range of this anomalous dispersion of snow is higher than that of pure ice and is almost comparable to that of the second anomalous dispersion of sea ice. The anomalous dispersion shifts to a lower frequency range with decreasing temperature.

IV. Discussions

As the characteristics of the dielectric properties of sea ice, the followings may be particularly mentioned: 1) The grouping related to temperature is observed corresponding to the temperature at about -22.5 and -55°C . 2) No remarkable difference is found between the parallel and the vertical application of the electric field to the brine cells. 3) Two anomalous dispersions in the permittivity are observed and the extremely large values of the permittivity are obtained in the first anomalous dispersion range. 4) A high conductivity is observed

in a high temperature range.

Discussions concerning these characteristics of sea ice will be given in the following.

1. THE GROUPING RELATED TO TEMPERATURE

Since sea ice is considered to be a dielectric mixture of ice and brine, it may be easily expected that the behaviour of the dielectric properties of sea ice may be affected by the behaviours of brine cells, that is, their arrangement, shape, volume and salt content.

The behaviours of these factors related to temperature have been investigated by many workers from various points of view (Lewis, 1950; Nelson, 1954; Assur, 1958; Richardson, 1966). When the temperature of sea ice is close to the freezing point, brine cells have a large volume and are connected each other. With decreasing temperature, the volume of each brine cell decreases rapidly and the connection among brine cells is gradually cut off and finally the brine cells are isolated completely from each other. At the temperature of -22.9°C , sodium chloride, the main component of brine, begins to precipitate, and then, brine may be considered as a mixture of liquid and solid. At the temperature of -55°C , the entire salt content of brine turns to an eutectic mixture so that, below this temperature, liquid state brine may not be present in sea ice.

Two kinks which are observed in the curves of the permittivity and the conductivity versus temperature seem to correspond to the precipitation point of sodium chloride and the eutectic point and may have their origin in the phase change of brine content related to temperature (Figs. 7, 8 and 9).

The presence of liquid state

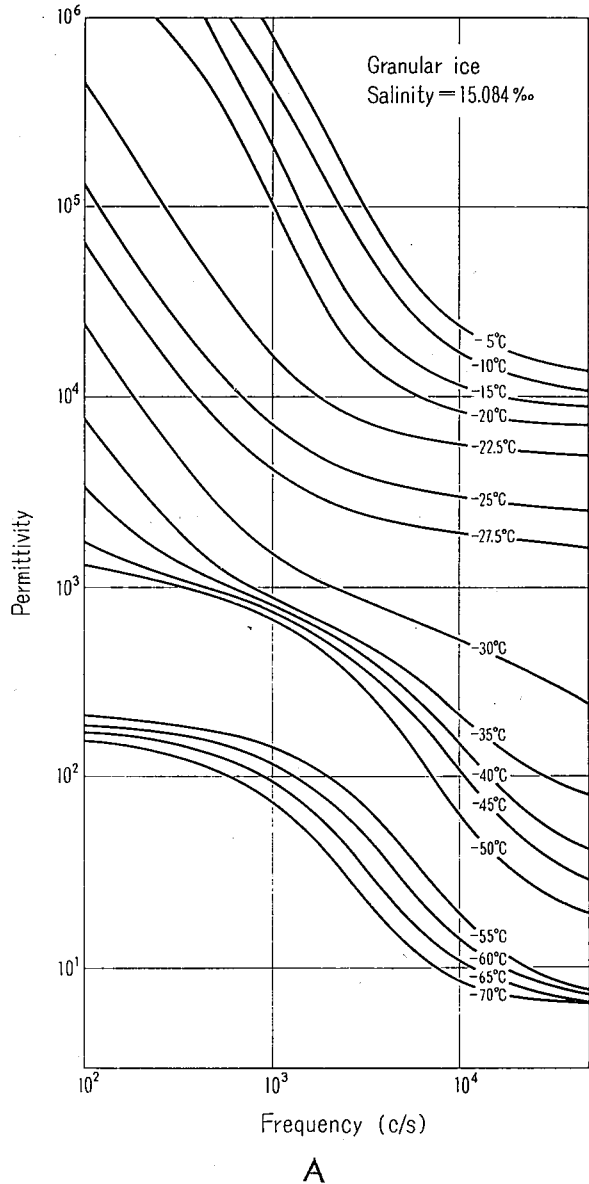


Fig. 4. Dispersions of the permittivity (Fig. 4A) and the conductivity (Fig. 4B) of the granular ice

brine may have effects on both the permittivity and the conductivity. In the permittivity, since that of liquid remains high to centimeter wavelength band, the apparent permittivity of sea ice with liquid state brine may be higher than that of sea ice in which the liquid state brine is not present, in the entire frequency range used in this measurement. On the other hand, in the conductivity, the effective conductivity may be increased in a low frequency range when the liquid state brine is present. So that the contribution of the liquid state brine to the behaviour of the permittivity and the

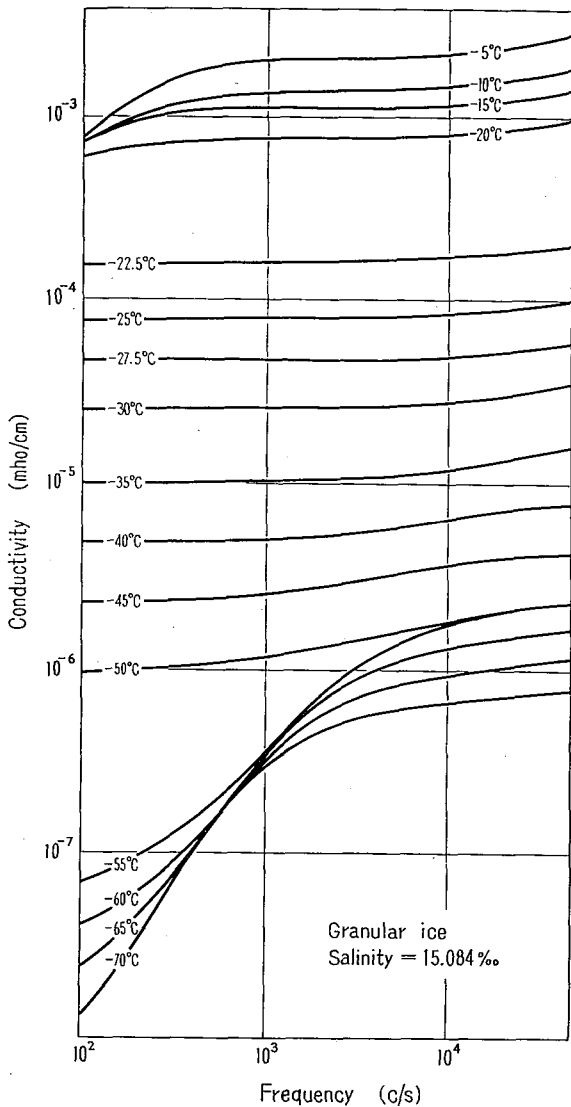
conductivity is considered to be very large.

Therefore, the kink on the curves which corresponds to the precipitation point may be explained from the difference of the phase in brine content. The same consideration may be made on the kink which corresponds to the eutectic point.

2. THE EFFECT OF THE ARRANGEMENT OF BRINE CELLS

The arrangement of brine cells in mosaic ice may be expressed by the models of cylindrical shaped or film like layered cells arranged in parallel with each other and, those in granular ice, by a model of three dimensional network. Therefore, if the electric field is applied in parallel or vertically to the lines of brine cells in mosaic ice, different values of the permittivity should be expected, provided that the total brine content in sea ice is constant (Wiener, 1910; Wagner, 1914; Sillars, 1937). According to the theories of mixture dielectrics, composed of laminated materials, the parallel application of the electric field to the direction of layers creates more higher apparent permittivity than that produced by the vertical application of the field to the direction of layers.

From the obtained results, we



B

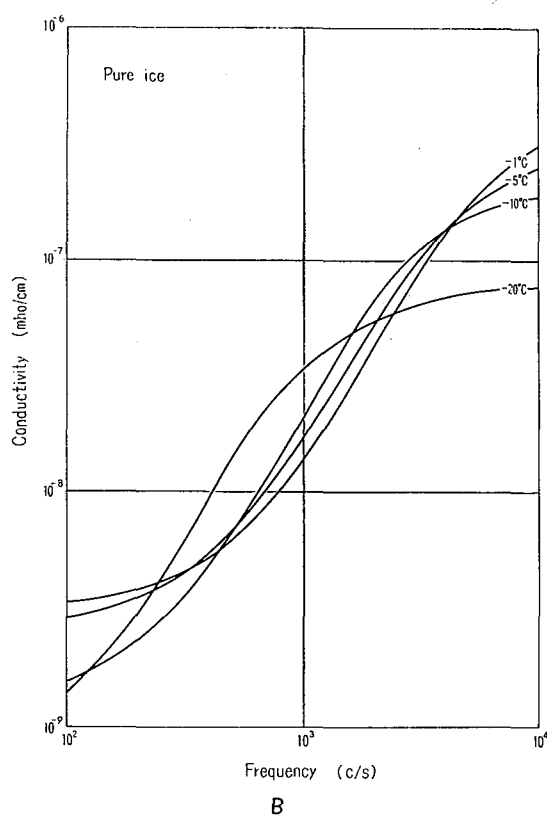
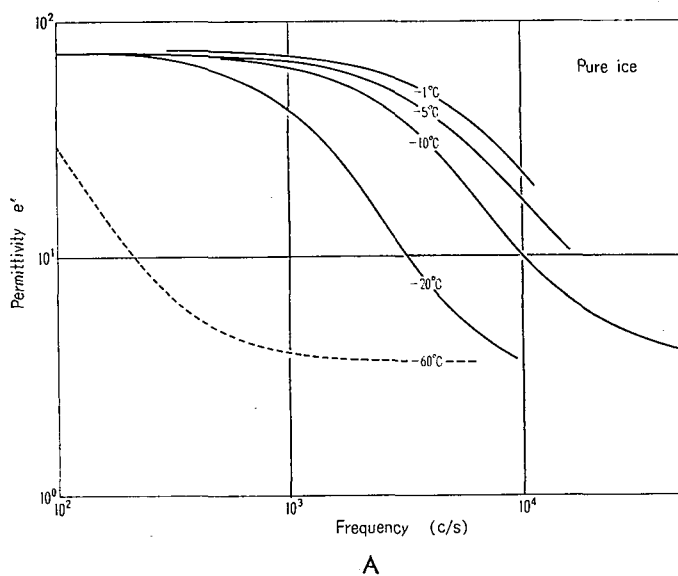


Fig. 5. Dispersions of the permittivity (Fig. 5A) and the conductivity (Fig. 5B) of pure ice

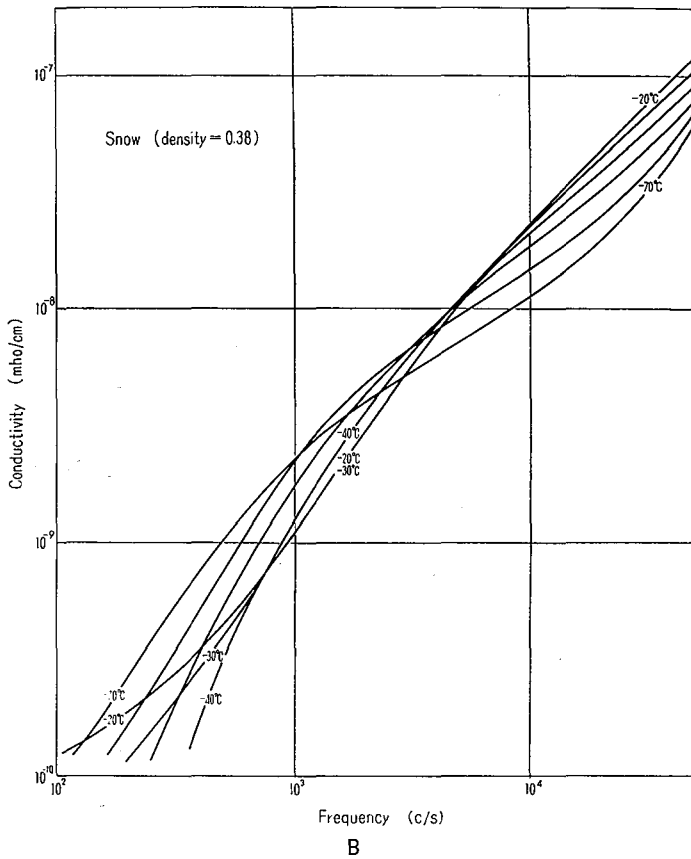
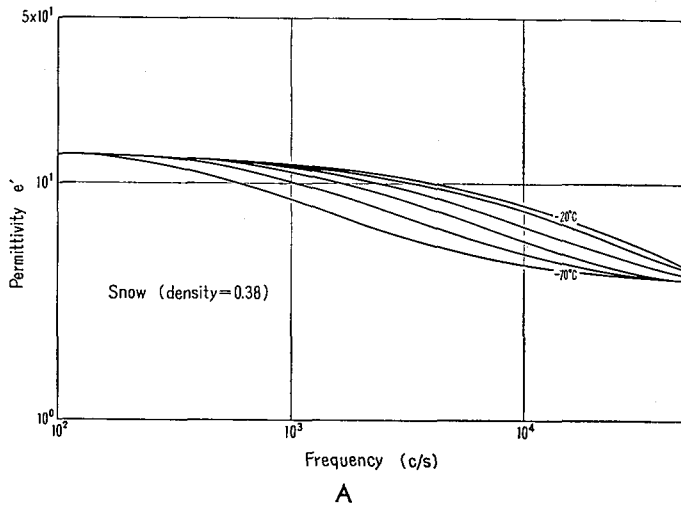


Fig. 6. Dispersions of the permittivity (Fig. 6 A) and the conductivity (Fig. 6 B) of snow

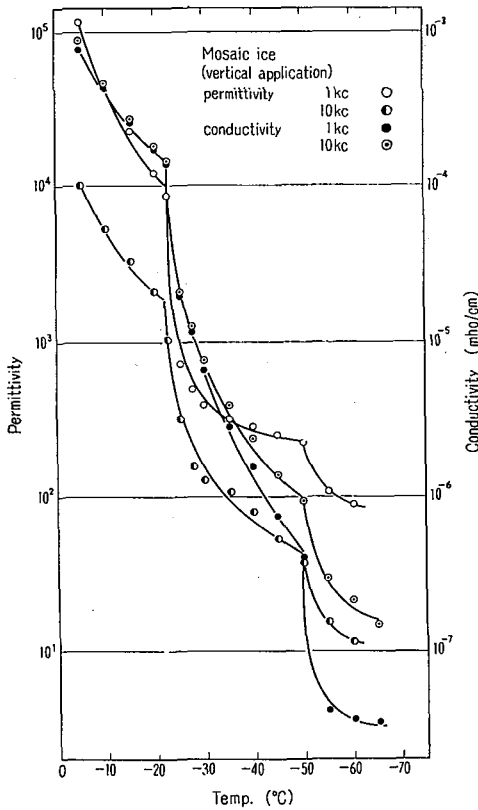


Fig. 7. Temperature dependence of the permittivity and the conductivity of the mosaic ice. The field is applied vertically

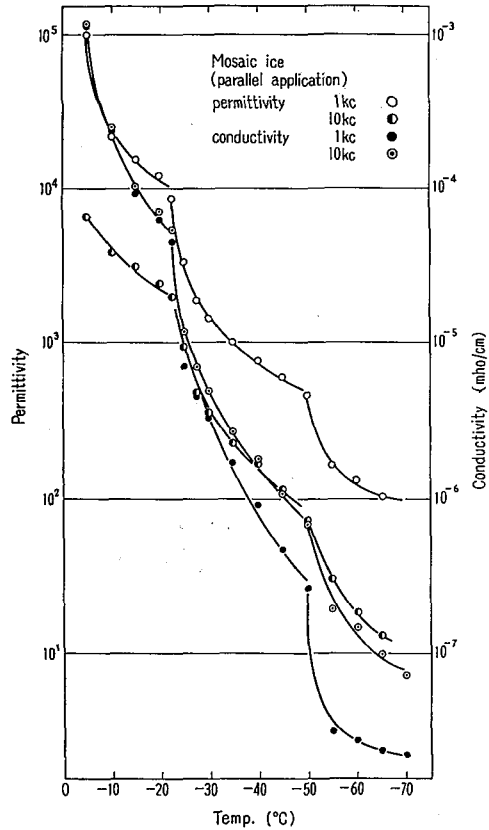


Fig. 8. Temperature dependence of the permittivity and the conductivity of the mosaic ice. The field is applied in parallel

can not find such a clear heterogeneity in the apparent permittivity due to the different field application. It may be considered that the arrangements of brine cells in mosaic ice can not be expressed by such simple models as mentioned above in which each brine cell is arranged in parallel and isolated like a cylinder or a layer.

Comparing the behaviour of each specimen, it may be considered that, in a high temperature range, brine cells may be connected with each other as in a three dimensional network but, in a low temperature range, brine cells may be isolated from each other and arranged like dispersed spheres.

3. THE CHARACTERISTICS OF THE PERMITTIVITY

In the permittivity of sea ice two anomalous dispersions are observed, namely, the first anomalous dispersion in low frequency range and the second anomalous dispersion in a high frequency range.

i) *The second anomalous dispersion*

The frequency range of the second anomalous dispersion in sea ice shifts to a

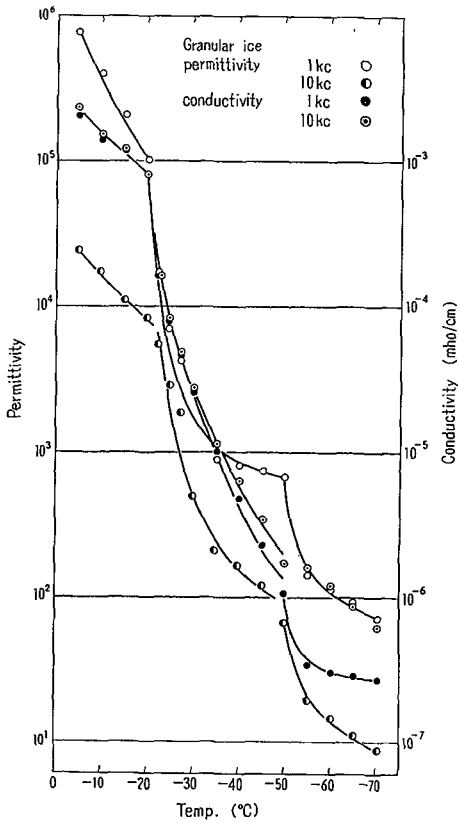


Fig. 9. Temperature dependence of the permittivity and the conductivity of the granular ice

the relaxation time of sea ice beyond that of snow as was observed.

The temperature dependence of the anomalous dispersion in the permittivity may also be explained from the relaxation mechanism of ice molecules.

ii) *The first anomalous dispersion*

The second anomalous dispersion in the permittivity of sea ice may be derived from the orientation polarization in ice molecules but the first anomalous dispersion in the permittivity may not be derived from such a molecular structure but may rather be derived from the geometrical structure of sea ice.

In a heterogeneous material, an additional type of polarization, interfacial polarization, arises from the accumulation of charge at the interface between phases, when the ratios of the permittivity to the conductivity of each phase differ from each other, that is, $\epsilon'_1/\delta_1 \neq \epsilon'_2/\delta_2$. This accumulation of charge requires a flow of current through the dielectric phase, a process which usually may require seconds or minutes, so that it may be observed only at a relatively low frequency. But if one phase has a high conductivity, this polarization may occur so rapidly as to be observed up to a relatively high frequency (Mizushima, 1946; Smyth, 1955).

higher range than that of pure ice but is almost comparable to that of snow. The anomalous dispersion of pure ice and snow has been explained to be derived from orientation polarization of ice molecules (Kuroiwa, 1951; Auty, 1952). Considering the frequency range, the second anomalous dispersion of sea ice may also be derived from the same mechanism as in pure ice and snow.

The obtained relaxation time of snow is in the order of 10^{-4} sec and almost comparable to that of sea ice, while that of pure ice is in the order of 10^{-2} sec at the same temperature. It has been pointed out (Kuroiwa, 1951; Gräni-cher, 1957) that the presence of impurity makes the relaxation time shorten because of the interaction between ice molecules and impurities which exist in the molecular structure of ice. The quantity of impurity contained in sea ice may be larger than that in snow, so that a shorter relaxation time might be expected in sea ice but impurities existing in the molecular structure of ice may be limited under a certain saturation value defined by the molecular structure of ice, so that the quantity of the impurity which exceeds the saturation value is segregated from the molecular structure of ice and is contained in the brine, and may not shorten

Since brine in sea ice is an electrolyte solution and has a very high conductivity, the interfacial polarization may be observed up to a frequency of 4~5 kc/s. The first anomalous dispersion in the permittivity of sea ice, therefore, may be derived from interfacial polarization.

If this interfacial polarization arises in the layered structure its effect on the permittivity is equivalent to that the distance between the conducting plates of the condenser is shortened to the thickness of low conductive phase, so that the value of the apparent permittivity is increased. The thickness of the ice platelet in sea ice is nearly 1 mm and that of brine layer is less than 0.1 mm (at -5°C), so that the values of the apparent permittivity are increased only 10% by this effect. For obtaining the same order of value as observed in this measurement, it is necessary not only to shorten the distance between conducting plates, but enlarging the area of the those plates. A possible mechanism for such enlarging of the area is that the ice platelets and film like brine make a kind of foliated structure in sea ice. This may be reasonable, considering the random orientation of grains in sea ice. Therefore, the first anomalous dispersion in the permittivity may be explained by interfacial polarization which is derived from the geometrical structure in sea ice.

4. THE CHARACTERISTICS OF THE CONDUCTIVITY

The conductivity in this measurement is obtained from measurements of the parallel leakage resistance of the condenser containing sea ice. This apparent conductivity is composed of the contributions to total dissipation of the effective conductivity and the a-c conductivity.

The effective conductivity in sea ice may be derived from free ions in brine and ice. The conduction due to free ions may be limited in a relatively low frequency range because the ions can not follow the change of the electric field in high frequency and this range in brine is relatively higher than that in ice.

Sea ice has a three dimensional network structure of either brine or ice. At high temperature, since brine cells are connected with each other, the network is composed of brine. While, at low temperatures, brine cells are isolated from each other and the network is composed of ice. Therefore, the effective conduction in sea ice may be produced, in a high temperature range, mainly through the brine network and can be predominant in the apparent conductivity up to a relatively high frequency range, so that the values of the apparent conductivity may increase up to almost that of brine in that frequency range. With decreasing temperature, the composition of this network changes from brine to ice and, in a low temperature range, the effective conduction may be made through the ice network. Therefore, the predominant range may be relatively low and the values of the apparent conductivity may decrease down to almost that of ice.

At the same time, with decreasing temperature, as mentioned above, the phase of the brine component which affects the behaviour of the effective conductivity changes.

The contribution of the a-c conductivity to the apparent conductivity is remarkably observed in the high frequency range of the second group and the third group with the decreasing effective conductivity. The behaviour of the a-c conductivity may be explained from the orientation polarization in ice molecules as in the second anomalous dispersion of the permittivity.

The temperature dependence of the apparent conductivity may be explained from the effects of the brine component and the network on the effective conductivity and the relaxation mechanism in the a-c conductivity.

The apparent conductivity at low frequency and high temperature in laboratory measurements are in good agreement with that *in situ* (Fujino, 1960, 1963). Concerning the electrical anisotropy in mosaic ice, clear results are not observed from the measurements between the parallel and the vertical. The same temperature dependence in the conductivity is also observed and concerning this dependence, some considerations related to the behaviour of brine with temperature are presented in the discussion.

V. Concluding Remarks

Qualitatively, dielectric properties of sea ice can be explained by considering sea ice as a mixture of ice and brine of the electrolyte. At high temperatures, the effects of brine are predominant, while, with decreasing temperature, the effects of ice become gradually predominant.

For quantitative discussions of the dielectric properties, more data and further measurements are needed.

Acknowledgments

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